CS 3430: S20: SciComp with Py
Practice Problems for Midterm 2

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The second midterm will cover the following topics:

1. Newton-Raphson Algorithm;
2. Central Divided Difference (CDD);
3. Richardson's Extrapolation;
4. Romberg Integration;
5. Edge Detection;
6. Hough Transform (HT);
7. Direct Correlation in DPIV;
8. Image Histograms.

Problem 1

Compute the NRA approximations for the following values, numbers of iterations, and initial guesses. State which functions, methods, and classes and from which assignments you used to compute the values.

1. $\sqrt{3}$ for 10 iterations and an initial guess of 1;
2. $\sqrt{5}$ for 10 iterations and an initial guess of 1;
3. $\sqrt{7}$ for 10 iterations and an initial guess of 1;
4. $\sqrt{11}$ for 10 iterations and an initial guess of 1;
5. $5^{1/3}$ for 10 iterations and an initial guess of 1;
6. $7^{1/3}$ for 10 iterations and an initial guess of 1;
7. $10^{1/3}$ for 10 iterations and an initial guess of 1.

Problem 2

Use the central divided difference (CDD) formulas of the specified order to compute the approximations of the derivatives of the following functions at given points and values of h. State which functions, methods, and classes and from which assignments you used to compute the values.

1. $f(x) = \texttt{sin}(x)$, $f'(0.7)$, $h = 0.001$, CDD of order 2 = ?;
2. $f(x) = \texttt{sin}(x)$, $f'(0.7)$, $h = 0.001$, CDD of order 4 = ?;
3. $f(x) = e^x$, $f'(0.3)$, $h = 0.001$, CDD of order 2 = ?;
4. $f(x) = e^x$, $f'(0.3)$, $h = 0.001$, CDD of order 4 = ?;
5. $f(x) = e^{3x}$, $f'(3.0)$, $h = 0.00001$, CDD of order 2 = ?;
6. $f(x) = e^{3x}$, $f'(3.0)$, $h = 0.00001$, CDD of order 4 = ?;

Problem 3

Let $f(x) = 5x^4 - 4x^3 + 2x^2 - 10x$. Use Richardson's Extrapolation with the CDD of order 2 with $h = 0.0001$ to compute $f'(0.5)$. State which functions, methods, and classes and from which assignments you used to compute the values.

Problem 4

Draw the lattice for Romberg$_{3,3}$ ($R_{3,3}$) and state its error. Use the notation $T_{i,j}$ to denote the calls to the trapezoidal rule.

Problem 5

Consider the following 5x5 grayscale image **I** where the top left corner is at $(0, 0)$ and the bottom right corner is at $(4, 4)$. Compute the gradient's magnitude and orientation at the pixel $I(2, 1) = 0$, where 2 is a row number and 1 is a column number. State which functions, methods, and classes and from which assignments you used to compute the values.

0	1	9	255	25
0	0	2	3	53
0	0	10	0	15
255	250	10	9	95
155	200	15	0	0

Problem 6

Suppose that HT is a Hough Transform table where $HT[\rho_1, \theta_1] > HT[\rho_2, \theta_2]$ for some values of $\rho_1, \theta_1, \rho_2, \theta_2$. Explain what the values $HT[\rho_1, \theta_1]$ and $HT[\rho_2, \theta_2]$ signify and what $HT[\rho_1, \theta_1] > HT[\rho_2, \theta_2]$ means.

Problem 7

Write a Py function `center_xy(x, y, w, h)`, where `x` and `y` are the coordinates of a pixel in a PIL image whose width is `w` and whose height is `h`, that converts `x` and `y` into their equivalents in the coordinate system with the origin in the center of the image (i.e., at `w/2` and `h/2`). A unit test on a 5x5 image and its output are given below.

```
def to_pil_xy_ut0():
    w, h = 5, 5
    for x in range(5):
        for y in range(5):
            cx, cy = center_xy(x, y, w, h)
            print('x={},y={}: cx={},cy={}'.format(x, y, cx, cy))
```

```
x=0,y=0: cx=-2,cy=2
x=0,y=1: cx=-2,cy=1
x=0,y=2: cx=-2,cy=0
x=0,y=3: cx=-2,cy=-1
x=0,y=4: cx=-2,cy=-2
x=1,y=0: cx=-1,cy=2
x=1,y=1: cx=-1,cy=1
x=1,y=2: cx=-1,cy=0
x=1,y=3: cx=-1,cy=-1
x=1,y=4: cx=-1,cy=-2
x=2,y=0: cx=0,cy=2
x=2,y=1: cx=0,cy=1
x=2,y=2: cx=0,cy=0
x=2,y=3: cx=0,cy=-1
x=2,y=4: cx=0,cy=-2
x=3,y=0: cx=1,cy=2
```

```
x=3,y=1: cx=1,cy=1
x=3,y=2: cx=1,cy=0
x=3,y=3: cx=1,cy=-1
x=3,y=4: cx=1,cy=-2
x=4,y=0: cx=2,cy=2
x=4,y=1: cx=2,cy=1
x=4,y=2: cx=2,cy=0
x=4,y=3: cx=2,cy=-1
x=4,y=4: cx=2,cy=-2
```

Problem 8

Suppose

$$\mathbf{M_1} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

and

$$\mathbf{M_2} = \begin{bmatrix} 0 & 10 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

1. Compute the direct correlation matrix C_1 where M_1 is the fixed matrix and M_2 is the dancing matrix.
2. Compute the direct correlation matrix C_2 where M_1 is the dancing matrix and M_2 is the fixed matrix.

Problem 9

Suppose we have a 3x3 grayscale image **I**

$$\mathbf{I} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 3 & 4 \\ 4 & 5 & 5 \end{bmatrix}.$$

Compute a histogram of this image with 3 bins so that bin 1 is $[0, 2)$, bin 2 is $[2, 4)$, and bin 3 is $[4, 6)$.