Let G be a group. Let $H \triangleleft G$. Let $K \triangleleft G$ with $K \subseteq H$.
Then:

$$N_G(K) = \{g \in G \mid gKg^{-1} = K\}$$
$$N_H(K) = \{h \in H \mid hKh^{-1} = K\}$$

We have $N_H(K) = N_G(K) \cap H$.
Since $K \triangleleft G$, we have $N_G(K) = G$.
So $N_H(K) = G \cap H = H$.
Therefore $K \triangleleft H$.