EE501 Kogan Ucak

4.1) Eqn 4.5 = $\dfrac{n_+}{n_-} = e^{\frac{-\Delta\varepsilon}{kT}} \equiv r \rightarrow N = n_+ + n_-$

$n_+ = r \cdot n_-$

$N = r n_- + n_- = n_-(r+1)$

$n_- = \dfrac{N}{r+1}$

$n_+ - n_- = n_-(r-1)$

$$\boxed{n_+ - n_- = \frac{N(r-1)}{r+1}} \quad \text{where } r = e^{\frac{-\Delta\varepsilon}{kT}}$$

b-) $kT \gg \Delta\varepsilon$

$n_+ - n_- = \dfrac{N(r-1)}{r+1}, \quad r = e^{\frac{-\Delta\varepsilon}{kT}}$

$= N \cdot \tanh\left(\dfrac{-\Delta\varepsilon}{2kT}\right)$

For this case
$\tanh(x) \approx x \rightarrow n_+ - n_- = N \cdot \left(\dfrac{-\Delta\varepsilon}{2kT}\right)$

$\varepsilon = -\gamma \hbar I_z B, \quad \Delta\varepsilon = \varepsilon_- - \varepsilon_+ = \gamma \hbar B$

$n_+ - n_- = N \cdot \left(\dfrac{-\gamma\hbar B}{2kT}\right)$

4.2) $M = [0,0,0]$ initially. After $90°$ $\rightarrow M = \left[M_0 e^{-t/T_2}, 0, M_0\left(1 - e^{-t/T_1}\right)\right]$

$|M| = M_0 \sqrt{e^{-2t/T_2} + 1 - 2e^{-t/T_1} + e^{-2t/T_1}}$

$T_2 < T_1 \rightarrow$ then $|M(t)| \leq M_0$

$M_x > M_z \rightarrow 1 > M_x \rightarrow |M| = M_0 \cdot \underbrace{\sqrt{M_x^2 + M_z^2}}_{<1}$

$\boxed{|M| < M_0 \text{ always}}$

4.4) After 90° excitation $\rightarrow$ $M_{xy}(0) = 0$, $M_z(0) = 0$

For $A \rightarrow M_{xy} = M_0 \cdot e^{-t/T_{2a}}$, $M_z = M_0\left(1 - e^{-t/T_{1a}}\right)$

$B \rightarrow M_{xy} = M_0 \cdot e^{-t/T_{2b}}$, $M_z = M_0\left(1 - e^{-t/T_{2a}}\right)$

So, $\Delta S_{xy} = M_0\left(e^{-t/T_{2a}} - e^{-t/T_{2B}}\right)$

$\Delta S_z = M_0\left(e^{-t/T_{1B}} - e^{-t/T_{1A}}\right)$

to maximize this $d(\Delta S_{xy})/dt = 0$

$\hookrightarrow 0 = \frac{-1}{T_{2a}} e^{-t/T_{2a}} + \frac{1}{T_{2B}} e^{-t/T_{2B}} \rightarrow \frac{e^{-t/T_{2A}}}{T_{2A}} = \frac{e^{-t/T_{2B}}}{T_{2B}}$

$$\boxed{t^A = \frac{T_{2A} T_{2B}}{T_{2A} - T_{2B}} \cdot \ln\left(\frac{T_{2A}}{T_{2B}}\right)}$$

b-) maximize $|\Delta S_2|$ $\rightarrow$ again $d(\Delta S_2)/dt = 0$, same math but $2b \rightarrow 1a$, $2a \rightarrow 1b$

$$t^* = \frac{T_{1A} \cdot T_{1B}}{T_{1B} - T_{1A}} \cdot \ln\left(\frac{T_{1b}}{T_{1a}}\right)$$

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c-) From part B, for a given values of T_{1A}, T_{1B}, T_{2A}, T_{2B}

the ΔS_{xy}, ΔS_Z can be calculated.
