Magnetic Helicity

- Magnetic helicity is defined as

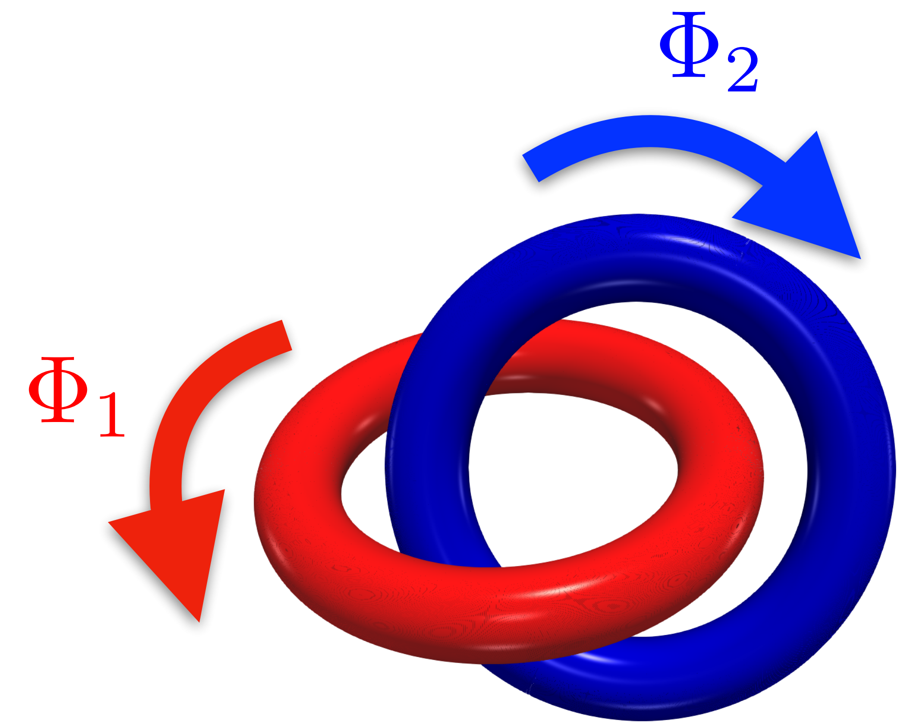
$$H = \int_{\Omega} \mathbf{A} \cdot \mathbf{B} \mathrm{d}^3 \vec{x}$$

  - Coulomb gauge =>

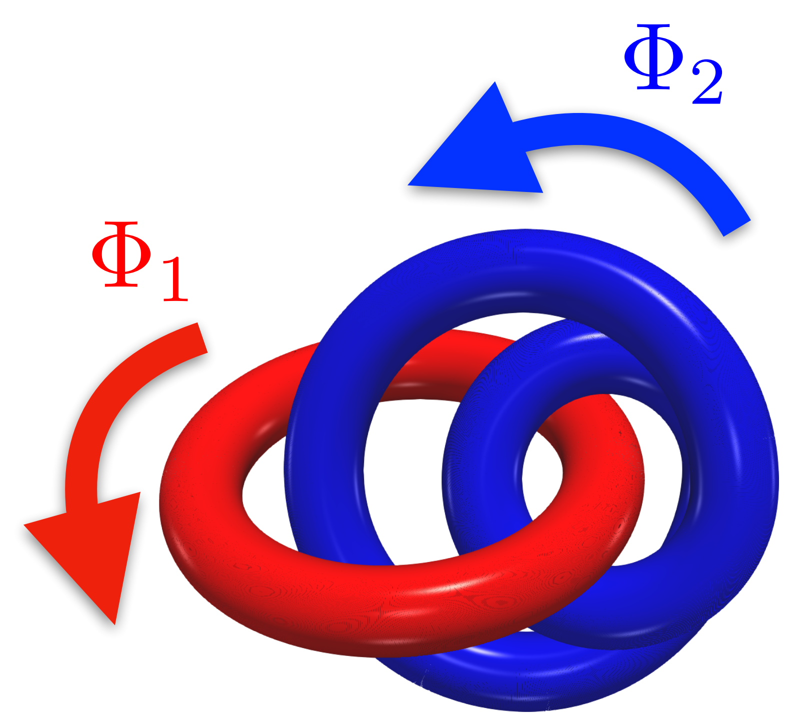
$$H = \int_{\mathcal{C}_1, \mathcal{C}_2} \mathcal{L}_{1,2} \mathrm{d}\Phi_1 \mathrm{d}\Phi_2$$

  - $\mathcal{L}_{1,2}$ is a topology invariant, which lead the helicity be a topology invariant.

- Quasi-/invariant under resistive/idea MHD process (Taylor 1986, Berger 1992).



$$H = 2\Phi_1\Phi_2$$



$$H = -4\Phi_1\Phi_2$$

Relative Magnetic Helicity

- Gauge dependent was solved by relative magnetic helicity (Berger & Field 1984).
  - Defined as $H_R = \int_\Omega (\mathbf{A} + \mathbf{A}_R) \cdot (\mathbf{B} - \mathbf{B}_R) \mathrm{d}^3\vec{x}$ Finn & Antonsen1985.
- **B** field can be separated into two part $\mathbf{B}_\mathrm{j}$ + $\mathbf{B}_\mathrm{p}$,
  - $(\mathbf{B}_\mathrm{p} - \mathbf{B}) \cdot \hat{\mathbf{n}}|_{\partial\Omega} = 0$ and $\mathbf{B}_\mathrm{j} \cdot \hat{\mathbf{n}}|_{\partial\Omega} = 0$
- Current-free field ($\mathbf{B}_\mathrm{p}$) is usually chosen as $\mathbf{B}_\mathrm{R}$, It is the minimum energy state.