Notes - January 8th

Math 32A

Today we will work in $\mathbb{R}^3 := \{(x,y,z) \mid x,y,z \in \mathbb{R}\}$.

The coordinates planes are defined by setting one coordinate equal to zero. The xy-plane corresponds to $z=0$, the xz-plane to $y=0$ and finally the yz-plane is $x=0$. The sectors of $\mathbb{R}^3$ are called octants. The most important one is the first octant, given by the points (a,b,c) such that $a,b,c>0$.

The distance between two points $P=(a_1,b_1,c_1)$ and $Q=(a_2,b_2,c_2)$ is given by

$$|P-Q| = \sqrt{(a_1-a_2)^2+(b_1-b_2)^2+(c_1-c_2)^2}$$

Some surfaces.

Planes. The most basic surface in $\mathbb{R}^3$ are planes. They are given by an equation of the form $ax + by + cz = n$.

Spheres. The equation of a sphere centered at a point $P = (a, b, c)$ and of radius R is

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = R^2$$

Cylinders. The equation of a (right circular) cylinder of radius R whose central axis is the vertical line through $(a, b, 0)$ is

$$(x - a)^2 + (y - b)^2 = R^2$$

Vectors in $\mathbb{R}^3$. A vector in $\mathbb{R}^3$ is, as before, defined by two points $P = (a_1, b_1, c_1)$ and $Q = (a_2, b_2, c_2)$. We write $\mathbf{v} = \overrightarrow{PQ} = \langle a_2 - a_1, b_2 - b_1, c_2 - c_1 \rangle = \langle a, b, c \rangle$.

The length of $\mathbf{v}$ is

$$\|\mathbf{v}\| = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2}.$$

The properties and definitions of vectors in $\mathbb{R}^3$ are the same as the ones in $\mathbb{R}^2$.

The standard basis vectors in $\mathbb{R}^3$ are $\mathbf{i} = \langle 1, 0, 0 \rangle$, $\mathbf{j} = \langle 0, 1, 0 \rangle$, $\mathbf{k} = \langle 0, 0, 1 \rangle$

Curves and lines. A parametric curve in $\mathbb{R}^3$ is a curve represented by a set of three equations for $x(t), y(t), z(t)$. Equivalently, we can represent the curve by a vector $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$.

In particular, a line can be described by a point and a direction. Given a point $P_0 = (x_0, y_0, z_0)$ and a vector $\mathbf{v} = \langle a, b, c \rangle$, the equation of the line that contains the point P_0 in the direction of $\mathbf{v}$ is

$$\mathbf{r}(t) = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$

As paramentric equations it can be written as

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct$$

Exercise. Find a vector parametrization for the line through $P = (2, 0, -1)$ and $Q = (0, 3, 3)$.

Exercise. Show that

$$\mathbf{r}_1(t) = \langle -1, 1, 1 \rangle + t\langle 1, -2, 3 \rangle \quad \mathbf{r}_2(t) = \langle 0, -1, 4 \rangle + t\langle -2, 4, -6 \rangle$$

define the same line.

Exercise. Determine if the following lines interesect.

$$\mathbf{r}_1(t) = \langle 1, 0, -1 \rangle + t\langle 1, 2, 0 \rangle \quad \mathbf{r}_2(t) = \langle 0, 1, 1 \rangle + t\langle -2, -1, 2 \rangle$$

If they do, find the intersection point.