HW3: Optimal Economic Dispatch in Distribution Feeders with Renewables

Due: Wednesday Mar. 15, 2023 at 11:59pm PT

This assignment will provide hands-on practice for optimization with application to the economic dispatch problem in power systems. The assignment is organized in a tutorial fashion, thereby allowing you to practice optimization theory on a relevant real-world energy system example.

Background
In this assignment you will practice optimization with the DistFlow equations of Baran & Wu [1] (1)-(9). The DistFlow equations model power flow through radial distribution power networks. The beauty of the DistFlow equations is that they are relatively simple (i.e. they yield convex programs). Yet, they model active & reactive (AC) power, branch power flow limits, node voltage limits, etc. Despite their simplicity, the DistFlow equations are much more sophisticated that the "supply = demand" single equation used by many energy policy researchers. In this assignment, your objective is to optimally schedule distributed energy generators in a distribution feeder to minimize economic costs, while maintaining safe operating constraints.

Consider the IEEE 13-node Test Feeder shown in Fig. 1, adopted from [2]. We mathematically represent this test feeder by a radial undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$. Symbol $\mathcal{N} = \{0, 1, 2, \cdots, 12\}$ represents the set of nodes (a.k.a. "buses" in power systems jargon). Symbol $\mathcal{L} \subset \mathcal{N} \times \mathcal{N}$ represents the set of edges (a.k.a. "lines" in power systems jargon). For notational convenience, we introduce the concepts of parent nodes and the adjacency matrix. The parent of j, $\rho(j)$, is the adjacent node in the direction toward node 0. The adjacency matrix, A, encodes the network topology. Mathematically, it is a 13×13 matrix of zeros and ones. $A_{ij} = 1$ if node i is the parent node of node j. Otherwise, $A_{ij} = 0$.

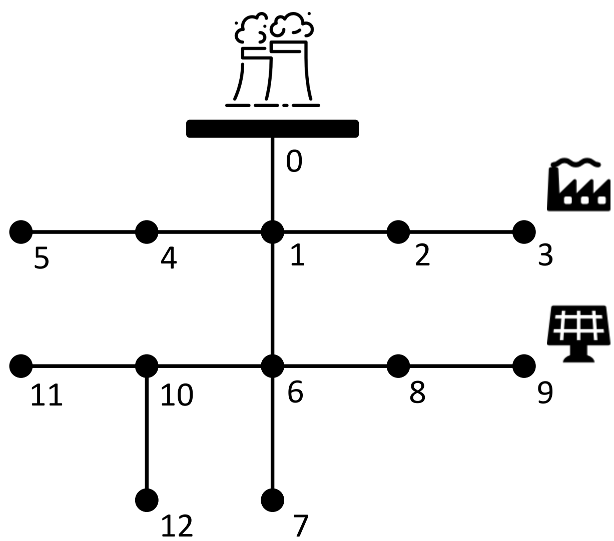

Figure 1: IEEE 13-Node Test Feeder

At each node $j \in \mathcal{N}$, we have l_j^P, l_j^Q active and reactive power consumed, respectively. Additionally, p_j, q_j active and reactive power are generated, respectively. Note that total power is complex number (due to AC power flow), where the real part is given by active power p_j and the imaginary part is given by reactive power q_j. The magnitude of complex power is given by s_j in equation (6).

Voltage at each node is a complex number. We mathematically model this by the squared voltage magnitude

V_j. In general, we must schedule generators to regulate nodal voltages around a nominal value (see (8)). Each edge (i,j) (or "line") has a characteristic impedance, decomposed into resistance r_{ij} and reactance x_{ij}. The active and reactive power flowing along line (i,j) is given by P_{ij} and Q_{ij}, respectively. Finally, the squared magnitude of complex current on line (i,j) is given by L_{ij}. Lines have a maximum current capacity, given by constraint (9).

