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The impact of virus carrier screening and actively seeking treatment on dynamical behavior of a stochastic HIV/AIDS infection model

Kai Qi [a], Daqing Jiang [a,b,c,*]

[a] *College of Science, China University of Petroleum (East China), Qingdao 266580, P. R. China*
[b] *Key Laboratory of Unconventional Oil & Gas Development, China University of Petroleum (East China), Ministry of Education, Qingdao 266580, P. R. China*
[c] *Nonlinear Analysis and Applied Mathematics (NAAM)-Research Group, King Abdulaziz University, Jeddah, Saudi Arabia*

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A B S T R A C T

To screen for virus carriers and for infected individuals to actively seek treatment are the key factors to curb the spread of HIV/AIDS. We propose a stochastic HIV/AIDS model to evaluate their effect. First, we theoretically prove that the solution of the stochastic model is positive and global. Second, we obtain the existence of an ergodic stationary distribution if $R_0^S > 1$, and we establish sufficient conditions $R_0^{'} < 1$ for disease extinction. We provide examples and numerical simulations to verify our theoretical results. Finally, we analyze the impact of the above two factors on the dynamical behavior of the stochastic system, and draw conclusions regrading disease prevention and elimination.

1. Introduction

The human immunodeficiency virus-1 (HIV-1, or simply HIV) caused a global epidemic that had affected approximately 36.9 million people by the end of 2017, according to the World Health Organization (WHO). Lacking awareness and treatment, the HIV virus slowly causes the collapse of the body's immune system and makes people sick [1]. Antiretroviral therapy (ART) is suggested for infected individuals to inhibit the virus, but there is no vaccine or cure thus far [2–5]. Although some people can obtain effective treatment, medical aid still remains a dream for millions.

In practice, many factors prevent the elimination of AIDS. For example, asymptomatic HIV virus carriers induce the complexity of disease transmission. Seemingly healthy individuals who are infected can transmit the virus [6]. The danger posed by carriers may not be apparent if they are not under disease surveillance. Thus it is essential to promote the screening of virus carriers [7]. There are numerous models for monitoring while taking account of intervention strategies [4,8–11]. Tripathi [10] evaluated the role of screening unaware sick individuals.

Many factors contribute to the inhibition of AIDS. Education and media play an important role [12,13] by increasing the number of infected individuals who actively seek treatment, and they help government to control the spread of HIV/AIDS [2]. Individuals under treatment have a lower probability of infecting others. We account for rates at which different types

* Corresponding author.
E-mail addresses: upcqikai@hotmail.com (K. Qi), daqingjiang2010@hotmail.com (D. Jiang).

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of infected persons seek treatment. These rates can also be criteria to evaluate the adequacy of publicity through education and media.

Epidemic models are inevitably subject to environmental noise. Mao [14] maintained that a small number of environmental disturbances in a deterministic model can effectively restrain a potential population explosion. Noise is essential in reality, hence models including environmental noise can describe the disease transmission progress exactly, and can draw reasonable conclusions [15–17]. Randomness has been introduced in various biological models and epidemic models of human populations [18–23]. Thus it is significant to construct a stochastic model that contains both random and deterministic terms [24].

This paper develops a stochastic model including virus carrier screening, education, and media (which prompt more individuals to actively seek treatment). We concentrate on two points: (i) to analyze the dynamical behavior of the stochastic model that we constructed, i.e., the positive and global solutions of the model, the existence of an ergodic stationary distribution, and disease extinction; and (ii) to investigate how the rates of virus carrier screening and seeking of treatment affect the model's behavior.

The rest of this paper is organized as follows. Section 2 shows the mathematical model and necessary lemmas. We obtain the existence and uniqueness of the positive solution of our model in Section 3. The existence of a unique ergodic stationary distribution is proved in Section 4. In Section 5, we establish sufficient conditions for disease extinction. Section 6 shows a numerical simulation to verify our theoretical results, and to specifically explore how virus carrier screening and the seeking of treatment actively impact the dynamical behavior of the stochastic system. In Section 7, we discuss our conclusions, the deficiencies of this paper, and details on how the above two factors affect the system.

2. Mathematical model and necessary lemmas

Most epidemic models are ordinary differential equations (ODEs). For HIV/AIDS epidemics, we usually divide the population into several clusters, such as susceptible and infectious individuals. Including the rate of transformation between different kinds of individuals, ODE modelling can reflect the spread of disease in a region. Based on the dynamics behavior of ODE, we can obtain the development process of HIV/AIDS epidemics and predict their variation trends. After considering environmental noise, we improve the ODE to obtain stochastic differential equation (SDE), which can describe disease transmission progress exactly and provide reasonable conclusions.

HIV/AIDS models with screened disease carriers have been studied in recent years. Hove-Musekwa [6] formulated a deterministic differential equation model with active screening of disease carriers and seeking of treatment, which can divide the population into seven classes and reasonably consider the relationship among them. The infection rate is measured by the probability parameters β_i (i=1,2,3,4) and risk behavior (average number of sexual partners k), as follows:

$$k\left(\beta_1\frac{I(t)}{N(t)}+\beta_2\frac{C(t)}{N(t)}+\beta_3\frac{T(t)}{N(t)}+\beta_4\frac{A(t)}{N(t)}\right).$$

Many researchers of HIV/AIDS apply the bilinear incidence to their models [1,25–30]. Without considering standard incidence, we take account of bilinear incidence in our model, which is

$$k\Big(\beta_1 I(t)+\beta_2 C(t)+\beta_3 T(t)+\beta_4 A(t)\Big).$$

Other model assumptions are the same as [6], and we obtain the deterministic model of HIV/AIDS in the next subsection.

2.1. Deterministic HIV/AIDS model with disease carrier screening and active seeking of treatment

The population $N(t)$ is divided into seven clusters, which include susceptible individuals $S(t)$, infectious and symptomatic primary HIV-infected individuals $I(t)$, asymptomatic and infectious disease carriers $C(t)$, randomly screened disease carriers $C_s(t)$, individuals under treatment $T(t)$, individuals with full blown AIDS $A(t)$ and those with AIDS but under treatment $A_r(t)$. The size of the population is $N(t)=S(t)+I(t)+C(t)+C_s(t)+T(t)+A(t)+A_r(t)$, which varies with time, and

$$\begin{cases}\dot S(t) = \Pi-k(\beta_1 I(t)+\beta_2 C(t)+\beta_3 T(t)+\beta_4 A(t))S(t)-\mu S(t),\\ \dot I(t) = k(\beta_1 I(t)+\beta_2 C(t)+\beta_3 T(t)+\beta_4 A(t))S(t)-(\mu+\sigma+\rho_1+\gamma_1)I(t),\\ \dot C(t) = \sigma I(t)-(\mu+\rho_2+\psi)C(t),\\ \dot C_s(t) = \psi C(t)-(\mu+\rho_3+\gamma_2)C_s(t),\\ \dot T(t) = \gamma_1 I(t)+\gamma_2 C_S(t)-(\mu+\rho_4)T(t),\\ \dot A(t) = \rho_1 I(t)+\rho_2 C(t)+\rho_3 C_s(t)+\rho_4 T(t)-(\mu+\gamma_3+\delta_1)A(t),\\ \dot A_r(t) = \gamma_3 A(t)-(\mu+\delta_2)A_r(t).\end{cases} \tag{2.1}$$

The parameter Π is the recruitment rate. The average number of sexual partners k can measure risk behavior. β_i (i=1,2,3,4) are the infection probabilities for different infectious individuals $I(t)$, $C(t)$, $T(t)$ and $A(t)$, respectively, and β_i satisfy the relation $\beta_1>\beta_4>\beta_2>\beta_3$ in order to conform to reality. The parameter μ is the natural death rate, and σ is the rate from the infected individuals $I(t)$ to carriers $C(t)$. Different infectious individuals $I(t)$, $C(t)$, $C_s(t)$ and $T(t)$ become the full-blown AIDS patients $A(t)$ at the rate of ρ_i (i=1,2,3,4) in order. The rates at which the infected seek treatment are γ_i (i=1,2,3) in sequence. The rate at which carriers are screened is ψ, where δ_i (i=1,2) represent the disease-caused mortality rates. All of the above parameters are positive.

2.2. Analysis of deterministic model

We obtain the derivative of the total population $N(t)$ from the sum of each equation in model (2.1), $\frac{dN(t)}{dt} = \Pi - \mu N(t) - \delta_1 A(t) - \delta_2 A_r(t)$. Then we get the positive invariant set from the comparison theorem,

$$\Theta = \left\{ (S(t), I(t), C(t), C_s(t), T(t), A(t), A_r(t)) | \{S(t), I(t), C(t), C_s(t), T(t), A(t), A_r(t)\} \geq 0, N \leq \frac{\Pi}{\mu} \right\}.$$

The region Θ ensures that the model (2.1) is well posed and biologically meaningful. We will ignore the last equation of model (2.1), since the $A_r(t)$ is not involved in the dynamics of the disease [6].

Although we changed the incidence, we can draw on the work of [6] in terms of the basic reproduction number and steady state of the deterministic model. The basic reproduction number can be given by:

$$R_0 = \frac{\Pi k(\beta_1 + \beta_2\omega_1 + \beta_3\omega_3 + \beta_4\omega_4)}{\mu(\mu + \sigma + \rho_1 + \gamma_1)},$$

where

$$\omega_1 = \frac{\sigma}{\mu + \rho_2 + \psi}, \quad \omega_2 = \frac{\psi}{\mu + \gamma_2 + \rho_3}\omega_1,$$

$$\omega_3 = \frac{\gamma_1}{\mu + \rho_4} + \frac{\gamma_2\sigma\psi}{(\mu + \rho_4)(\mu + \gamma_2 + \rho_3)(\mu + \rho_2 + \psi)}, \quad \omega_4 = \frac{1}{\mu + \gamma_3 + \delta_1}(\rho_1 + \rho_2\omega_1 + \rho_3\omega_2 + \rho_4\omega_3).$$

Setting the right hand sides of model (2.1) to zero, we obtain that the model has two possible steady states. The disease-free equilibrium point is $E^0 = \left(\frac{\Pi}{\mu}, 0, 0, 0, 0, 0\right)$. In this case, the total population approaches the steady value $\frac{\Pi}{\mu}$. The other state is the endemic equilibrium point $E^* = (S^*, I^*, C^*, C_s^*, T^*, A^*)$, where

$$S^* = \frac{\Pi}{\mu R_0}, \quad I^* = \frac{\Pi(R_0 - 1)}{R_0(\mu + \sigma + \rho_1 + \gamma_1)}, \quad C^* = \omega_1 I^*, \quad C_s^* = \omega_2 I^*, \quad T^* = \omega_3 I^*, \quad A^* = \omega_4 I^*.$$

We have the following results of the existence and stability of the equilibrium points E^0 and E^* [6].

(i) If $R_0 \leq 1$, then system (2.1) has a unique disease-free equilibrium point E^0. If $R_0 > 1$, then there exists a unique endemic equilibrium point E^*;

(ii) If $R_0 < 1$, then E^0 is globally asymptotically stable. If $R_0 > 1$, then E^0 is unstable;

(iii) The endemic equilibrium point E^* is locally asymptotically stable for $R_0 > 1$, when R_0 is close to 1.

Thus R_0 is a threshold quantity, which is the basic reproduction number of the deterministic system (2.1). R_0 is the mean number of secondary cases which one case would produce in a completely susceptible population.

2.3. Stochastic HIV/AIDS model with disease carrier screening and active seeking of treatment

The deterministic system does not include unpredictable biological conditions, while all of the parameters in system (2.1) can be perturbed by environmental noise. Our theory that introduces stochastic linear perturbations is similar to that in section 3.1 of Imhof and Walcher [31]. The construction of the stochastic model is similar to that in the appendix of Liu and Jiang [32]. We derive our stochastic model in Appendix C.

Linear perturbation, which is the simplest and most intuitive hypothesis, is convenient for theoretical analysis, and its influence on a deterministic model has been studied [25,33–35]. Nonlinear perturbation has also been introduced to a deterministic model [36,37], but this is rare in the literature because of the technical difficulties and model feasibility. We adopt linear perturbation to a deterministic model in this paper, with plans to study nonlinear perturbation in the future.

After considering the impact of random environmental disturbance, we obtain the following stochastic model (see Appendix C for details on construction of our model):

$$\begin{cases} dS(t) = [\Pi - k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t))S(t) - \mu S(t)]dt + \sigma_1 S(t)dB_1(t), \\ dI(t) = [k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t))S(t) - (\mu + \sigma + \rho_1 + \gamma_1)I(t)]dt + \sigma_2 I(t)dB_2(t), \\ dC(t) = [\sigma I(t) - (\mu + \rho_2 + \psi)C(t)]dt + \sigma_3 C(t)dB_3(t), \\ dC_s(t) = [\psi C(t) - (\mu + \rho_3 + \gamma_2)C_s(t)]dt + \sigma_4 C_s(t)dB_4(t), \\ dT(t) = [\gamma_1 I(t) + \gamma_2 C_s(t) - (\mu + \rho_4)T(t)]dt + \sigma_5 T(t)dB_5(t), \\ dA(t) = [\rho_1 I(t) + \rho_2 C(t) + \rho_3 C_s(t) + \rho_4 T(t) - (\mu + \gamma_3 + \delta_1)A(t)]dt + \sigma_6 A(t)dB_6(t). \end{cases} \tag{2.2}$$

Let I={1,2,3,4,5,6}, and let $B_i(t)(i\in I)$ be independent standard Brownian motions, where $B_i(0)=0\ (i\in I)$, and $\sigma_i^2>0\ (i\in I)$ are the intensities of the white noise.

We define $x(t)=(S(t),I(t),C(t),C_s(t),T(t),A(t))^T$ as the solution of system (2.2) with initial value $x_0=(S(0),I(0),C(0),C_s(0),T(0),A(0))^T$.

2.4. Necessary lemmas

Unless otherwise specified, we work on a complete probability space $(\Omega,\mathscr{F},p)$ with filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $B_i(t)(i\in I)$ are independent Brownian motions defined on $(\Omega,\mathscr{F},p)$. We denote $\mathbb{R}_+^d=\left\{y\in R^d: y_i>0,\quad i=1,2,...,d\right\}$.

