

Let G be a group. Let $H \leq G$. Let $K \leq G$ such that $K/H = G/H$.
We have:

$$\begin{aligned} |G| &= |H| \cdot |K/H| \\ &= |H| \cdot |G/H| \\ &= |H| \cdot \frac{|G|}{|H|} \\ &= |G| \end{aligned}$$

Now consider:

$$\begin{aligned} N_G(K) &= \{g \in G \mid gKg^{-1} = K\} \\ N_H(K) &= \{h \in H \mid hKh^{-1} = K\} \end{aligned}$$

Since $K \leq G$ and $H \leq G$, we have $K \cap HK$.
Also, since $K/H = G/H$, we have $KH = G$.
Now, if $k \in K$ and $h \in H$, then:

$$khk^{-1} \in H \quad (\text{since } H \leq G)$$

and

$$khk^{-1} \in K \quad (\text{since } K \leq G \text{ and } h \in H \leq G)$$

Therefore $khk^{-1} \in K \cap H$.

So K normalizes H , and H normalizes K .

Thus $N_G(K) \geq H$ and $N_H(K) = H \cap N_G(K)$.

Since $K/H = G/H$, we have $|G/K| = |H/(K \cap H)|$.

Therefore:

$$|N_G(K)| = |H| \cdot |N_G(K)/H|$$