INTERNAL ASSESSMENT: PHYSICS SL

Exploring Momentum Dynamics: Studying the impact of applied force on DigiCart and its analogy to financial markets

"How does changing the force exerted on a DigiCart affect its momentum, and how can this relationship be understood analogously through the dynamics of financial market investments?"

INTRODUCTION

Personal interest

At Wuhan Zhongrun Fuxing, I learned how they use quantitative analysis for stock price predictions, sparking my interest in the physics of finance. Such concepts, although subjective, are explored in papers like "Predicting Price Trends Using Momentum and Deep Reinforcement Learning." My Physics IA is an exploration of these principles, linking them to real-life market understanding. With a background in Python and insights from my time at Zhongrun Fuxing, I aim to delve deeper into quantitative finance. With increasing discussions around Bitcoin's predicted halving in April and uncertainties post-halving[1], analyzing kinetic energy data from the last halving in May 2020 could provide a deeper understanding of market behaviors in line with physical theories.

Objective and research question

The interaction between force and kinetic energy forms the cornerstone of classical mechanics, a field that has long interested me for its profound applications in understanding the physical world. This interest naturally extends to exploring the nuances of energy transfer within dynamic systems, particularly in educational tools like DigiCart, which provide concrete representations of these fundamental principles."Predicting Price Trends Combining Kinetic Energy and Deep Reinforcement Learning" Drawing on the innovative works of Khrennikov (2010)[2] and Fakult (2017)[3], who established intriguing analogies between the dynamical systems in physics and financial markets, this research seeks to extend the metaphor to the concept of kinetic energy as it pertains to the movement within the cryptocurrency market. The traditional physics equation[4] :

$$v_j(t) = \lim_{t \to 0} \frac{q_j(t+\Delta t) - q_j(t)}{\Delta t}$$

[1] Krisztian Sandor (2024) Bitcoin Could Hit $150K This Year, Says Fundstrat's Tom Lee, retrieved January 10, 2024, from https://www.coindesk.com/markets/2024/02/21/bitcoin-could-hit-150k-this-year-says-fundstrats-tom-lee/

[2] Khrennikov (2010) Ubiquitous Quantum Structure, retrieved January 10, 2024, from https://link.springer.com/book/10.1007/978-3-642-05101-2

[3] Fakult (2017) Kinetic Modeling of Financial Market Models, from https://d-nb.info/1162629665/34

[4] Morteza Zahedi (2023) Predicting Price Trends Combining Kinetic Energy and Deep Reinforcement Learning, retrieved January 10, 2024, from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4502473

which describes the dynamics of price as an analog to velocity in mechanics, offers a theoretical framework for understanding how forces—akin to market pressures—impact the kinetic energy, or the vigorous activity, of an asset's price movements over time.

The purpose of this research work is to study how the kinetic energy of a DigiCart changes with different magnitudes of forces. We try to understand this relationship from a physical perspective and draw an analogy with the investment dynamics of financial markets to illustrate how the basic principles of physics are reflected in other fields such as finance.

This survey is rooted in the research questions: "How does changing the force exerted on a DigiCart affect its momentum, and how can this relationship be understood analogously through the dynamics of financial market investments?"

THEORETICAL FRAMEWORK

Kinetic energy:

The investigation into the DigiCart's kinetic energy as influenced by varying applied forces is anchored in classical mechanics, which provides robust models for predicting and analyzing motion. Central to this exploration is Newton's Second Law, which directly relates the force exerted on an object to its acceleration, a determinant of kinetic energy. This law underpins the experimental design, where varying masses will produce different forces on the DigiCart, leading to measurable changes in acceleration and consequently, kinetic energy.

The concept of kinetic energy, a cornerstone in the study of classical mechanics, is predicated on the equation:

$$k = \frac{1}{2}mv^2 \text{ with}$$
$$\Delta E_{kin,1\to 2} = E_{kin,1} - E_{kin,2}$$

Where k represents the kinetic energy of a system, m denotes the mass of an individual particle within the system, and v symbolizes the velocity of that particle. In this investigation, it serves as a metric for evaluating how energy translates into motion. The work-energy principle offers a pathway to understand the transfer of energy into the system, providing a method to quantify the force's effect on the DigiCart through the lens of energy conservation.

Analogy with the financial markets:

In a broader context, the theoretical framework extends to consider the dynamics of financial markets investments. Here, forces are conceptualized as the various financial pressures that drive market behavior, and kinetic energy is analogous to the vigor of price movements. While the domains of physics and finance differ in substance, they share mathematical symmetry in their descriptions of dynamic change.

This symmetry present with pioneering work of Khrennikov (2010) introduced a novel interpretation of kinetic energy within economic systems, positing the 'mass' m as a representation of the financial 'mass', and the velocity v as the dynamics of the price changes over time. The 'volume' of trades, reflecting the number of shares exchanged within a specified period, is analogous to the 'mass' in physical systems, indicating market participation and liquidity, both of which are influential in the movement of prices.

Fakult (2017) further explored this analogy, demonstrating how kinetic energy models can encapsulate features of the financial market with notable fidelity. By equating the volume of trades to the 'mass' m in the kinetic energy equation, our approach seeks to quantify the vibrancy and liquidity of the market, which are integral to the price momentum.

However, financial markets exhibit discrete rather than continuous time intervals, challenging the direct application of the kinetic energy equation. In these markets, price changes are often recorded at distinct time frames, requiring a modified interpretation of velocity. The price dynamics vj (t), therefore, are defined by:

$$v_j(t) = \lim_{t \to 0} \frac{q_j(t+\Delta t) - q_j(t)}{\Delta t}$$

where $v_j(t)$ is the price at time t, and Δt is the infinitesimally small period of time over which price change is measured.