Let $X(t)$ be a homogeneous Markov process in E_d (d-dimensional Euclidean space), satisfying the stochastic differential equation $dX(t)=b(X)dt+\sum_{r=1}^k g_r(X)dB_r(t)$. The diffusion matrix is defined as $A(x)=(a_{ij}(x)),\quad a_{ij}(x)=\sum_{r=1}^k g_r^i(x)g_r^j(x)$.

We will give three lemmas that will be used in this paper.

Lemma 2.1 [38]**.** *Assume there exists a bounded open domain $G\subset R^l$ with regular boundary Γ, having the following properties:*

- (I): *In the domain G and some neighborhood thereof, the smallest eigenvalue of the diffusion matrix $A(x)$ is bounded away from zero.*
- (II): *If $z\in R^l\backslash G$, then mean time τ at which a path issuing from z reaches the set G is finite, and $\sup_{z\in K}E^z\tau<\infty$ for every compact subset $K\in R^l$.*

Then the Markov process $X(t)$ has a unique stationary distribution $\pi(\cdot)$. Let $f(x)$ be a function integrable with respect to the measure π. For all $x\in R^l$, the following formula holds:

$$\mathbb{P}\left\{\lim_{t\to\infty}\frac{1}{t}\int_0^t f(X(s))ds=\int_{R^l}f(x)\pi(dx)\right\}=1.$$

Remark 2.1. Lemma 2.1 comes from Theorem 4.1, which is stated and proved on pages 108-109 in [38]. The assumption (I) means that there exists a positive number M such that $\sum_{i,j=1}^l a_{ij}(x)\xi_i\xi_j\geq M|\xi|^2,\ z\in K,\ \xi\in\mathbb{R}^l$ (see Chapter 3, [39], and Rayleigh's principle, Chapter 6 in [40]). If we show that there exists a nonnegative C^2-function V such that LV is negative for any $R^l\backslash G$, then the assumption (II) holds (see page 1163, [41]).

Lemma 2.2 [35]**.** *Let $x(t)$ be the solution of system (2.2) with initial value $x_0\in\mathbb{R}_+^6$. Then*

$$\lim_{t\to\infty}\frac{S(t)+I(t)+C(t)+C_s(t)+T(t)+A(t)}{t}=0,\quad a.s.$$

In addition,

$$\lim_{t\to\infty}\frac{S(t)}{t}=0,\ \lim_{t\to\infty}\frac{I(t)}{t}=0,\ \lim_{t\to\infty}\frac{C(t)}{t}=0,\ \lim_{t\to\infty}\frac{C_s(t)}{t}=0,\ \lim_{t\to\infty}\frac{T(t)}{t}=0,\ \lim_{t\to\infty}\frac{A(t)}{t}=0,\quad a.s.$$

Lemma 2.3 [35]**.** *Assuming that $\mu>(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)/2$, let $x(t)$ be the solution of system (2.2) with initial value $x_0\in\mathbb{R}_+^6$. Then*

$$\lim_{t\to\infty}\frac{\int_0^t S(s)dB_1(s)}{t}=0,\quad \lim_{t\to\infty}\frac{\int_0^t I(s)dB_2(s)}{t}=0,\quad \lim_{t\to\infty}\frac{\int_0^t C(s)dB_3(s)}{t}=0,\quad a.s.$$

$$\lim_{t\to\infty}\frac{\int_0^t C_s(s)dB_4(s)}{t}=0,\quad \lim_{t\to\infty}\frac{\int_0^t T(s)dB_5(s)}{t}=0,\quad \lim_{t\to\infty}\frac{\int_0^t A(s)dB_6(s)}{t}=0,\quad a.s.$$

Remark 2.2. Lemmas 2.2 and 2.3 come from 2.1 (page 719) and 2.2 (page 722) of Zhao and Jiang [35]. We skip the proofs, which are similar.

3. Existence and uniqueness of positive solution

The existence of a global positive solution is required before we investigate the dynamical behavior of stochastic epidemic model (2.2). In practical terms, because S, I, C, C_s, T, and A represent numbers of different kinds of individuals, they should be nonnegative. We will show that system (2.2) has a unique and global positive solution with any initial value.

Theorem 3.1. *For any initial value $x_0\in\mathbb{R}_+^6$, system (2.2) has a unique solution $x(t)$ on $t\geq 0$ and it will remain in $\mathbb{R}_+^6$ with probability 1, i.e., $x(t)\in\mathbb{R}_+^6$ a.s.*

Proof. *We omit the beginning of proof, which is similar to [42]. We merely show the key to these proofs, the Lyapunov function.*

Define a C^2-function $U:\mathbb{R}_+^6\to\mathbb{R}$ by

$$U(x)=\left(S-\theta-\theta\ln\frac{S}{\theta}\right)+(I-1-\ln I)+(C-1-\ln C)+(C_s-1-\ln C_s)+(T-1-\ln T)+(A-1-\ln A),$$

where θ is a positive constant to be determined later. The nonnegativity of the function can be obtained from the above definition. Applying the Itô formula to $U(x)$, we have

$$dU(x(t)) = LU(x(t))dt + \sigma_1(S(t)-\theta)dB_1(t) + \sigma_2(I(t)-1)dB_2(t) + \sigma_3(C(t)-1)dB_3(t) + \sigma_4(C_s(t)-1)dB_4(t) + \sigma_5(T(t)-1)dB_5(t) + \sigma_6(A(t)-1)dB_6(t),$$

where $LU(x): \mathbb{R}^6_+ \to \mathbb{R}$ is defined by

$$\begin{aligned}
LU(x(t)) &= -\frac{\theta\Pi}{S(t)} + k\theta(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) + \theta\mu + \frac{\theta}{2}\sigma_1^2 - k\beta_1 S(t) - k\beta_2\frac{C(t)S(t)}{I(t)} \\
&\quad - k\beta_3\frac{T(t)S(t)}{I(t)} - k\beta_4\frac{A(t)S(t)}{I(t)} + \mu + \sigma + \rho_1 + \gamma_1 + \frac{1}{2}\sigma_2^2 - \frac{\sigma I(t)}{C(t)} + \mu + \rho_2 + \psi + \frac{1}{2}\sigma_3^2 - \frac{\psi C(t)}{C_s(t)} \\
&\quad + \mu + \rho_3 + \gamma_2 + \frac{1}{2}\sigma_4^2 - \frac{\gamma_1 I(t)}{T(t)} - \frac{\gamma_2 C_s(t)}{T(t)} + \mu + \rho_4 + \frac{1}{2}\sigma_5^2 - \frac{\rho_1 I(t)}{A(t)} - \frac{\rho_2 C(t)}{A(t)} - \frac{\rho_3 C_s(t)}{A(t)} \\
&\quad - \frac{\rho_4 T(t)}{A(t)} + \mu + \gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2 + \Pi - \mu(S(t)+I(t)+C(t)+C_s(t)+T(t)+A(t)) - (\gamma_3+\delta_1)A(t) \\
&\le k\theta(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) - \mu(S(t)+I(t)+C(t)+C_s(t)+T(t)+A(t)) + \Pi + \theta\mu \\
&\quad + \frac{\theta}{2}\sigma_1^2 + 5\mu + \sigma + \rho_1 + \rho_2 + \rho_3 + \rho_4 + \gamma_1 + \gamma_2 + \gamma_3 + \delta_1 + \psi + \frac{1}{2}\left(\sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2 + \sigma_6^2\right),
\end{aligned}$$

If we choose

$$\theta = \frac{\mu}{k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)},$$

then we get

$$k\theta(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) \le \mu(S(t)+I(t)+C(t)+C_s(t)+T(t)+A(t)),$$

so we have

$$\begin{aligned}
LU(x(t)) &\le \Pi + \frac{\mu(\mu+\frac{1}{2}\sigma_1^2)}{k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)} + 5\mu + \sigma + \rho_1 + \rho_2 + \rho_3 + \rho_4 + \gamma_1 + \gamma_2 + \gamma_3 + \delta_1 + \psi \\
&\quad + \frac{1}{2}\left(\sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2 + \sigma_6^2\right) \\
&:= Q,
\end{aligned}$$

where Q is a positive constant which is independent of $S(t)$, $I(t)$, $C(t)$, $C_s(t)$, $T(t)$ and $A(t)$. The remainder of the proof is similar to that of Theorem 3.1 of Mao [20] and is omitted, and this completes the proof. □

4. Existence of unique and ergodic stationary distribution

We will show the sufficient conditions for the existence of an ergodic stationary distribution, which also means that the disease will be persistent in the mean.

Define

$$\omega_1' = \frac{\sigma}{\mu+\rho_2+\psi+\frac{1}{2}\sigma_3^2}, \quad \omega_2' = \frac{\psi}{\mu+\rho_3+\gamma_2+\frac{1}{2}\sigma_4^2}\omega_1',$$

$$\omega_3' = \frac{\gamma_1}{\mu+\rho_4+\frac{1}{2}\sigma_5^2} + \frac{\gamma_2\sigma\psi}{(\mu+\rho_4+\frac{1}{2}\sigma_5^2)(\mu+\rho_3+\gamma_2+\frac{1}{2}\sigma_4^2)(\mu+\rho_2+\psi+\frac{1}{2}\sigma_3^2)},$$

$$\omega_4' = \frac{1}{\mu+\gamma_3+\delta_1+\frac{1}{2}\sigma_6^2}(\rho_1+\rho_2\omega_1'+\rho_3\omega_2'+\rho_4\omega_3').$$

and

$$R_0^S = \frac{\Pi k(\beta_1+\beta_2\omega_1'+\beta_3\omega_3'+\beta_4\omega_4')}{(\mu+\frac{1}{2}\sigma_1^2)(\mu+\sigma+\rho_1+\gamma_1)}.$$

Theorem 4.1. *Assuming that $R_0^S > 1$, the solution $x(t) = (S(t), I(t), C(t), C_s(t), T(t), A(t))^T$ of system (2.2) has an ergodic unique stationary distribution.*

The proof of Theorem 4.1 is shown in Appendix A.

Table 1
Range of the parameter values from the CSZ and the literature [6,11].

Parameter	Range of values (/year)	Parameter	Range of values(/year)
Π	$\Pi \geq 10,000$ people	γ_3	[0.1,0.4]
μ	[0.01,0.025] (CSZ data)	δ_1	[0.3,0.36]
β_1 [a]	$[0,1]\times 10^{-5}$	k	[0.25,2.5] (Assumed)
β_2 [a]	$[0,1]\times 10^{-5}$	ρ_1	[0.03,0.1]
β_3 [a]	$[0,1]\times 10^{-5}$	ρ_2	[0.06,0.45]
β_4 [a]	$[0,1]\times 10^{-5}$	ρ_3	[0.2,0.4]
γ_1	[0.06,0.2]	σ	[0.05,0.3]
γ_2	[0.1,0.3]	ψ	[0.3,0.6]
σ_i^2 (i=1,...,6)	[0,2] (Assumed)	ρ_4	[0.1,0.5] (Assumed)

[a] In Section 2, our model adopts bilinear incidence instead of standard incidence [11]. So, there is an order of magnitude difference between β_i (i=1,2,3,4) in our paper and in [11].

5. Extinction

We establish sufficient conditions for the extinction of disease, i.e., its disappearance in the long term.
For simplicity, define

$$\langle S(t)\rangle = \frac{1}{t}\int_0^t S(r)dr,$$

$$R_0' = \frac{\Pi k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)}{\mu^2 + \frac{1}{5}\mu[(\gamma_3+\delta_1+\frac{1}{2}\sigma_6^2)\wedge\frac{1}{2}\sigma_2^2\wedge\frac{1}{2}\sigma_3^2\wedge\frac{1}{2}\sigma_4^2\wedge\frac{1}{2}\sigma_5^2]}.$$

Theorem 5.1. *Assuming that $\mu > (\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)/2$, for any initial value $x_0 \in \mathbb{R}_+^6$, if $R_0' < 1$, then the disease will die out exponentially with probability 1, i.e., for the solution of system (2.2), we have*

$$\limsup_{t\to\infty}\frac{\ln(I(t)+C(t)+C_s(t)+T(t)+A(t))}{t} < 0, \quad a.s.$$

Then

$$\lim_{t\to\infty} I(t)=0,\ \lim_{t\to\infty} C(t)=0,\ \lim_{t\to\infty} C_s(t)=0,\ \lim_{t\to\infty} T(t)=0,\ \lim_{t\to\infty} A(t)=0, \quad a.s.$$

and

$$\lim_{t\to\infty}\langle S(t)\rangle = \frac{\Pi}{\mu}, \quad a.s.$$

The proof of Theorem 5.1 is shown in Appendix B.

6. Examples and Numerical simulations

We introduce Milstein's higher order method [43] to illustrate our theoretical results. The discretizing equations of model (2.2) are:

$$\begin{cases} S(k+1) = [\Pi - k(\beta_1 I(k)+\beta_2 C(k)+\beta_3 T(k)+\beta_4 A(k))S(k) - \mu S(k)]\Delta t + \sigma_1 S(k)\eta_{1k}\sqrt{\Delta t} + \dfrac{\sigma_1^2 S(k)}{2}\Delta t(\eta_{1k}^2-1), \\ I(k+1) = [k(\beta_1 I(k)+\beta_2 C(k)+\beta_3 T(k)+\beta_4 A(k))S(k) - (\mu+\sigma+\rho_1+\gamma_1)I(k)]\Delta t + \sigma_2 I(k)\eta_{1k}\sqrt{\Delta t} + \dfrac{\sigma_2^2 I(k)}{2}\Delta t(\eta_{1k}^2-1), \\ C(k+1) = [\sigma I(k) - (\mu+\rho_2+\psi)C(k)]\Delta t + \sigma_3 C(k)\eta_{1k}\sqrt{\Delta t} + \dfrac{\sigma_3^2 C(k)}{2}\Delta t(\eta_{1k}^2-1), \\ C_s(k+1) = [\psi C(k) - (\mu+\rho_3+\gamma_2)C_s(k)]\Delta t + \sigma_4 C_s(k)\eta_{1k}\sqrt{\Delta t} + \dfrac{\sigma_4^2 C_s(k)}{2}\Delta t(\eta_{1k}^2-1), \\ T(k+1) = [\gamma_1 I(k) + \gamma_2 C_s(k) - (\mu+\rho_4)T(k)]\Delta t + \sigma_5 T(k)\eta_{1k}\sqrt{\Delta t} + \dfrac{\sigma_5^2 T(k)}{2}\Delta t(\eta_{1k}^2-1), \\ A(k+1) = [\rho_1 I(k) + \rho_2 C(k) + \rho_3 C_s(k) + \rho_4 T(k) - (\mu+\gamma_3+\delta_1)A]\Delta t + \sigma_6 A(k)\eta_{1k}\sqrt{\Delta t} + \dfrac{\sigma_6^2 A(k)}{2}\Delta t(\eta_{1k}^2-1). \end{cases} \tag{6.1}$$

where η_{ik}, i=1,2,3,4,5,6; k=1,2,3,...,n. η_i (i=1,2,3,4,5,6) are independent Gaussian random variables with distribution $N(0, 1)$, and η_{ik} (k=1,2,3,...n) is the k-th value of η_i. The n is the total number of iterations, i.e., the time t in the following diagrams. We choose the length of one iteration step as $\Delta t = 0.01$. The diagrams are shown in Appendix D. The values of parameters in Table 1 are chosen from the Central Statistical Office of Zimbabwe (CSZ) and [6,11].

