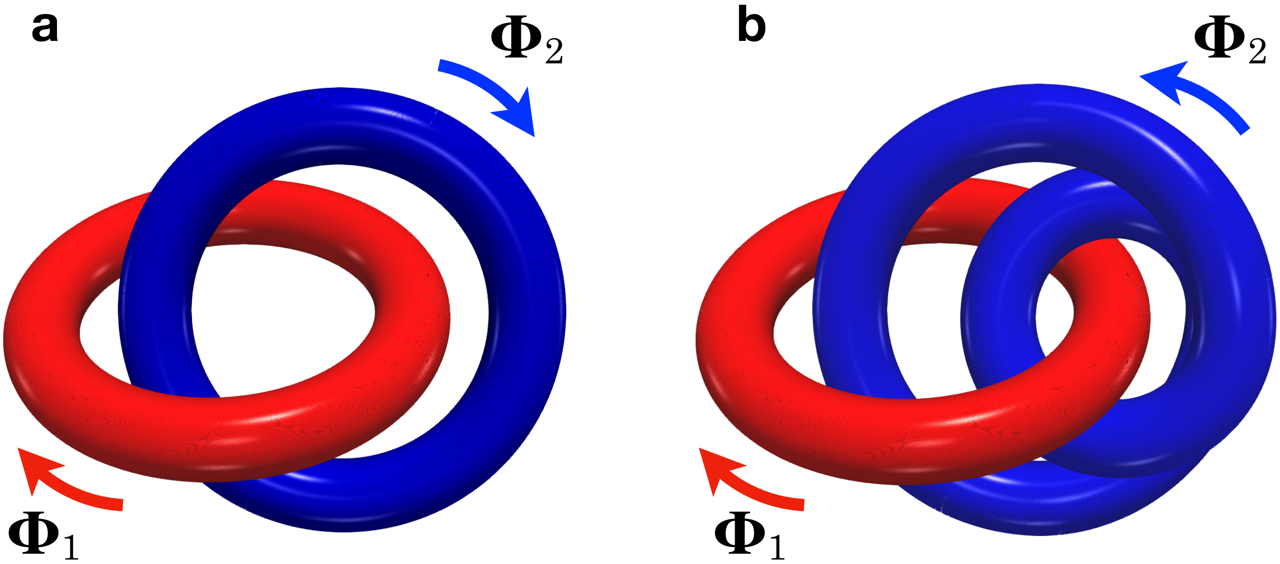
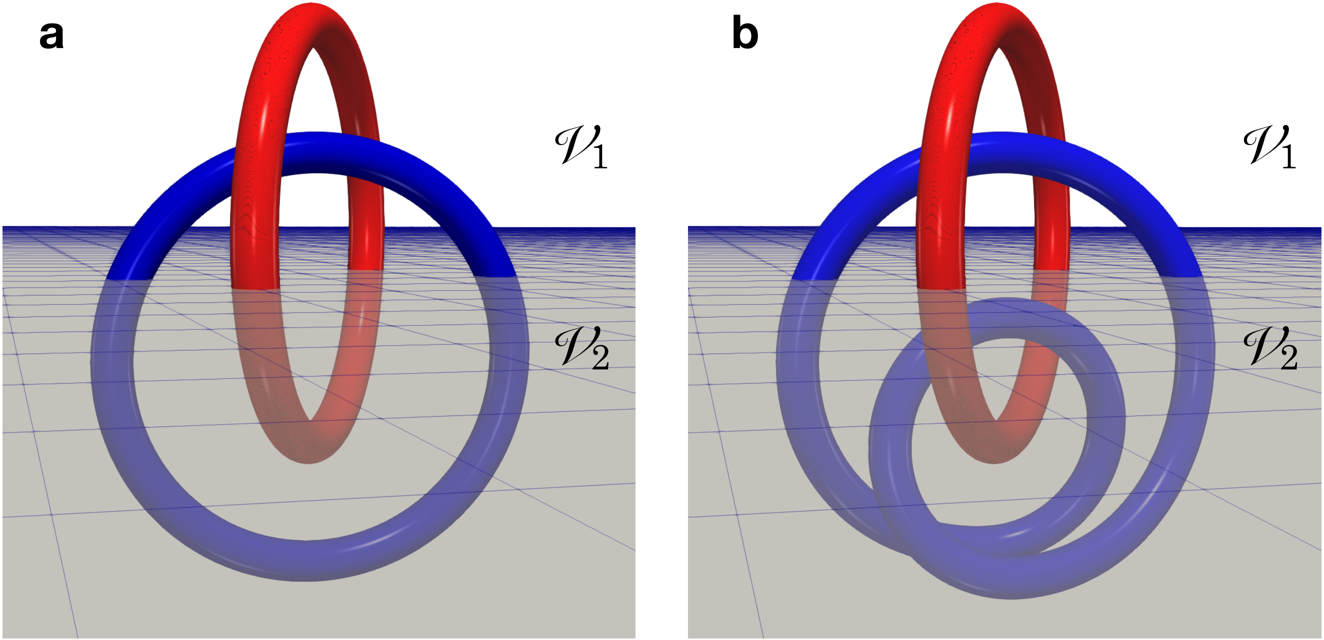

