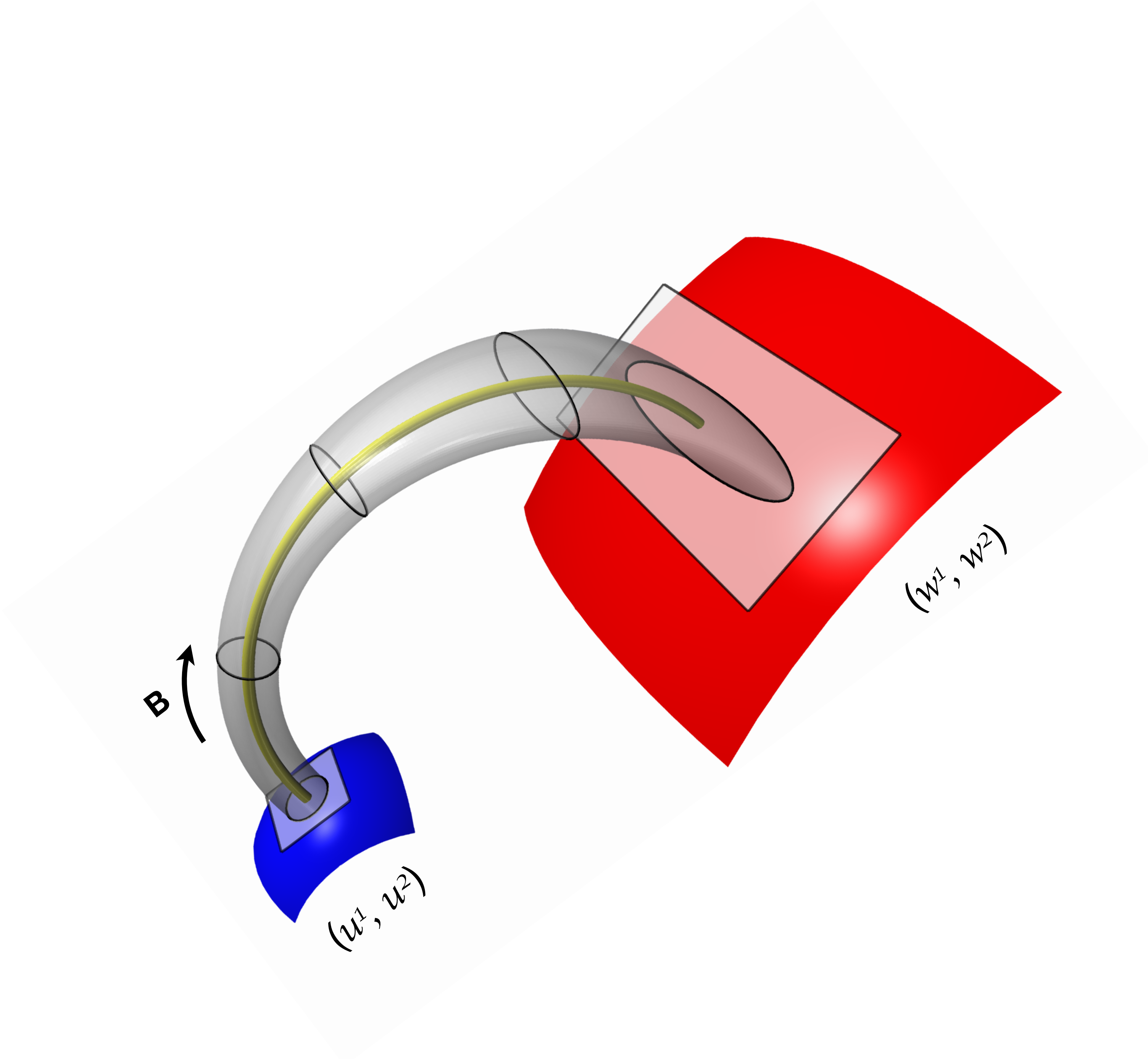

B
(w^1, w^2)
(u^1, u^2)

Introduction: Specific Topology Quantities 1

Topological Boundary

Gradient of the field line mapping f_B: $\mathbf{r} \rightarrow \mathbf{R}$.