This paper focuses on the impact of Bitcoin halving on cryptocurrency market momentum. The May 2020 halving, a significant reduction in mining rewards, provides critical insights into market volatility and trading volume, revealing participants' behavior and sentiment during such pivotal events. This theoretical framework, thus, serves as the bedrock for our investigation into the force applied to a DigiCart and its resultant kinetic energy, drawing parallels with the dynamics of financial markets investments, and providing a comprehensive lens through which to understand both physical and financial systems.

Variables and hypotheses:

For an investigation into how varying the force applied to a DigiCart affects its kinetic energy, we need to define the variables and formulate a hypothesis.

Variables

Independent Variable: The force applied to the DigiCart. This can be manipulated by changing the mass of the hanging weights that exert the force via a pulley system.

Dependent Variable: The kinetic energy of the DigiCart, calculated from its velocity which is measured directly during the experiment.

Controlled Variables:

- The track's surface and incline, to maintain consistent friction and gravitational force components.
- The air resistance, which should be minimized and kept constant.

- The starting position of the DigiCart for each trial, to ensure that the distance covered during acceleration is consistent.

Additional Variable: The mass of the DigiCart. Conducting experiments with carts of different masses allows for the observation of mass influence on the force-kinetic energy relationship.

Hypothesis

"As the force applied to the DigiCart increases, the kinetic energy of the DigiCart will also increase proportionally, assuming that all other conditions remain constant. This is expected because, according to Newton's second law, an increase in force results in an increase in acceleration, and since kinetic energy is a function of the square of velocity, any increase in acceleration should lead to a greater increase in kinetic energy."

On Financial Data "As the effective 'force' exerted on the Bitcoin market (market pressure from the halving event) increases, we assume that the 'momentum' (market momentum) of the Bitcoin price will also increase. This' Momentum's build-up should increase," reflecting heightened market activity and potentially greater price volatility. By analogy with DigiCart’s momentum, we hypothesize that the market’s “velocity” (rate of price change) and “quality” (volume) can combine to reflect changes in “momentum” resulting from the halving. Empirical evidence gathered by analyzing market data before and after the halving will support or challenge this assumption.

EXPERIMENTATION

<u>Materials and Equipment</u>

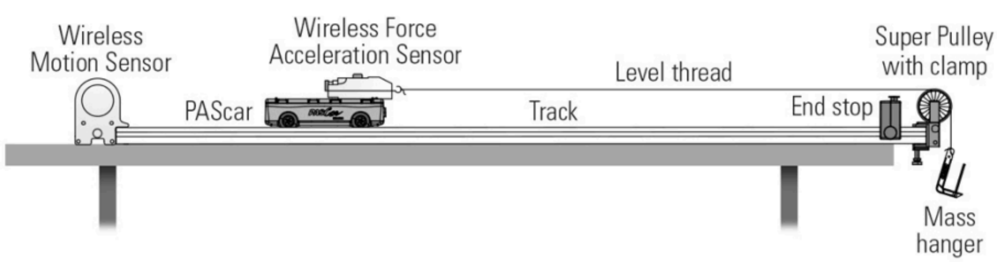

Figura 1, Cobra DigiCart Set Básico[5]

Cobra DigiCart Set Básico:
- Data Collection System
- Dynamics Cart
- Wireless Force Acceleration Sensor
- Dynamics Track End Stop
- Wireless Motion Sensor
- Mass and Hanger Set
- Dynamics Track with Feet
- Thread

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[5] Change in Kinetic Energy, retrieved January 10, 2024, from https://cdn.pasco.com/lab_experiment/l_856/DeltaKinetic_S.pdf

- Super Pulley with Clamp
- Balance (0.1-g resolution, 2,000-g capacity)

Cobra DigiChartAPP:
Calibrate all equipment as directed, ensure the track is stable and level, and check that wireless sensors are charged and connected.

Methodology

1.Preparation:

- Assemble the dynamics track, ensuring it is level so the cart does not roll when placed on the track.
- Attach the wireless force acceleration sensor to the cart using the included thumbscrew.

2.Mass Measurement:

- Measure the mass of each cart along with the attached wireless force acceleration sensor using a balance with a 0.1-g resolution. Record the combined mass in kilograms.

3.Equipment Setup. Arrange the equipment as shown in Figure 1, using a dynamics track, super pulley with clamp, and end stop, following these guidelines:

- Adjust the track to be level.
- Determine and cut the length of the thread needed for the mass hanger to be near the floor when the cart reaches the track's end.
- Attach one end of the thread to the wireless force sensor's hook and tie the other end to the mass hanger.
- Adjust the pulley angle to ensure the thread is parallel to the track.
- Attach the motion sensor at the track's end, aiming it directly at the cart.

4.Sensor Connection:

- Connect the wireless force acceleration sensor and the wireless motion sensor to the data collection system.

5.Calibration:

- Ensure the thread is slack, not pulling on the force sensor, and zero the force using the data collection software.

6.Trial Runs:

- Place a 10 g mass on the hanger for the first trial.
- Hold the cart at the starting position, ready to catch it at the track's end. Start data collection and release the cart. Stop collecting data as soon as it hits the end stop.
- Verify that the data appears smooth, with a parabolic position graph and a linearly increasing velocity graph. Adjust the motion sensor if necessary.

7.Repetition for Different Masses:

- Repeat the above procedures for the second cart with a different mass to compare the effects on kinetic energy.

Error Analysis and Safety

As with the previous methodology, ensure safety measures are in place and conduct an error analysis to identify potential sources of systematic or random errors.