6.1. Ergodic stationary distribution and disease extinction

The parameter values are chosen as the caption of Fig 1.1. We compute that $R_0^S = 80.7569 > 1$. According to Theorem 4.1, the solution of system (2.2) has an ergodic unique stationary distribution (as shown in Fig 1.2), which also means the disease is persistent (as shown in Fig 1.1). From Fig 1.1, the population sizes of different types of individuals in the stochastic model undulate compared to the deterministic model, which is reasonable due to inescapable environmental white noise. Fig 1.2 displays the results of 60,000 numerical simulations using the same initial values and parameter values. We plot the density curves at time points $k = 20000, 40000$ and 60000, each in a different color. The three density curves almost coincide, and these overlapping curves are the stationary distribution in Theorem 4.1.

We use the same initial values, and the parameter values are chosen as the caption of Fig 1.3. We compute $R_0^{'} = 0.8956 < 1$, and $0.0250 = \mu > \frac{1}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2) = 0.0005$. According to Theorem 5.1, the disease will die out exponentially with probability 1, as shown in Fig 1.3.

6.2. Impact of screening rate ψ

As shown in Fig 2.1, R_0 and R_0^S decrease as the screening rate increases as $\psi = 0.3, 0.301, 0.302, \ldots, 0.6$ (step size 0.001). Under the assumed parameter values, we observe that a screening rate satisfying $R_0 < 1$ or $R_0^S < 1$, should give 55.4% coverage for the given parameter values in deterministic model (2.1), and 38.3% coverage in stochastic model (2.2). According to Theorem 4.1, $R_0^S > 1$ is only the sufficient condition for the existence of a stationary distribution. This means that if $\psi > 38.3\%$, then the disease in system (2.2) is likely to be extinct, and it may also persist. In short, $\psi > 38.3\%$ makes the extinction of diseases possible. The parameter values come from the laboratory and the literature, and are not those estimated by rigorous statistical methods in a certain region, so the screening rates $\psi > 55.4\%$ and $\psi > 38.3\%$ are just a comparison between the deterministic and stochastic models, and they cannot be used as a strict standard for disease control in a certain region.

To further investigate the significance of the screening rate ψ, we assume that the interventions to an ongoing epidemic are implemented. We introduce a control measure after a period of time, which is reflected in $\psi = 0.3, 0.4, 0.5, 0.6$. Then the variation trends of carriers $C(t)$ are as shown in Fig 2.2. We do not draw the trend of susceptible individuals or other disease carriers because there is no obvious change.

6.3. Impact of seeking treatment rate γ_i $(i = 1, 2, 3)$

Similar to Fig 2.1, we investigate the impact of γ_i $(i = 1, 2, 3)$ on R_0 and R_0^S in Fig 3.1-3.3.

From the expression of $R_0^{'}$, if $(\gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2) < (\frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\sigma_3^2 \wedge \frac{1}{2}\sigma_4^2 \wedge \frac{1}{2}\sigma_5^2)$, then

$$R_0^{'} = \frac{\Pi k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)}{\mu^2 + \frac{1}{5}\mu(\gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2)},$$

and if $(\gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2) \geq (\frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\sigma_3^2 \wedge \frac{1}{2}\sigma_4^2 \wedge \frac{1}{2}\sigma_5^2)$, then $R_0^{'}$ is a constant. This is shown in Fig 3.4.

We introduce a change of γ_1 after a period of time, i.e., $\gamma_1 = 0.06, 0.10, 0.15, 0.20$. The variation trend of individuals $I(t)$, $C(t)$, $C_s(t)$, and $T(t)$ is shown in Fig 3.5. The same operation is implemented in Fig 3.6 and 3.7.

A more detailed analysis of Fig 2.1-3.7 is provided below.

7. Conclusions and Discussions

We propose a stochastic HIV/AIDS model including the screening rate for virus carriers and the rate at which infected individuals actively seek treatment. After proving that the solution of the stochastic model is positive and global, we obtain sufficient conditions for the existence of ergodic stationary distribution and disease extinction.

7.1. Discussion of R_0^S and $R_0^{'}$ (disadvantages of our work)

A value gap exists between the parameters R_0^S and $R_0^{'}$ for sufficient conditions,and this is due to the limitation of our mathematical method for complex and high-dimensional models. For some low-dimensional models [23,35,42,45], our technique can get the threshold behavior between the existence of a stationary distribution and disease extinction. Namely, there is no value gap between the two sufficient condition parameters R_0^S and $R_0^{'}$. It is a pity to say that it is extremely difficult at present for the model (2.2) presented in this paper to have a threshold result.

There is no obvious association between R_0^S and $R_0^{'}$ in this paper. When investigating the sufficient condition for disease extinction of low-dimensional models, we use another method, the spectral radius method (refer to the proof of Theorem 2.2 in Liu and Jiang [46]). Although this method enables the association of R_0^S and $R_0^{'}$, it still fails due to the high dimension of model (2.2) in this paper.

7.2. Comparison of asymptotic behaviors of deterministic and stochastic models

If $R_0 \leq 1$, then deterministic model (2.1) has a unique disease-free equilibrium point $E^0 = (\frac{\Pi}{\mu}, 0, 0, 0, 0, 0)$. According to Theorem 5.1, if $R_0^{'} < 1$, then disease in the stochastic model (2.2) will die out exponentially with probability 1, i.e.,

$$\lim_{t\to\infty} \langle S(t)\rangle = \lim_{t\to\infty} \frac{1}{t}\int_0^t S(r)dr = \frac{\Pi}{\mu}, \quad a.s.,$$

and $I(t) \to 0$ when $t \to \infty$ (the same as for $C(t)$, $C_s(t)$, $T(t)$, $A(t)$, refer to Theorem 5.1). This indicates some consistency between the deterministic and stochastic systems in terms of disease extinction. Although the solution of the stochastic model is not a fixed value, it equals to $\frac{\Pi}{\mu}$ a.s., in the time mean of integral value, corresponding to the fixed value of the deterministic system.

If $R_0 > 1$, then there exists a unique endemic equilibrium point E^* for deterministic model (2.1). According to Theorem 4.1, if $R_0^S > 1$, then the solution of stochastic system (2.2) has an ergodic unique stationary distribution, which means the disease will be persistent in the mean. After considering environmental noise, the solution of stochastic system (2.2) is not a fixed equilibrium point, but fluctuates around a point. This promotes the possible existence of a stationary distribution.

From Fig 2.1 and 3.1-3.3, we know that $R_0^S < R_0$ under the same parameter values, which is easily verified from their expressions. The intensities of the white noise σ_i^2 (i=1,3,4,5,6) are included in R_0^S, and play an important role in the existence of an ergodic stationary distribution of the stochastic model in Theorem 4.1. If $\sigma_i = 0$ (i=1,3,4,5,6), then $R_0^S = R_0$, and the stationary distribution degenerates into the fixed equilibrium point E^*. That is, the stochastic model (2.2) involves partial results of the deterministic model (2.1).

From the expression of $R_0^{'}$ in Theorem 5.1, we know that very large σ_i (i=2,3,4,5,6) may promote $R_0^{'} < 1$ from $R_0^{'} \geq 1$. That is, big white noise may promote the extinction of disease in some circumstances.

7.3. Impact of ψ and γ_i on the existence of stationary distribution and disease extinction

A higher screening rate ψ contributes to the inhibition of AIDS. From Fig 2.2, a higher ψ can cause a significant decline in $C(t)$ (asymptomatic and infectious disease carriers). If $C(t)$ decreases, then $I(t)$ will decrease, and the disease may be effectively controlled. Screening helps raise awareness of the importance of the disease, educate carriers to take safe sexual interaction measures, and encourage them to be active in treatment. If carriers are isolated, then they will not spread the disease, which is verified by the second sub-equation in model (2.2). People can learn about various ways to prevent further infection after screening, which is important in regions with less advanced medical technology.

Higher rates γ_1 and γ_3 of seeking treatment also contribute to the inhibition of AIDS. From Fig 3.5, a higher γ_1 can significantly reduce $I(t)$, $C(t)$ and $C_s(t)$, and increase $T(t)$. From Fig 3.7, a higher γ_3 can significantly reduce $A(t)$. Some carriers will not receive treatment, since the side effects lead to much fear of ARV therapy or HAART for people living with HIV/AIDS. With publicity and guidance from the government, more people will actively seek treatment, and then a part of $I(t)$ will become $T(t)$ and a part of $A(t)$ will become $A_r(t)$. Both changes can decrease $I(t)$, according to model (2.2), and $\beta_3 < \beta_1$.

Higher rates of seeking treatment γ_2 may cause the AIDS not to be eliminated. From Fig 3.6, a higher γ_2 can significantly pull down the size of $C_s(t)$ and increase $T(t)$. A higher γ_2 can make part of $C_s(t)$ become $T(t)$. $C_s(t)$ does not have the ability to spread disease, but $T(t)$ does. In short, a higher γ_2 is a negative factor for the inhibition of AIDS.

Finally, this study shows that screening for virus carriers and actively seeking of treatment by infected individuals (except γ_2) contribute to the inhibition of AIDS, and they are important supplementary means to lower the incidence and prevalence levels. This encourages positive attitude towards preventive methods against infection, and eliminates the fear of disease. They should be the main tools in places where ARVs are not readily available, especially in developing countries. In conclusion, multiple strategies, including ARVs, virus carriers screening, and the active seeking of treatment (except γ_2), should be implemented in the process of eliminating AIDS.

Declaration of Competing Interest

The authors declare that they do not have any financial or nonfinancial conflict of interests

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Appendix A. (Proof of Theorem 4.1)

To prove Theorem 4.1, it suffices to verify conditions (I) and (II) in Lemma 2.1. Since the diffusion matrix $D(S, I, C, C_s, T, A) = diag\{\sigma_1^2 S^2, \sigma_2^2 I^2, \sigma_3^2 C^2, \sigma_4^2 C_s^2, \sigma_5^2 T^2, \sigma_6^2 A^2\}$ of system (2.2) is positive definite, condition (I) holds. Hence we must only prove that condition (II) holds.

For convenient calculation, let $\bar{S}=\frac{\Pi}{\mu+\frac{1}{2}\sigma_1^2}$. $\bar{I}$, $\bar{C}$, $\bar{C}_s$, $\bar{T}$ and $\bar{A}$ satisfy the following equations:

$$\begin{cases}\sigma\bar{I}=\left(\mu+\rho_2+\psi+\frac{1}{2}\sigma_3^2\right)\bar{C},\\ \psi\bar{C}=\left(\mu+\rho_3+\gamma_2+\frac{1}{2}\sigma_4^2\right)\bar{C}_s,\\ \gamma_1\bar{I}+\gamma_2\bar{C}_s=\left(\mu+\rho_4+\frac{1}{2}\sigma_5^2\right)\bar{T},\\ \rho_1\bar{I}+\rho_2\bar{C}+\rho_3\bar{C}_s+\rho_4\bar{T}=\left(\mu+\gamma_3+\delta_1+\frac{1}{2}\sigma_6^2\right)\bar{A}.\end{cases} \tag{A.1}$$

In fact, we only need the proportional values of $\frac{\bar{C}}{\bar{I}}$, $\frac{\bar{C}_s}{\bar{I}}$, $\frac{\bar{T}}{\bar{I}}$ and $\frac{\bar{A}}{\bar{I}}$ below. Without loss of generality, we can assume that $\bar{I}=1$.