Separatrix: $|\nabla f_B| = \infty$.

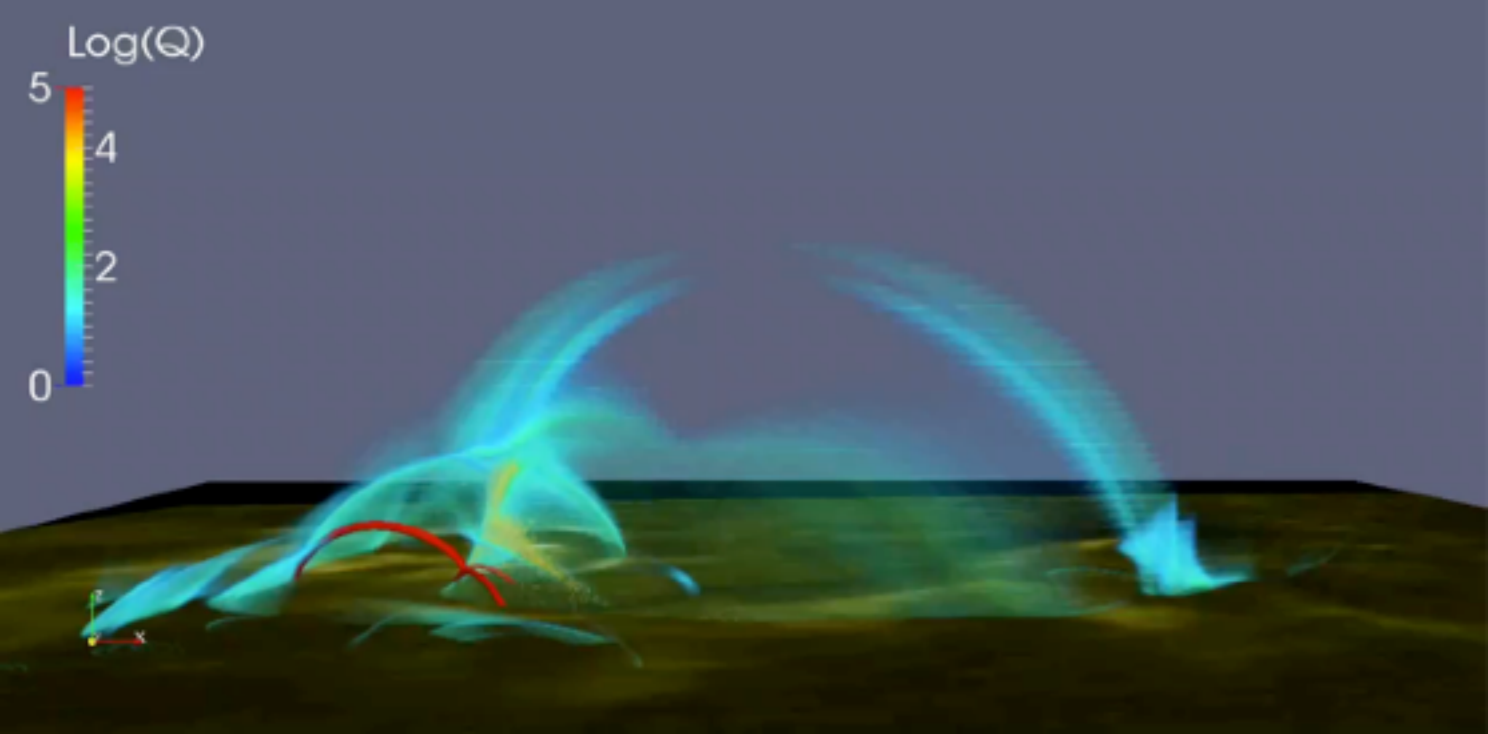
Quasi-separatrix: $|\nabla f_B| \gg 1$.