Data Collection and Calculations

To investigate the relationship between the force applied to a DigiCart and its kinetic energy, we performed a series of controlled experiments. The DigiCart was equipped with varying added masses and propelled along a track to measure how these changes affected its kinetic energy. Two trials were conducted for each mass increment to ensure the reliability of the data.

Data Collection Procedure:

- We began by placing the DigiCart at a consistent starting point on the dynamics track for each trial.
- A known mass was added to the cart, and the system was released, allowing the cart to accelerate along the track under the influence of the gravitational force acting on the added mass.
- The initial and final velocities of the DigiCart were recorded using a wireless motion sensor.
- Each trial was performed twice to account for random errors and validate the consistency of the results.

Calculations: The force applied to the DigiCart was calculated using the formula

$$F = m \cdot g$$

where *m* is the added mass and g is the acceleration due to gravity (9.81 m/s²).The kinetic energy for the DigiCart at both the initial and final velocities was calculated using the equation

$$k = \frac{1}{2}mv^2$$

where m is the total mass of the cart including the added mass, and v is the velocity.The change in kinetic energy (Δ*KE*) was determined by subtracting the initial kinetic energy from the final kinetic energy for each trial.The calculations were performed for each trial, and the results were tabulated as seen in the provided tables.

Example Calculation for a Single Trial: For a DigiCart with an added mass of 17.80 g (0.01780 kg) and a cart mass of 220 g (0.220 kg), the initial velocity (v_i) was 0.42 m/s, and the final velocity (v_f) was 0.93 m/s. Initial kinetic energy (EK_i) was calculated as

$$EK_i = \frac{1}{2} \cdot (0.220 + 0.1780) \cdot (0.42)^2$$

Final kinetic energy (EK_f) was calculated as

$$EK_f = \frac{1}{2} \cdot (0.220 + 0.1780) \cdot (0.92)^2$$

The change in kinetic energy $: \Delta EK = EK_i - EK_f$

These calculations were systematically applied to all collected data points for each mass increment and trial conducted.

Added Mass (g) ± $1 \times 10^{-}$	Applied Force (N)	Kinetic Energy and Velocity Data for Cart with Mass of 220g									
		Trial 1					Trial 2				
		Initial Velocity (m/s)	Final Velocity (m/s)	Initial KE (J)	Final KE (J)	ΔKE (J)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial KE (J)	Final KE (J)	ΔKE (J)
17.80	174.44	0.42	0.93	0.02	0.10	0.08	0.43	0.95	0.02	0.10	0.08
28.19	276.26	0.50	1.11	0.03	0.13	0.11	0.53	1.14	0.03	0.14	0.11
38.30	375.34	0.70	1.33	0.05	0.19	0.14	0.86	1.40	0.08	0.22	0.13
48.70	477.26	1.02	1.47	0.11	0.24	0.12	1.14	1.56	0.14	0.27	0.12

[Table 1] - Kinetic Energy and Velocity Data for Cart with Mass of 220g

Added Mass (g) ± $1 \times 10^{-}$	Applied Force (N)	Kinetic Energy and Velocity Data for Cart with Mass of 271g									
		Trial 1					Trial 2				
		Initial Velocity (m/s)	Final Velocity (m/s)	Initial KE (J)	Final KE (J)	ΔKE (J)	Initial Velocity (m/s)	Final Velocity (m/s)	Initial KE (J)	Final KE (J)	ΔKE (J)
17.80	174.44	0.37	0.87	0.01	0.08	0.07	0.40	0.86	0.02	0.08	0.06
28.19	276.26	0.47	1.12	0.02	0.14	0.11	043	1.11	0.02	0.14	0.12
38.30	375.34	0.83	1.32	0.08	0.19	0.12	0.79	1.30	0.07	0.19	0.12
48.70	477.26	0.86	1.43	0.08	0.23	0.14	0.88	1.36	0.09	0.21	0.13

[Table 2] - Kinetic Energy and Velocity Data for Cart with Mass of 271g

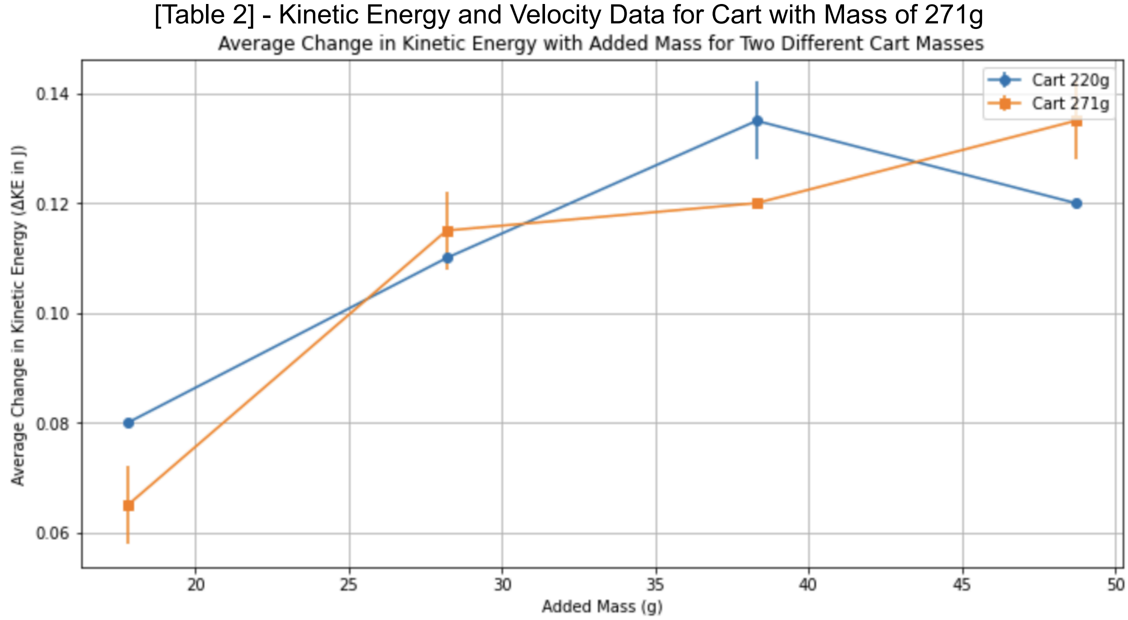

Figura 2, Change in Kinetic Energy with Added Mass for Two Different Cart Masses from Python

