Let G be a group. Let $H \triangleleft G$. Let $K \leq G$ such that $K/H = G/H$. Let $f : G \to G/H$ be the quotient map.

1. $\ker(f) = H$

2. $f(K) = K/H = G/H$ since $K/H = G/H$

3. $f^{-1}(f(K)) = K$? Actually $f^{-1}(f(K)) = KH = K$ since $H \leq K$

4. $K/H = G/H$ implies $K = G$? Not necessarily, but $KH = G$

5. Since $H \triangleleft G$ and $K \leq G$, then $KH \leq G$

6. $KH = G$ since $K/H = G/H$

7. Therefore $G = KH$

8. If $K \cap H = \{e\}$, then $G = K \times H$

9. Otherwise, G is a semidirect product