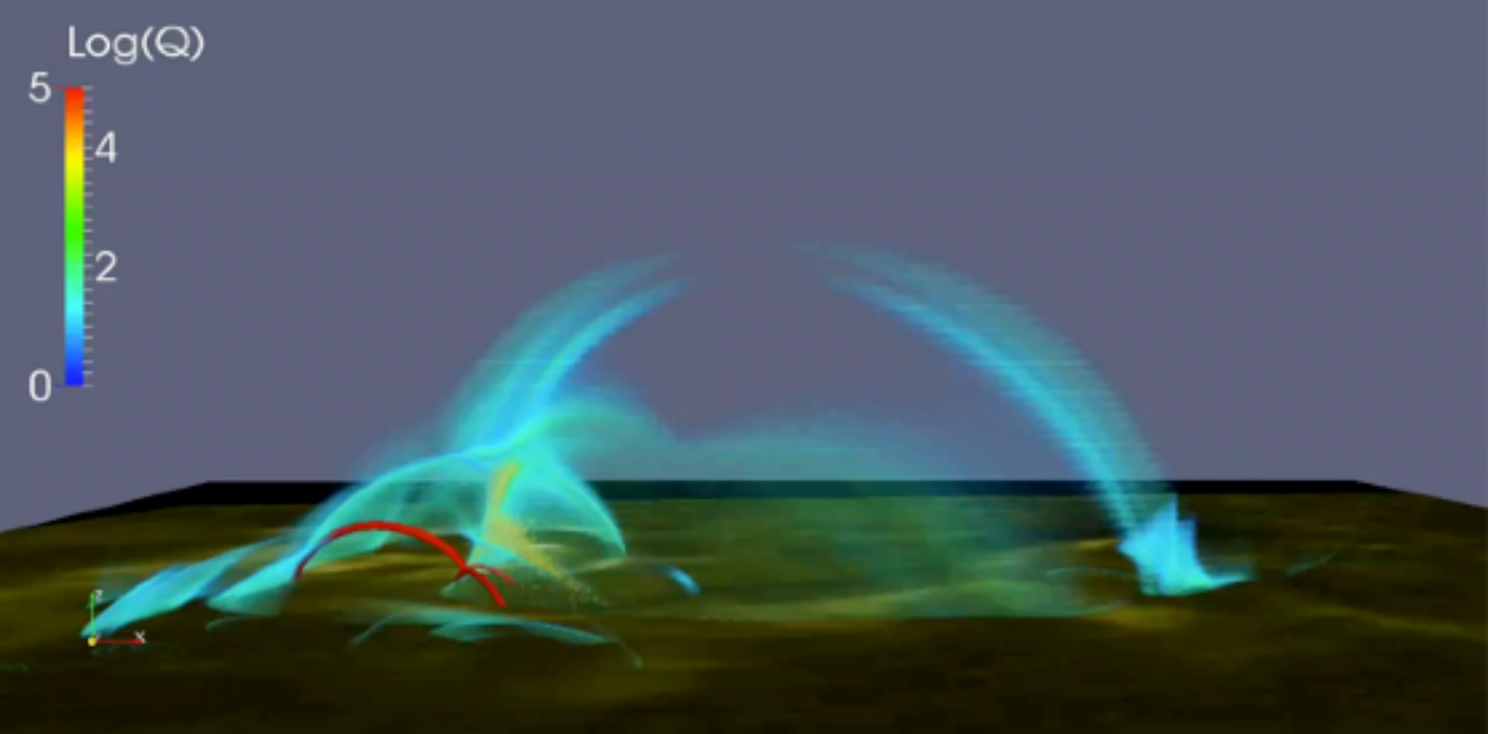
Widely used measurement of the gradient is:

