ODE B Quiz 5

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1 Problem 1

Use the exponential intrinsic function $e^{\omega t}$.

$$\begin{aligned} 9\ddot{y} + 9\dot{y} - 4y = 0 \\ e^{\omega t}(9\omega^2 + 9\omega - 4) = 0 \\ \Rightarrow \omega_1, \omega_2 = \left(\frac{1}{3}, -\frac{4}{3}\right) \end{aligned} \tag{1}$$

Therefore, the general solution of the differential equation is

$$y(t) = A\mathrm{e}^{\frac{t}{3}} + B\mathrm{e}^{-\frac{4}{3}t} \tag{2}$$

2 Problem 2

Apply the order reduction on the given equation. Take

$$y_2 = u(t)y_1(t) = u(t)t^{-\frac{1}{2}}\sin(t) \tag{3}$$

Then the equation can be rearranged into the standard 2nd order form $\ddot{y} + p(t)\dot{y} + q(t)y = 0$:

$$\begin{aligned} p(t) &= t^{-1} \\ q(t) &= 1 - 0.25t^{-2} \\ K(t) &= \dot{c} = t^{-\frac{1}{2}}\cos(t) - \frac{1}{2}t^{-\frac{3}{2}}\sin(t) \end{aligned} \tag{4}$$

$$\Rightarrow 2K(t)u'(t) + (p(t)K(t) + K'(t))u(t) = 0$$

Solve the equation 4, the coefficient $u(t)$ and the another linearly independent specified solution $y_2(t) = u(t)y_1(t)$ can be obtained:

$$\begin{aligned} u(t) &= C_1 \exp\left(\frac{\ln(t) - 2\ln(2t\cos(t) - \sin(t))}{4}\right) \\ y_2(t) &= C_1 t^{-\frac{1}{2}}\sin(t)\exp\left(\frac{\ln(t) - 2\ln(2t\cos(t) - \sin(t))}{4}\right) \end{aligned} \tag{5}$$

3 Problem 3

Solve the non-homogeneous equation with Laplace transform.

$$\begin{aligned} (1+s^2)Y(s) &= \frac{12s}{(4+s^2)^2} + \frac{s}{4+s^2} \\ &= \frac{s}{4+s^2}\frac{16+s^2}{4+s^2} \\ \Rightarrow Y(s) &= \frac{s(16+s^2)}{(4+s^2)(1+s^2)} \end{aligned} \tag{6}$$

Inverse Laplace transform gives out the general solution:

$$y(t) = \sin(t)y'(0) + \cos(t)\left(-2t\sin(t) + y(0) + \frac{5}{3}\right) - \frac{5}{3}\cos(2t) \tag{7}$$