Power is supplied in two ways. First, power can be imported from the transmission grid connected to node 0. Second, power can be generated from distributed generators within the feeder network. We have a gas generator on node 3, and solar generator on node 9.

Your objective is to supply all electricity demand at minimum cost. Simultaneously, you must ensure all generator output powers, nodal voltages, and line currents are within safe operating bounds.

Table 1: Nomenclature

Symbol	**Description**	**Units**
$\mathcal{N}$	Set of nodes (a.k.a. buses)	[-]
$\mathcal{E}$	Set of edges (a.k.a. lines)	[-]
$\rho(i)$	Parent of node i, e.g. $\rho(6) = \{1\}$, $\rho(12) = \{10\}$	[-]
A	Adjacency Matrix (13×13) encoding network structure	[-]
p_i	active power generated at node i	[MW]
q_i	reactive power generated at node i	[MVAr]
s_i	apparent power generated at node i	[MVA]
$s_{i,\max}$	apparent power generation capacity at node i	[MVA]
l_i^P, l_i^Q	active, reactive power consumed at node i	[MW], [MVAr]
V_i	Squared voltage magnitude at node i	[p.u.]
$v_{\min}, v_{\max}$	Minimum, maximum nodal voltage	[p.u.]
c_i	Marginal cost of *apparent* power generation at node i	[USD/MVA]
r_{ij}	resistance of line (i,j)	[p.u.]
x_{ij}	reactance of line (i,j)	[p.u.]
P_{ij}	active power flowing on line (i,j)	[MW]
Q_{ij}	reactive power flowing on line (i,j)	[MVAr]
L_{ij}	Squared magnitude of complex current on line (i,j)	[p.u.]
I_{ij}	Maximum magnitude of complex current on line (i,j)	[p.u.]
W_A, W_B	Uncertain & weather-dependent power capacity of PV panels A,B	[MVA]
σ_A, σ_B	Percentage power output of PV panels A,B	[%]

Note: The acronym "p.u." stands for per-unit. It's power systems jargon for "normalized to a unitless quantity". For your convenience, all the data has been normalized to simplify your analysis. For interested readers, the base parameters for normalization in this HW are $S_{base} = 1$ MW, $V_{base} = 4.17$ kV.

$$P_{ij} = (l_j^P - p_j) + r_{ij} L_{ij} + \sum_{k \in \mathcal{N}} A_{jk} P_{jk} \qquad \forall\, j \in \mathcal{N},\ i = \rho(j) \tag{1}$$

$$Q_{ij} = (l_j^Q - q_j) + x_{ij} L_{ij} + \sum_{k \in \mathcal{N}} A_{jk} Q_{jk} \qquad \forall\, j \in \mathcal{N},\ i = \rho(j) \tag{2}$$

$$V_j = V_i + (r_{ij}^2 + x_{ij}^2) L_{ij} - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) \qquad \forall\, j \in \mathcal{N},\ i = \rho(j) \tag{3}$$

$$L_{ij} = \frac{{P_{ij}}^2 + {Q_{ij}}^2}{V_j} \qquad \forall\, j \in \mathcal{N},\ i = \rho(j) \tag{4}$$

$$p_j \geq 0, \quad q_j \geq 0 \qquad \forall\, j \in \mathcal{N} \tag{5}$$

$$\sqrt{{p_j}^2 + {q_j}^2} = \|[p_j,\ q_j]\|_2 = s_j \qquad \forall\, j \in \mathcal{N} \tag{6}$$

$$s_j \leq s_{j,\max} \qquad \forall\, j \in \mathcal{N} \tag{7}$$

$$v_{\min}^2 \leq V_j \leq v_{\max}^2 \qquad \forall\, j \in \mathcal{N} \tag{8}$$

$$L_{ij} \leq I_{ij,\max}^2 \qquad \forall\, j \in \mathcal{N},\ i = \rho(j) \tag{9}$$

Problem 1: Network Parameters
Download and open the data file `HW_Data.xls`. Copy over the test network parameters from the `xls` file into the Matlab/Python skeleton code file.

