

Let G be a group. Let $H \triangleleft G$. Let $K = G/H$.

We have: $|G| = |H| \cdot |K|$.

Let $a \in G, h \in H, k \in K$ such that $k \in K$ and $K \triangleleft G$ so $aK = Ka$ so $K \triangleleft G$.

Then: $K \triangleleft G$ so $ka = ak$ for all $k \in K$, so $(a)(k) = (k)(a)$.

$\Leftrightarrow K \triangleleft G$ so $ka = ak, k^{-1} \in K$, so $(a)(k^{-1}) = (k^{-1})(a)$.

$k \in K$ so $aK = Ka$ and $K \triangleleft G$.

If $H \triangleleft G$ then $|K| = |G/H| = |G|/|H|$.