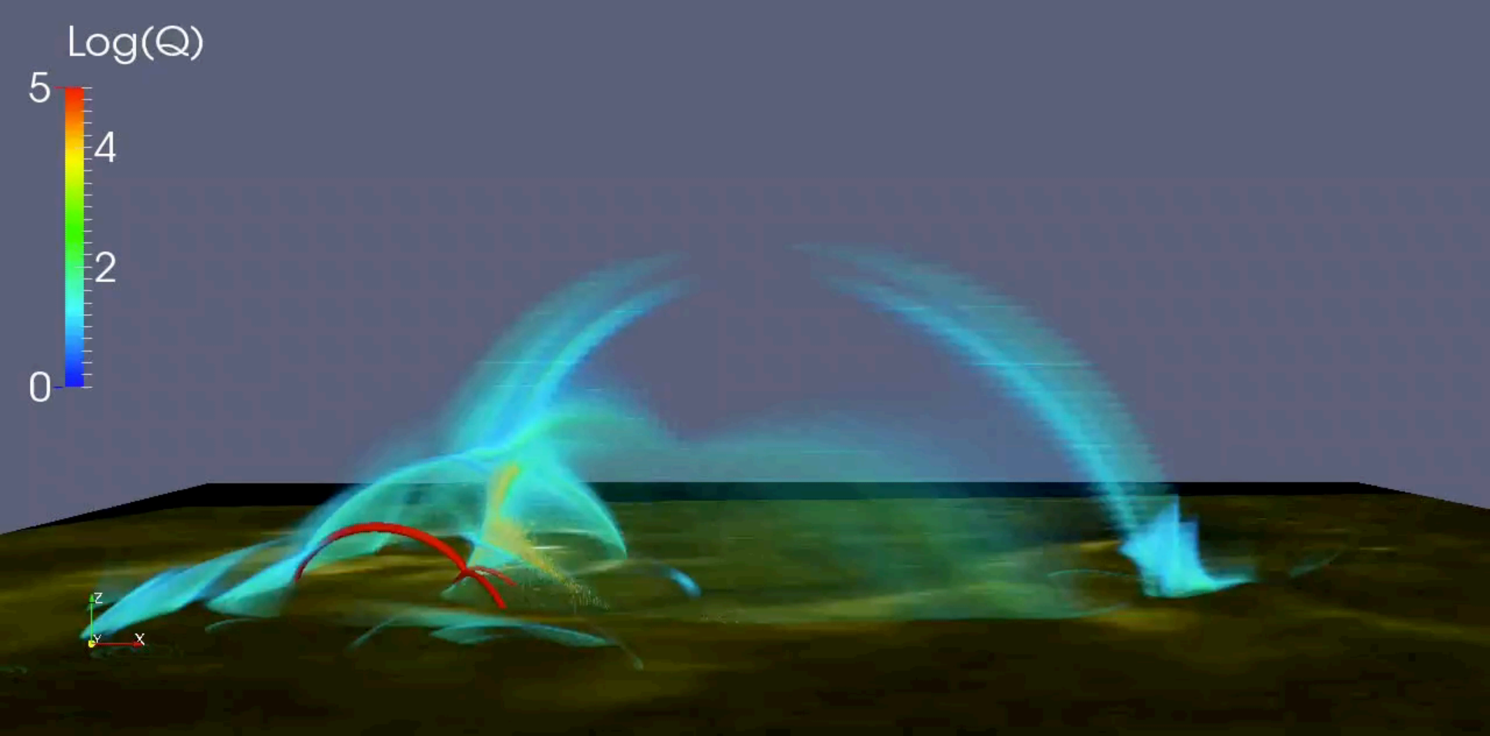
$$Q = \frac{\|\mathbf{J}\|^2}{|\det \mathbf{J}|}$$

,where J is the Jacobian matrix of the map f.

