CALCULUS

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ABSTRACT. In this report, we see how calculus had emerged and evolved through centuries.Many mathematicians worked on calculus and gave fundamental results.

INTRODUCTION

Calculus became prominent from the 17th century and it replaced method of exhaustion which was used by people back then. Method of exhaustion involves polygons to find areas of curved figures that is, finding areas of circles by inscribing polygons of certain number of sides. Integration, differentiation, finding lengths,areas,volumes,tangents,normals of curves all these are included in calculus. Calculus is also helpful in solving problems of mechanics. In order to understand mathematical physics, one must understand calculus first. Chapters like number theory,combinatorial proofs, probability are also linked with calculus.Calculus got a huge hype as it replaced method of exhaustion. Calculus means "rules for calculating results." **Huygens (1659)**, p. 337, wrote,

"Mathematicians will never have enough time to read all the discoveries in Geometry (a quantity which is increasing from day to day and seems likely in this scientific age to develop to enormous proportions) if they continue to be presented in a rigorous form according to the manner of the ancients."

There is a reason behind Huygen's words that is,the progress in geometry was too high. Whereas, the case is opposite with calculus. People in the early seventeenth century barely had knowledge about calculus. They only knew differentiation and integration of terms which are in powers of x and implicit differentiation of polynomials which contains variables x and y. Later on, mathematicians gained interest in this particular topic. We can integrate and differentiate algebraic functions which can be expressed as power series (for instance, Newton's infinite series $(1+x)^r$ is integrable and differentiable as well).

Mathematicians gave a complete set of rules for differentiation whereas set of rules for integration is incomplete. For instance, the rules were not sufficient to integrate algebraic functions like $\sqrt{1+x^3}$ and rational functions which contain undetermined constants like $1/(x^5-x-A)$. **Davneport(1981)** gave us an idea about integrable functions,that is to distinguish the functions which can be integrable using the rules they gave and the functions which are not integrable. Books such as Boyer (1959), Baron (1969), Edwards (1979), and Bressoud (2019) gives idea of history of calculus. In the 17th century, people used method of exhaustion, in the 19th century, people gave a logical justification for their results. Robinson(1966) gave new theory of infinitesimals in the 20th century.

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The *Arithmetica Infinitorum* of Wallis

Wallis(1655), in his book *Arithmetica Infinitorum*, made efforts to arithmetize geometry, that is to arithmetize theories of areas and volumesof curved figures. For instance, he gave a proof for the following result

$$\int_0^1 x^p dx = \frac{1}{p+1}$$

which is valid for all positive integers p by showing that

$$\frac{0^p + 1^p + 2^p + \cdots + n^p}{n^p + n^p + n^p + \cdots + n^p} \to \frac{1}{p+1}$$

as $n \to \infty$.

Wallis tried to find integration of expressions which involve fractional powers, say $\int_0^1 x^{\frac{m}{n}} dx$ without using the substitution $y^n = x^m$, which was given by Fermat. Firstly, he found $\int_0^1 x^{\frac{1}{2}} dx$, $\int_0^1 x^{\frac{1}{3}}, ...dx$ by finding areas complementary to those under $y = x^2, y = x^3, ...$ From these results, he obtained the results for other fractional powers of x. There were quantities that tended to zero and Wallis was ambivalent about such quantities. Wallis treated them as nonzero once and zero the next. His arch-enemy Thomas Hobbes commented Wallis very badly regarding this:

"Your scurvy book of Arithmetica infinitorum; where your indivisibles have nothing to do, but as they are supposed to have quantity, that is to say, to be divisibles."

Hobbes (1656), p. 301.

However, the reasoning of Wallis is incomplete by today's standards because it is not correct to estimate a formula for all positive integers p "by induction" and for all fractional p "by interpolation" just by taking formulas for $p = 1, 2, 3, ...$ He neglected these flaws. Wallis gave infinite product formula,

$$\frac{\pi}{4} = \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdots$$

The reasoning for Wallis's result was mentioned in **Edwards (1979)**, pp. 171–176, and it was described as *"one of the more audacious investigations by analogy and intuition that has ever yielded a correct result."* In the year 1593, Viete had discovered

$$\begin{aligned} \frac{2}{\pi} &= \cos\tfrac{\pi}{4}\cos\tfrac{\pi}{8}\cos\tfrac{\pi}{16}\cdots \\ &= \sqrt{\tfrac{1}{2}} \cdot \sqrt{\tfrac{1}{2}\left(1 + \sqrt{\tfrac{1}{2}}\right)} \cdots \end{aligned}$$

Later on, based on rational operations, Wallis gave a result which involves π:

and

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots.$$

Brouncker obtained the continued fraction using Wallis's result. This series is a special case of the following series

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

which was discovered by the Indian mathematician named Madhava in the 15th century and the same was rediscovered by Newton, Gregory, and Leibniz. Brouncker's continued fraction is linked with Euler's transformation of the series for $\pi/4$ ((1748a), p. 311). Using Wallis's interpolation, Newton discovered the binomial theorem for fractional powers r for the infinite series $(1+x)^r$.

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