	Mean Volatility	Mean Volume	Mean Price_Dynamics	Mean Kinetic Energy
Before Halving	0.059959	81223.41766	53.32948	290.72401
After Halving	0.025721	58299.808723	15.65366	31.09015

[Table 3] - Comparative Market Statistics Before and After Bitcoin Halving Event

Using the Binance API for cryptocurrency, we accessed daily price data for Bitcoin spanning four months before and after its last halving. This dataset includes high, low, closing prices, and trading volume. Subsequently, through Python code, we calculated Mean Volatility and Mean Volume, along with Mean Price Dynamics and Mean Kinetic Energy, as derived from the formulas mentioned earlier.

ANALYSIS

The provided graph delineates a comparative study of kinetic energy changes in response to varying added masses for two distinct DigiCart masses—220 grams and 271 grams. The upward trajectories of both lines unveil a positive correlation, suggesting that the kinetic energy imparted to the carts increases with the additional mass.

Upon closer examination, the initial steeper ascent of the line for the 271-gram cart implies a greater sensitivity to mass changes when the cart is heavier, particularly within the lower mass range. This could be attributed to the fact that a heavier cart has more inertia, and the addition of mass has a more pronounced effect on its kinetic energy. In contrast, the 220-gram cart exhibits a steeper slope at higher added masses, hinting at a complex relationship where factors such as the proportion of mass added to the system's total mass become significant. The phenomenon of diminishing returns is markedly observable in the 271-gram cart's curve, where beyond a critical point, the increase in kinetic energy decelerates despite the continuous addition of mass. This suggests a saturation point beyond which the system's efficiency in converting added force into kinetic energy wanes.

An intriguing anomaly surfaces with the 220-gram cart at the highest added mass (48.70 grams), where a slight decrement in ΔKE is observed rather than the anticipated increase. This could be indicative of experimental limits or perhaps an inherent system characteristic, such as the cart reaching an optimal point of force application efficiency beyond which additional mass does not translate to proportional increases in kinetic energy.

Added Mass (g) ± 1×10^{-3}	Applied Force (N)	Rate of Change ΔKE Cart 220g	Rate of Change ΔKE Cart 271g
17.80	174.44	0.002887	0.004812
28.19	276.26	0.002677	0.002624
38.30	375.34	0.000543	0.000962
48.70	477.26	-0.001442	0.001442

[Table 4] - Rate of Change of KE with Added Mass for Two Cart Masses

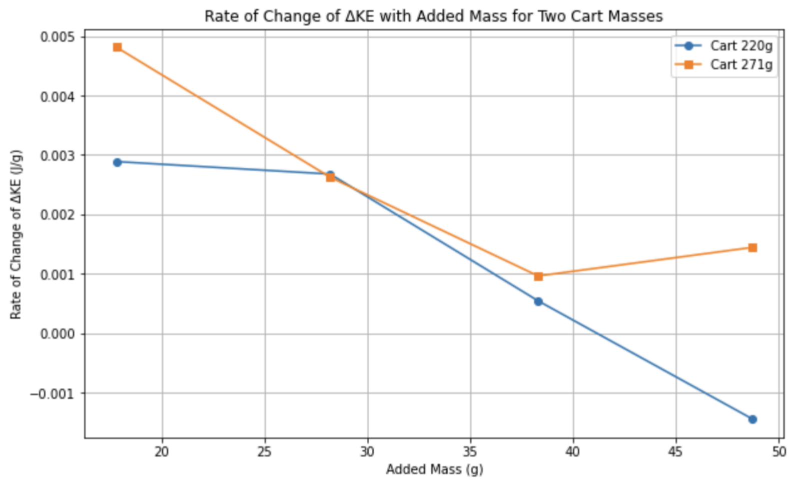

Figura 3, Rate of Change of KE with Added Mass for Two Cart Masses

The data presented in the graph and table illustrates a nuanced relationship between the added mass and the rate of change in kinetic energy for two carts of different masses. For both the 220g and 271g carts, we observe a general trend where the rate of change in kinetic energy initially decreases as more mass is added. This trend, represented by the downward slope in the graph for added masses ranging from 17.80g to 38.30g, suggests that the system's efficiency in converting applied force into kinetic energy diminishes with increased mass.

Interestingly, the 220g DigiCart exhibits a negative rate of change in kinetic energy after the added mass surpasses 38.30g. Such a downturn implies that the kinetic energy of the cart actually begins to decrease as additional mass is added, which could be a manifestation of physical constraints such as increasing effects of friction or air resistance. Alternatively, this could signal experimental errors that might necessitate a review of the data collection process.

Contrastingly, the 271g cart's rate of change remains relatively stable and even shows a slight increase once the added mass exceeds 38.30g. This behavior could indicate that the cart has reached a threshold where additional mass has a negligible impact on kinetic energy change, or it might suggest the presence of compensatory mechanisms at play within this specific mass range.

