Quiz 3b

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1 Problem 1

Sol: True.

Since $f(t,y) = y^2$ is continuous and differentiable on $\mathbb{R}$, apply the existence and uniqueness theorem over the region

$$\Omega = (-a,a) \times (-b,b) \tag{1}$$

beneath the desired initial value $y(0) = 0$:

$$\begin{aligned} |f(t,y)||_{(t,y)=(0,0)} &\leq b^2 \\ \Rightarrow M &= b^2 \end{aligned} \tag{2}$$

Therefore, a unique solution exists at the desired point $(t,y) = (0,0)$ over the interval $[-h,h], h = |\min\{a,b\}|$.

2 Problem 2

Sol:

Use the existence and uniqueness theorem over the region

$$\Omega = (-2,2) \times (-b,b) \tag{3}$$

including the initial value $y(0) = 0$:

$$\begin{aligned} |f(t,y)||_{(0,0)} &\leq |b|^{\frac{1}{3}} \\ \Rightarrow M &= |b|\, b^{\frac{1}{3}} \\ h &= \min\{2, b^{\frac{2}{3}}\} \end{aligned} \tag{4}$$

Therefore, the set of values that the solutions satisfying the IVP can have contains only one single element $\{(8\sqrt{3})/9\}$since the uniqueness is present.

$$\begin{aligned} y'(t) &= y^{\frac{1}{3}} \\ \Rightarrow y^{-\frac{1}{3}}\,\mathrm{d}y &= \mathrm{d}t \\ \frac{3}{2}y^{\frac{2}{3}} &= t + C_1 \\ y(0) = 0 \Rightarrow C_1 &= 0 \end{aligned} \tag{5}$$

3 Problem 3

Sol:

(i)

If the (x, y) satisfies $\sin x = 0$, the equation degenerates to

$$n \sin y = 0 \tag{6}$$

Therefore, $(x, y) = \pi(k_1, k_2), \{k_1, k_2\} \subset \mathbb{Z}$ is a solution.

(ii)

If the (x, y) satisfies $\sin y = 0$, the equation degenerates to

$$\begin{aligned} \sin x \frac{\mathrm{d}y}{\mathrm{d}x} &= 0 \\ \int \frac{1}{\sin x}\,\mathrm{d}x &= y + C_1 \\ &= k\pi + C_1 \\ \Rightarrow \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) &= k\pi + C_1 \end{aligned} \tag{7}$$

This equation gives out all the possible x regarding some initial conditions.

(iii)

If $\sin x \sin y \neq 0$, the equation is separable.

$$\begin{aligned} \cot y\,\mathrm{d}y &= -n \cot x\,\mathrm{d}x \\ \Rightarrow \ln(\sin(y)) &= -n \ln(\sin(x)) + C_1 \\ y &= \pm \cos^{-1}\left(\frac{1}{2} c_1 \sec^n(x)\right) \end{aligned} \tag{8}$$

4 Problem 4

Sol:

Use the Euler's method

$$\begin{cases} y[n+1] &= y[n] + f(t[n], y[n])\delta \\ t[n+1] &= t[n] + \delta \end{cases} \tag{9}$$

with the provided equation

$$y'(t) = y(3 - ty), y(0) = 0.5, \delta = 0.1 \tag{10}$$

Use the MATLAB to obtain the solution sequence:

```
x=[0,0.1,0.2,0.3,0.4,0.5];
y=[0.5,0.65,0.8407,1.078,1.368,1.703]
```