Homework 2

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September 23, 2022

1 Problem 2.2

1.1 No.3

$$
\begin{aligned}
y' &= \cos^2(x)\cos^2(2y) \\
\Rightarrow \frac{\mathrm{d}y}{\cos^2(2y)} &= \cos^2 x\,\mathrm{d}x \\
\text{LHS: } \int \frac{\mathrm{d}y}{\cos^2(2y)} &= \frac{\tan 2y}{2} + C_1 \\
\text{RHS: } \int \cos^2 x\,\mathrm{d}x &= \frac{x}{2} + \frac{\sin 2x}{4} + C_2 \\
y = \pm\frac{1}{2}\tan^{-1}&\left(\pm\left(x + \frac{\sin 2x}{2} + C\right)\right)
\end{aligned} \tag{1}
$$

1.2 No.5

$$
\begin{aligned}
y' &= \frac{x - e^{-x}}{y + e^y} \\
(y + e^y)\,\mathrm{d}y &= (x - e^{-x})\,\mathrm{d}x \\
\frac{y^2}{2} + e^y &= \frac{x^2}{2} + e^{-x} + C
\end{aligned} \tag{2}
$$

Since the LHS of eq(2) is transcendent, the further simplification is not possible.

1.3 No.9

(a)

General solution:

$$
\begin{aligned}
y^{-2}\,\mathrm{d}y &= (1 - 2x)\,\mathrm{d}x \\
-y^{-1} &= x - x^2 + C_1 \\
y &= \frac{1}{x(x-1) + C_2}
\end{aligned} \tag{3}
$$

Specific solution:

$$
\begin{aligned}
y(0) &= \frac{1}{C_2} = -\frac{1}{6} \\
C_2 &= -6 \\
y_p &= \frac{1}{x^2 - x - 6} \\
&= \frac{1}{(x+2)(x-3)}
\end{aligned} \tag{4}
$$

Plot:

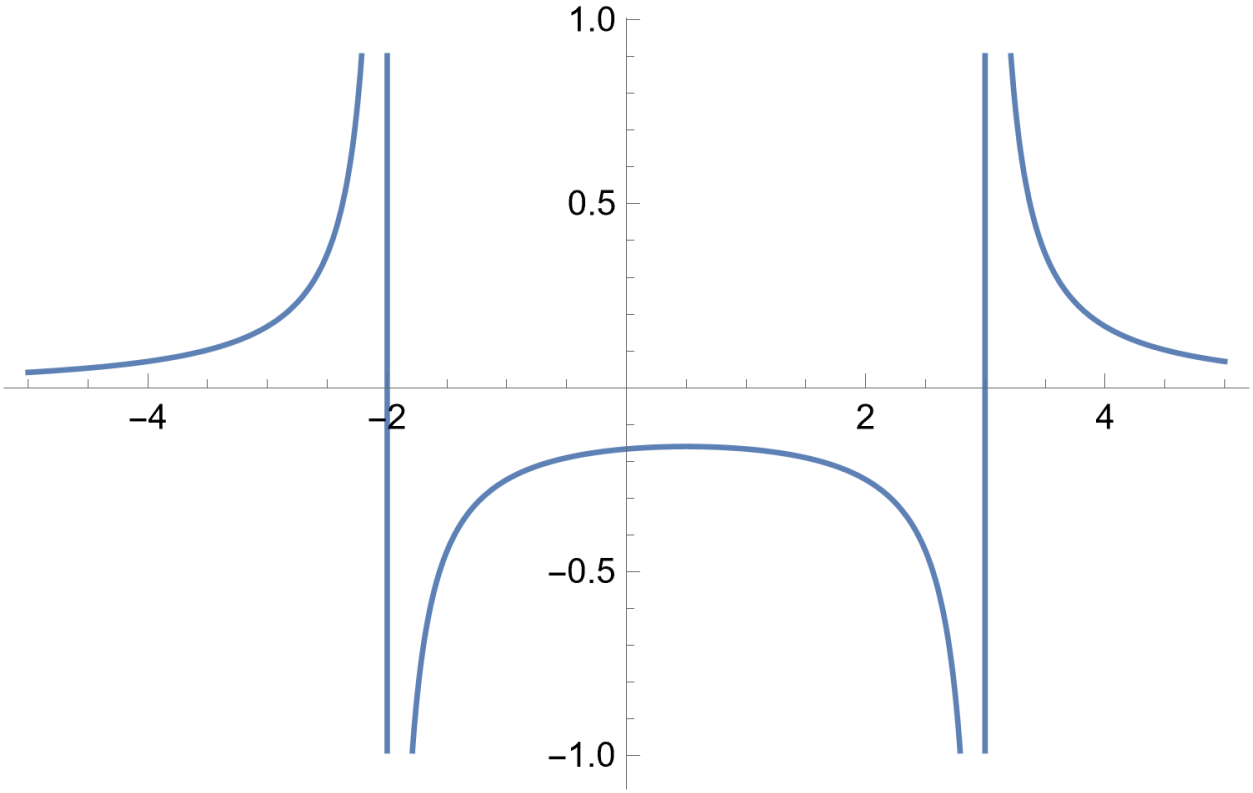

Figure 1: Specific Solution at $y(0) = -1/6$