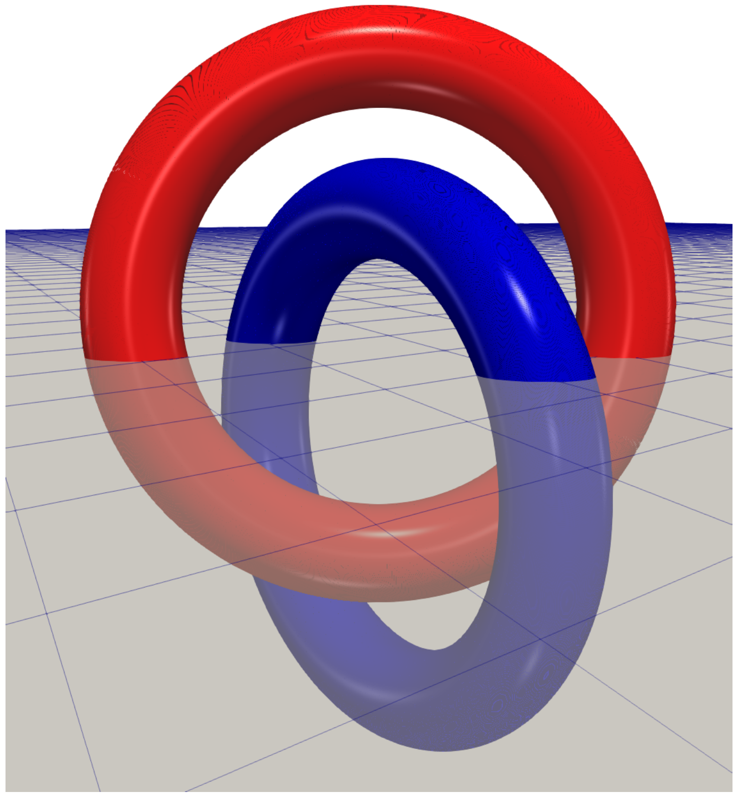
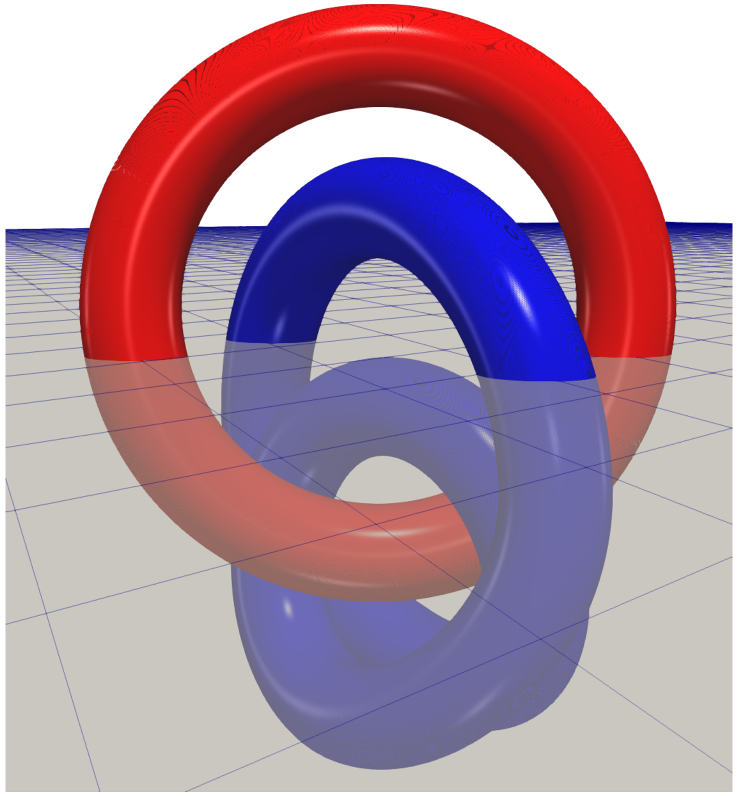
Relative Magnetic Helicity

- Gauge dependent was solved by relative magnetic helicity (Berger & Field 1984).
  - Defined as $H_R = \int_\Omega (\mathbf{A} + \mathbf{A}_R) \cdot (\mathbf{B} - \mathbf{B}_R) \mathrm{d}^3\vec{x}$ Finn & Antonsen1985.
- **B** field can be separated into two part **B**$_j$ + **B**$_p$,
  - $(\mathbf{B}_\mathrm{p} - \mathbf{B}) \cdot \hat{\mathbf{n}}|_{\partial\Omega} = 0$ and $\mathbf{B}_\mathrm{j} \cdot \hat{\mathbf{n}}|_{\partial\Omega} = 0$
- Current-free field (**B**$_p$) is usually chosen as **B**$_R$, It is the minimum energy state.





Relative Magnetic Helicity

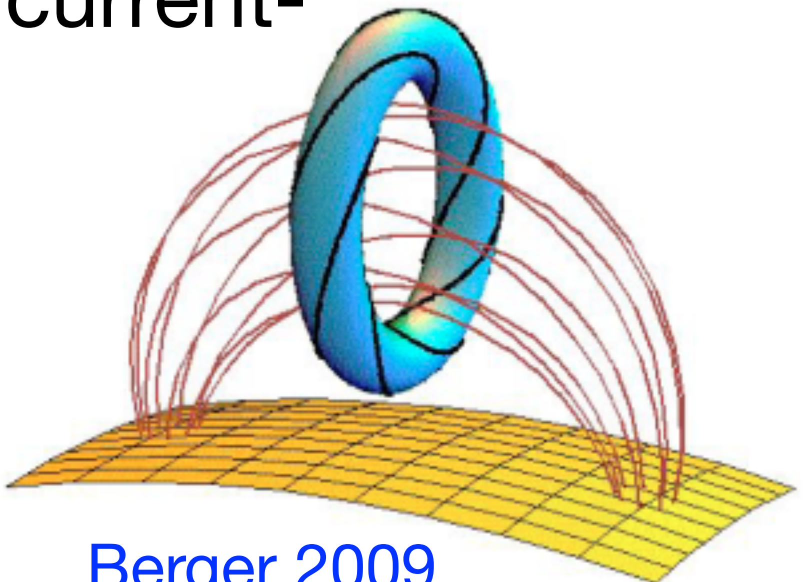
- The relative magnetic helicity can be separated into two parts

- $H_{\mathrm{j}} = \int_{\Omega} \mathbf{B}_{\mathrm{j}} \cdot \mathbf{A}_{\mathrm{j}} \mathrm{d}^3\vec{x}$ and $H_{\mathrm{pj}} = 2\int_{\Omega} \mathbf{B}_{\mathrm{j}} \cdot \mathbf{A}_{\mathrm{p}} \mathrm{d}^3\vec{x}$

  - H_j is the self helicity of the current-carrying field.

  - H_{pj} is the mutual helicity between the current-carrying and the reference field.

  - Both parts are gauge independent.



Berger 2009