Let G be a group. Let $H \triangleleft G$. Let $K = G/H$. Then:

$$\begin{aligned} \phi : G &\to K \\ g &\mapsto gH \end{aligned}$$

Let $R = \ker \phi$. Then $\phi : G \to K$ with $\ker \phi = R$.
Let $H = \ker \phi$. Then:

$$\begin{aligned} N(H) &= \{g \in G \mid gHg^{-1} = H\} \\ &= \{g \in G \mid gH = Hg\} \end{aligned}$$

Now:

$$\begin{aligned} g \in N(H) &\Leftrightarrow gH = Hg \\ &\Leftrightarrow gHg^{-1} = H \\ &\Leftrightarrow g \in N(H) \end{aligned}$$

Also:

$$\begin{aligned} g \in N(H) &\Leftrightarrow gH = Hg \\ &\Leftrightarrow (gH)(g^{-1}H) = H \\ &\Leftrightarrow gHg^{-1} = H \end{aligned}$$

Since $H \triangleleft G$, we have $N(H) = G$.
Now consider:

$$\begin{aligned} &K = G/H \\ &k \in K \Rightarrow k = gH \text{ for some } g \in G \end{aligned}$$

Then:

$$\begin{aligned} &kH = (gH)H = gH = k \\ &Hk = H(gH) = (Hg)H = (gH)H = gH = k \end{aligned}$$

So $H \triangleleft K$.
Now:

$$\begin{aligned} &|K| = |G/H| = [G : H] \\ &|H| = |\ker \phi| \end{aligned}$$

By the first isomorphism theorem:

$$\begin{aligned} G/\ker \phi &\cong \phi(G) \\ G/H &\cong K \end{aligned}$$

So $|G| = |H| \cdot |K|$.

Now let $K < G$ and $H < G$ with $H \triangleleft G$. Then:

$$KH = \{kh \mid k \in K, h \in H\}$$
$$HK = \{hk \mid h \in H, k \in K\}$$

Since $H \triangleleft G$, we have $KH = HK$.
Also:

$$K \cap H \triangleleft K$$
$$K/(K \cap H) \cong KH/H$$

By the second isomorphism theorem:

$$K/(K \cap H) \cong KH/H$$

Now if $K < N(H)$, then:

$$KH = HK$$
$$|KH| = \frac{|K||H|}{|K \cap H|}$$

And:

$$[KH : H] = [K : K \cap H]$$