a
Φ_2
Φ_1
b
Φ_2
Φ_1

Introduction: Specific Topology Quantities 2

Topological Invariant

Helicity


a
$\mathcal{V}_1$
$\mathcal{V}_2$
b
$\mathcal{V}_1$
$\mathcal{V}_2$

Ideal cases

Coronal cases

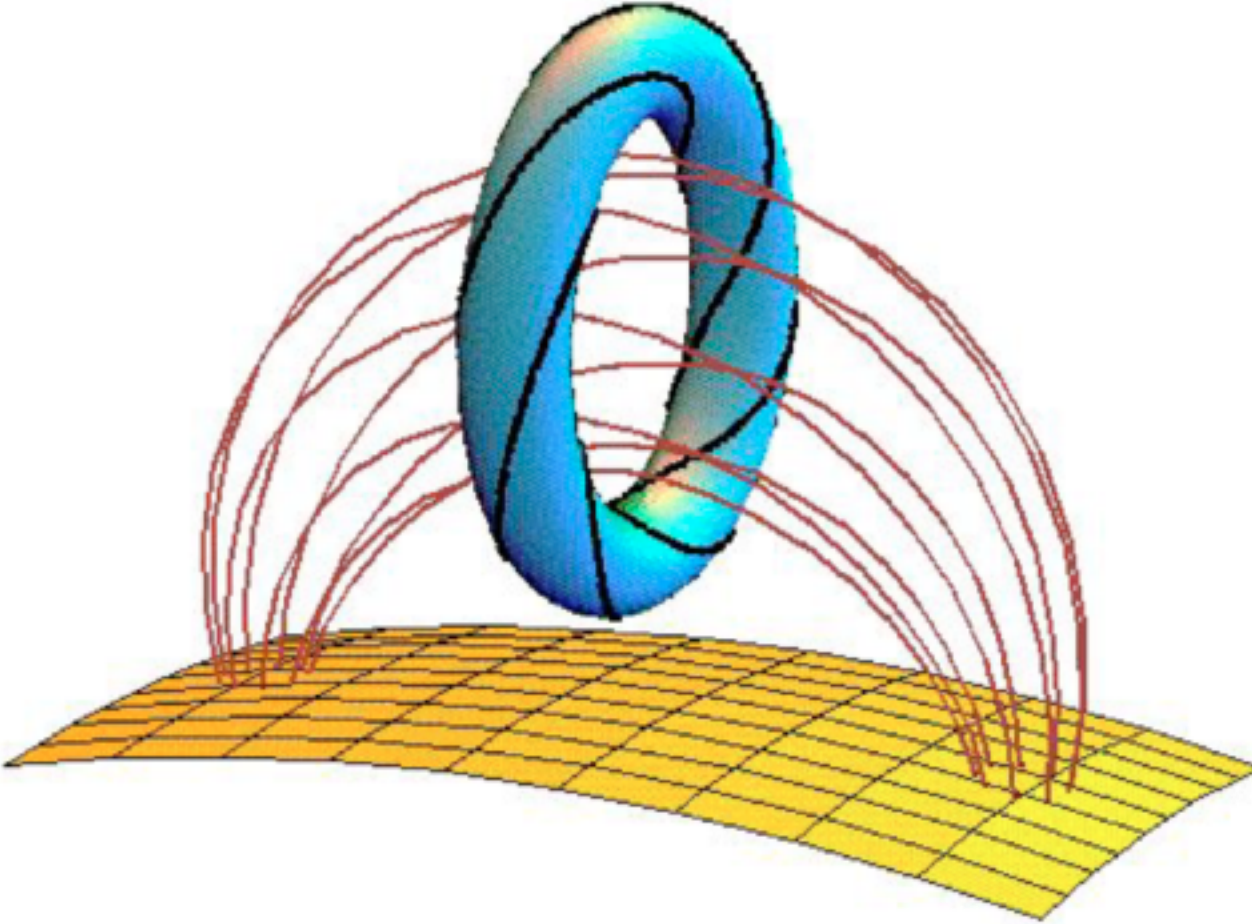
Relative Helicity

$$
\begin{aligned}
H &= \int_\Omega (\mathbf{A} \cdot \mathbf{B}) \mathrm{d}^3\mathbf{x} \\
&= \int_\Omega \int_\Omega \mathbf{B}(\mathbf{x}) \times \mathbf{B}'(\mathbf{x}') \cdot \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \mathrm{d}^3\mathbf{x}' \mathrm{d}^3\mathbf{x} \\
&= \iint \mathrm{d}\Phi \mathrm{d}\Phi' \mathcal{L}
\end{aligned}
$$

$$\mathcal{L} = \oint \oint \mathrm{d}\mathbf{l} \times \mathrm{d}\mathbf{l}' \cdot \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}$$