When comparing the two carts, it is initially apparent that the 271g cart has a higher rate of change in kinetic energy for the same added mass, potentially indicating a lower initial efficiency in transforming additional force into kinetic energy. However, beyond the critical mass of 38.30g, the rate of change diminishes more rapidly for the 220g cart than for the 271g cart.

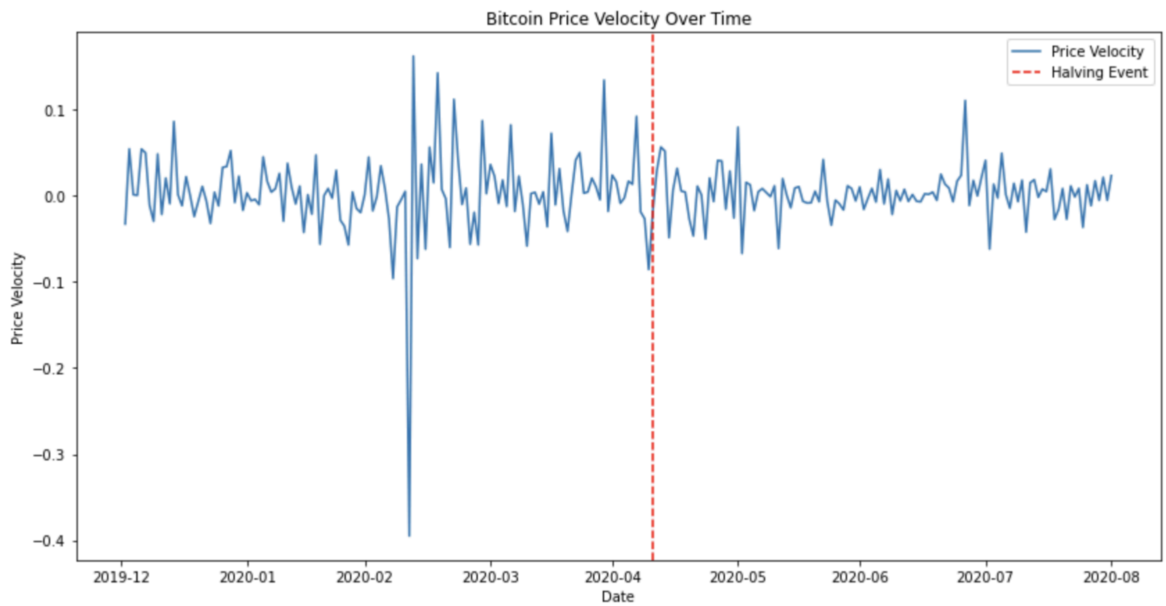

Figura 4, Bitcoin Price Velocity Over Time

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Average Kinetic Energy Before Halving: 290.724014538092
Average Kinetic Energy After Halving: 31.090151144159282
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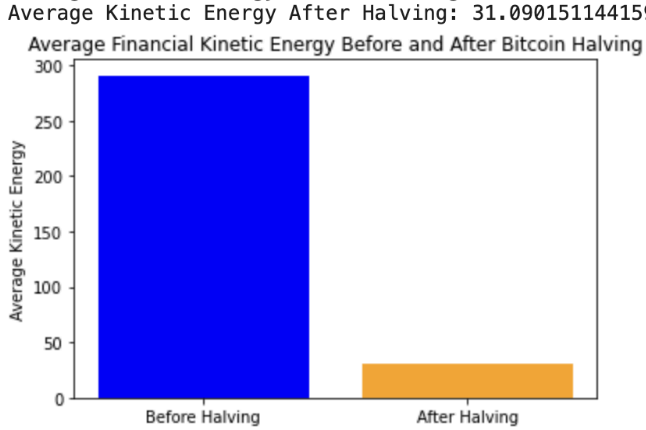

Figura 5,Average Financial Kinetic Energy Before and After Bitcoin Halving

Synthesized Market Analysis Post-Bitcoin Halving

The May 2020 Bitcoin halving, reducing mining rewards, offers key insights into market dynamics through volatility, volume, and a metaphor for momentum.

Volatility Analysis, the notable decrease in volatility, from 0.059959 to 0.025721, signals a shift in market dynamics from a pre-halving environment rife with uncertainty and speculation to a post-halving period that possibly reflects a market adjustment to the new norm. This change suggests that the anticipated event may have been factored into the market, resulting in a more stable and less speculative period following the halving.

Volume Analysis, a significant reduction in trading volume—from 81,223.41766 to 58,299.808723—highlights a cooling-off in market activity. The initial surge in volume before the halving could be attributed to traders positioning themselves to capitalize on the expected impact of the halving on Bitcoin's value. The subsequent decline may be a market-wide recalibration in response to the updated mining reward structure, hinting at a cautious approach by investors as they navigate the post-halving landscape.

Kinetic Energy Analysis, the concept of kinetic energy, repurposed here to quantify market activity, shows marked fluctuations around the halving event. The pronounced peaks pre-halving could be interpreted as periods of intense trading activity, possibly driven by the fervor of speculative trading and strategic portfolio adjustments in anticipation of the halving. The dampened post-halving "kinetic energy" suggests a period of market stabilization as the initial frenzy subsides and the market begins to establish a new equilibrium.

The comparative analysis before and after the halving underscores the event's role in invigorating market activity, as participants speculated on the ramifications of the reduced Bitcoin supply. The decline in both volatility and volume post-halving is emblematic of a market that has absorbed the halving's immediate effects and is transitioning into a phase characterized by more deliberate and potentially less speculative trading behaviors.

