

# CS/CS 316/365 Deep Learning

## Activity 4

September 22, 2024

### Loss Functions

Activity needs to be handwritten. Submission will be online on canvas only.

- Show that the logistic sigmoid function  $\text{sig}[z]$  becomes 0 as  $z \rightarrow -\infty$ , is 0.5 when  $z = 0$ , and becomes 1 when  $z \rightarrow \infty$ . Sigmoid's equation is given below. Showing doesn't require lengthy proof. Try putting in these values and show that result reaches to where it should be.

$$\text{sig}[z] = \frac{1}{1 + \exp[-z]}$$

- The loss  $L$  for binary classification for a single training pair  $\{\mathbf{x}, y\}$  is:

$$L = -(1 - y) \log[1 - \text{sig}[f[\mathbf{x}, \phi]]] - y \log[\text{sig}[f[\mathbf{x}, \phi]]]$$

where  $\text{sig}[z]$  is given above in first task. Plot this loss as a function of the transformed network output  $\text{sig}[f[\mathbf{x}, \phi]] \in [0, 1]$  when the training label

- $y = 0$
- $y = 1$

- Consider a multivariate regression problem where we predict ten outputs, so  $\mathbf{y} \in \mathbb{R}^{10}$ , and model each with an independent normal distribution where the means  $\mu_d$  are predicted by the network, and variances  $\sigma^2$  are constant. Write an expression for the likelihood  $\Pr(\mathbf{y} \mid f[\mathbf{x}, \phi])$ . Show that minimizing the negative log-likelihood of this model is still equivalent to minimizing a sum of squared terms if we don't estimate the variance  $\sigma^2$ .