

ODE B Quiz 6

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1 Problem 1

1.1 Issue 1

For the solution $y_1(t) = 1 + t$,

$$t\ddot{y} - (1+t)\dot{y} + y = 0 \Rightarrow 0 - (1+t) + (1+t) = 0 \quad (1)$$

For the solution $y_2(t) = e^t$,

$$te^t - (1+t)e^t + e^t = 0 \quad (2)$$

Thus, the two specific solutions to the homogeneous equation are verified as satisfying solutions.

1.2 Issue 2

Use the constant variation method.

$$\begin{aligned} W(y_1, y_2) &= \det \begin{pmatrix} y_1 & y_2 \\ \dot{y}_1 & \dot{y}_2 \end{pmatrix} \\ &= te^t \\ \Rightarrow -y_1(t) \int_0^t \frac{y_2(s)g(s)}{W(y_1, y_2)(s)} ds &= -(1+t) \int_0^t \frac{e^s s^2 e^{2s}}{se^s} ds \\ &= -\frac{1}{4}(1+t)(e^{2t}(2t-1) + 1) \\ y_2(t) \int_0^t \frac{y_1(s)g(s)}{W(y_1, y_2)(s)} ds &= e^t \int_0^t \frac{(1+t)s^2 e^{2s}}{se^s} ds \\ &= e^{2t}(t^2 - t + 1) - e^t \\ \Rightarrow y_p(t) &= -y_1(t) \int_0^t \frac{y_2(s)g(s)}{W(y_1, y_2)(s)} ds + y_2(t) \int_0^t \frac{y_1(s)g(s)}{W(y_1, y_2)(s)} ds \\ &= \frac{1}{4}(e^{2t}(2t^2 - 5t + 5) - t - 4e^t - 1) \end{aligned} \quad (3)$$