$$H_R = \int_\Omega (\mathbf{A} + \mathbf{A}_R) \cdot (\mathbf{B} - \mathbf{B}_R) \mathrm{d}^3\mathbf{x}$$

$$\mathrm{B}_R = -\nabla\Phi$$



$$(\mathbf{B}_R - \mathbf{B}) \cdot \mathbf{n}\Big|_{\partial\Omega} = 0$$

Introduction: Specific Topology Quantities 2

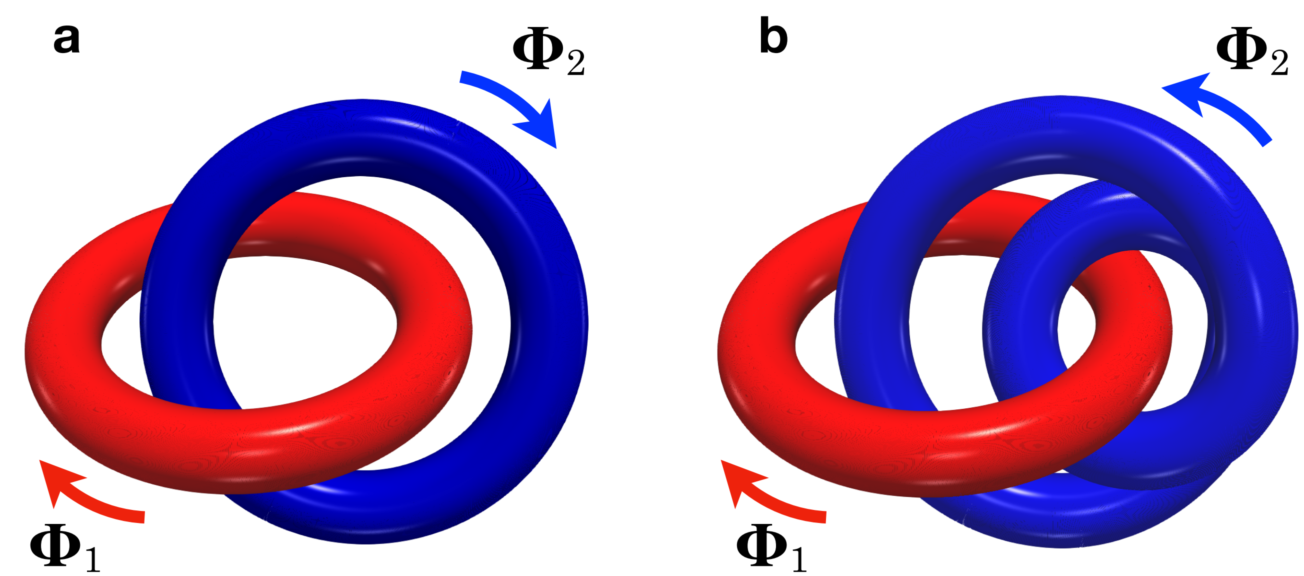
Topological Invariant

Helicity

$$
\begin{aligned}
H &= \int_\Omega (\mathbf{A}\cdot\mathbf{B})\mathrm{d}^3\mathbf{x} \\
&= \int_\Omega\int_\Omega \mathbf{B}(\mathbf{x})\times\mathbf{B}'(\mathbf{x}')\cdot\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\mathrm{d}^3\mathbf{x}'\mathrm{d}^3\mathbf{x} \\
&= \iint \mathrm{d}\Phi\mathrm{d}\Phi'\mathcal{L}
\end{aligned}
$$

$$
\mathcal{L} = \oint\oint \mathrm{d}\mathbf{l}\times\mathrm{d}\mathbf{l}'\cdot\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}
$$

