hw1

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1 ODE B HW1

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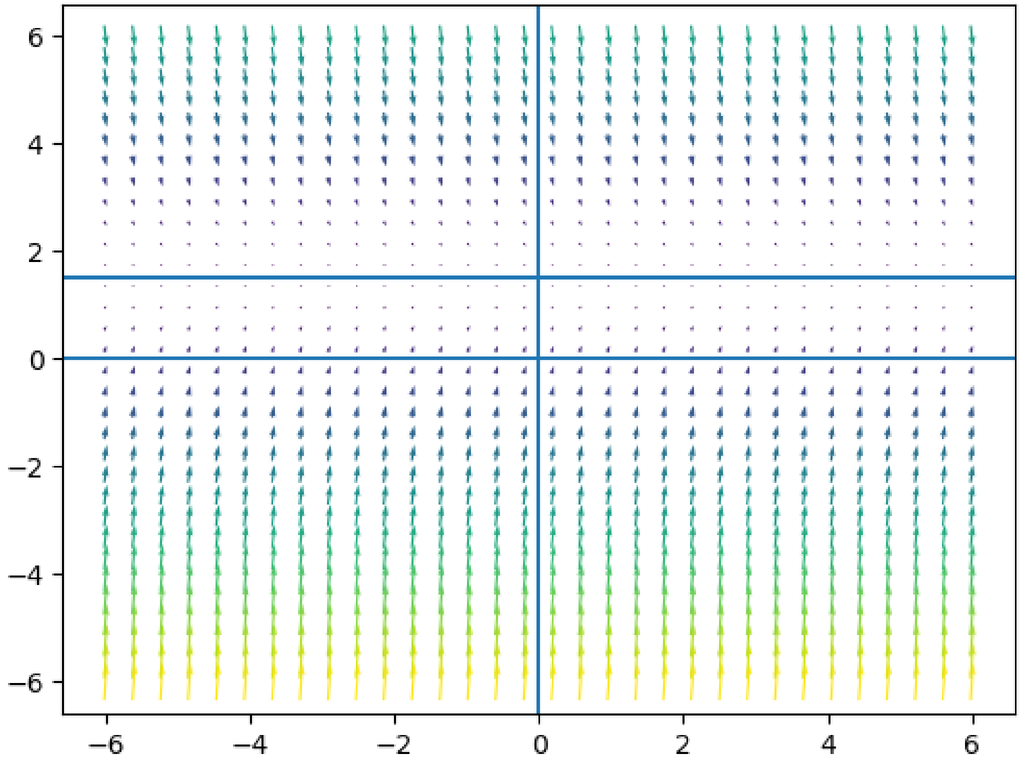
Problems: - 1.1 1 - 1.1 22 - 1.2 3 - 1.2 5 - 2.1 9 - 2.1 11 - 2.1 23