$Q=e+1/e$


Log(Q)
5
4
2
0
z
x

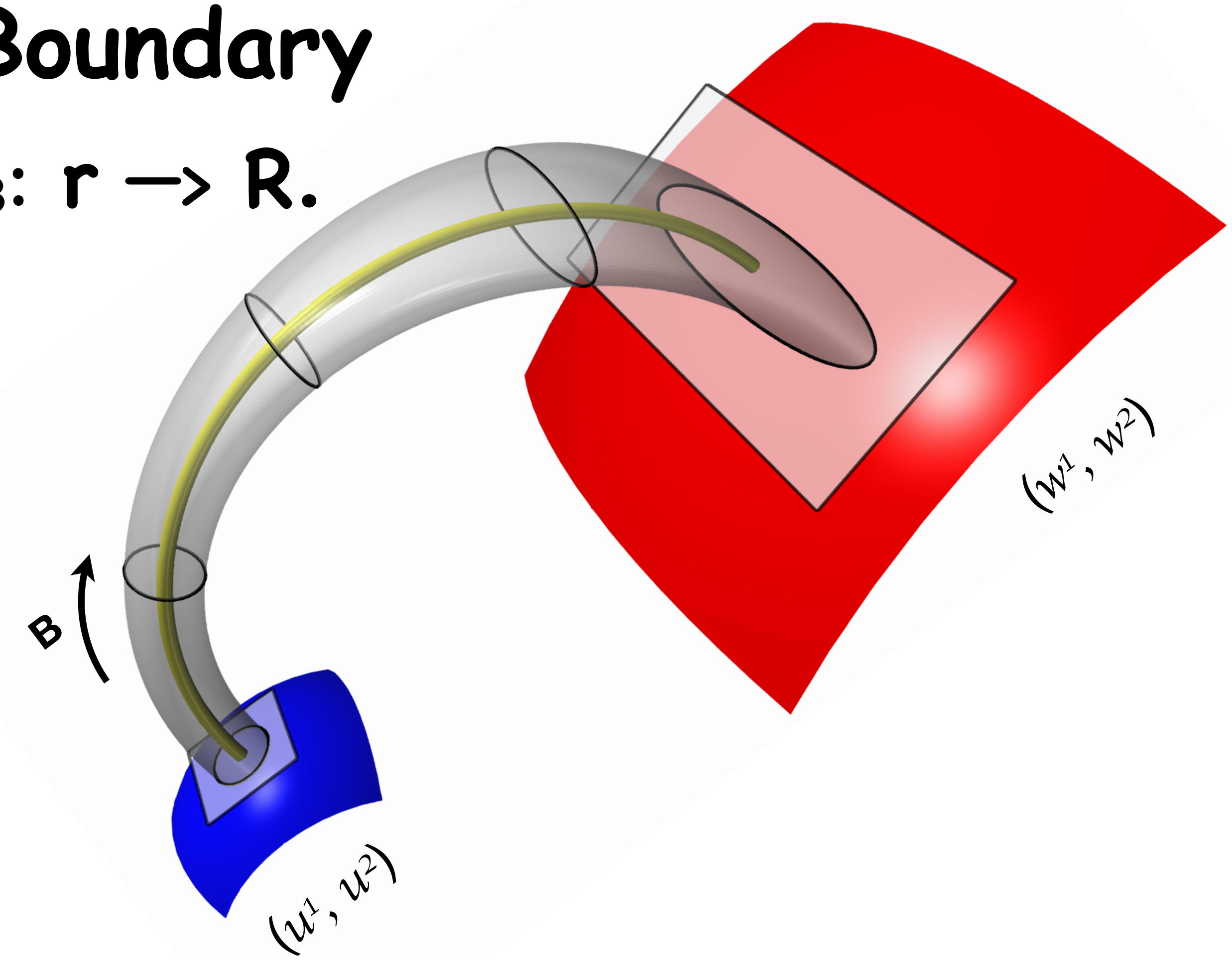

Log(Q)
5
4
2
0
z
x


Log(Q)
5
4
2
0
Z
Y
X

Introduction: Specific Topology Quantities 1

Topological Boundary

Gradient of the field line mapping f_B: **r** —> **R**.
Separatrix: $|\nabla f_B| = \infty$.
Quasi-separatrix: $|\nabla f_B| >> 1$.