The empirical data reflects a market that is responsive to the structural shifts imposed by the Bitcoin halving. While the event precipitated immediate changes in market volatility and trading volume, its long-term influence remains subject to the broader tapestry of market trends, global economic conditions, and evolving investor sentiment. The utility of the kinetic energy analogy in this context serves as a conceptual tool, providing a fresh perspective on market momentum and investor activity, but it is not a standalone metric for investment decision-making. A holistic approach that considers a spectrum of market indicators and analyses will offer a more comprehensive and nuanced understanding of the market's trajectory post-halving.

Conclusion

The DigiCart experiments and Bitcoin's market analysis around its May 2020 halving event illustrate a fascinating alignment between physical laws and financial market behaviors. Observations from the DigiCart experiments revealed a direct relationship between the force applied and the kinetic energy generated, mirroring the financial market's reaction to the Bitcoin halving. This parallel suggests that both physical objects and financial markets respond predictably to applied forces, whether they stem from physical interactions or economic pressures.

Hypothesis testing

Our findings confirm the hypothesis that an increase in applied force leads to a proportional rise in kinetic energy for the DigiCart, analogous to how market pressures preceding the Bitcoin halving amplified market momentum. The subsequent decrease in market volatility and trading volume post-halving mirrors a physical system stabilizing after an external force is applied and then removed, offering a unique lens through which to view and analyze financial market fluctuations.

This study's conclusions underscore the utility of applying physical principles, such as the work-energy theorem, to comprehend complex financial market dynamics. By drawing parallels between the straightforward, measurable responses of a DigiCart to applied forces and the more abstract, yet observable, reactions of financial markets to economic events, we gain deeper insights into the inherent order governing both realms.

The initial hypothesis posited that the Bitcoin halving event would lead to increased market activity and volatility due to speculative behavior and strategic adjustments by traders and investors. The data before the halving confirms this hypothesis, as evidenced by the higher volatility and trading volume. However, post-halving data contradicts the expectation of sustained high volatility, instead suggesting that the market absorbed the impact of the event, leading to decreased volatility and volume.

Error analysis

Several factors limit the direct application of physical concepts to financial markets. In the DigiCart experiment, elements such as air resistance and friction introduced deviations from ideal predictions, analogous to how external economic factors and investor sentiment influence financial markets beyond the simplified models. Additionally, the potential for experimental error and the challenge of accurately measuring market 'kinetic energy' necessitate cautious interpretation of the results.

It's important to consider the limitations of drawing direct parallels between financial and physical systems. The 'kinetic energy' of the market is a metaphorical concept and may not account for all variables influencing market behavior. Additionally, market sentiment is susceptible to external influences beyond the scope of this analysis, such as global economic news, regulatory changes, and technological advancements.

Improvements and extensions

For future studies, it would be beneficial to incorporate a broader range of market indicators and to analyze longer periods to better understand the long-term effects of significant events like the Bitcoin halving. Further exploration could include a more detailed examination of the interplay between trading volume, price movements, and market liquidity. Extending the analogy, one could investigate how other principles of physics apply to financial markets, potentially uncovering new insights into market dynamics and investor behavior.

In conclusion, while the physical analogy serves as a powerful tool for conceptualizing market behavior, it is crucial to approach such comparisons with caution and complement them with comprehensive financial analysis. The insights gleaned from this approach must be integrated into a broader investment strategy that considers a multitude of factors affecting the financial markets.

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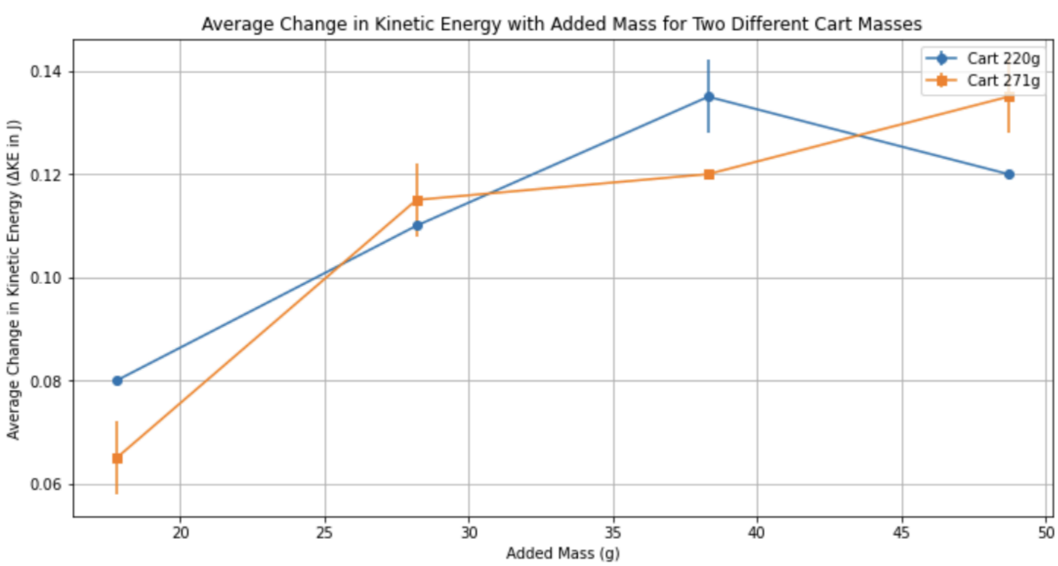
Appendices