From the above equations, we can obtain that

$$\frac{\bar{C}}{\bar{I}}=\omega_1',\quad \frac{\bar{C}_s}{\bar{I}}=\omega_2',\quad \frac{\bar{T}}{\bar{I}}=\omega_3',\quad \frac{\bar{A}}{\bar{I}}=\omega_4',$$

where ω_1', ω_2', ω_3' and ω_4' are defined in Section 4. Beyond that, let

$$\widetilde{S}(t)=\frac{S(t)}{\bar{S}},\quad \widetilde{I}(t)=\frac{I(t)}{\bar{I}},\quad \widetilde{C}(t)=\frac{C(t)}{\bar{C}},\quad \widetilde{C}_s(t)=\frac{C_s(t)}{\bar{C}_s},\quad \widetilde{T}(t)=\frac{T(t)}{\bar{T}},\quad \widetilde{A}(t)=\frac{A(t)}{\bar{A}}.$$

Define a C^2 function $\mathbb{R}_+^6\to\mathbb{R}$ by

$$G(x)=M[V_1(x)+V_2(x)]+V_3(x)+V_4(x),$$

We assume that $\tilde{G}$ is the minimum value of G. Then we define a nonnegative C^2 function,

$$V(x)=G(x)-\tilde{G},$$

where

$$V_1(x)=-\frac{\bar{I}}{\bar{S}}\ln I-c_1\bar{C}\ln C-c_2\bar{C}_s\ln C_s-c_3\bar{T}\ln T-c_4\bar{A}\ln A-\frac{c_5}{\mu+\frac{1}{2}\sigma_1^2}\ln S,$$

$$V_2(x)=d_1C+d_2C_s+d_3T+d_4A,$$

$$V_3(x)=-\ln S-\ln C-\ln C_s-\ln T-\ln A,$$

$$V_4(x)=\frac{1}{m+1}(S+I+C+C_s+T+A)^{m+1},$$

and c_i (i=1,2,3,4,5), d_j (j=1,2,3,4) are positive constants to be determined later. $m>0$ is a sufficiently small number, and M is a sufficiently large number satisfying $M>0$, such that

$$\mu-\tfrac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)>0, \tag{A.2}$$

$$-M\tfrac{\bar{I}}{\bar{S}}\Lambda+E\leq-2, \tag{A.3}$$

where

$$\Lambda=-(R_0^S-1)(\mu+\sigma+\rho_1+\gamma_1+\tfrac{1}{2}\sigma_2^2), \tag{A.4}$$

$$B=6\mu+\rho_2+\rho_3+\rho_4+\psi+\delta_1+\tfrac{1}{2}(\sigma_1^2+\sigma_3^2+\sigma_4^2+\sigma_5^2+\sigma_6^2), \tag{A.5}$$

$$E=\sup_{(S,I,C,C_s,T,A)\in\mathbb{R}_+^6}\left\{-\frac{1}{4}\left[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\right](I+C+T+A)^{m+1}+k(\beta_1I+\beta_2C+\beta_3T+\beta_4A)+B+D\right\}. \tag{A.6}$$

B, D, and E are positive constants, and D is defined in (A.17). By applying the Itô formula to $-\ln I(t)$, L denotes the differential operator, and we obtain

$$\begin{aligned}
L(-\ln I(t)) &= -k\beta_1 S(t) - k\beta_2\frac{C(t)S(t)}{I(t)} - k\beta_3\frac{T(t)S(t)}{I(t)} - k\beta_4\frac{A(t)S(t)}{I(t)} + \mu + \sigma + \rho_1 + \gamma_1 + \frac{1}{2}\sigma_2^2\\
&= -k\beta_1\bar{S}\widetilde{S}(t) - k\beta_2\frac{\bar{C}\,\bar{S}}{\bar{I}}\frac{\widetilde{C}(t)\widetilde{S}(t)}{\widetilde{I}(t)} - k\beta_3\frac{\bar{T}\,\bar{S}}{\bar{I}}\frac{\widetilde{T}(t)\widetilde{S}(t)}{\widetilde{I}(t)} - k\beta_4\frac{\bar{A}\,\bar{S}}{\bar{I}}\frac{\widetilde{A}(t)\widetilde{S}(t)}{\widetilde{I}(t)} + \mu + \sigma + \rho_1 + \gamma_1 + \frac{1}{2}\sigma_2^2\\
&= -k\beta_1\bar{S} - k\beta_2\frac{\bar{C}\,\bar{S}}{\bar{I}} - k\beta_3\frac{\bar{T}\,\bar{S}}{\bar{I}} - k\beta_4\frac{\bar{A}\,\bar{S}}{\bar{I}} + \mu + \sigma + \rho_1 + \gamma_1 + \frac{1}{2}\sigma_2^2 - k\beta_1\bar{S}(\widetilde{S}(t) - 1)\\
&\quad - k\beta_2\frac{\bar{C}\,\bar{S}}{\bar{I}}\left(\frac{\widetilde{C}(t)\widetilde{S}(t)}{\widetilde{I}(t)} - 1\right) - k\beta_3\frac{\bar{T}\,\bar{S}}{\bar{I}}\left(\frac{\widetilde{T}(t)\widetilde{S}(t)}{\widetilde{I}(t)} - 1\right) - k\beta_4\frac{\bar{A}\,\bar{S}}{\bar{I}}\left(\frac{\widetilde{A}(t)\widetilde{S}(t)}{\widetilde{I}(t)} - 1\right).
\end{aligned}$$

Applying the inequality $y - 1 \geq \ln y$ ($\forall\ y > 0$), we obtain

$$\begin{aligned}
L(-\ln I(t)) &\leq -\frac{\Pi k}{\mu + \frac{1}{2}\sigma_1^2}(\beta_1 + \beta_2\omega_1' + \beta_3\omega_3' + \beta_4\omega_4') + \mu + \sigma + \rho_1 + \gamma_1 + \frac{1}{2}\sigma_2^2 - k\beta_1\bar{S}\ln\widetilde{S}(t)\\
&\quad - k\beta_2\frac{\bar{C}\,\bar{S}}{\bar{I}}\ln\frac{\widetilde{C}(t)\widetilde{S}(t)}{\widetilde{I}(t)} - k\beta_3\frac{\bar{T}\,\bar{S}}{\bar{I}}\ln\frac{\widetilde{T}(t)\widetilde{S}(t)}{\widetilde{I}(t)} - k\beta_4\frac{\bar{A}\,\bar{S}}{\bar{I}}\ln\frac{\widetilde{A}(t)\widetilde{S}(t)}{\widetilde{I}(t)}\\
&= -(R_0^S - 1)(\mu + \sigma + \rho_1 + \gamma_1 + \frac{1}{2}\sigma_2^2) + \left(-k\beta_1\bar{S} - k\beta_2\frac{\bar{C}\,\bar{S}}{\bar{I}} - k\beta_3\frac{\bar{T}\,\bar{S}}{\bar{I}} - k\beta_4\frac{\bar{A}\,\bar{S}}{\bar{I}}\right)\ln\widetilde{S}(t)\\
&\quad + \left(k\beta_2\frac{\bar{C}\,\bar{S}}{\bar{I}} + k\beta_3\frac{\bar{T}\,\bar{S}}{\bar{I}} + k\beta_4\frac{\bar{A}\,\bar{S}}{\bar{I}}\right)\ln\widetilde{I}(t) - k\beta_2\frac{\bar{C}\,\bar{S}}{\bar{I}}\ln\widetilde{C}(t) - k\beta_3\frac{\bar{T}\,\bar{S}}{\bar{I}}\ln\widetilde{T}(t) - k\beta_4\frac{\bar{A}\,\bar{S}}{\bar{I}}\ln\widetilde{A}(t),
\end{aligned}$$

and combining the definition of Λ with (A.4) gives us

$$\begin{aligned}
L\left(-\frac{\bar{I}}{\bar{S}}\ln I(t)\right) &= -\frac{\bar{I}}{\bar{S}}\Lambda + \left(-k\beta_1\bar{I} - k\beta_2\bar{C} - k\beta_3\bar{T} - k\beta_4\bar{A}\right)\ln\widetilde{S}(t) + \left(k\beta_2\bar{C} + k\beta_3\bar{T} + k\beta_4\bar{A}\right)\ln\widetilde{I}(t)\\
&\quad - k\beta_2\bar{C}\ln\widetilde{C}(t) - k\beta_3\bar{T}\ln\widetilde{T}(t) - k\beta_4\bar{A}\ln\widetilde{A}(t).
\end{aligned}\tag{A.7}$$

Applying the Itô formula to $-\frac{1}{\mu+\frac{1}{2}\sigma_1^2}\ln S(t)$ and combining $-y + 1 \leq -\ln y$ ($\forall\ y > 0$), we get

$$\begin{aligned}
L\left(-\frac{1}{\mu + \frac{1}{2}\sigma_1^2}\ln S(t)\right) &= -\frac{1}{\widetilde{S}(t)} + 1 + \frac{k}{\mu + \frac{1}{2}\sigma_1^2}(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t))\\
&\leq -\ln\frac{1}{\widetilde{S}(t)} + \frac{k}{\mu + \frac{1}{2}\sigma_1^2}(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t))\\
&= \ln\widetilde{S}(t) + \frac{k}{\mu + \frac{1}{2}\sigma_1^2}(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)).
\end{aligned}\tag{A.8}$$

Applying the Itô formula to $-\ln C(t)$, we obtain

$$L(-\ln C(t)) = -\frac{\sigma I(t)}{C(t)} + \mu + \rho_2 + \psi + \frac{1}{2}\sigma_3^2 = -\frac{\sigma\bar{I}}{\bar{C}}\frac{\widetilde{I}(t)}{\widetilde{C}(t)} + \mu + \rho_2 + \psi + \frac{1}{2}\sigma_3^2.$$

We combine the first equation in (A.1) with $-y + 1 \leq -\ln y$ ($\forall\ y > 0$), to obtain

$$\begin{aligned}
L(-\bar{C}\ln C(t)) &= -\sigma\bar{I}\frac{\widetilde{I}(t)}{\widetilde{C}(t)} + (\mu + \rho_2 + \psi + \frac{1}{2}\sigma_3^2)\bar{C}\\
&= -\sigma\bar{I}\frac{\widetilde{I}(t)}{\widetilde{C}(t)} + \sigma\bar{I} = \sigma\bar{I}(-\frac{\widetilde{I}(t)}{\widetilde{C}(t)} + 1)\\
&\leq \sigma\bar{I}(-\ln\frac{\widetilde{I}(t)}{\widetilde{C}(t)}) = -\sigma\bar{I}\ln\widetilde{I}(t) + \sigma\bar{I}\ln\widetilde{C}(t).
\end{aligned}\tag{A.9}$$

Applying the Itô formula to $-\ln C_s(t)$, we obtain

$$L(-\ln C_s(t)) = -\frac{\psi C(t)}{C_s(t)} + \mu + \rho_3 + \gamma_2 + \frac{1}{2}\sigma_4^2 = -\frac{\psi\bar{C}}{\bar{C}_s}\frac{\widetilde{C}(t)}{\widetilde{C}_s(t)} + \mu + \rho_3 + \gamma_2 + \frac{1}{2}\sigma_4^2.$$

Then, combining the second equation in (A.1) with $-y+1 \le -\ln y$ ($\forall\ y>0$), we get

$$\begin{aligned} L(-\bar{C}_s \ln C_s(t)) &= -\psi\bar{C}\frac{\widetilde{C}(t)}{\widetilde{C}_s(t)} + (\mu+\rho_3+\gamma_2+\frac{1}{2}\sigma_4^2)\bar{C}_s \\ &= -\psi\bar{C}\frac{\widetilde{C}(t)}{\widetilde{C}_s(t)} + \psi\bar{C} = \psi\bar{C}(-\frac{\widetilde{C}(t)}{\widetilde{C}_s(t)}+1) \\ &\le \psi\bar{C}\left(-\ln\frac{\widetilde{C}(t)}{\widetilde{C}_s(t)}\right) = -\psi\bar{C}\ln\widetilde{C}(t) + \psi\bar{C}\ln\widetilde{C}_s(t). \end{aligned} \tag{A.10}$$

Similarly, we get

$$\begin{aligned} L(-\bar{T}\ln T(t)) &\le \gamma_1\bar{I}\left(-\ln\frac{\widetilde{I}(t)}{\widetilde{T}(t)}\right) + \gamma_2\bar{C}_s\left(-\ln\frac{\widetilde{C}_s(t)}{\widetilde{T}(t)}\right) \\ &= -\gamma_1\bar{I}\ln\widetilde{I}(t) + (\gamma_1\bar{I}+\gamma_2\bar{C}_s)\ln\widetilde{T}(t) - \gamma_2\bar{C}_s\ln\widetilde{C}_s(t). \end{aligned} \tag{A.11}$$

$$\begin{aligned} L(-\bar{A}\ln A(t)) &\le \rho_1\bar{I}\left(-\ln\frac{\widetilde{I}(t)}{\widetilde{A}(t)}\right) + \rho_2\bar{C}\left(-\ln\frac{\widetilde{C}(t)}{\widetilde{A}(t)}\right) + \rho_3\bar{C}_s\left(-\ln\frac{\widetilde{C}_s(t)}{\widetilde{A}(t)}\right) + \rho_4\bar{T}\left(-\ln\frac{\widetilde{T}(t)}{\widetilde{A}(t)}\right) \\ &= -\rho_1\bar{I}\ln\widetilde{I}(t) + (\rho_1\bar{I}+\rho_2\bar{C}+\rho_3\bar{C}_s+\rho_4\bar{T})\ln\widetilde{A}(t) - \rho_3\bar{C}_s ln\widetilde{C}_s(t) - \rho_4\bar{T}\ln\widetilde{T}(t) - \rho_2\bar{C}\ln\widetilde{C}(t). \end{aligned} \tag{A.12}$$

Combining (A.7)-(A.12), we obtain

$$\begin{aligned} LV_1 \le& -\frac{\bar{I}}{\bar{S}}\Lambda + \left[-k\beta_1\bar{I} - k\beta_2\bar{C} - k\beta_3\bar{T} - k\beta_4\bar{A} + c_5\right]\ln\widetilde{S}(t) + \left[k\beta_2\bar{C} + k\beta_3\bar{T} + k\beta_4\bar{A} - c_1\sigma\bar{I} - c_3\gamma_1\bar{I} - c_4\rho_1\bar{I}\right]\ln\widetilde{I}(t) \\ &+ \left[-k\beta_2\bar{C} + c_1\sigma\bar{I} - c_2\psi\bar{C} - c_4\rho_2\bar{C}\right]\ln\widetilde{C}(t) + \left[c_2\psi\bar{C} - c_3\gamma_2\bar{C}_s - c_4\rho_3\bar{C}_s\right]\ln\widetilde{C}_s(t) \\ &+ \left[c_3(\gamma_1\bar{I}+\gamma_2\bar{C}_s) - c_4\rho_4\bar{T} - k\beta_3\bar{T}\ \right]\ln\widetilde{T}(t)\left[-k\beta_4\bar{A} + c_4(\rho_1\bar{I}+\rho_2\bar{C}+\rho_3\bar{C}_s+\rho_4\bar{T})\right]\ln\widetilde{A}(t) \\ &+ \frac{c_5k}{\mu+\frac{1}{2}\sigma_1^2}(\beta_1I(t)+\beta_2C(t)+\beta_3T(t)+\beta_4A(t)). \end{aligned} \tag{A.13}$$