Ideal cases

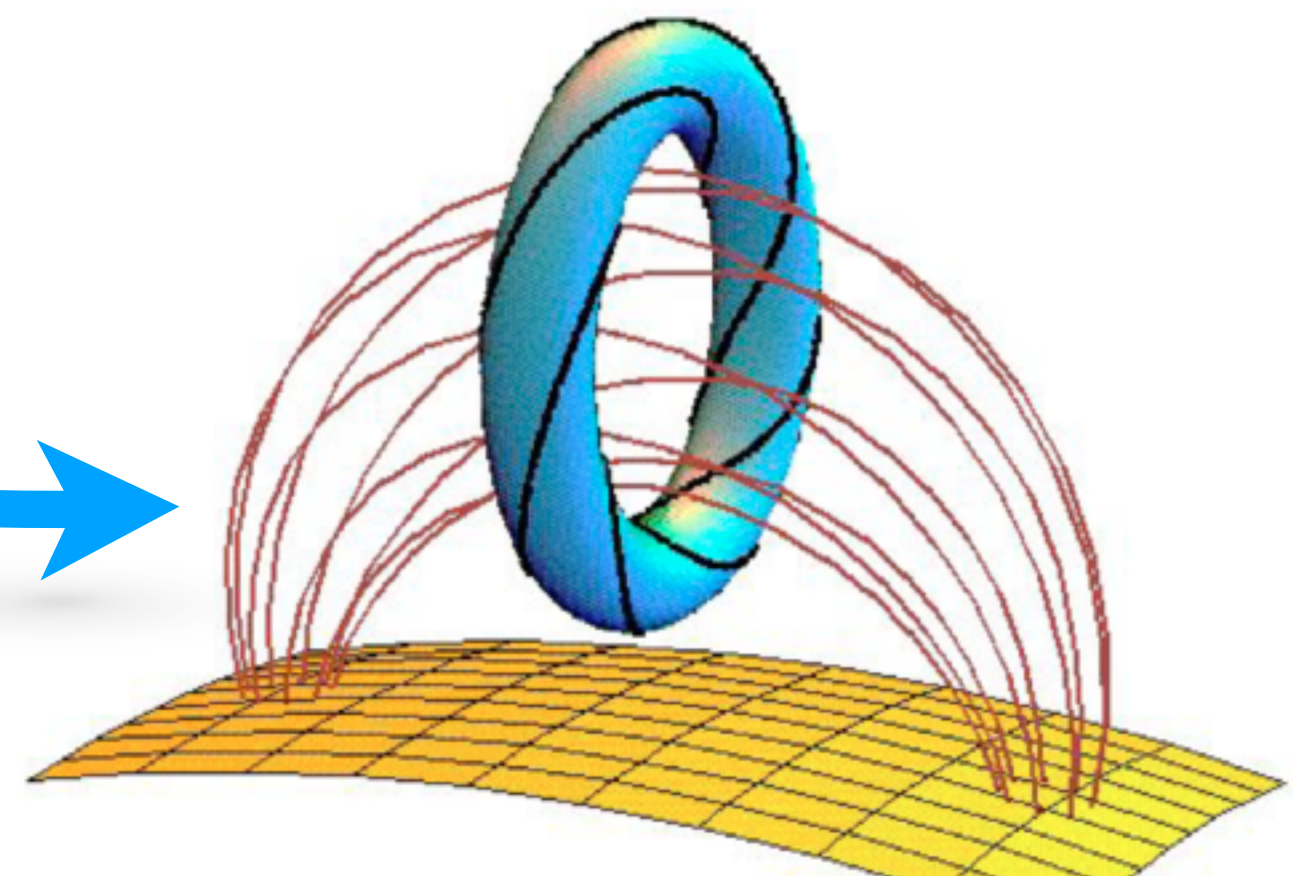


Relative Helicity

$$
H_R = \int_\Omega (\mathbf{A}+\mathbf{A}_R)\cdot(\mathbf{B}-\mathbf{B}_R)\mathrm{d}^3\mathbf{x}
$$