(a) Create a bar plot of the active and reactive power consumption. Make the x-axis the node index number, while the y-axis is the power consumption. Place the active & reactive powers side-by-side for each node. Add a legend.

(b) Fill out the adjacency matrix with zeros and ones. Include in your report.

Problem 2: Balancing Supply & Demand without a Network
Start simple: In this problem, we will optimally dispatch our generators to minimize cost, while disregarding the network completely. That is, we seek to balance active & reactive power supply & demand, while minimizing generation cost and completely ignoring line losses and constraints.

(a) What are the optimization variables?

(b) Write down the objective function, using the notation in Table 1.

(c) Write down ALL the constraints, using the notation from Table 1. Label the physical meaning of each constraint. For this problem, ignore voltages and all constraints associated with line flows.

(d) Is this a linear program (LP), quadratic program (QP), or convex program (CP)? Why or why not? What happens when we relax (6) from an equality constraint to an inequality (≤) constraint?

(e) Code and numerically solve this problem in Matlab or Python using `cvx` or `cvxpy`, respectively. Use the relaxed version of (6), so your optimization program is convex. In your report provide (i) the optimal active & reactive generator powers, and (ii) the minimum generating cost.

Problem 3: Add Line Power Flows
Next, we add line power flows P_{ij}, Q_{ij} but still neglect the nodal voltage V_j and L_{ij} terms.

(a) What are the optimization variables?

(b) Write down ALL the constraints, using the notation from Table 1. Label the physical meaning of each constraint. For this problem, ignore (3)-(4) and drop the L_{ij} terms from (1)-(2).

(c) Code and numerically solve this problem in Matlab or Python using `cvx` or `cvxpy`, respectively. In your report provide (i) the optimal active & reactive generator powers, and (ii) the minimum generating cost. Should the minimum and minimizers be different or the same as Problem 2? Why?

(d) In your code, declare a dual variable μ_s corresponding to the inequality (7). Re-compute the optimal solution and dual variable. If we could increase the solar power capacity by 1MW, then how much money would we save?

Problem 4: The Complete Optimal Economic Dispatch with DistFlow Equations
Now we add the nodal voltages V_j, squared current magnitudes L_{ij}, and their bounds (8)-(9). This incorporates impedance (i.e. losses) across the network, along with nodal voltage and line transmission limits.

(a) What are the optimization variables?

(b) Write down ALL the constraints, using the notation from Table 1. Label the physical meaning of each constraint.

(c) Is this a convex program (CP)? Why or why not? What happens when we relax (4) from an equality constraint to an inequality ($\geq$) constraint?

(d) Code and numerically solve this problem in Matlab or Python using `cvx` or `cvxpy`, respectively. In your report provide (i) the optimal active & reactive generator powers, and (ii) the minimum generating cost. Use $v_{\min} = 0.95, v_{\max} = 1.05$ as your voltage limits. Is the solution equivalent to the solution in Problem 3? Why or why not?

(e) Use the dual variables to determine which constraints are active. Specifically, identify at which nodes the voltage constraint (8) and line current constraint (9) are active.

(f) Now re-solve the problem with $v_{\min} = 0.98, v_{\max} = 1.02$ as your voltage limits. In your report provide (i) the optimal active & reactive generator powers, and (ii) the minimum generating cost. Why did the solution change?

Problem 5: Robust Economic Dispatch with Renewables. Next, we explore robust economic dispatch in the face of <u>uncertain</u> renewable generation. Imagine the solar generator at node 9 is comprised of two solar panels, A and B. The output of each panel is uncertain, and weather dependent. Mathematically, we write:

$$s_9 \leq W_A \cdot \sigma_a + W_B \cdot \sigma_b \tag{10}$$

where $\sigma_a, \sigma_b \in [0, 1]$ represent the percentage output of each panel. Symbols W_A, W_B are random variables that represent the uncertain power capacity of each panel, due to weather. We hypothesize that W_A, W_B

vary between 1 MW and 1.5 MW. Our goal, in this problem, is to optimally schedule the generators in the face of uncertain solar capacity. Note, this problem is similar to Example 3.4 in the CH3 notes.