Let

$$\begin{cases} -k\beta_1\bar{I} - k\beta_2\bar{C} - k\beta_3\bar{T} - k\beta_4\bar{A} + c_5 = 0, \\ k\beta_2\bar{C} + k\beta_3\bar{T} + k\beta_4\bar{A} - c_1\sigma\bar{I} - c_3\gamma_1\bar{I} - c_4\rho_1\bar{I} = 0, \\ -k\beta_2\bar{C} + c_1\sigma\bar{I} - c_2\psi\bar{C} - c_4\rho_2\bar{C} = 0, \\ c_2\psi\bar{C} - c_3\gamma_2\bar{C}_s - c_4\rho_3\bar{C}_s = 0, \\ c_3(\gamma_1\bar{I}+\gamma_2\bar{C}_s) - c_4\rho_4\bar{T} - k\beta_3\bar{T} = 0, \\ -k\beta_4\bar{A} + c_4(\rho_1\bar{I}+\rho_2\bar{C}+\rho_3\bar{C}_s+\rho_4\bar{T}) = 0. \end{cases}$$

The solution of above equations is unique, and it is

$$c_5 = k(\beta_1\bar{I}+\beta_2\bar{C}+\beta_3\bar{T}+\beta_4\bar{A}), \quad c_4 = \frac{k\beta_4\bar{A}}{\rho_1\bar{I}+\rho_2\bar{C}+\rho_3\bar{C}_s+\rho_4\bar{T}},$$

$$c_3 = \frac{\bar{T}(c_4\rho_4+k\beta_3)}{\gamma_1\bar{I}+\gamma_2\bar{C}_s}, \quad c_2 = \frac{\bar{C}_s(c_3\gamma_2+c_4\rho_3)}{\psi\bar{C}}, \quad c_1 = \frac{\bar{C}(k\beta_2+c_2\psi+c_4\rho_2)}{\sigma\bar{I}}.$$

So,

$$LV_1(x(t)) \le -\frac{\bar{I}}{\bar{S}}\Lambda + \frac{c_5k}{\mu+\frac{1}{2}\sigma_1^2}(\beta_1I(t)+\beta_2C(t)+\beta_3T(t)+\beta_4A(t)).$$

By using the Itô formula to $V_1(x)+V_2(x)$, we obtain

$$\begin{aligned} L(V_1(x(t))+V_2(x(t))) \le& -\frac{\bar{I}}{\bar{S}}\Lambda + k_1I(t) + \left[\frac{c_5k\beta_2}{\mu+\frac{1}{2}\sigma_1^2} - d_1(\mu+\rho_2+\psi) + d_2\psi + d_4\rho_2\right]C(t) \\ &+ [-d_2(\mu+\rho_3+\gamma_2) + d_3\gamma_2 + d_4\rho_3]C_s(t) + \left[\frac{c_5k\beta_3}{\mu+\frac{1}{2}\sigma_1^2} - d_3(\mu+\rho_4) + d_4\rho_4\right]T(t) \\ &+ \left[\frac{c_5k\beta_4}{\mu+\frac{1}{2}\sigma_1^2} - d_4(\mu+\gamma_3+\delta_1)\right]A(t). \end{aligned}$$

where

$$k_1 = \frac{c_5 k\beta_1}{\mu + \frac{1}{2}\sigma_1^2} + d_1\sigma + d_3\gamma_1 + d_4\rho_1,$$

Let

$$\begin{cases} \dfrac{c_5 k\beta_2}{\mu + \frac{1}{2}\sigma_1^2} - d_1(\mu + \rho_2 + \psi) + d_2\psi + d_4\rho_2 = 0, \\ -d_2(\mu + \rho_3 + \gamma_2) + d_3\gamma_2 + d_4\rho_3 = 0, \\ \dfrac{c_5 k\beta_4}{\mu + \frac{1}{2}\sigma_1^2} - d_4(\mu + \gamma_3 + \delta_1) = 0, \\ \dfrac{c_5 k\beta_3}{\mu + \frac{1}{2}\sigma_1^2} - d_3(\mu + \rho_4) + d_4\rho_4 = 0. \end{cases}$$

The solution of above equations is unique, and the unique solution is given as follows.

$$d_4 = \frac{c_5 k\beta_4}{(\mu + \frac{1}{2}\sigma_1^2)(\mu + \gamma_3 + \delta_1)}, \qquad d_3 = \frac{c_5 k\beta_3}{(\mu + \frac{1}{2}\sigma_1^2)(\mu + \rho_4)} + \frac{d_4\rho_4}{\mu + \rho_4},$$

$$d_2 = \frac{d_3\gamma_2 + d_4\rho_3}{\mu + \rho_3 + \gamma_2}, \qquad d_1 = \frac{c_5 k\beta_2}{(\mu + \frac{1}{2}\sigma_1^2)(\mu + \rho_2 + \psi)} + \frac{d_2\psi + d_4\rho_2}{\mu + \rho_2 + \psi}.$$

Then we have

$$L(V_1(x(t)) + V_2(x(t))) \quad \leq -\tfrac{\bar{I}}{\bar{S}}\Lambda + k_1 I(t). \tag{A.14}$$

Applying the Itô formula to $V_3(x)$ and combining this with the definition of B in (A.5), we obtain

$$\begin{aligned} LV_3(x(t)) = &-\frac{\Pi}{S(t)} + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) + \mu + \frac{1}{2}\sigma_1^2 - \sigma\frac{I(t)}{C(t)} + \mu + \rho_2 + \psi + \frac{1}{2}\sigma_3^2 \\ &- \psi\frac{C(t)}{C_s(t)} + \mu + \rho_3 + \gamma_2 + \frac{1}{2}\sigma_4^2 - \gamma_1\frac{I(t)}{T(t)} - \gamma_1\frac{C_s(t)}{T(t)} + \mu + \rho_4 + \frac{1}{2}\sigma_5^2 - \rho_1\frac{I(t)}{A(t)} - \rho_2\frac{C(t)}{A(t)} \\ &- \rho_3\frac{C_s(t)}{A(t)} - \rho_4\frac{T(t)}{A(t)} + \mu + \gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2 \\ \leq &-\frac{\Pi}{S(t)} + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) - \sigma\frac{I(t)}{C(t)} - \psi\frac{C(t)}{C_s(t)} - \gamma_1\frac{I(t)}{T(t)} - \rho_1\frac{I(t)}{A(t)} + B. \end{aligned} \tag{A.15}$$

Applying the Itô formula to V_4 and combining this with (A.2), we obtain

$$\begin{aligned} LV_4(x(t)) = &\ (S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t))^m\Big[\Pi - \mu(S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t)) \\ &- (\gamma_3 + \delta_1)A(t)\Big] + \frac{m}{2}(S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t))^{m-1}[\sigma_1^2(S(t))^2 + \sigma_2^2(I(t))^2 \\ &+ \sigma_3^2(C(t))^2 + \sigma_4^2(C_s(t))^2 + \sigma_5^2(T(t)£©^2 + \sigma_6^2(A(t))^2] \\ \leq &\ (S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t))^m[\Pi - \mu(S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t))] \\ &+ \frac{m}{2}(S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t))^{m+1}\big[\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2\big] \\ \leq &\ D - \frac{1}{2}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t))^{m+1}, \end{aligned} \tag{A.16}$$

where

$$D = \sup_{(S,I,C,C_s,T,A)\in\mathbb{R}_+^6}\Big\{\Pi(S + I + C + C_s + T + A)^m - \frac{1}{2}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](S + I + C + C_s + T + A)^{m+1}\Big\}. \tag{A.17}$$

Then, combining (A.14), (A.15), and (A.16), we obtain:

$$\begin{aligned}
LV(x(t)) \le\ & -M\frac{\bar{I}}{\bar{\bar{S}}}\Lambda + Mk_1I(t) + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) \\
& - \frac{\Pi}{S(t)} - \sigma\frac{I(t)}{C(t)} - \psi\frac{C(t)}{C_s(t)} - \gamma_1\frac{I(t)}{T(t)} - \rho_1\frac{I(t)}{A(t)} + B + D \\
& - \frac{1}{2}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t))^{m+1} \\
\le\ & -M\frac{\bar{I}}{\bar{\bar{S}}}\Lambda + Mk_1I(t) + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) - \frac{\Pi}{S(t)} - \sigma\frac{I(t)}{C(t)} - \psi\frac{C(t)}{C_s(t)} - \gamma_1\frac{I(t)}{T(t)} \\
& - \rho_1\frac{I(t)}{A(t)} - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](S(t) + I(t) + C(t) + C_s(t) + T(t) + A(t))^{m+1} \\
& + B + D - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I(t) + C(t) + T(t) + A(t))^{m+1}.
\end{aligned} \tag{A.18}$$

We define a bounded closed set as follows, and $x(t) = (S(t), I(t), C(t), C_s(t), T(t), A(t))^T$.

$$D_\varepsilon = \Big\{x(t) \in \mathbb{R}_+^6 : \varepsilon < S(t) < \frac{1}{\varepsilon}, \varepsilon < I(t) < \frac{1}{\varepsilon}, \varepsilon^2 < C(t) < \frac{1}{\varepsilon^2}, \varepsilon^3 < C_s(t) < \frac{1}{\varepsilon^3}, \varepsilon^2 < T(t) < \frac{1}{\varepsilon^2}, \varepsilon^2 < A(t) < \frac{1}{\varepsilon^2}\Big\},$$

where ε are sufficiently small and satisfy the following conditions:

$$-\Pi/\varepsilon + F \le -1, \tag{A.19}$$

$$Mk_1\varepsilon \le 1, \tag{A.20}$$

$$-\tfrac{\sigma}{\varepsilon} + F \le -1, \tag{A.21}$$

$$-\tfrac{\psi}{\varepsilon} + F \le -1, \tag{A.22}$$

$$-\tfrac{\gamma_1}{\varepsilon} + F \le -1, \tag{A.23}$$

$$-\tfrac{\rho_1}{\varepsilon} + F \le -1, \tag{A.24}$$

$$-\tfrac{1}{4}\big[\mu - \tfrac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\big]\tfrac{1}{\varepsilon^{m+1}} + F \le -1, \tag{A.25}$$

$$-\tfrac{1}{4}\big[\mu - \tfrac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\big]\tfrac{1}{\varepsilon^{2m+2}} + F \le -1, \tag{A.26}$$

$$-\tfrac{1}{4}\big[\mu - \tfrac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\big]\tfrac{1}{\varepsilon^{3m+3}} + F \le -1, \tag{A.27}$$

where F are positive constants defined in (A.28).

For convenience, we can divide $\mathbb{R}_+^6 \setminus D_\varepsilon$ into 12 domains,

$$D_\varepsilon^1 = \big\{x(t) \in \mathbb{R}_+^6 : 0 < S(t) \le \varepsilon\big\}, D_\varepsilon^2 = \big\{x(t) \in \mathbb{R}_+^6 : 0 < I(t) \le \varepsilon\big\},$$

$$D_\varepsilon^3 = \big\{x(t) \in \mathbb{R}_+^6 : 0 < C(t) \le \varepsilon^2,\ I(t) \ge \varepsilon\big\}, D_\varepsilon^4 = \big\{x(t) \in \mathbb{R}_+^6 : 0 < C_s(t) \le \varepsilon^3,\ C(t) \ge \varepsilon^2\big\},$$

$$D_\varepsilon^5 = \big\{x(t) \in \mathbb{R}_+^6 : 0 < T(t) \le \varepsilon^2,\ I(t) \ge \varepsilon\big\}, D_\varepsilon^6 = \big\{x(t) \in \mathbb{R}_+^6 : 0 < A(t) \le \varepsilon^2,\ I(t) \ge \varepsilon\big\},$$

$$D_\varepsilon^7 = \Big\{x(t) \in \mathbb{R}_+^6 : S(t) \ge \frac{1}{\varepsilon}\Big\}, \quad D_\varepsilon^8 = \Big\{x(t) \in \mathbb{R}_+^6 : I(t) \ge \frac{1}{\varepsilon}\Big\},$$

$$D_\varepsilon^9 = \Big\{x(t) \in \mathbb{R}_+^6 : C(t) \ge \frac{1}{\varepsilon^2}\Big\}, \quad D_\varepsilon^{10} = \Big\{x(t) \in \mathbb{R}_+^6 : C_s(t) \ge \frac{1}{\varepsilon^3}\Big\},$$

$$D_\varepsilon^{11} = \Big\{x(t) \in \mathbb{R}_+^6 : T(t) \ge \frac{1}{\varepsilon^2}\Big\}, \quad D_\varepsilon^{12} = \Big\{x(t) \in \mathbb{R}_+^6 : A(t) \ge \frac{1}{\varepsilon^2}\Big\}.$$

Then $\mathbb{R}_+^6 \setminus D_\varepsilon = D_\varepsilon^1 \cup D_\varepsilon^2 \cup D_\varepsilon^3 \cup \ldots \cup D_\varepsilon^{12}$. Below we show that $LV(x) \le -1$ on $\mathbb{R}_+^6 \setminus D_\varepsilon$, which is equivalent to verifying it on the above 12 domains.