$$
\mathrm{B}_R = -\nabla\Phi
$$

$$
(\mathbf{B}_R-\mathbf{B})\cdot\mathbf{n}\big|_{\partial\Omega} = 0
$$



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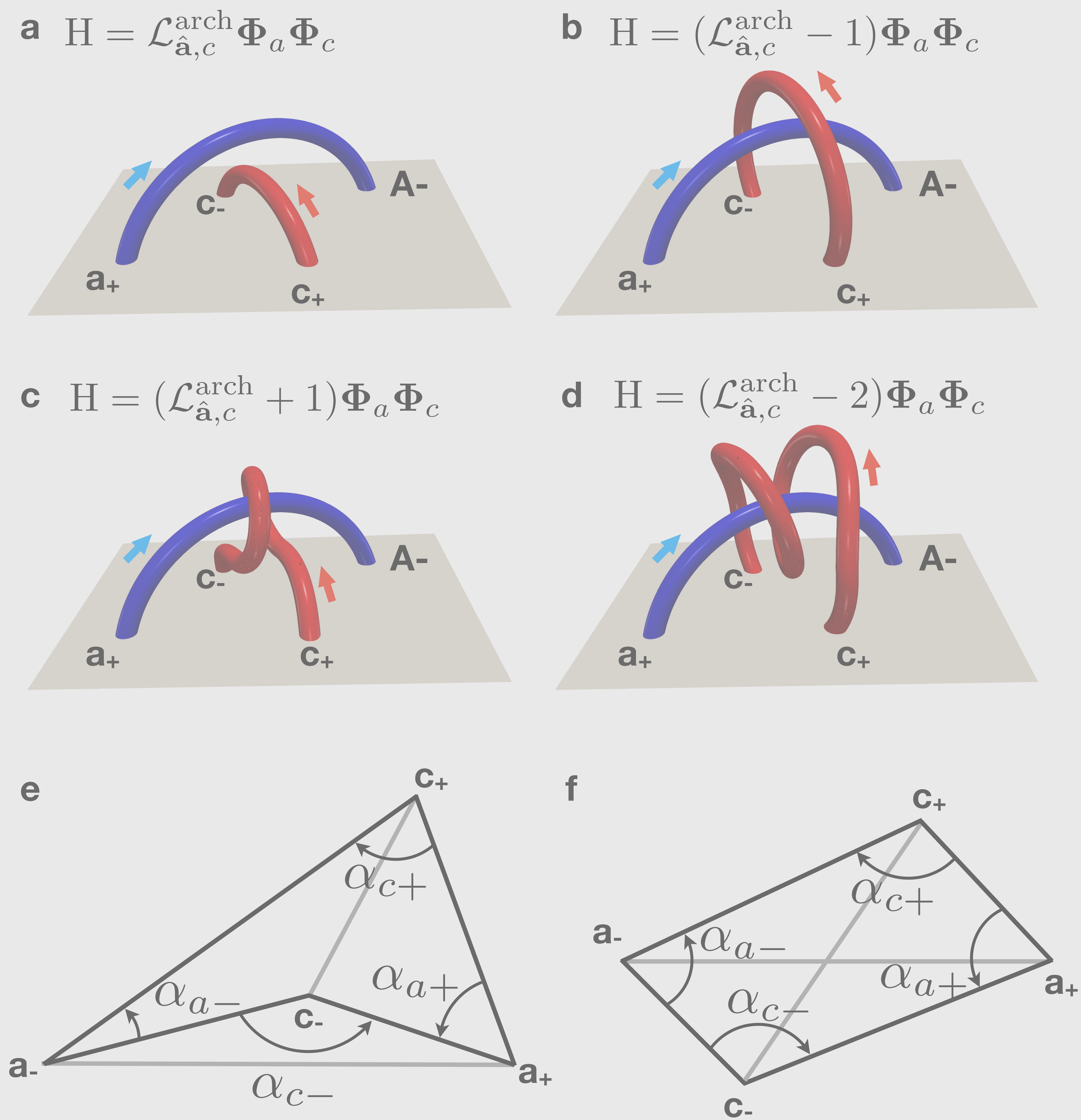

a $\mathrm{H} = \mathcal{L}^{\mathrm{arch}}_{\hat{\mathbf{a}},c}\mathbf{\Phi}_a\mathbf{\Phi}_c$
b $\mathrm{H} = (\mathcal{L}^{\mathrm{arch}}_{\hat{\mathbf{a}},c} - 1)\mathbf{\Phi}_a\mathbf{\Phi}_c$
c $\mathrm{H} = (\mathcal{L}^{\mathrm{arch}}_{\hat{\mathbf{a}},c} + 1)\mathbf{\Phi}_a\mathbf{\Phi}_c$
d $\mathrm{H} = (\mathcal{L}^{\mathrm{arch}}_{\hat{\mathbf{a}},c} - 2)\mathbf{\Phi}_a\mathbf{\Phi}_c$
a_+
c_-
c_+
A_-
e
f
a_-
a_+
c_-
c_+
α_{c+}
α_{a-}
α_{a+}
α_{c-}