1. (a) 是 lattice. 对任意两个元素都存在 LUB 与 GLB, 故为格.

(b) 是子格.

2. ① ∵ $C=(a\cap c)\cup(b\cap c)\cup(a\cap b)$, $(a\cap b)$ 与另任一个集合的并大于等于 $a\cap b$,

故 $C\geq(a\cap b)$.

若 C 中有 $\alpha\in C$, $\alpha\notin a$, $\alpha\notin b$, 则 $\alpha\notin a\cap c$, $\alpha\notin b\cap c$, $\alpha\notin a\cap b$,

故 $\alpha\notin C$, 矛盾. 故 $\therefore C\leq a\cup b$.

② ∵ $C\leq a\cup b$, ∴ $(a\cup b)\cap c=C=(a\cap c)\cup(b\cap c)$, ∵ $C\geq a\cap b$,

∴ L is a distributive lattice, ∴ $C=(a\cap c)\cup(b\cap c)\cup(a\cap b)$.

3. ¬ 为 unary operator, 故只有一个 propositional letter, 每次加一个 ¬(), 则加了两个符号, 不能得证符号个数为奇数, 故不成立.

5 ① $T\downarrow T=F$, $T\downarrow F=F$, $F\downarrow T=F$, $F\downarrow F=T$

② $\alpha=(\neg A_1\wedge\neg A_2)$.

4. $\neg A=A\downarrow F$. $A\vee B=\neg(A\downarrow B)$.

$A\wedge B=\neg(\neg A\vee\neg B)=\neg((A\downarrow F)\vee(B\downarrow F))=(A\downarrow F)\downarrow(B\downarrow F))$.

$A\to B=A\vee\neg B=A\vee(B\downarrow F)=\neg(A\downarrow(B\downarrow F))$.

$A\leftrightarrow B=(A\wedge B)\vee(\neg A\wedge\neg B)=((A\downarrow F)\downarrow(B\downarrow F))\downarrow(A\downarrow B)$. ∴ ↓ is adequate.

6. (a) $F((q\to r)\to((\neg q)\to(\neg p)\to(p\to r)))$.

$T(q\to r)$

$F((\neg q)\to(\neg p)\to(p\to r))$

Fq Tr

$T(\neg q)$ $F(p\to r)$

Fq Tp

$F((\neg p)\to(p\to r))$ Fr

$T\neg p$ ⊗.

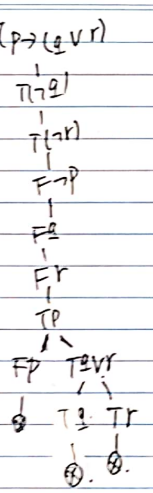
Fp Provable.

$F(p\to r)$ True.

Tp

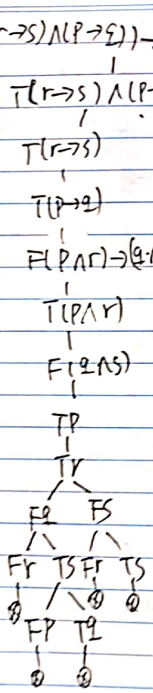
⊗

(b) $T(p \to (q \vee r))$



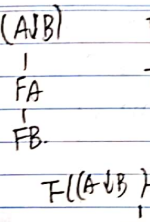
Provable.

(c) $F((r \to s) \wedge (p \to q)) \to ((p \wedge r) \to (q \wedge s))$.



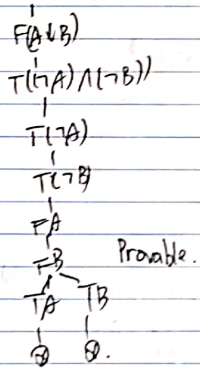
Provable.

1. ① $T(A \vee B)$



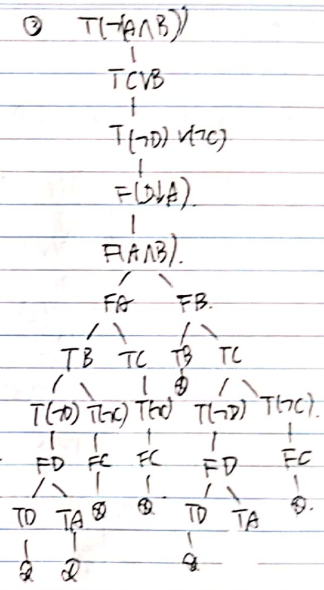
$F(A \vee B)$

② $F((A \vee B) \to ((\neg A) \wedge (\neg B)))$



Provable.

③ $T((\neg A \wedge B) \vee (\neg C) \vee (\neg D) \vee A)$



Provable.

反之 $A=T, B=F, C=T, D=F$ 则 $\neg(A \wedge B)=T, C \vee B=T, (\neg D) \vee (\neg C)=T, (A \vee D)=F$，结为 false.

8. (a) $K(x)$: x is kind; $C(x)$: x has a child; $P(x)$: x has cat
$L(x)$: he loves his child; $A(x)$: x is married; $F(x)$: x has a family.

$$(P(x)\to(K(x)\to((C(x)\leftrightarrow L(x))\to(A(x)\to F(x)))))\to(\neg(P(x))\to(\neg(F(x))))$$

(b) T

$P(x)\to(K(x)\to((C(x)\leftrightarrow L(x))\to(A(x)\to F(x))))\to(\neg(P(x))\to\neg(F(x)))$

$F(\neg(P(x))\to F(x))$

$T(\neg P(x))$

$F\,P(x)$

$F\,F(x)$

$T\,F(x)$

$T\,P(x)$ | $F(K(x)\to(C(x)\to L(x)))\to(A(x)\to F(x))$

$\otimes$ | $T(K(x))$

$F(C(x)\leftrightarrow L(x))\to(A(x)\to F(x))$

$T(C(x)\leftrightarrow L(x))$

$F(A(x)\to F(x))$

$T(A(x))$

$F\,F(x)$

$\otimes$.

$\therefore$ provable.

9. (a) $(A<B)\Leftrightarrow(A\to B)\wedge\neg(B\to A)$.

(b) 假设对于 $\{P_n\mid n\in\mathbb{N}\}$ 有: ① $R_{P_iP_j}: P_i<P_j$ ② A_{P_i}: P_i 在链 A 上 $(B_{P_i}, C_{P_i}$ 同)

则构以下条件构造集合 X:

① A, B, C 均为 chain

② $A_{P_i}\vee B_{P_i}\vee C_{P_i}=T$

③ 该集合的 width 为 3.

则存在无穷命题集合 X 满足条件, $\because X$ 中每个有限子集均为 valid.

故原命题成立, 得证.