Case 1. If $x(t) \in D_\varepsilon^1$, given (A.2), (A.18), and (A.19), we have

$$\begin{aligned} LV(x(t)) &\le Mk_1 I(t) - \frac{\Pi}{S(t)} + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) + B + D \\ &\quad - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I(t) + C(t) + T(t) + A(t))^{m+1} \\ &\le -\frac{\Pi}{S} + F \le -\frac{\Pi}{\varepsilon} + F \\ &\le -1, \end{aligned}$$

where

$$\begin{aligned} F = \sup_{(S,I,C,C_s,T,A)\in\mathbb{R}_+^6} &\Big\{ Mk_1 I + k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A) + B + D \\ &- \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I + C + T + A)^{m+1}\Big\}. \end{aligned} \tag{A.28}$$

Case 2. If $x(t) \in D_\varepsilon^2$, given (A.2), (A.3), (A.18), and (A.20), we have

$$\begin{aligned} LV(x(t)) &\le -M\frac{\bar{I}}{\bar{\bar{S}}}\Lambda + Mk_1 I(t) + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) + B + D \\ &\quad - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I(t) + C(t) + T(t) + A(t))^{m+1} \\ &\le -M\frac{\bar{I}}{\bar{\bar{S}}}\Lambda + E + Mk_1\varepsilon \\ &\le -2 + 1 = -1. \end{aligned}$$

Case 3. If $x(t) \in D_\varepsilon^3$, from (A.2), (A.18), and (A.21), we have

$$\begin{aligned} LV(x(t)) &\le Mk_1 I(t) + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) - \sigma\frac{I(t)}{C(t)} + B + D \\ &\quad - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I(t) + C(t) + T(t) + A(t))^{m+1} \\ &\le -\sigma\frac{I(t)}{C(t)} + F \le -\frac{\sigma}{\varepsilon} + F \\ &\le -1. \end{aligned}$$

Case 4. If $x(t) \in D_\varepsilon^4$, given (A.2), (A.18), and (A.22), we have

$$\begin{aligned} LV(x(t)) &\le Mk_1 I + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) - \psi\frac{C(t)}{C_s(t)} + B + D \\ &\quad - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I(t) + C(t) + T(t) + A(t))^{m+1} \\ &\le -\psi\frac{C(t)}{C_s(t)} + F \le -\frac{\psi}{\varepsilon} + F \\ &\le -1. \end{aligned}$$

Case 5. If $x(t) \in D_\varepsilon^5$, then, from (A.2), (A.18), and (A.23), we have

$$\begin{aligned} LV(x(t)) &\le Mk_1 I(t) + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) - \gamma_1\frac{I(t)}{T(t)} + B + D \\ &\quad - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I(t) + C(t) + T(t) + A(t))^{m+1} \\ &\le \gamma_1\frac{I(t)}{T(t)} + F \le -\frac{\gamma_1}{\varepsilon} + F \\ &\le -1. \end{aligned}$$

Case 6. If $x(t)\in D_\varepsilon^6$, from (A.2), (A.18), and (A.24), we have

$$\begin{aligned}
LV(x(t)) &\le Mk_1I(t)+k(\beta_1I(t)+\beta_2C(t)+\beta_3T(t)+\beta_4A(t))-\rho_1\frac{I(t)}{A(t)}+B+D\\
&\quad-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](I(t)+C(t)+T(t)+A(t))^{m+1}\\
&\le\rho_1\frac{I(t)}{A(t)}+F\le-\frac{\rho_1}{\varepsilon}+F\\
&\le-1.
\end{aligned}$$

Case 7. If $x(t)\in D_\varepsilon^7$, from (A.2), (A.18), and (A.25), we have

$$\begin{aligned}
LV(x(t)) &\le Mk_1I(t)+k(\beta_1I(t)+\beta_2C(t)+\beta_3T(t)+\beta_4A(t))+B+D-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big]\\
&\quad\times(S(t))^{m+1}-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](I(t)+C(t)+T(t)+A(t))^{m+1}\\
&\le-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](S(t))^{m+1}+F\\
&\le-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big]\frac{1}{\varepsilon^{m+1}}+F\\
&\le-1.
\end{aligned}$$

Case 8. If $x(t)\in D_\varepsilon^8$, given (A.2), (A.18), and (A.25), we have

$$\begin{aligned}
LV(x(t)) &\le Mk_1I(t)+k(\beta_1I(t)+\beta_2C(t)+\beta_3T(t)+\beta_4A(t))+B+D-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big]\\
&\quad\times(I(t))^{m+1}-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](I(t)+C(t)+T(t)+A(t))^{m+1}\\
&\le-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](I(t))^{m+1}+F\\
&\le-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big]\frac{1}{\varepsilon^{m+1}}+F\\
&\le-1.
\end{aligned}$$

Case 9. If $x(t)\in D_\varepsilon^9$, from (A.2), (A.18), and (A.26), we have

$$\begin{aligned}
LV(x(t)) &\le Mk_1I(t)+k(\beta_1I(t)+\beta_2C(t)+\beta_3T(t)+\beta_4A(t))+B+D-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big]\\
&\quad\times(C(t))^{m+1}-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](I(t)+C(t)+T(t)+A(t))^{m+1}\\
&\le-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](C(t))^{m+1}+F\\
&\le-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big]\frac{1}{\varepsilon^{2m+2}}+F\\
&\le-1.
\end{aligned}$$

Case 10. If $x(t)\in D_\varepsilon^{10}$, from (A.2), (A.18), and (A.27), we have

$$\begin{aligned}
LV(x(t)) &\le Mk_1I(t)+k(\beta_1I(t)+\beta_2C(t)+\beta_3T(t)+\beta_4A(t))+B+D-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big]\\
&\quad\times(C_s(t))^{m+1}-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](I(t)+C(t)+T(t)+A(t))^{m+1}\\
&\le-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big](C_s(t))^{m+1}+F\\
&\le-\frac{1}{4}\Big[\mu-\frac{m}{2}(\sigma_1^2\vee\sigma_2^2\vee\sigma_3^2\vee\sigma_4^2\vee\sigma_5^2\vee\sigma_6^2)\Big]\frac{1}{\varepsilon^{3m+3}}+F\\
&\le-1.
\end{aligned}$$

Case 11. If $x(t) \in D_\varepsilon^{11}$, from (A.2), (A.18), and (A.26), we have

$$\begin{aligned} LV(x(t)) &\le Mk_1 I(t) + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) + B + D - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big] \\ &\quad \times (T(t))^{m+1} - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I(t) + C(t) + T(t) + A(t))^{m+1} \\ &\le -\frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](T(t))^{m+1} + F \\ &\le -\frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big]\frac{1}{\varepsilon^{2m+2}} + F \\ &\le -1. \end{aligned}$$

Case 12. If $x(t) \in D_\varepsilon^{12}$, from (A.2), (A.18), and (A.26), we have

$$\begin{aligned} LV(x(t)) &\le Mk_1 I(t) + k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t)) + B + D - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big] \\ &\quad \times (A(t))^{m+1} - \frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](I(t) + C(t) + T(t) + A(t))^{m+1} \\ &\le -\frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big](A(t))^{m+1} + F \\ &\le -\frac{1}{4}\Big[\mu - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2)\Big]\frac{1}{\varepsilon^{2m+2}} + F \\ &\le -1. \end{aligned}$$

Given the above, one can see that for sufficiently small ε,

$$LV(x) \le -1, \text{ for any } x \in \mathbb{R}_+^6 \setminus D_\varepsilon.$$

Condition (II) in Lemma 2.1 is satisfied. According to Lemma 2.1, we can know that system (2.2) has an ergodic stationary distribution. This completes the proof.

Appendix B. (Proof of Theorem 5.1)

Since for any initial value $x_0 \in \mathbb{R}_+^6$, the solution of system (2.2) is positive, we obtain

$$dS \le (\Pi - \mu S)dt + \sigma_1 S dB_1(t).$$

Then we consider the following one-dimensional stochastic differential equation:

$$dX = (\Pi - \mu X)dt + \sigma_1 X dB_1(t). \tag{B.1}$$

Eq. (B.1) has a stationary solution with density

$$\pi(x) = D^{-1}x^{-2}x^{-\frac{2\mu}{\sigma_1^2}} \exp\left(-\frac{2\Pi}{\sigma_1^2 x}\right),$$

where $D = \left(\frac{2\Pi}{\sigma_1^2}\right)^{-\frac{2\mu+\sigma_1^2}{\sigma_1^2}} \cdot \Gamma\left(\frac{2\mu+\sigma_1^2}{\sigma_1^2}\right)$. By the ergodic theorem, it follows that

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t X(s)ds = \int_0^\infty x\pi(x)dx, \qquad a.s.,$$

and from Theorem 4.1 of [21], we obtain

$$\int_0^\infty x\pi(x)dx = EX = \frac{\Pi}{\mu}.$$

Let $X(t)$ be the solution of (B.1), with initial value $X(0) = S(0) > 0$. Then, applying the comparison theorem of one-dimensional stochastic differential equations [19], we have $S(t) \le X(t)$ for any $t \ge 0$ a.s. So, we have

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t S(s)ds \le \lim_{t\to\infty} \frac{1}{t}\int_0^t X(s)ds = \frac{\Pi}{\mu}, \qquad a.s. \tag{B.2}$$

Define $P(x) = I + C + C_s + T + A$. Then applying the Itô formula to $\ln P(x)$ gives us

$$\begin{aligned} d\ln P(x(t)) = &\bigg\{\frac{1}{P(x(t))}\Big[k(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t))S - \mu P(x(t)) - (\gamma_3 + \delta_1)A(t)\Big] \\ &- \frac{1}{2P(x(t))^2}\big(\sigma_2^2(I(t))^2 + \sigma_3^2(C(t))^2 + \sigma_4^2(C_s(t))^2 + \sigma_5^2(T(t))^2 + \sigma_6^2(A(t))^2\big)\bigg\}dt \\ &+ \frac{1}{P}\Big[\sigma_2 I(t)dB_2(t) + \sigma_3 C(t)dB_3(t) + \sigma_4 C_s(t)dB_4(t) + \sigma_5 T(t)dB_5(t) + \sigma_6 A(t)dB_6(t)\Big]. \end{aligned} \tag{B.3}$$

Considering that

$$\frac{(\beta_1 I(t) + \beta_2 C(t) + \beta_3 T(t) + \beta_4 A(t))}{P(x(t))} \le (\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4),$$

$$-\frac{(\gamma_3 + \delta_1)A(t)}{P(x(t))} \le -\frac{(\gamma_3 + \delta_1)(A(t))^2}{P(x(t))^2},$$

$$\frac{I(t)}{P(x(t))} \le 1, \quad \frac{C(t)}{P(x(t))} \le 1, \quad \frac{C_s(t)}{P(x(t))} \le 1, \quad \frac{T(t)}{P(x(t))} \le 1, \quad \frac{A(t)}{P(x(t))} \le 1,$$

we have

$$\begin{aligned} d\ln P(x(t)) \le &\bigg\{k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)S(t) - \mu - \frac{1}{P(x(t))^2}\Big[(\gamma_3 + \delta_1)(A(t))^2 + \frac{1}{2}\sigma_6^2(A(t))^2\Big] \\ &- \frac{1}{2P(x(t))^2}\big(\sigma_2^2(I(t))^2 + \sigma_3^2(C(t))^2 + \sigma_4^2(C_s(t))^2 + \sigma_5^2(T(t))^2\big)\bigg\}dt \\ &+ \frac{1}{P(x(t))}\Big(\sigma_2 I(t)dB_2(t) + \sigma_3 C(t)dB_3(t) + \sigma_4 C_s(t)dB_4(t) + \sigma_5 T(t)dB_5(t) + \sigma_6 A(t)dB_6(t)\Big) \\ \le &\bigg\{k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)S(t) - \mu - \frac{1}{P(x(t))^2}\Big[(\gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2) \wedge \frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\sigma_3^2 \wedge \frac{1}{2}\sigma_4^2 \wedge \frac{1}{2}\sigma_5^2\Big]((I(t))^2 \\ &+ (C(t))^2 + (C_s(t))^2 + (T(t))^2 + (A(t))^2)\bigg\}dt \\ &+ \frac{1}{P(x(t))}\Big[\sigma_2 I(t)dB_2(t) + \sigma_3 C(t)dB_3(t) + \sigma_4 C_s(t)dB_4(t) + \sigma_5 T(t)dB_5(t) + \sigma_6 A(t)dB_6(t)\Big] \\ \le &\bigg\{k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)S(t) - \mu - \frac{1}{5}\Big[(\gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2) \wedge \frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\sigma_3^2 \wedge \frac{1}{2}\sigma_4^2 \wedge \frac{1}{2}\sigma_5^2\Big]\bigg\}dt \\ &+ \sigma_2 dB_2(t) + \sigma_3 dB_3(t) + \sigma_4 dB_4(t) + \sigma_5 dB_5(t) + \sigma_6 dB_6(t). \end{aligned} \tag{B.4}$$

Integrating from 0 to t and dividing by t on both sides of (B.4) leads to

$$\begin{aligned} \frac{\ln P(x(t)) - \ln P(x(0))}{t} \le &\, k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)\frac{1}{t}\int_0^t S(r)dr - \mu - \frac{1}{5}\Big[(\gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2) \wedge \frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\sigma_3^2 \wedge \frac{1}{2}\sigma_4^2 \wedge \frac{1}{2}\sigma_5^2\Big] \\ &+ \frac{\sigma_2}{t}B_2(t) + \frac{\sigma_3}{t}B_3(t) + \frac{\sigma_4}{t}B_4(t) + \frac{\sigma_5}{t}B_5(t) + \frac{\sigma_6}{t}B_6(t). \end{aligned} \tag{B.5}$$

Applying the strong law of large numbers [44], we have

$$\lim_{t\to\infty}\frac{B_i(t)}{t} = 0, \quad a.s. \quad (i = 2, 3, 4, 5, 6). \tag{B.6}$$

Taking the superior limit on the both sides of (B.5), and combining with (B.6) and $R_0' < 1$, we obtain

$$\begin{aligned} \limsup_{t\to\infty}\frac{\ln\ P(x(t))}{t} &\le k(\beta_1 \vee \beta_2 \vee \beta_3 \vee \beta_4)\frac{\Pi}{\mu} - \mu - \frac{1}{5}\Big[(\gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2) \wedge \frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\sigma_3^2 \wedge \frac{1}{2}\sigma_4^2 \wedge \frac{1}{2}\sigma_5^2\Big] \\ &= (R_0' - 1)\Big\{\mu + \frac{1}{5}\Big[(\gamma_3 + \delta_1 + \frac{1}{2}\sigma_6^2) \wedge \frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\sigma_3^2 \wedge \frac{1}{2}\sigma_4^2 \wedge \frac{1}{2}\sigma_5^2\Big]\Big\} \\ &< 0, \quad a.s. \end{aligned}$$

That is,

$$\lim_{t\to\infty} I(t) = 0,\ \lim_{t\to\infty} C(t) = 0,\ \lim_{t\to\infty} C_s(t) = 0,\ \lim_{t\to\infty} T(t) = 0,\ \lim_{t\to\infty} A(t) = 0 \ \ a.s. \tag{B.7}$$

Moreover,

$$\frac{S(t)-S(0)}{t}+\frac{I(t)-I(0)}{t}+\frac{C(t)-C(0)}{t}+\frac{C_s(t)-C_s(0)}{t}+\frac{T(t)-T(0)}{t}+\frac{A(t)-A(0)}{t}$$
$$=\Pi-\mu\Big[\langle S(t)\rangle+\langle I(t)\rangle+\langle C(t)\rangle+\langle C_s(t)\rangle+\langle T(t)\rangle\Big]-(\mu+\gamma_3+\delta_1)\langle A(t)\rangle+\frac{\sigma_1}{t}\int_0^t S(s)dB_1(s)$$
$$+\frac{\sigma_2}{t}\int_0^t I(s)dB_2(s)+\frac{\sigma_3}{t}\int_0^t C(s)dB_3(s)+\frac{\sigma_4}{t}\int_0^t C_s(s)dB_4(s)+\frac{\sigma_5}{t}\int_0^t T(s)dB_5(s)+\frac{\sigma_6}{t}\int_0^t A(s)dB_6(s).$$

Then, combining Lemma 2.2, 2.3, and (B.7) implies that

$$\lim_{t\to\infty}\langle S(t)\rangle=\frac{\Pi}{\mu},\quad a.s.$$

This completes the proof.

