Notes - January 22nd

Math 32A

Today we will learn how to use vectors to describe a curve.

Consider the following three parametric equations in $\mathbb{R}^2$, $c_1(t) = (t, t^2)$, $c_2(t) = (2t, 4t^2)$, $c_3(t) = (-t, t^2)$.

Vector-valued functions. Consider a particle moving in $\mathbb{R}^3$ with coordinates $(x(t), y(t), z(t))$ at time t. We can represent this path by the vector-valued function

$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$$

More precisely, a vector-valued function is a function from $\mathbb{R}$ to the set of vectors of $\mathbb{R}^3$. The variable t is called a parameter (and usually is referred as time). The vector-valued function $\mathbf{r}(t)$ defines a path in $\mathbb{R}^3$ as t varies.

We call the projections of the path to the restriction of the vector-valued function to the coordinate planes.

Example. Sketch the curve traced by $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ for $t \geq 0$. Describe the projections to the coordinate planes.

Example. We can parametrize a circle in $\mathbb{R}^2$, of radius R and centered at (a, b) as

$$\mathbf{r}(t) = \langle a + R\cos t, b + R\sin t\rangle.$$

Exercise. Find a parametrization of the circle of radius 2 centered at $P = (1, 5, -2)$ located on a plane parallel to the xy-plane. Do the same, but when the circle lies on a plane parallel to the yz-plane.

Exercise. Find a parametrization of the intersection of the cylinder $(x-1)^2+(y-1)^2 = 9$ and the plane $2x-3y+z = 2$.