[8]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

# Sample data based on the provided tables
data = {
    'Added Mass (g)': [17.80, 28.19, 38.30, 48.70],
    'Trial 1 ΔKE Cart 220g (J)': [0.08, 0.11, 0.14, 0.12],
    'Trial 2 ΔKE Cart 220g (J)': [0.08, 0.11, 0.13, 0.12],
    'Trial 1 ΔKE Cart 271g (J)': [0.07, 0.11, 0.12, 0.14],
    'Trial 2 ΔKE Cart 271g (J)': [0.06, 0.12, 0.12, 0.13]
}

# Create DataFrame
df = pd.DataFrame(data)

# Calculate average ΔKE for each cart and the standard deviation which will be used for error bars
df['Avg ΔKE Cart 220g (J)'] = df[['Trial 1 ΔKE Cart 220g (J)', 'Trial 2 ΔKE Cart 220g (J)']].mean(axis=
df['Std ΔKE Cart 220g (J)'] = df[['Trial 1 ΔKE Cart 220g (J)', 'Trial 2 ΔKE Cart 220g (J)']].std(axis=1

df['Avg ΔKE Cart 271g (J)'] = df[['Trial 1 ΔKE Cart 271g (J)', 'Trial 2 ΔKE Cart 271g (J)']].mean(axis=
df['Std ΔKE Cart 271g (J)'] = df[['Trial 1 ΔKE Cart 271g (J)', 'Trial 2 ΔKE Cart 271g (J)']].std(axis=1

# Show the DataFrame with average and std values
df_avg_std = df[['Added Mass (g)', 'Avg ΔKE Cart 220g (J)', 'Std ΔKE Cart 220g (J)',
                 'Avg ΔKE Cart 271g (J)', 'Std ΔKE Cart 271g (J)']]
df_avg_std

# Plotting the results with error bars
plt.figure(figsize=(12, 6))

# Error bars represent the standard deviation of the trials
plt.errorbar(df['Added Mass (g)'], df['Avg ΔKE Cart 220g (J)'],
             yerr=df['Std ΔKE Cart 220g (J)'], fmt='o-', label='Cart 220g')
plt.errorbar(df['Added Mass (g)'], df['Avg ΔKE Cart 271g (J)'],
             yerr=df['Std ΔKE Cart 271g (J)'], fmt='s-', label='Cart 271g')

# Adding titles and labels
plt.title('Average Change in Kinetic Energy with Added Mass for Two Different Cart Masses')
plt.xlabel('Added Mass (g)')
plt.ylabel('Average Change in Kinetic Energy (ΔKE in J)')
plt.legend()
plt.grid(True)

# Display the plot with error bars
plt.show()
```

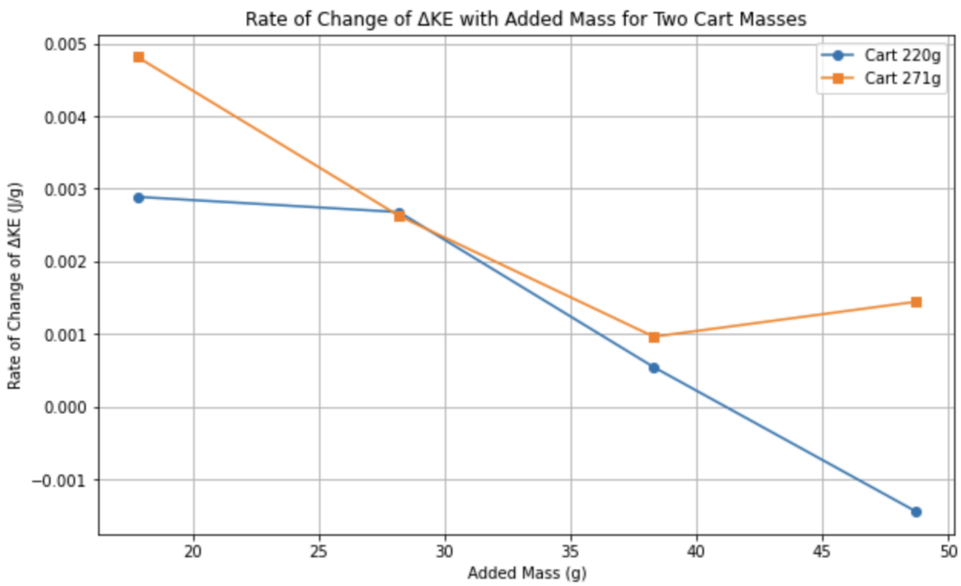


[2]:
```python
import pandas as pd
import numpy as np

# Assuming the data provided is correct, we will use that data to calculate the percentage change.
# Here is the given data structured in a dictionary for pandas DataFrame creation.

data = {
    'Added Mass (g)': [17.80, 28.19, 38.30, 48.70],
    'Trial 1 ΔKE Cart 220g (J)': [0.08, 0.11, 0.14, 0.12],
    'Trial 2 ΔKE Cart 220g (J)': [0.08, 0.11, 0.13, 0.12],
    'Trial 1 ΔKE Cart 271g (J)': [0.07, 0.11, 0.12, 0.14],
    'Trial 2 ΔKE Cart 271g (J)': [0.06, 0.12, 0.12, 0.13]
}

# Create DataFrame
df = pd.DataFrame(data)

# Calculate the mean ΔKE for each mass for both carts
df['Mean ΔKE Cart 220g (J)'] = df[['Trial 1 ΔKE Cart 220g (J)', 'Trial 2 ΔKE Cart 220g (J)']].mean(axis=1)
df['Mean ΔKE Cart 271g (J)'] = df[['Trial 1 ΔKE Cart 271g (J)', 'Trial 2 ΔKE Cart 271g (J)']].mean(axis=1)

# Calculate the rate of change of kinetic energy with respect to added mass
# This will be the slope of the line connecting the points in the ΔKE vs. Added Mass graph

