Let G be a group, $H \leq G$, and KG. Let $\phi : G \to G/K$ be the natural homomorphism.

Then:

1. $\phi(H) = HK/K$
2. $H \cap KH$
3. $H/(H \cap K) \cong HK/K$

Proof:

- Since KG, we have $kK = K$ for all $k \in K$
- For $h \in H$, $\phi(h) = hK$
- Thus $\phi(H) = \{hK : h \in H\} = HK/K$
- $H \cap KH$ because KG
- By the First Isomorphism Theorem, $H/(H \cap K) \cong HK/K$