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Characterization of input data of NTG – OUTPUTS

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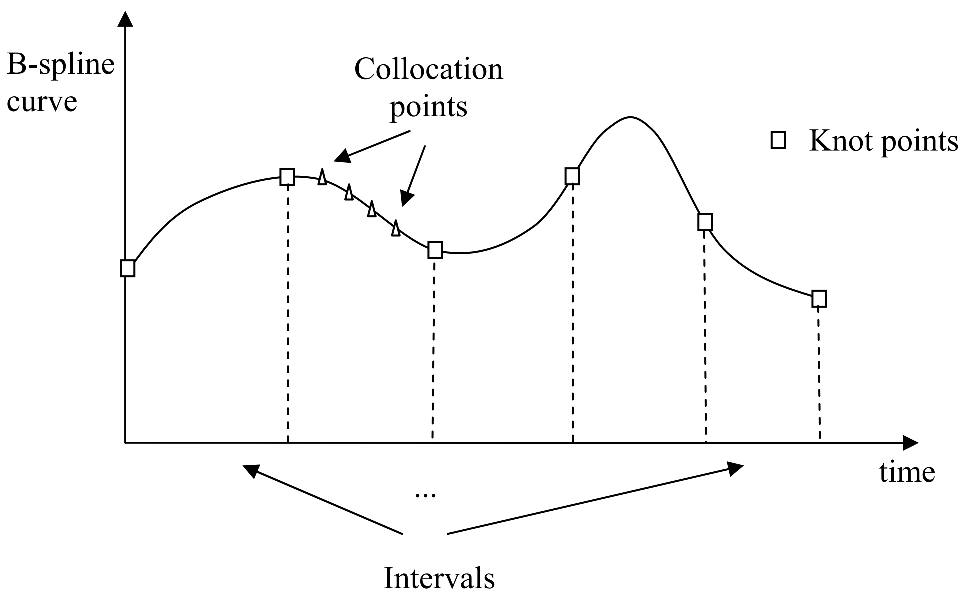
Introduction

This document defines parameters used to specify the B-splines in the input data for the NTG software library. The input data is referred to in NTGML documentation as OUTPUT.
The document is organized as follows. In the first section we define the parameters used to define the output trajectory in terms of B-splines. The last section presents illustrative examples.

Definitions

Consider the following figure:



Guidelines for input data configuration

The following table describes the parameters used to specify an output trajectory:

Output trajectory input data definitions

NTG	Meaning	Relationship
ORDER	Degree of each piecewise polynomial segment (t^k)	ORDER = MULT + 1
NINTERVAL	Number of intervals in the B-Spline curve	
MULT	Continuity of the B-Spline at the knot points (C^{MULT-1})	C^{MULT-1}
NAME	Name of the trajectory	
MAXDERIV	Maximum derivative +1	
LENGTH	Length of the horizon in seconds	
NBPS	Number of collocation points between each knot point.	

Where:

MULT stands for the level of continuity of the B-spline. Thus, if we require the trajectory to be a C^k function at the knot points then MULT=k+1 (Mark Milam PhD thesis page 48).

ORDER is the degree of each piecewise polynomial and should be defined to be ORDER = MULT + 1 (Mark Milam PhD thesis page 48).

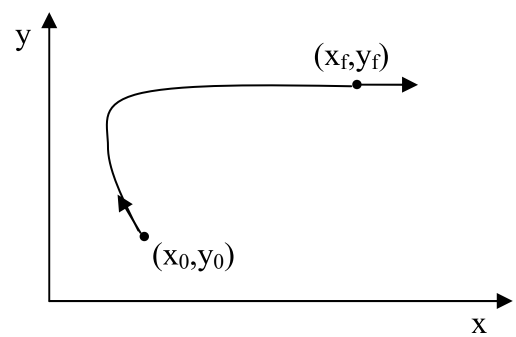
NINTERVAL is the number of intervals of the B-spline and can be chosen freely. Nevertheless, we should keep in mind that increasing the number of intervals leads to better results.