1.1 Section 1.1

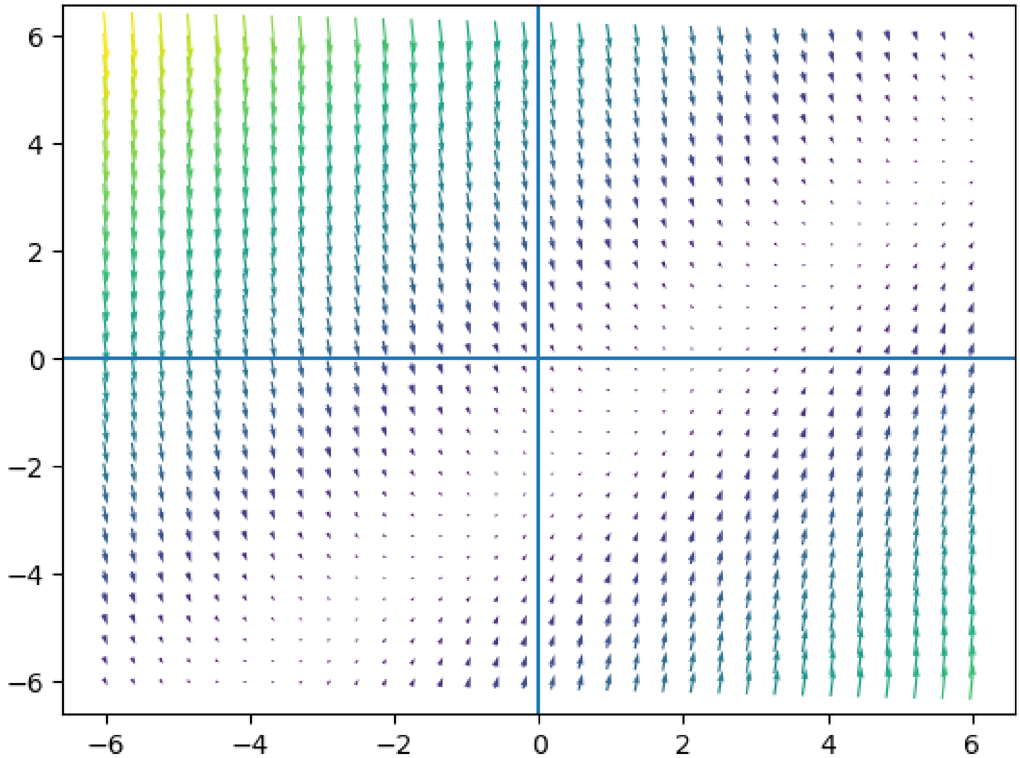
1.1.1 Problem 1

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt

def dy_1(y:np.ndarray)->np.ndarray:
    return 3-2*y

x = np.linspace(-6,6,32)
y = np.linspace(-6,6,32)
hX,hY=np.meshgrid(x,y)

dy=dy_1(hY)
dx=np.ones(dy.shape)
Mcolor=np.hypot(dx,dy)

plt.quiver(hX,hY,dx,dy,Mcolor,pivot='mid')
plt.axhline(0)
plt.axhline(1.5)
plt.axvline(0)
plt.show()
```



1.1.2 Problem 22

[ ]:
```
def dy_2(y:np.ndarray,t:np.ndarray) -> np.ndarray:
    return -y+t-2

dy=dy_2(hY,hX)
dx=np.ones(dy.shape)
Mcolor=np.hypot(dx,dy)

plt.quiver(hX,hY,dx,dy,Mcolor,pivot='mid')
plt.axhline(0)
plt.axvline(0)
plt.show()
```



The direction field of the 1st order ODE $y'(t) = -y(t) + t - 2$ indicates that the line

$$-y + t - 2 = 0$$

Is an attractor line for the specific solutions with determined initial condition (t_0, y_0). Therefore, all the specific solutions converges to the attractor line as $t \to \infty$.

1.2 Section 1.2

1.2.1 Problem 3

(a) This is a **separable** equation, therefore the general solution can be easily obtained below.

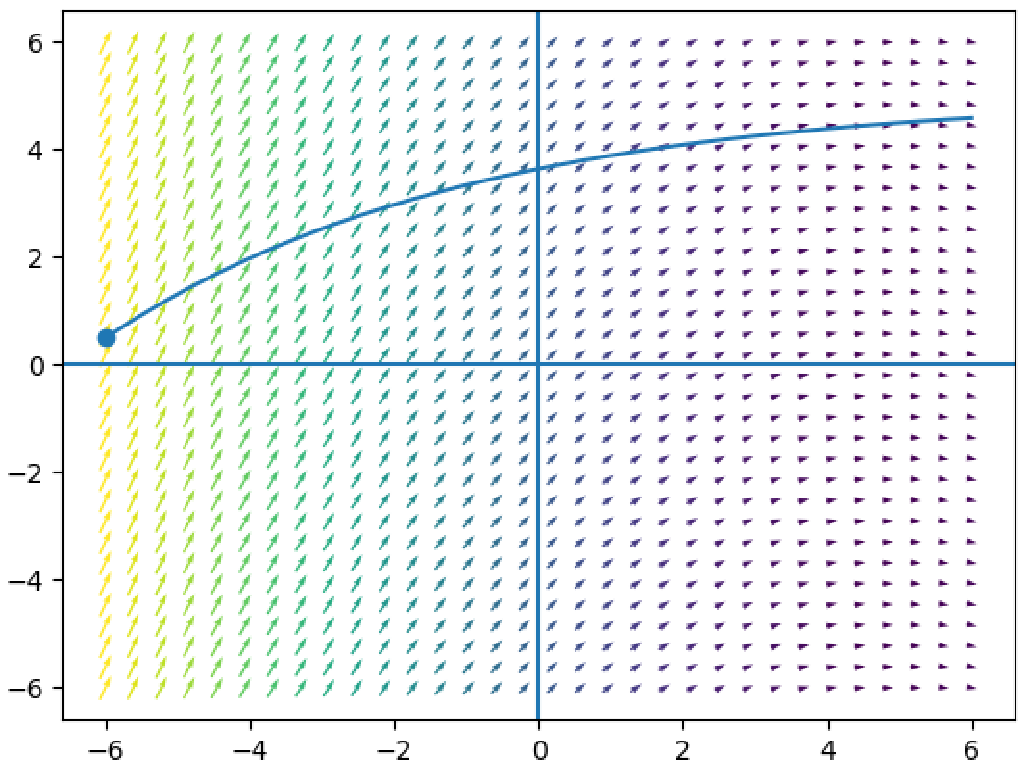
$$\begin{aligned} \frac{\mathrm{d}y}{y - b/a} &= -a\mathrm{d}t \\ \ln(y - b/a) &= -at + C \\ y &= Ce^{-at} + b/a \end{aligned}$$

(b) Use a certain coefficient `[a,b]` and initial condition `[t,y]` to generate the specific solutions.

[ ]:
```
from scipy.integrate import odeint

