ODE B Quiz 6

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1 Problem 1

1.1 Issue 1

For the solution $y_1(t) = 1 + t$,

$$t\ddot{y} - (1+t)\dot{y} + y = 0 \Rightarrow 0 - (1+t) + (1+t) \quad = 0 \tag{1}$$

For the solution $y_2(t) = e^t$,

$$te^t - (1+t)e^t + e^t = 0 \tag{2}$$

Thus, the two specific solutions to the homogeneous equation are verified as satisfying solutions.

1.2 Issue 2

Use the constant variation method.

$$\begin{aligned}
W(y_1, y_2) &= \det\left(\begin{bmatrix} y_1 & y_2 \\ \dot{y}_1 & \dot{y}_2 \end{bmatrix}\right) \\
&= te^t \\
\Rightarrow -y_1(t)\int_0^t \frac{y_2(s)g(s)}{W(y_1,y_2)(s)}\,\mathrm{d}s &= -(1+t)\int_0^t \frac{e^s s^2 e^{2s}}{se^s}\,\mathrm{d}s \\
&= -\frac{1}{4}(1+t)(e^{2t}(2t-1)+1) \\
y_2(t)\int_0^t \frac{y_1(s)g(s)}{W(y_1,y_2)(s)}\,\mathrm{d}s &= e^t\int_0^t \frac{(1+t)s^2e^{2s}}{se^s}\,\mathrm{d}s \\
&= e^{2t}\left(t^2 - t + 1\right) - e^t \\
\Rightarrow y_p(t) &= -y_1(t)\int_0^t \frac{y_2(s)g(s)}{W(y_1,y_2)(s)}\,\mathrm{d}s + y_2(t)\int_0^t \frac{y_1(s)g(s)}{W(y_1,y_2)(s)}\,\mathrm{d}s \\
&= \frac{1}{4}\left(e^{2t}\left(2t^2 - 5t + 5\right) - t - 4e^t - 1\right)
\end{aligned} \tag{3}$$

2 Problem 2

Solve the homogeneous differential equation first:

$$\begin{aligned}
\ddot{y} + 4\dot{y} + 4y &= 0 \\
\omega^2 + 4\omega + 4 &= 0 \\
y(t) &= C_1te^{-2t} + C_2e^{-2t}
\end{aligned} \tag{4}$$

Apply the constant variation on the homogeneous basis $y_1(t) = te^{-2t}, y_2(t) = e^{-2t}$:

$$\begin{aligned}
W(y_1, y_2) &= \det\left(\begin{bmatrix} y_1(t) & y_2(t) \\ \dot{y}_1(t) & \dot{y}_2(t) \end{bmatrix}\right) \\
&= e^{-4t} \\
\Rightarrow y_p(t) &= -y_1(t) \int_0^t \frac{y_2(s)g(s)}{W(y_1, y_2)(s)} \,\mathrm{d}s + y_2(t) \int_0^t \frac{y_1(s)g(s)}{W(y_1, y_2)(s)} \,\mathrm{d}s \\
&= C_1 e^{-2t} \int \frac{\mathrm{d}s}{s} + C_2 e^{-2t} \\
&= C_1 e^{-2t} \ln(t) + C_2 e^{-2t}
\end{aligned} \tag{5}$$