Feature of the Newly Defined Helicity

- We can define a relative helicity for the field $\mathbf{B}_\mathrm{c}$ as

$$H_{\mathrm{cr}} = \int_\Omega (\mathbf{A}_\mathrm{c} + \mathbf{A}_{\mathrm{p1}}) \cdot (\mathbf{B}_\mathrm{c} - \mathbf{B}_{\mathrm{p1}}) \mathrm{d}^3\vec{x}$$

- From the field decomposition $$\begin{aligned} \mathbf{B} &= \mathbf{B}_\mathrm{p} + \mathbf{B}_\mathrm{j} \\ &= \mathbf{B}_0 + \mathbf{B}_\mathrm{c} \\ &= \mathbf{B}_0 + \mathbf{B}_{\mathrm{p1}} + \mathbf{B}_{\mathrm{c1}} \end{aligned}$$

- Clearly, $\mathbf{B}_\mathrm{p} = \mathbf{B}_0 + \mathbf{B}_{\mathrm{p1}}$ and $\mathbf{B}_\mathrm{j} = \mathbf{B}_{\mathrm{c1}}$, in most case, $\mathbf{B}_{\mathrm{p1}}$ is very small.

Feature of the Newly Defined Helicity

- The self helicity of the current-carrying field

$$H_{\mathrm{cj}} = \int_{\Omega} (\mathbf{A}_{\mathrm{c}} - \mathbf{A}_{\mathrm{p1}}) \cdot (\mathbf{B}_{\mathrm{c}} - \mathbf{B}_{\mathrm{p1}}) \mathrm{d}^3\vec{x}$$

- The mutual helicity of the current-carrying field and referenced field

$$H_{\mathrm{cpj}} = 2\int_{\Omega} \mathbf{A}_{\mathrm{p1}} \cdot (\mathbf{B}_{\mathrm{c}} - \mathbf{B}_{\mathrm{p1}}) \mathrm{d}^3\vec{x}$$

- $\mathbf{B}_{p1}$ is very small —> small $\mathbf{A}_{p1}$ —> small H_{CPJ} and $H_{cj} \sim H_j$

- $H_{cr} \sim H_{cj} = H_j$