def dy_dt(y:np.ndarray,t,a,b)->np.ndarray:
    return -a*y+b
```

```
a = 0.2
b=1.0
y0=[0.5]
y = odeint(dy_dt, y0, x,args=(a,b))

dy=dy_dt(hX,1,a,b)
Mcolor=np.hypot(dx,dy)
print("C1=",(y[0]-b/a)*np.exp(a*x[0]))

plt.quiver(hX,hY,dx,dy,Mcolor,pivot='mid')
plt.axhline(0)
plt.axvline(0)
plt.scatter(x[0],y0)
plt.plot(x,y)
plt.show()
```

```
C1= [-1.35537395]
```



(c) For a given initial condition pair (y_0, t_0), the specific solution is

$$\left(y_0 - \frac{b}{a}\right) e^{-a(t-t_0)} + \frac{b}{a}$$

i. When a increases, the constant $(y_0 - b/a)$ and the time constant a increases, while the terminal value b/a decreases. Therefore, the solution approaches to zero more rapidly.

ii. When b increases, the constant $(y_0 - b/a)$ decreases and the terminal value b/a increases. Thus, the terminal value of the solution increases.

iii. When a and b both increases while preserving b/a constant, only the time constant a increases. Therefore, the solution approaches to its original terminal value more rapidly.

1.2.2 Problem 5

(a)

$$y_1(t) = Ce^{at}$$

(b) Use the undetermined coefficient method.

$$\begin{aligned} \frac{\mathrm{d}y}{ay-b} &= \mathrm{d}t + k \\ \frac{\mathrm{d}y_1}{ay_1} &= \mathrm{d}t_1 \\ \Rightarrow y_1 &= \frac{ay-b}{a} \\ &= y - \frac{b}{a}, \\ k &= -\frac{b}{a} \end{aligned}$$

(c)

The solution of the problem (b) is identical to the solution of the textbook (17).

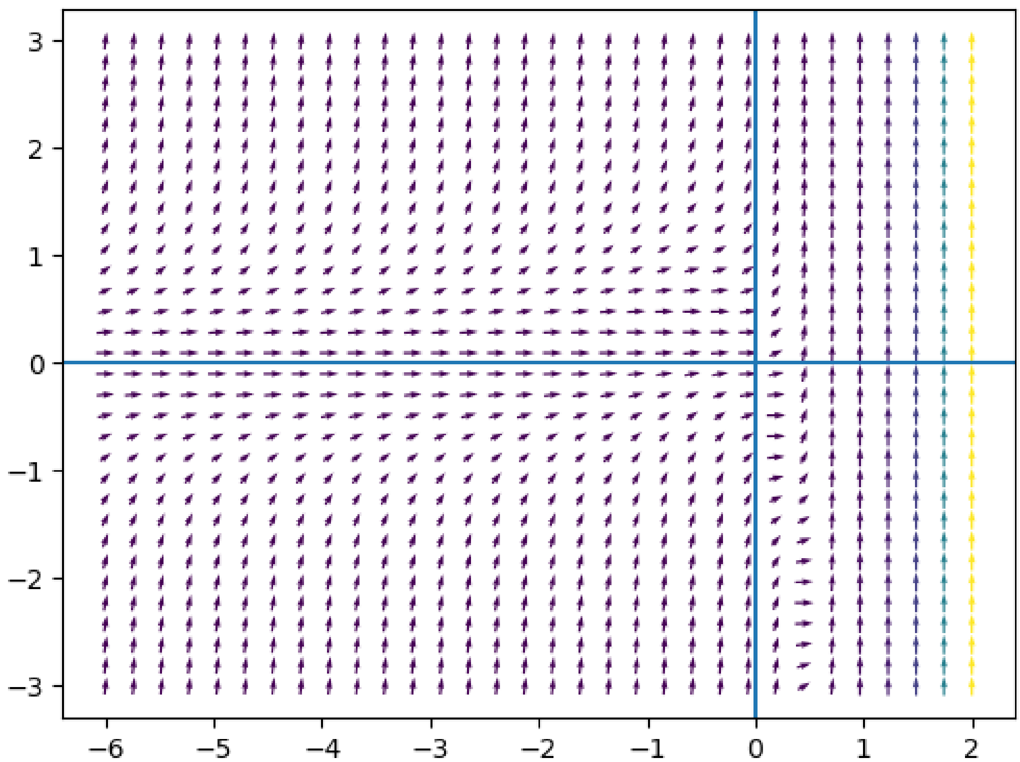
1.3 Section 2.1

1.3.1 Problem 9

(a)(b)

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt

def dy_9(y:np.ndarray,t:np.ndarray)->np.ndarray:
    return 2*t*np.exp(2*t)+y

x = np.linspace(-6,2,32)
y = np.linspace(-3,3,32)
hX,hY=np.meshgrid(x,y)

slope_9=dy_9(hY,hX)
dx9=1/(1+slope_9**2)
dy9=(slope_9**2)/(1+slope_9**2)
Mcolor=np.hypot(dx,dy)
```

```
plt.quiver(hX,hY,dx9,dy9,Mcolor,pivot='mid')
plt.axhline(0)
plt.axvline(0)
plt.show()
```