Appendix C. (Construction of stochastic model (2.2))

First, we consider a discrete time Markov chain. For a fixed time increment $\triangle t > 0$, we define a process $X^{(\Delta t)}(t) = (S^{(\Delta t)}(t), I^{(\Delta t)}(t), C^{(\Delta t)}(t), C_s^{(\Delta t)}(t), T^{(\Delta t)}(t), A^{(\Delta t)}(t))^T$, $t = 0, \Delta t, 2\triangle t, 3\triangle t, \ldots,$ and S, I, C, C_s, T, A have the meanings in model (2.1). The initial value $X^{(\Delta t)}(0) = \nu \in \mathbb{R}_+^6$ is a constant. Six sequences of random variables are denoted by $\left\{P_i^{(\Delta t)}(k)\right\}_{k=1}^{\infty}$, $i = 1, 2, 3, 4, 5, 6$. We assume that these variables are jointly independent and are identically distributed within each sequence such that

$$\mathbb{E}P_i^{(\Delta t)}(k)=0,\quad \mathbb{E}[P_i^{(\Delta t)}(k)]^2=\sigma_i^2\triangle t,\quad \mathbb{E}[P_i^{(\Delta t)}(k)]^4=o(\Delta t) \tag{C.1}$$

for $i = 1, 2, 3, 4, 5, 6$ and $k = 1, 2, \ldots,$ where $\sigma_i^2 > 0 (i = 1, 2, 3, 4, 5, 6)$ are the intensities of the white noise.

The $P_i^{(\Delta t)}(k)$ represent the effects of random influences on S, I, C, C_s, T, and A during the period $[k\triangle t, (k+1)\triangle t)$. We also assume that $S^{(\Delta t)}, I^{(\Delta t)}, C^{(\Delta t)}, C_s^{(\Delta t)}, T^{(\Delta t)}$ and $A^{(\Delta t)}$ change within that time period according to the deterministic model (2.1), and the random amounts are $P_1^{(\Delta t)}(k)S^{(\Delta t)}(k\triangle t)$, $P_2^{(\Delta t)}(k)I^{(\Delta t)}(k\triangle t)$, $P_3^{(\Delta t)}(k)C^{(\Delta t)}(k\triangle t)$, $P_4^{(\Delta t)}(k)C_s^{(\Delta t)}(k\triangle t)$, $P_5^{(\Delta t)}(k)T^{(\Delta t)}(k\triangle t)$, and $P_6^{(\Delta t)}(k)A^{(\Delta t)}(k\triangle t)$, respectively. Specifically, for $k = 0, 1, \ldots,$ we obtain

$$S^{(\Delta t)}((k+1)\triangle t)=S^{(\Delta t)}(k\triangle t)+P_1^{(\Delta t)}(k)S^{(\Delta t)}(k\triangle t)+\Big\{\Pi-k[\beta_1 I^{(\Delta t)}(k\triangle t)+\beta_2 C^{(\Delta t)}(k\triangle t)$$
$$+\beta_3 T^{(\Delta t)}(k\triangle t)+\beta_4 A^{(\Delta t)}(k\triangle t)]S^{(\Delta t)}(k\triangle t)-\mu S^{(\Delta t)}(k\triangle t)\Big\}\triangle t,$$

$$I^{(\Delta t)}((k+1)\triangle t)=I^{(\Delta t)}(k\triangle t)+P_2^{(\Delta t)}(k)I^{(\Delta t)}(k\triangle t)+\Big\{k[\beta_1 I^{(\Delta t)}(k\triangle t)+\beta_2 C^{(\Delta t)}(k\triangle t)$$
$$+\beta_3 T^{(\Delta t)}(k\triangle t)+\beta_4 A^{(\Delta t)}(k\triangle t)]S^{(\Delta t)}(k\triangle t)-(\mu+\sigma+\rho_1+\gamma_1)I^{(\Delta t)}(k\triangle t)\Big\}\triangle t,$$

$$C^{(\Delta t)}((k+1)\triangle t)=C^{(\Delta t)}(k\triangle t)+P_3^{(\Delta t)}(k)C^{(\Delta t)}(k\triangle t)+\Big\{\sigma I^{(\Delta t)}(k\triangle t)-(\mu+\rho_2+\psi)C^{(\Delta t)}(k\triangle t)\Big\}\triangle t,$$

$$C_s^{(\Delta t)}((k+1)\triangle t)=C_s^{(\Delta t)}(k\triangle t)+P_4^{(\Delta t)}(k)C_s^{(\Delta t)}(k\triangle t)+\Big\{\psi C^{(\Delta t)}(k\triangle t)-(\mu+\rho_3+\gamma_2)C_s^{(\Delta t)}(k\triangle t)\Big\}\triangle t,$$

$$T^{(\Delta t)}((k+1)\triangle t)=T^{(\Delta t)}(k\triangle t)+P_5^{(\Delta t)}(k)T^{(\Delta t)}(k\triangle t)+\Big\{\gamma_1 I^{(\Delta t)}(k\triangle t)+\gamma_2 C_S^{(\Delta t)}(k\triangle t)-(\mu+\rho_4)T^{(\Delta t)}(k\triangle t)\Big\}\triangle t,$$

and

$$A^{(\Delta t)}((k+1)\triangle t)=A^{(\Delta t)}(k\triangle t)+P_6^{(\Delta t)}(k)A^{(\Delta t)}(k\triangle t)+\Big\{\rho_1 I^{(\Delta t)}(k\triangle t)+\rho_2 C^{(\Delta t)}(k\triangle t)$$
$$+\rho_3 C_s^{(\Delta t)}(k\triangle t)+\rho_4 T^{(\Delta t)}(k\triangle t)-(\mu+\gamma_3+\delta_1)A^{(\Delta t)}(k\triangle t)\Big\}\triangle t.$$

Next, we prove that $X^{(\Delta t)}(t)$ converges to a diffusion process as $\triangle t \to \infty$. We only need to determine the drift coefficient and diffusion coefficients of the diffusion process. Let $\mathbb{P}^{(\Delta t)}(x, dz)$ denote the transition probabilities of the homogeneous Markov chain $\{X^{(\Delta t)}(k\triangle t)\}_{k=0}^{\infty}$, i.e.,

$$\mathbb{P}^{(\Delta t)}(x,Z)=Prob\{X^{(\Delta t)}((k+1)\triangle t)\in Z:X^{(\Delta t)}(k\triangle t)=x\}$$

for all $x = (S, I, C, C_s, T, A)^T \in \mathbb{R}_+^6$ and all Borel sets $Z \subset \mathbb{R}_+^6$. Let

$$F^{(\triangle t)}(x) = (f_1^{(\triangle t)}(x), f_2^{(\triangle t)}(x), f_3^{(\triangle t)}(x), f_4^{(\triangle t)}(x), f_5^{(\triangle t)}(x), f_6^{(\triangle t)}(x))^T$$

and

$$G^{(\triangle t)}(x) = (g_{ij}^{(\triangle t)}(x))_{6\times 6}$$

denote the drift coefficient and diffusion coefficient, respectively. From Eq. (C.1), we have

$$\begin{aligned} f_1^{(\triangle t)}(x) &= \frac{1}{\triangle t}\int (z_1 - S)\mathbb{P}^{(\triangle t)}(x, dz) = \Pi - k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - \mu S + \frac{S}{\triangle t}\mathbb{E}P_1^{(\triangle t)}(0) \\ &= \Pi - k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - \mu S, \end{aligned} \tag{C.2}$$

$$\begin{aligned} f_2^{(\triangle t)}(x) &= \frac{1}{\triangle t}\int (z_2 - I)\mathbb{P}^{(\triangle t)}(x, dz) = k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - (\mu + \sigma + \rho_1 + \gamma_1)I + \frac{S}{\triangle t}\mathbb{E}P_2^{(\triangle t)}(0) \\ &= k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - (\mu + \sigma + \rho_1 + \gamma_1)I, \end{aligned} \tag{C.3}$$

$$\begin{aligned} f_3^{(\triangle t)}(x) &= \frac{1}{\triangle t}\int (z_3 - I)\mathbb{P}^{(\triangle t)}(x, dz) = \sigma I - (\mu + \rho_2 + \psi)C + \frac{S}{\triangle t}\mathbb{E}P_3^{(\triangle t)}(0) \\ &= \sigma I - (\mu + \rho_2 + \psi)C, \end{aligned} \tag{C.4}$$

$$\begin{aligned} f_4^{(\triangle t)}(x) &= \frac{1}{\triangle t}\int (z_4 - I)\mathbb{P}^{(\triangle t)}(x, dz) = \psi C - (\mu + \rho_3 + \gamma_2)C_s + \frac{S}{\triangle t}\mathbb{E}P_4^{(\triangle t)}(0) \\ &= \psi C - (\mu + \rho_3 + \gamma_2)C_s \end{aligned} \tag{C.5}$$

$$\begin{aligned} f_5^{(\triangle t)}(x) &= \frac{1}{\triangle t}\int (z_5 - I)\mathbb{P}^{(\triangle t)}(x, dz) = \gamma_1 I + \gamma_2 C_S - (\mu + \rho_4)T + \frac{S}{\triangle t}\mathbb{E}P_5^{(\triangle t)}(0) \\ &= \gamma_1 I + \gamma_2 C_S - (\mu + \rho_4)T, \end{aligned} \tag{C.6}$$

$$\begin{aligned} f_6^{(\triangle t)}(x) &= \frac{1}{\triangle t}\int (z_6 - I)\mathbb{P}^{(\triangle t)}(x, dz) = \rho_1 I + \rho_2 C + \rho_3 C_s + \rho_4 T - (\mu + \gamma_3 + \delta_1)A + \frac{S}{\triangle t}\mathbb{E}P_6^{(\triangle t)}(0) \\ &= \rho_1 I + \rho_2 C + \rho_3 C_s + \rho_4 T - (\mu + \gamma_3 + \delta_1)A. \end{aligned} \tag{C.7}$$

$$\begin{aligned} g_{11}^{(\triangle t)}(x) &= \frac{1}{\triangle t}\int (z_1 - S)^2\mathbb{P}^{(\triangle t)}(x, dz) = \frac{1}{\triangle t}\mathbb{E}\Big[\Big(\Pi - k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - \mu S\Big)\triangle t + P_1^{(\triangle t)}(0)S\Big]^2 \\ &= \Big(\Pi - k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - \mu S\Big)^2 \triangle t + \frac{S^2}{\triangle t}\mathbb{E}[P_1^{(\triangle t)}(0)]^2 \\ &\quad + 2\Big(\Pi - k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - \mu S\Big)S\mathbb{E}[P_1^{(\triangle t)}(0)] \\ &= \Big(\Pi - k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - \mu S\Big)^2 \triangle t + \sigma_1^2 S^2. \end{aligned}$$

The calculation processes of $g_{ii}^{(\triangle t)}(x) (i = 2, 3, 4, 5, 6)$ are similar to that of $g_{11}^{(\triangle t)}(x)$, hence we omit this.

$$g_{22}^{(\triangle t)}(x) = \Big(k(\beta_1 I + \beta_2 C + \beta_3 T + \beta_4 A)S - (\mu + \sigma + \rho_1 + \gamma_1)I\Big)^2 \triangle t + \sigma_2^2 I^2,$$

$$g_{33}^{(\triangle t)}(x) = \Big(\sigma I - (\mu + \rho_2 + \psi)C\Big)^2 \triangle t + \sigma_3^2 C^2,$$

$$g_{44}^{(\triangle t)}(x) = \Big(\psi C - (\mu + \rho_3 + \gamma_2)C_s\Big)^2 \triangle t + \sigma_4^2 C_s^2,$$

$$g_{55}^{(\triangle t)}(x) = \Big(\gamma_1 I + \gamma_2 C_S - (\mu + \rho_4)T\Big)^2 \triangle t + \sigma_5^2 T^2,$$

$$g_{66}^{(\triangle t)}(x) = \Big(\rho_1 I + \rho_2 C + \rho_3 C_s + \rho_4 T - (\mu + \gamma_3 + \delta_1)A\Big)^2 \triangle t + \sigma_6^2 A^2.$$

Therefore,

$$\begin{aligned}\lim_{\triangle t\to 0^+}\sup_{\|x\|\le\bar K}|g_{11}^{(\triangle t)}(x)-\sigma_1^2S^2| &= \lim_{\triangle t\to 0^+}\sup_{\|x\|\le\bar K}|g_{22}^{(\triangle t)}(x)-\sigma_2^2I^2| = \lim_{\triangle t\to 0^+}\sup_{\|x\|\le\bar K}|g_{33}^{(\triangle t)}(x)-\sigma_3^2C^2|\\ &= \lim_{\triangle t\to 0^+}\sup_{\|x\|\le\bar K}|g_{44}^{(\triangle t)}(x)-\sigma_4^2C_s^2| = \lim_{\triangle t\to 0^+}\sup_{\|x\|\le\bar K}|g_{55}^{(\triangle t)}(x)-\sigma_5^2T^2|\\ &= \lim_{\triangle t\to 0^+}\sup_{\|x\|\le\bar K}|g_{66}^{(\triangle t)}(x)-\sigma_6^2A^2|\\ &= 0 \end{aligned} \tag{C.8}$$

for $\bar K \in (0,\infty)$. We can similarly prove that

$$\lim_{\triangle t\to 0^+}\sup_{\|x\|\le\bar K}|g_{ij}^{(\triangle t)}(x)| = 0 \tag{C.9}$$

for $i, j = 1, 2, 3, 4, 5$ and $i \neq j$. In addition, by Eq. (C.1), we can obtain that for all $\bar K \in (0,\infty)$,

$$\lim_{\triangle t\to 0^+}\sup_{\|x\|\le\bar K}\frac{1}{\Delta t}\int \| z-x\|^3\, \mathbb{P}^{(\triangle t)}(x,dz) = 0. \tag{C.10}$$

From Imhof and Walcher [31], the definition of $X^{(\triangle t)}(t)$ can be extended to all $t\ge 0$ by setting $X^{(\triangle t)}(t) = X^{(\triangle t)}(k\triangle t)$ for $t \in [k\triangle t, (k+1)\triangle t)$. By Theorem 7.1 and Lemma 8.2 in [47], and equation (C.2)-(C.10), we can conclude that as $\triangle t\to\infty$, $X^{(\triangle t)}(t)$ converges weakly to the solution of stochastic differential equation (2.2) with initial condition $X^{(\triangle t)}(0) = \nu \in \mathbb{R}_+^6$, provided its unique solution exists.

