Notes - January 17th

Math 32A

Today we will learn how to determine the equation of a plane in $\mathbb{R}^3$.

Given a plane $\mathcal{P}$ in $\mathbb{R}^3$, we can find a vector $\mathbf{n}$ with the following property: given two points P, Q in $\mathcal{P}$, let $\mathbf{v} = \overrightarrow{PQ}$. Then $\mathbf{v} \perp \mathbf{n}$. We call such vector $\mathbf{n}$ a normal vector to the plane $\mathcal{P}$.

Definition. Let $P_0 = (x_0, y_0, z_0)$ a point in $\mathbb{R}^3$ and $\mathbf{n} = \langle a, b, c \rangle$. The plane $\mathcal{P}$ that contains the point P_0 with normal vector $\mathbf{n}$ consists of the heads Q of all the vectors $\overrightarrow{P_0Q}$, based at P_0, that are perpendicular to $\mathbf{n}$.

From this description, how do we obtain an equation for the plane $\mathcal{P}$?

The plane through the point $P_0 = (x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is described by the equation

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Exercise. Find the equation of the plane that contains the point $P_0 = (2, 3, -1)$ and is perpendicular to the vector $\mathbf{n} = \langle 1, -4, 5 \rangle$.

Exercise. Find the equation of the plane that contains the point $P_0 = (1, 1, -1)$ and is parallel to the plane $\mathcal{P}$ with equation $3x - 5y + 2z = 3$.

Exercise. Find the equation of the plane that contains the points $P_1 = (1, 2, 3)$, $P_2 = (-1, 0, 1)$ and $P_3 = (0, 1, 1)$.

Exercise. Find the point of intersection of the plane $x + 2y - 2z = 3$ and the line $\mathbf{r}(t) = \langle 4, 0, -1 \rangle + t\langle 1, 1, 0 \rangle$.

Traces of a plane. The trace of a plane $\mathcal{P}$ with respect to a coordinate plane is the intersection of the plane with that coordinate plane. It might be empty if $\mathcal{P}$ is parallel to such coordinate plane.

Exercise. Find the traces of the plane $2x - y + 3z = 2$ in the coordinates planes.