CS 3430: S20: SciComp with Py
Midterm 02

March 26, 2020

Instructions

1. This exam has 8 problems worth a total of 10 points; you may use your class notes, class slides and screencasts on Canvas, reading materials distributed in class, class code samples, and, most importantly, your homework solutions in solving these problems; **you may not use any other materials while working on your solutions**.
2. You'll submit your solutions in the attached text file `midterm02_s20.py`. You'll type your answers to Problems 1 – 5, 7, and 8 inside multi-line Py comments and your answer to Problem 6 as Py code in that file. Don't copy and paste the problem statements from this pdf into the file and don't submit any other files. **Write your name and A-number at the top of the file**.
3. You may not talk to anyone during this exam orally, digitally, in writing, or by phone. Telepathy is also strictly prohibited and, if detected, will be considered as plagiarism!
4. You may use your Python IDE and/or a scientific calculator.
5. You must submit your `midterm02_s20.py` in Canvas by 11:59pm today (March 26, 2020).
6. Good luck!

Problem 1 (2 points)

Use the NRA to compute the approximations for the following values, numbers of iterations, and initial guesses. State which functions, methods, and classes and from which assignments you used to compute the approximations.

1. $\sqrt{13}$ for 5 iterations and an initial guess of 0.5;
2. $\sqrt{15}$ for 15 iterations and an initial guess of 0.75;
3. $\sqrt{171}$ for 7 iterations and an initial guess of 1;
4. $2112^{1/3}$ for 10 iterations and an initial guess of 3;
5. $15^{1/5}$ for 9 iterations and an initial guess of 1;
6. $3113^{1/7}$ for 100 iterations and an initial guess of 1.25;
7. $1000131^{1/17}$ for 200 iterations and an initial guess of 1.

Problem 2 (2 points)

Use the central divided difference (CDD) formulas of the specified order to compute the approximations of the derivatives of the following functions at given points and values of h. State which functions, methods, and classes and from which assignments you used to compute the approximations.

1. $f(x) = \texttt{sin}(x)$, $f'(1.7)$, $h = 0.001$, CDD of order 2 = ?;

2. $f(x) = \texttt{sin}(x)$, $f'(1.7)$, $h = 0.001$, CDD of order $4 = ?$;
3. $f(x) = e^x$, $f'(15.35)$, $h = 0.00001$, CDD of order $2 = ?$;
4. $f(x) = e^x$, $f'(15.35)$, $h = 0.00001$, CDD of order $4 = ?$;
5. $f(x) = e^{5x}$, $f'(1.5)$, $h = 0.00001$, CDD of order $2 = ?$;
6. $f(x) = e^{5x}$, $f'(1.5)$, $h = 0.00001$, CDD of order $4 = ?$;

Problem 3 (1 point)

Let $f(x) = 10x^4 - 5x^3 + 3x^2 - 15x$. Use the 2nd Richardson's Extrapolation with the CDD of order 2 and $h = 0.0001$ to compute $f'(0.5)$. State which functions, methods, and classes and from which assignments you used to compute the values.

Problem 4 (1 point)

Type-draw the lattice for Romberg$_{4,4}$ (i.e., $R_{4,4}$) and state the error at each level. Use the notation $T_{i,j}$ to denote the calls to the trapezoidal rule.

Problem 5 (1 point)

Consider the following 5x5 grayscale image $\mathbf{I}$ where the top left corner is at $(0, 0)$ and the bottom right corner is at $(4, 4)$. Compute the gradient's magnitude and orientation at the pixel $\mathbf{I}(1, 1) = 0$, where 1 is a row number and 1 is a column number. State which functions, methods, and classes and from which assignments you used to compute the two values.

$\mathbf{I} =$

0	1	9	255	25
0	0	2	3	53
255	250	10	9	95
155	200	15	0	0
0	0	10	0	15

Problem 6 (1 point)

Write a Py function `center_xy(x, y, w, h)`, where `x` and `y` are the coordinates of a pixel in a PIL image whose width is `w` and whose height is `h`, that converts `x` and `y` into their equivalents in the coordinate system with the origin in the center of the image (i.e., at `w/2` and `h/2`). A unit test on a 5x5 image and its output are given below.

```
def to_pil_xy_ut0():
    w, h = 5, 5
    for x in range(5):
        for y in range(5):
            cx, cy = center_xy(x, y, w, h)
            print('x={},y={}: cx={},cy={}'.format(x, y, cx, cy))
```

```
x=0,y=0: cx=-2,cy=2
x=0,y=1: cx=-2,cy=1
x=0,y=2: cx=-2,cy=0
x=0,y=3: cx=-2,cy=-1
x=0,y=4: cx=-2,cy=-2
x=1,y=0: cx=-1,cy=2
x=1,y=1: cx=-1,cy=1
x=1,y=2: cx=-1,cy=0
x=1,y=3: cx=-1,cy=-1
x=1,y=4: cx=-1,cy=-2
x=2,y=0: cx=0,cy=2
x=2,y=1: cx=0,cy=1
```

```
x=2,y=2: cx=0,cy=0
x=2,y=3: cx=0,cy=-1
x=2,y=4: cx=0,cy=-2
x=3,y=0: cx=1,cy=2
x=3,y=1: cx=1,cy=1
x=3,y=2: cx=1,cy=0
x=3,y=3: cx=1,cy=-1
x=3,y=4: cx=1,cy=-2
x=4,y=0: cx=2,cy=2
x=4,y=1: cx=2,cy=1
x=4,y=2: cx=2,cy=0
x=4,y=3: cx=2,cy=-1
x=4,y=4: cx=2,cy=-2
```

Problem 7 (1 point)

Suppose

$$\mathbf{M_1} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 5 & 0 \end{bmatrix}$$

and

$$\mathbf{M_2} = \begin{bmatrix} 0 & 0 & 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

1. Compute the direct correlation matrix C_1 where M_1 is the fixed matrix and M_2 is the dancing matrix.
2. Compute the direct correlation matrix C_2 where M_1 is the dancing matrix and M_2 is the fixed matrix.

Problem 8 (1 point)

Suppose we have a 3x3 grayscale image

$$\mathbf{I} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 3 & 3 \\ 4 & 4 & 5 \end{bmatrix}.$$

Compute the histogram of this image with 3 bins where bin 1 is $[0, 2)$, bin 2 is $[2, 4)$, and bin 3 is $[4, 6)$.