Let G be a group and $H \triangleleft G$. Let $K \leq G$ such that $H \subseteq K$. Then:

1. $K/H \triangleleft G/H$ if and only if $K \triangleleft G$
2. $(G/H)/(K/H) \cong G/K$

Proof of (1):
($\Rightarrow$) Assume $K/H \triangleleft G/H$. Then for all $gH \in G/H$, we have:

$$(gH)(K/H)(gH)^{-1} = K/H$$

This implies $gKg^{-1} \subseteq K$, so $K \triangleleft G$.
($\Leftarrow$) Assume $K \triangleleft G$. Then for all $g \in G$, we have $gKg^{-1} = K$. Therefore:

$$(gH)(K/H)(gH)^{-1} = (gKg^{-1})/H = K/H$$

So $K/H \triangleleft G/H$.
Proof of (2):
Define $\phi : G/H \to G/K$ by $\phi(gH) = gK$. This is well-defined since if $gH = g'H$, then $g^{-1}g' \in H \subseteq K$, so $gK = g'K$.
ϕ is a homomorphism:

$$\phi(gH \cdot g'H) = \phi(gg'H) = gg'K = (gK)(g'K) = \phi(gH)\phi(g'H)$$

ϕ is surjective: For any $gK \in G/K$, we have $\phi(gH) = gK$.
The kernel of ϕ is:

$$\ker\phi = \{gH \in G/H : \phi(gH) = K\} = \{gH : gK = K\} = \{gH : g \in K\} = K/H$$

By the First Isomorphism Theorem:

$$(G/H)/\ker\phi \cong \operatorname{im}\phi$$

So:

$$(G/H)/(K/H) \cong G/K$$