Let G be a group. Let HG (i.e., H is a normal subgroup of G).
Let $\phi : G \to G/H$ be the quotient map.
For $g \in G$, we have $\phi(g) = gH$.
Now, let $K \leq G$ be a subgroup.
Then $\phi(K) = \{kH : k \in K\} = KH/H$.
Also, $\ker(\phi|_K) = K \cap H$.
So by the first isomorphism theorem:

$$K/(K \cap H) \cong KH/H$$

Now, if $K \leq H$, then $K \cap H = K$ and $KH = H$, so:

$$K/K \cong H/H \Rightarrow \{1\} \cong \{1\}$$

If $H \leq K$, then $K \cap H = H$ and $KH = K$, so:

$$K/H \cong K/H$$

This is consistent.
Now, consider the case where KG as well.
Then KH is a subgroup of G.
Since HG and KG, we have $KH = HK$ and KHG.
Then:

$$G/KH \cong (G/H)/(KH/H)$$

This is the third isomorphism theorem.
Also, $K \cap HG$ and:

$$G/(K \cap H) \cong (G/K) \times (G/H)$$

if $G = KH$ and $K \cap H = \{1\}$ (direct product).
Let $a, b \in G$. Then:

$$ab = ba? \quad \text{Not necessarily, unless } G \text{ is abelian}$$

If $k \in K$, then $\phi(k) = kH$.
Since KG, we have $gkg^{-1} \in K$ for all $g \in G$.
In particular, for $h \in H$, we have $hkh^{-1} \in K$.
But since HG, we also have $khk^{-1} \in H$.
So K and H normalize each other.
Then $[K, H] \leq K \cap H$.
If $K \cap H = \{1\}$, then $[K, H] = \{1\}$, so K and H commute elementwise.
Then $KH \cong K \times H$.
This is a common situation in group theory.