The slope field indicates that the solution approaches infinity rapidly when $t \to \infty$.

(c)

$$y(t) = Ce^t + 2e^{2t}(t-1)$$

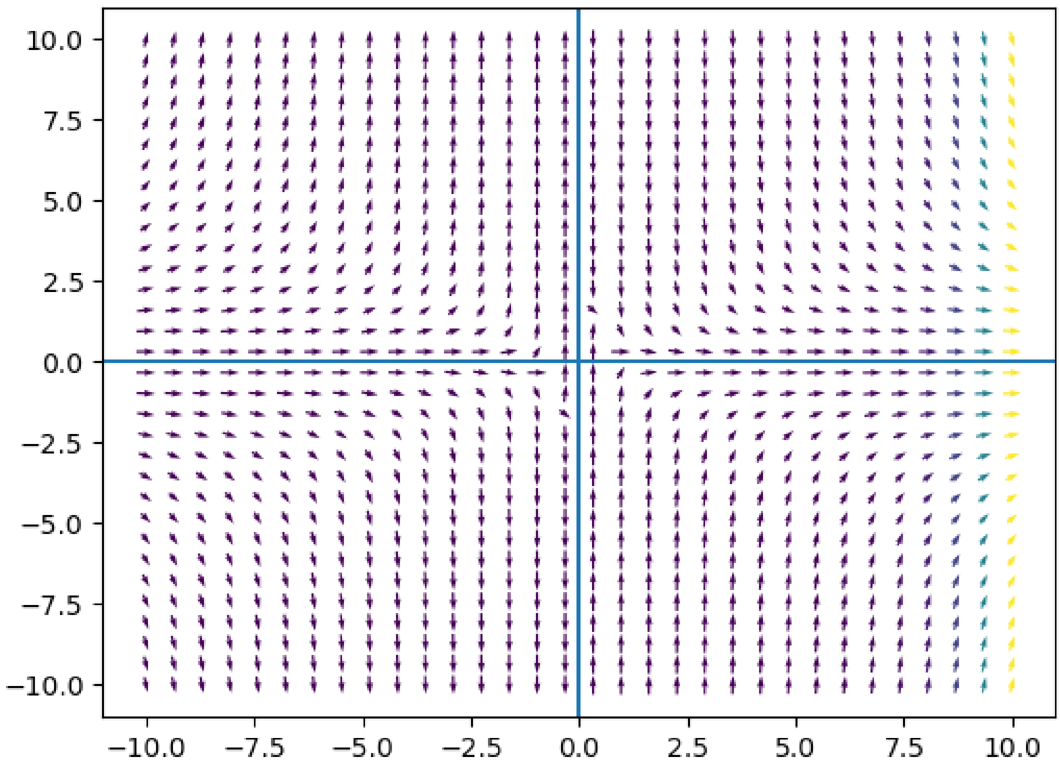
It's easy to verify the deduction from the direction field in (a) and (b).

1.3.2 Problem 11

(a)(b)

[ ]:
```
def dy_11(y:np.ndarray,t:np.ndarray)->np.ndarray:
    return -2*y/t+np.cos(t)/(t**2)

x = np.linspace(-10,10,32)
y = np.linspace(-10,10,32)
hX,hY=np.meshgrid(x,y)
```

```
slope_11=dy_11(hY,hX)
dx9=1/(1+slope_11**2)
dy9=(slope_11**2)/(1+slope_11**2)*np.sign(slope_11)
Mcolor=np.hypot(dx,dy)

plt.quiver(hX,hY,dx9,dy9,Mcolor,pivot='mid')
plt.axhline(0)
plt.axvline(0)
plt.show()
```



The slope field indicates that the solution approaches 0 when $t \to \infty$.

(c)

$$y(t) = \frac{c_1}{t^2} + \frac{\sin(t)}{t^2}$$

It's easy to verify the deduction from the direction field in (a) and (b).