Example

Consider the point mass problem where we are required to steer the mass between to configurations with velocity constraints at the initial and final instants and with the following dynamics:

$$\ddot{x} = f_1 \qquad |f_1| \leq 1$$

$$\ddot{y} = f_2 \qquad |f_2| \leq 1$$



The problem is formulated as follows (minimize actuation effort):

$$\min(\ddot{x}^2 + \ddot{y}^2)$$
$$s.t.$$
$$x(0) = y(0) = \dot{x}(0) = \dot{y}(0) = 0$$
$$x(10) = y(10) = 10$$
$$\dot{x}(10) = \dot{y}(10) = 0$$

With this example we show two different approaches:

1. **Allow the control (2^{nd} derivative of the trajectory) to be a piecewise continuous trajectory**:

Output trajectories should be a C^1 function then MULT = 2 and the ORDER = MULT + 1 = 3. LENGTH = 10.

a. NINTERVAL = 3 Final cost = 2.77 (figure 1)
b. NINTERVAL = 5 Final cost = 2.36 (figure 2)

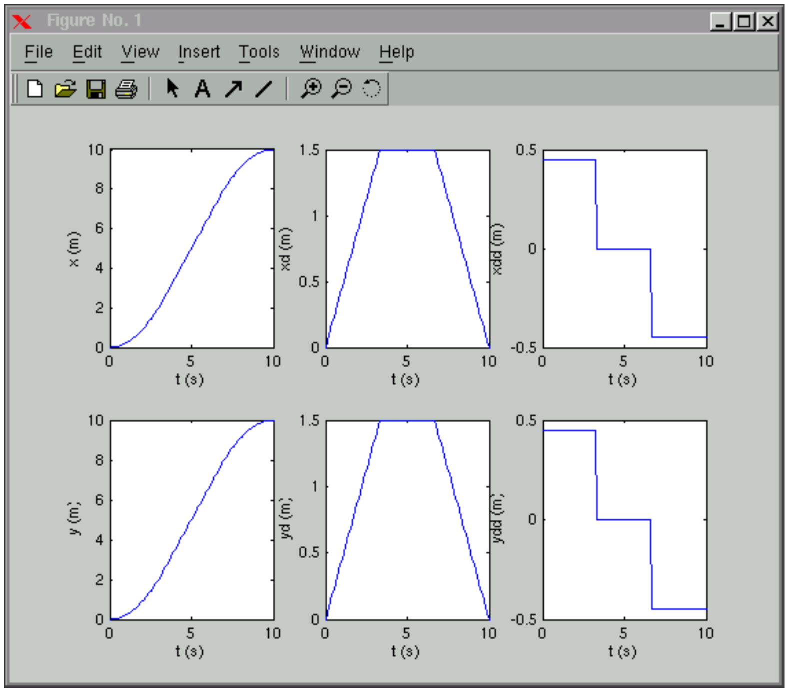


Fig 1 – Point mass steering problem. All the constraints are satisfied and the control is piecewise continuous. The x,y trajectories are C^1 functions. It is possible to notice the 3 intervals defined for the B-spline in this problem.