(a) Re-arrange (10) into the form $a^T y \leq b$. What are a, y, and b? Hint: $a \in \mathbb{R}^3$, $y \in \mathbb{R}^3$, $b \in \mathbb{R}$.

(b) We now hypothesize that vector a is uncertain, but lies within an ellipsoid

$$a \in \mathcal{E} = \{\bar{a} + Eu \mid \|u\|_2 \leq 1\} \tag{11}$$

Provide the values for $\bar{a}$ and E. Hint #1: $\bar{a} \in \mathbb{R}^3$ represents the center of the ellipsoid. Hint #2: $E \succeq 0 \in \mathbb{R}^{3\times 3}$ encodes the lengths of the semi-axes. If E is diagonal, then the diagonal elements represent the semi-axis lengths along each coordinate of a.

(c) The robust version of (10) is

$$a^T y \leq b, \quad \forall\, a \in \mathcal{E} \tag{12}$$

Convert this robust linear inequality into a second-order cone constraint. In your report, it is only necessary to provide the final written form of the second order cone constraint.

(d) In your report, write down the new robust optimization problem. What are the optimization variables? Write down ALL the constraints, using the notation from Table 1. Label the physical meaning of each constraint.

(e) Now solve the optimization program with $v_{\min} = 0.95, v_{\max} = 1.05$ as your voltage limits. In your report provide (i) the optimal active & reactive generator powers, and (ii) the minimum generating cost. How did the solution change?

Interesting Remarks

- Power system operators utilize day-ahead load predictions to solve the economic dispatch problem and procure generation on an hourly basis. Mismatch between predicted and actual load is compensated by “spinning reserves”.
- This homework is not trivial. However, by completing it, you have quickly acquired sophisticated knowledge and skills in power systems and optimization. DistFlow equations, convex optimization, and robust optimization are skills one normally finds in power systems / optimization PhD students. Well done!

Deliverables

Submit your report on GradeScope. Include your code as an Appendix at the end of your report. You can, for example, print/export your code/notebooks/livescripts to PDF, and append to your homework report.

How to Download and Install CVX

Matlab Instructions

- Go to http://cvxr.com/cvx/download/
- In the "Download Matrix", focus your attention on the "Standard bundles, including Gurobi and/or MOSEK".
- Click the package corresponding to your system. For example, I have a MacBook Pro Early 2015, so I'll select `cvx-mexmaci64`. You can download a `.zip` or `.tar.gz` file.
- Download and unpack anywhere you like. The Downloads folder is a reasonable option.
- Start Matlab.
- Change directories to the top of the CVX distribution. Hint: Use command » `cd /Users/scottmoura/Downloads/cvx`
- Run Matlab command » `cvx_setup`
- The `cvx_setup` command runs a variety of checks to verify your installation is correct.
- To confirm a successful understanding, try coding and solving the example shown at http://cvxr.com/cvx/

Python Instructions

- Ensure you have installed Anaconda, as recommended in this class
- Go to https://www.cvxpy.org/install/index.html
- Follow the instructions corresponding to your system
- After successfully installing CVXPY, fire-up the iPython notebook. You can try the Portfolio optimization example linked here.

References

[1] M.E. Baran and F.F. Wu. Network reconfiguration in distribution systems for loss reduction and load balancing. IEEE Transactions on Power Delivery, 4(2):1401–1407, 1989.

[2] J. Fuller, Y. Xu, B. Kersting, R. Dugan, and S. Carneiro, Jr. Ieee pes distribution test feeders, 2010.