HW1 - Kagan Ucak

Q1) 2.1 (a) $\rightarrow F(k_x) = \int_{-\infty}^{\infty} f(x) \cdot e^{-i2\pi k_x x} dx$

For $f(ax+b) \rightarrow u = ax+b \rightarrow f(u) = f(ax+b)$

$\hookrightarrow du = a \cdot dx \rightarrow dx = \frac{du}{a}$

$$F(k_x) = \int_{-\infty}^{\infty} f(u) \cdot e^{-i2\pi \cdot k_x (u-b)/a} \cdot \frac{du}{a} \rightarrow F(k_x) = \frac{1}{|a|} \cdot e^{i2\pi k_x \frac{b}{a}} \cdot \underbrace{\int f(u) e^{-i2\pi \frac{k_x}{a} u} \cdot du}$$

$$\boxed{F\{f(ax+b)\} = \frac{1}{|a|} e^{i2\pi k_x \frac{b}{a}} \cdot F\left(\frac{k_x}{a}\right)}$$

b-) $F\{f(-x)\} = \left.\begin{matrix} u = -x \\ du = -dx \end{matrix}\right\} F\{f(-x)\} = \underbrace{\int_{+\infty}^{-\infty} -f(u) \cdot e^{i2\pi k_x u} du}_{F(-k_x)} =$

$$\boxed{F\{f(-x)\} = F(-k_x)}$$

C-) $F\{F(x)\} = \int_{-\infty}^{\infty} F(x) \cdot e^{-i2\pi k_x x} dx$, $F(x) = \int f(t) e^{-i2\pi x t} dt$

$$\int f(t) \Big[\underbrace{\int e^{-2\pi x (t+k_x)} dx}_{\delta(t+k_x) \text{ delta function}} \Big] dt$$

$\int f(t)\, \delta(t+k_x)\, dt = f(-k_x)$

$$\boxed{F\{F(x)\} = f(-k_x)}$$

Q2) $x = x'\cdot\cos\theta - y'\cdot\sin\theta$, $y = \sin\theta\cdot x' + y'\cdot\cos\theta$

a.) $g(x,y) = e^{-\pi r^2}$, $r^2 = x^2 + y^2$

$$g_\theta(x') = \int_{-\infty}^{\infty} e^{-\pi(x'^2 + y'^2)}\,dy' = e^{-\pi x'^2} \underbrace{\int_{-\infty}^{\infty} e^{-\pi y'^2}\,dy'}_{=1}$$

$$\boxed{g_\theta(x') = e^{-\pi x'^2}}$$

b.) ${}^2\Pi(x,y)$ for $\theta = 45^\circ$

$${}^2\Pi(x,y) = \begin{cases} 1, & |x| \le \frac{1}{2},\ |y| \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases} \rightarrow \text{by } \theta = 45^\circ \text{ tilt} = L(x') = \sqrt{2} - 2|x'| \text{ for } |x'| \le \frac{1}{\sqrt{2}}$$

$$\boxed{g_{45^\circ}(x') = \begin{cases} \sqrt{2} - 2|x'|, & |x'| \le 1/\sqrt{2} \\ 0, & \text{otherwise} \end{cases}}$$

c.) jinc(r) is radially symmetric. Therefore no change in θ directions.

$$\boxed{g_\theta(x') = \operatorname{sinc}(x')}$$

d.) $x_0' = x_0\cdot\cos\theta + y_0\cdot\sin\theta \longrightarrow \boxed{g_\theta(x') = \delta(x' - (x_0\cos\theta + y_0\sin\theta))}$

2.2.4) $f_e(x) = f_o(x) + f_e(x) \rightarrow \boxed{g(x,y) = f_{o,o}(x,y) + f_{o,e}(x,y) + f_{e,o}(x,y) + f_{e,e}(x,y)}$

$$f_{ee} = \frac{1}{4}\left(f(x,y) + f(-x,y) + f(x,-y) + f(-x,-y)\right)$$

$$f_{oe} = \frac{1}{4}\left(f(x,y) - f(-x,y) + f(x,-y) - f(-x,-y)\right)$$

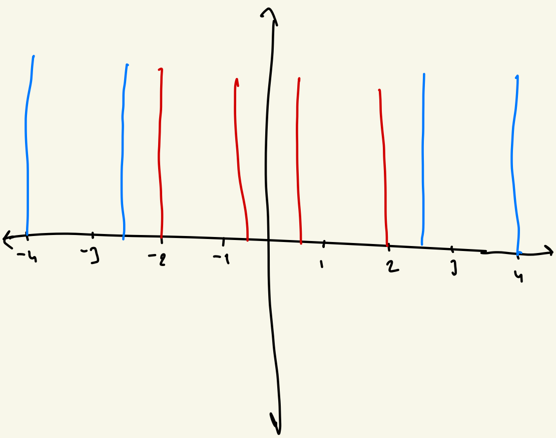
$$f_{eo} = \frac{1}{4}\left(f(x,y) + f(-x,y) - f(x,-y) - f(-x,-y)\right)$$

$$f_{o,o} = \frac{1}{4}\left(f(x,y) - f(-x,y) - f(x,-y) + f(-x,-y)\right)$$

2.5) $f(x) = \cos(2\pi f x)$, $f_s = \frac{3}{2} f$

2 spikes at $\pm f$

periodic replications occur at $\nu = \pm f \pm k \cdot \frac{3}{2} f$



$\nu = \pm f$, $k = 0$

$\nu = \pm \frac{5}{2} f$, $k = 1$

$\nu = \pm \frac{1}{2} f$, $k = -1$

$\nu = \pm 4f$, $k = 2$

$\nu = \pm 2f$, $k = -2$

2.6) $M_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix} \rightarrow R_x(90^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$$R_z(\alpha) = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow$ operations $\Rightarrow M_0' = R_x(90^\circ) \cdot R_z(\alpha) \, R_x(90^\circ) \cdot M_0$

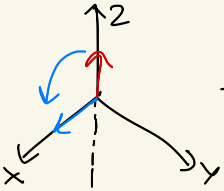
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \quad '' \quad '' \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$= \quad '' \quad \cdot \begin{bmatrix} \sin\alpha \\ -\cos\alpha \\ 0 \end{bmatrix}$$

$$\boxed{M_0' = \begin{bmatrix} \sin\alpha \\ 0 \\ -\cos\alpha \end{bmatrix}}$$

2.7)



→ So, a rotation axis of y with $\alpha = 90^\circ$