# For Cart 220g
df['Rate of Change ΔKE Cart 220g'] = np.gradient(df['Mean ΔKE Cart 220g (J)'], df['Added Mass (g)'])

# For Cart 271g
df['Rate of Change ΔKE Cart 271g'] = np.gradient(df['Mean ΔKE Cart 271g (J)'], df['Added Mass (g)'])

# Select only the relevant columns to display
df_rates = df[['Added Mass (g)', 'Rate of Change ΔKE Cart 220g', 'Rate of Change ΔKE Cart 271g']]
df_rates
```

[2]:

	Added Mass (g)	Rate of Change ΔKE Cart 220g	Rate of Change ΔKE Cart 271g
0	17.80	0.002887	0.004812
1	28.19	0.002677	0.002624
2	38.30	0.000543	0.000962
3	48.70	-0.001442	0.001442

[3]:
```python
import matplotlib.pyplot as plt
import pandas as pd

# Assuming this is the data provided by the user
data = {
    'Added Mass (g)': [17.80, 28.19, 38.30, 48.70],
    'Rate of Change ΔKE Cart 220g': [0.002887, 0.002677, 0.000543, -0.001442],
    'Rate of Change ΔKE Cart 271g': [0.004812, 0.002624, 0.000962, 0.001442]
}

# Create DataFrame
df = pd.DataFrame(data)

# Plotting the rate of change
plt.figure(figsize=(10, 6))

plt.plot(df['Added Mass (g)'], df['Rate of Change ΔKE Cart 220g'], marker='o', label='Cart 220g')
plt.plot(df['Added Mass (g)'], df['Rate of Change ΔKE Cart 271g'], marker='s', label='Cart 271g')

# Adding titles and labels
plt.title('Rate of Change of ΔKE with Added Mass for Two Cart Masses')
plt.xlabel('Added Mass (g)')
plt.ylabel('Rate of Change of ΔKE (J/g)')
plt.legend()
plt.grid(True)
plt.show()
```



[5]:
```
import numpy as np
import pandas as pd
import requests
import matplotlib.pyplot as plt

# Define the function to get Binance data
def get_binance_data(symbol, interval, start_str, end_str=None):
    endpoint = f"https://api.binance.com/api/v3/klines"
    params = {
        'symbol': symbol,
        'interval': interval,
        'startTime': int(pd.Timestamp(start_str).timestamp() * 1000),
    }
    if end_str:
        params['endTime'] = int(pd.Timestamp(end_str).timestamp() * 1000)
    response = requests.get(endpoint, params=params)
    data = response.json()

    df = pd.DataFrame(data)
    df.columns = ['OpenTime', 'Open', 'High', 'Low', 'Close', 'Volume', 'CloseTime', 'QuoteAssetVo
                  'NumberOfTrades', 'TakerBuyBaseAssetVolume', 'TakerBuyQuoteAssetVolume', 'Ignore
    df['OpenTime'] = pd.to_datetime(df['OpenTime'], unit='ms')
    df['CloseTime'] = pd.to_datetime(df['CloseTime'], unit='ms')
    df['Close'] = df['Close'].astype(float)
    df.set_index('OpenTime', inplace=True)

    return df
df = get_binance_data('BTCUSDT', '1d', '2020-01-01', '2020-09-01')
# Assuming 'df' is your DataFrame with Bitcoin prices obtained from the Binance API or loaded from

# Calculate the daily price change
df['Price_Change'] = df['Close'].diff()
df['Volume'] = df['Volume'].astype(float)
# Calculate the rate of change (velocity) as the price change divided by the previous day's close
# df['Price_Velocity'] = df['Price_Change'] / df['Close'].shift()

# This calculates the velocity for each day in your dataset


# Calculate the rate of change of price (velocity)
# Here we avoid division by zero by adding a small number (epsilon) to the denominator
epsilon = 1e-8
df['Price_Velocity'] = df['Price_Change'] / (df['Close'].shift() + epsilon)

# Calculate the 'kinetic energy' for the financial data
# We use the absolute value of volume, as it cannot be negative in this context
df['Financial_KE'] = 0.5 * np.abs(df['Volume']) * df['Price_Velocity'] ** 2

# The resulting 'Financial_KE' column is what we consider as the financial analogy to kinetic ener
print(df['Financial_KE'])
```

[5]:

```
import numpy as np
import pandas as pd
import requests
import matplotlib.pyplot as plt

# Define the function to get Binance data
def get_binance_data(symbol, interval, start_str, end_str=None):
    endpoint = f"https://api.binance.com/api/v3/klines"
    params = {
        'symbol': symbol,
        'interval': interval,
        'startTime': int(pd.Timestamp(start_str).timestamp() * 1000),
    }
    if end_str:
        params['endTime'] = int(pd.Timestamp(end_str).timestamp() * 1000)
    response = requests.get(endpoint, params=params)
    data = response.json()

    df = pd.DataFrame(data)
    df.columns = ['OpenTime', 'Open', 'High', 'Low', 'Close', 'Volume', 'CloseTime', 'QuoteAssetVo
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    df['OpenTime'] = pd.to_datetime(df['OpenTime'], unit='ms')
    df['CloseTime'] = pd.to_datetime(df['CloseTime'], unit='ms')
    df['Close'] = df['Close'].astype(float)
    df.set_index('OpenTime', inplace=True)

    return df
df = get_binance_data('BTCUSDT', '1d', '2020-01-01', '2020-09-01')
# Assuming 'df' is your DataFrame with Bitcoin prices obtained from the Binance API or loaded from

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print(df['Financial_KE'])
```