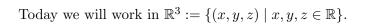
Notes - January 8th Math 32A



The coordinates planes are defined by setting one coordinate equal to zero. The xy-plane corresponds to z=0, the xz-plane to y=0 and finally the yz-plane is x=0. The sectors of \mathbb{R}^3 are called octants. The most important one is the first octant, given by the points (a,b,c) such that a,b,c>0.

The distance between two points $P=(a_1,b_1,c_1)$ and $Q=(a_2,b_2,c_2)$ is given by

$$|P - Q| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2}$$

Planes. The most basic surface in \mathbb{R}^3 are planes. They are given by an equation of the form ax + by + cz = n.

Spheres. The equation of a sphere centered at a point P = (a, b, c) and of radius R is

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$$

Cylinders. The equation of a (right circular) cylinder of radius R whose central axis is the vertical line through (a, b, 0) is

$$(x-a)^2 + (y-b)^2 = R^2$$

Vectors in \mathbb{R}^3 . A vector in \mathbb{R}^3 is, as before, defined by two points $P = (a_1, b_1, c_1)$ and $Q = (a_2, b_2, c_2)$. We write $\mathbf{v} = \overrightarrow{PQ} = \langle a_2 - a_1, b_2 - b_1, c_2 - c_1 \rangle = \langle a, b, c \rangle$.

The length of \mathbf{v} is

$$\|\mathbf{v}\| = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2}.$$

The properties and definitions of vectors in \mathbb{R}^3 are the same as the ones in \mathbb{R}^2 .

The standard basis vectors in \mathbb{R}^3 are $\mathbf{i}=\langle 1,0,0\rangle,\,\mathbf{j}=\langle 0,1,0\rangle,\,\mathbf{k}=\langle 0,0,1\rangle$

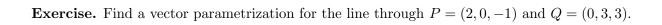
Curves and lines. A parametric curve in \mathbb{R}^3 is a curve represented by a set of three equations for x(t), y(t), z(t). Equivalently, we can represent the curve by a vector $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$.

In particular, a line can be described by a point and a direction. Given a point $P_0 = (x_0, y_0, z_0)$ and a vector $\mathbf{v} = \langle a, b, c \rangle$, the equation of the line that contains the point P_0 in the direction of \mathbf{v} is

$$\mathbf{r}(t) = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle$$

As paramentric equations it can be written as

$$x = x_0 + at$$
, $y = y_0 + bt$, $z = z_0 + ct$



Exercise. Show that

$$\mathbf{r}_1(t) = \langle -1, 1, 1 \rangle + t \langle 1, -2, 3 \rangle$$
 $\mathbf{r}_2(t) = \langle 0, -1, 4 \rangle + t \langle -2, 4, -6 \rangle$

define the same line.

Exercise. Determine if the following lines interesect.

$$\mathbf{r}_1(t) = \langle 1, 0, -1 \rangle + t \langle 1, 2, 0 \rangle \quad \mathbf{r}_2(t) = \langle 0, 1, 1 \rangle + t \langle -2, -1, 2 \rangle$$

If they do, find the intersection point.