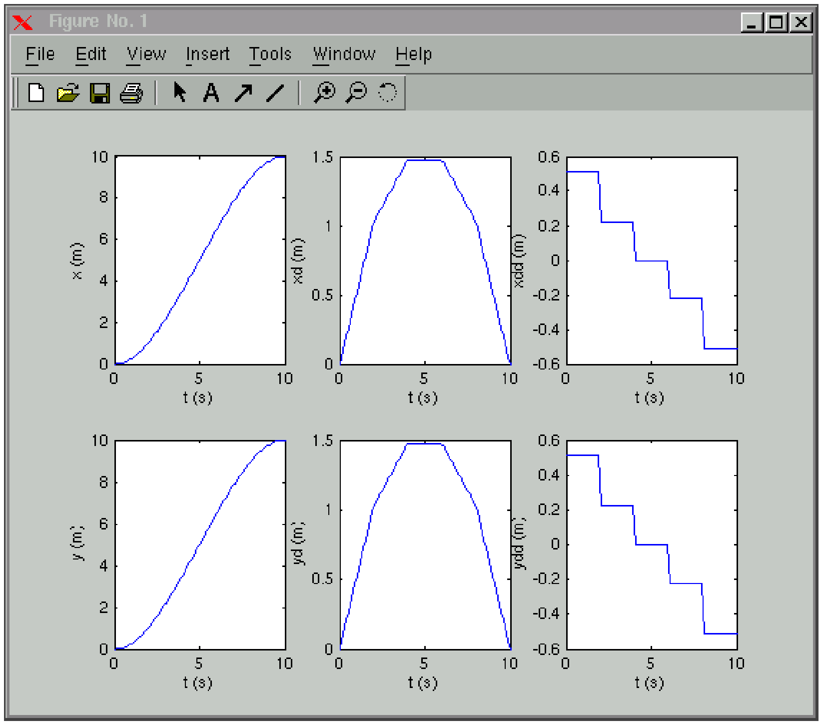


Fig. 2 – In this figure we present the same problem but with 5 intervals in the B-spline trajectory. The final cost is lower than in the previous example and the constraints are also satisfied.

2. Specify the control trajectory (2nd derivative of the trajectory) to be a continuous function.

Output trajectories should be C^2 functions then MULT = 3 and ORDER = 4.
LENGTH = 10.

NINTERVALS = 3 Final cost = 2.41 (figure 3)

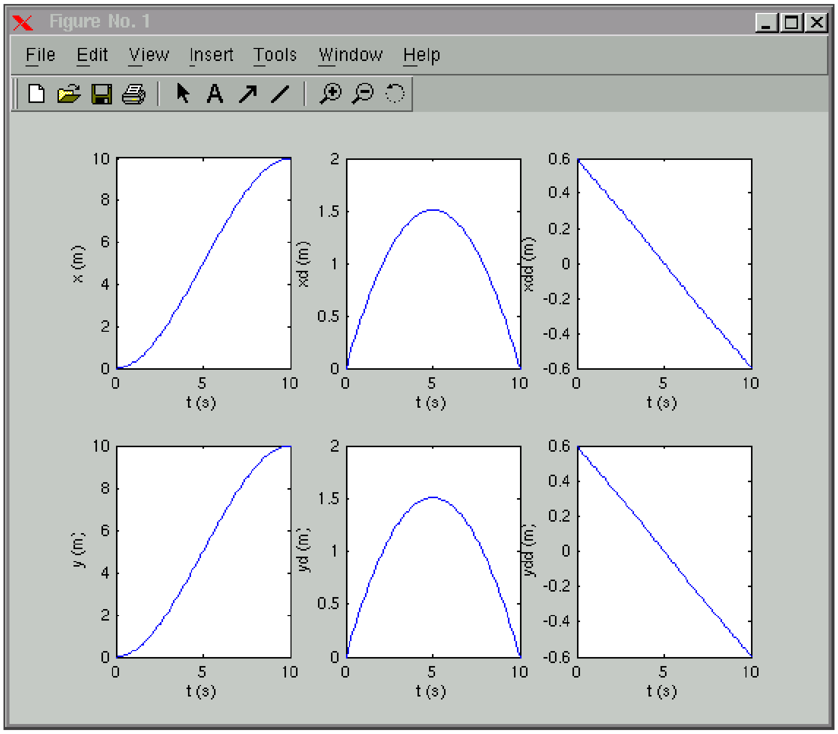


Fig 3 – Point mass steering problem. All the constraints are satisfied and the control is continuous. The x,y trajectories are C^2 functions.