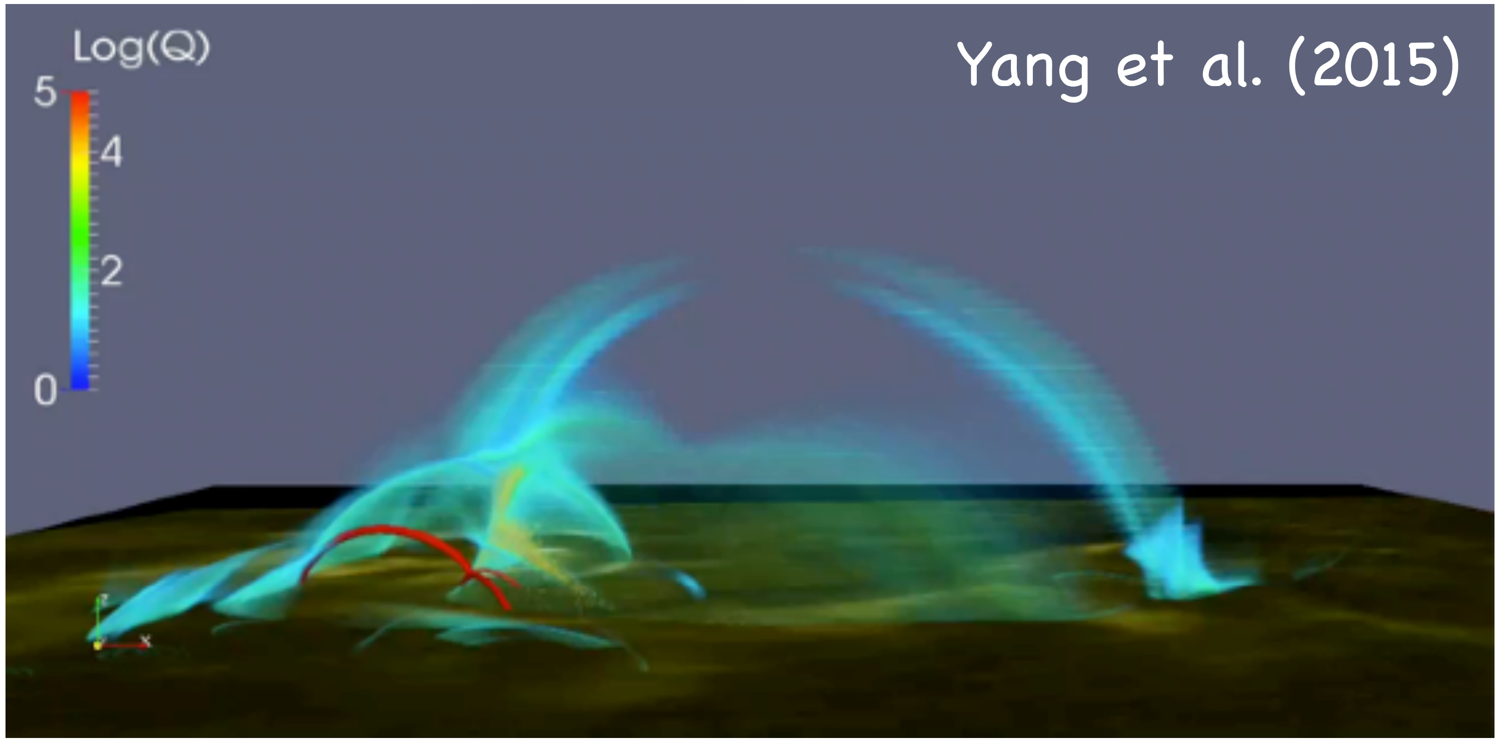


Widely used measurement of the gradient is:

$$Q = \frac{||\mathbf{J}||^2}{|\det \mathbf{J}|}$$

,where J is the Jacobian matrix of the map f.

Q=e+1/e



Introduction: Specific Topology Quantities 2

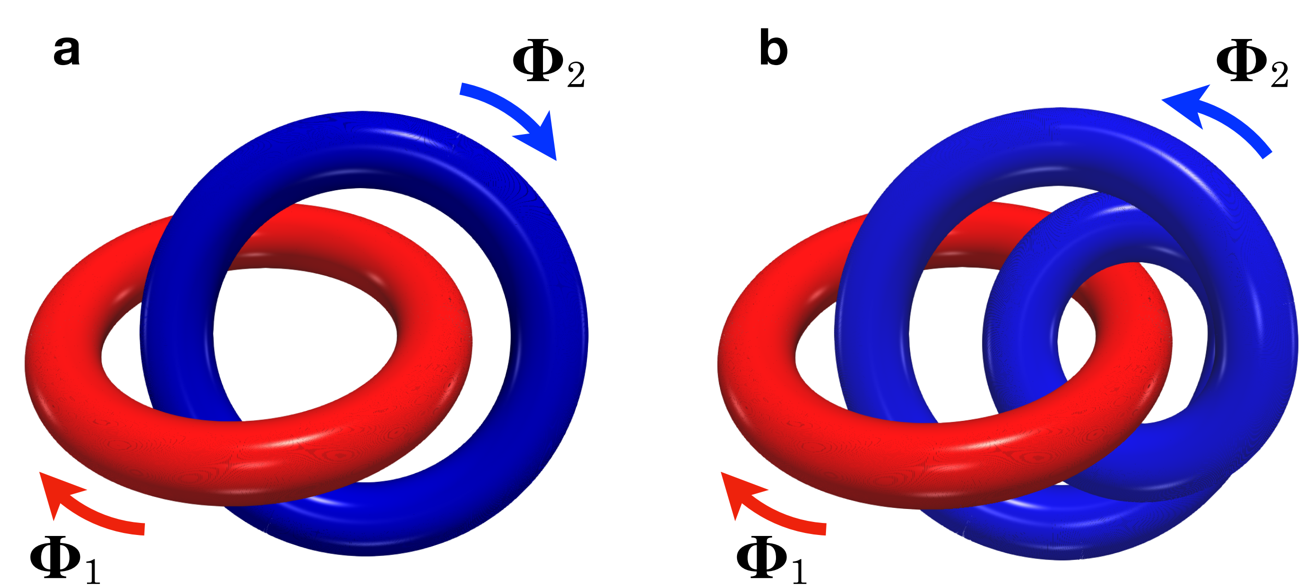
Topological Invariant

Helicity

$$
\begin{aligned}
H &= \int_\Omega (\mathbf{A}\cdot\mathbf{B})\mathrm{d}^3\mathbf{x} \\
&= \int_\Omega\int_\Omega \mathbf{B}(\mathbf{x})\times\mathbf{B}'(\mathbf{x}')\cdot\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\mathrm{d}^3\mathbf{x}'\mathrm{d}^3\mathbf{x} \\
&= \iint \mathrm{d}\Phi\mathrm{d}\Phi'\mathcal{L}
\end{aligned}
$$

$$
\mathcal{L} = \oint\oint \mathrm{d}\mathbf{l}\times\mathrm{d}\mathbf{l}'\cdot\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}
$$

Ideal cases



Relative Helicity

$$
H_R = \int_\Omega (\mathbf{A}+\mathbf{A}_R)\cdot(\mathbf{B}-\mathbf{B}_R)\mathrm{d}^3\mathbf{x}
$$

$$
\mathrm{B}_R = -\nabla\Phi
$$

$$
(\mathbf{B}_R-\mathbf{B})\cdot\mathbf{n}\big|_{\partial\Omega} = 0
$$

Coronal cases