Appendix D. (Diagrams of Section 6)

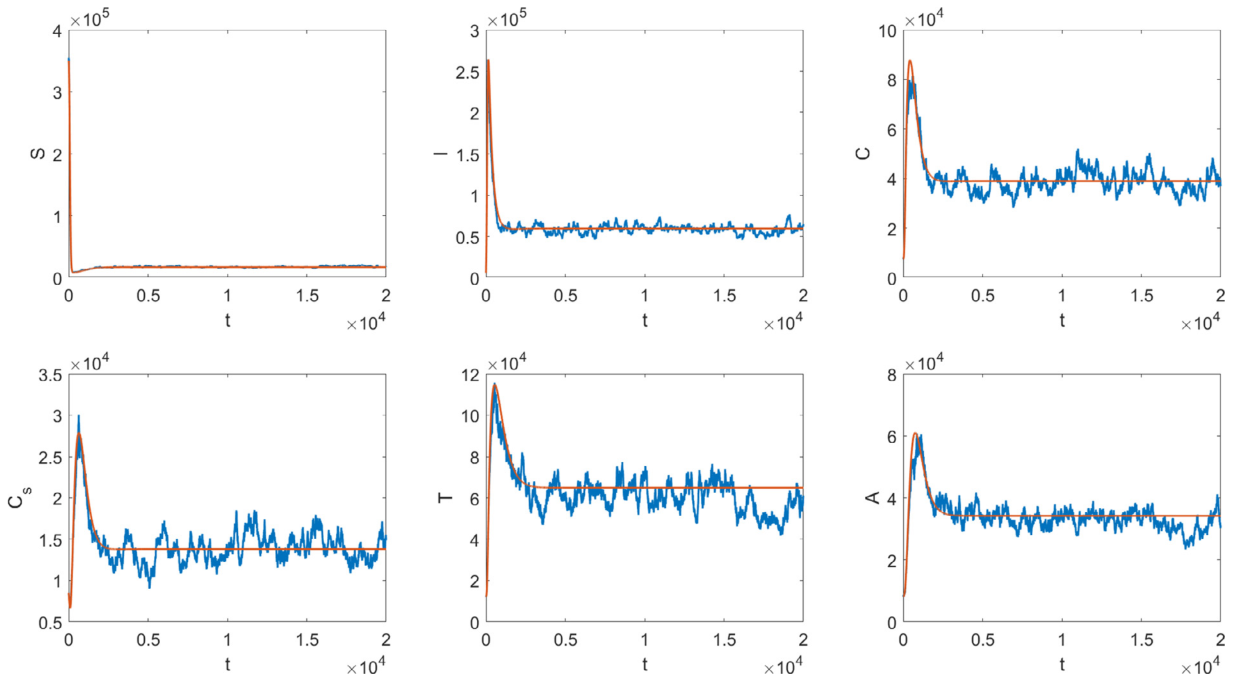

Fig. 1.1. The diagrams track the populations size of $S(t)$, $I(t)$, $C(t)$, $C_s(t)$, $T(t)$, and $A(t)$ over time. The blue line represents the result of stochastic model (2.2), while the red line represents the result of deterministic model (2.1). With parameters: $x_0 = (350,000, 6000, 7500, 8500, 12,000, 7500)^T$, $\Pi = 25000$, $k = 1.2$, $\beta_1 = 0.8\times 10^{-5}$, $\beta_2 = 0.6\times 10^{-5}$, $\beta_3 = 0.4\times 10^{-5}$, $\beta_4 = 0.7\times 10^{-5}$, $\mu = 0.015$, $\sigma = 0.2$, $\rho_1 = 0.001$, $\gamma_1 = 0.2$, $\rho_2 = 0.09$, $\psi = 0.2$, $\rho_3 = 0.4$, $\gamma_2 = 0.15$, $\rho_4 = 0.2$, $\gamma_3 = 0.3$, $\delta_1 = 0.33$, $\sigma_i = 0.0775$ $(i = 1, 2, 3, 4, 5, 6)$.

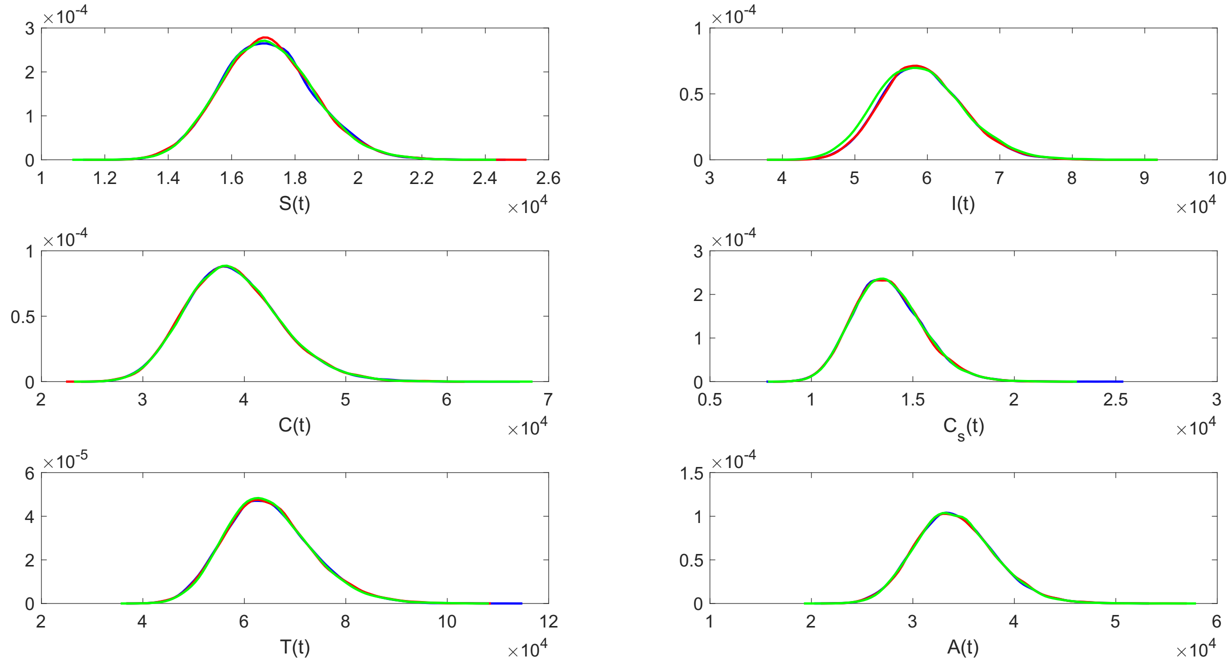

Fig. 1.2. The diagrams represent the density curves of different types of individuals in stochastic model (2.2). The blue, red, and green lines respectively represent the density curves of 20,000, 40,000, and 60,000 points-in-time, respectively. All of the parameter values are the same as in Fig 1.1.

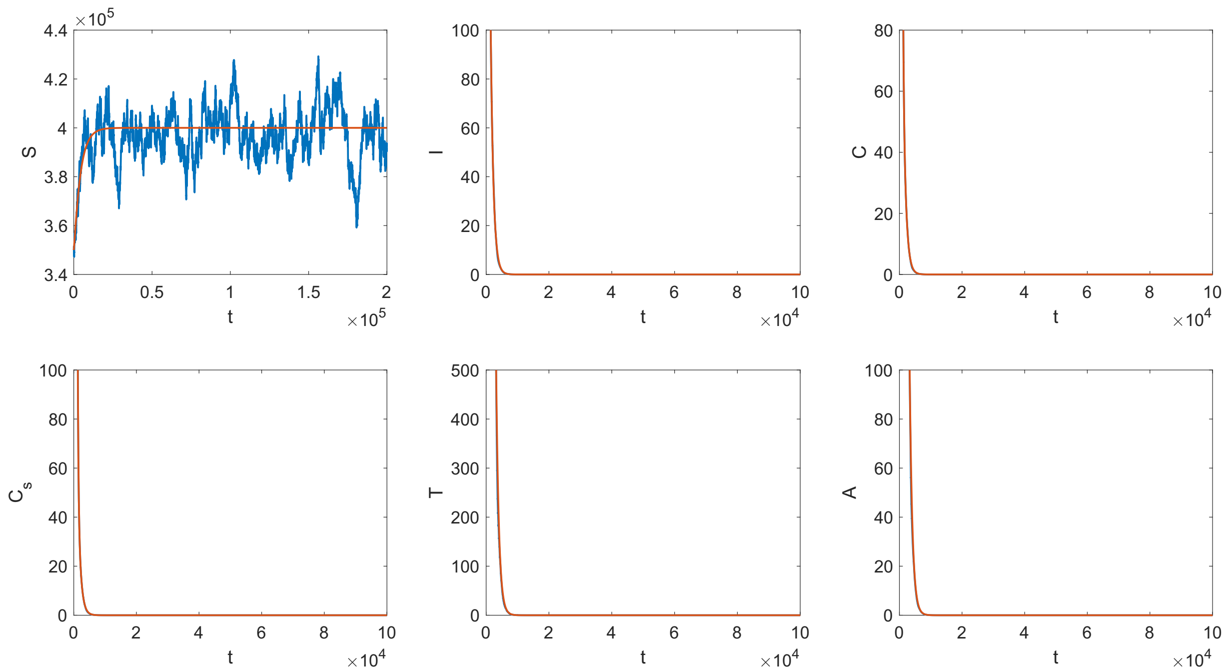

Fig. 1.3. The diagrams track the extinction of $I(t)$, $C(t)$, $C_s(t)$, $T(t)$, and $A(t)$. The blue line represents the result of stochastic model (2.2), while the red line represents the result of deterministic model (2.1). With parameters: $x_0 = (350000, 6000, 7500, 8500, 12000, 8000, 7500)^T$, $\Pi = 10000$, $k = 0.8$, $\beta_1 = 0.007 \times 10^{-5}$, $\beta_2 = 0.005 \times 10^{-5}$, $\beta_3 = 0.004 \times 10^{-5}$, $\beta_4 = 0.006 \times 10^{-5}$, $\mu = 0.025$, $\sigma = 0.2$, $\rho_1 = 0.1$, $\gamma_1 = 0.3$, $\rho_2 = 0.09$, $\psi = 0.4$, $\rho_3 = 0.4$, $\gamma_2 = 0.15$, $\rho_4 = 0.1$, $\gamma_3 = 0.3$, $\delta_1 = 0.33$, $\sigma_1 = 0.0063$, $\sigma_i = 0.0316$ $(i = 2, 3, 4, 5, 6)$.

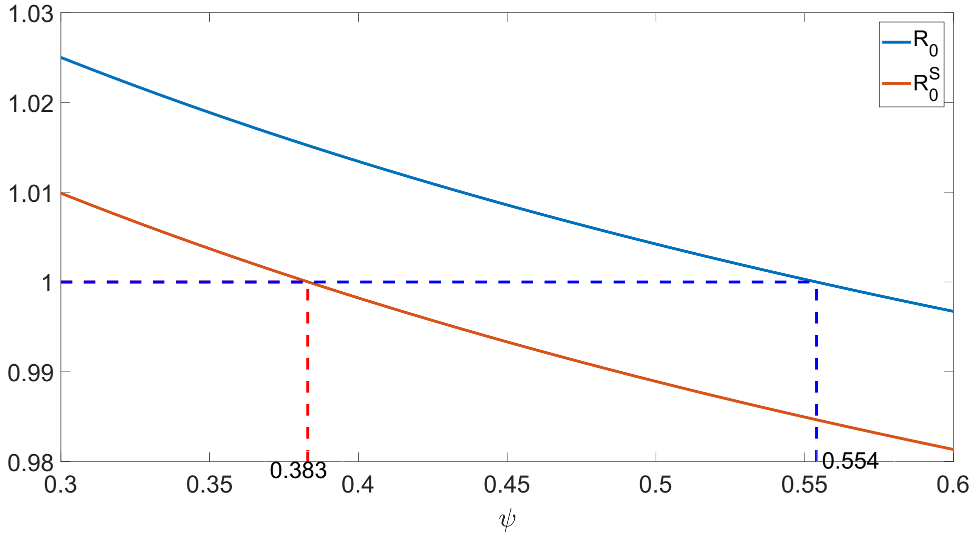

Fig. 2.1. The variation trends of R_0 and R_0^S with different ψ. With parameters: $\Pi = 10000$, $k = 0.8$, $\beta_1 = 0.09 \times 10^{-5}$, $\beta_2 = 0.06 \times 10^{-5}$, $\beta_3 = 0.035 \times 10^{-5}$, $\beta_4 = 0.08 \times 10^{-5}$, $\mu = 0.025$, $\sigma = 0.3$, $\rho_1 = 0.1$, $\gamma_1 = 0.2$, $\rho_2 = 0.45$, $\rho_3 = 0.4$, $\gamma_2 = 0.15$, $\rho_4 = 0.45$, $\gamma_3 = 0.3$, $\delta_1 = 0.33$, $\sigma_1 = 0.02$, $\sigma_i = 0.1$ $(i = 2, 3, 6)$, $\sigma_4 = 0.2$, $\sigma_5 = 0.01$, $\sigma_7 = 0.06$.

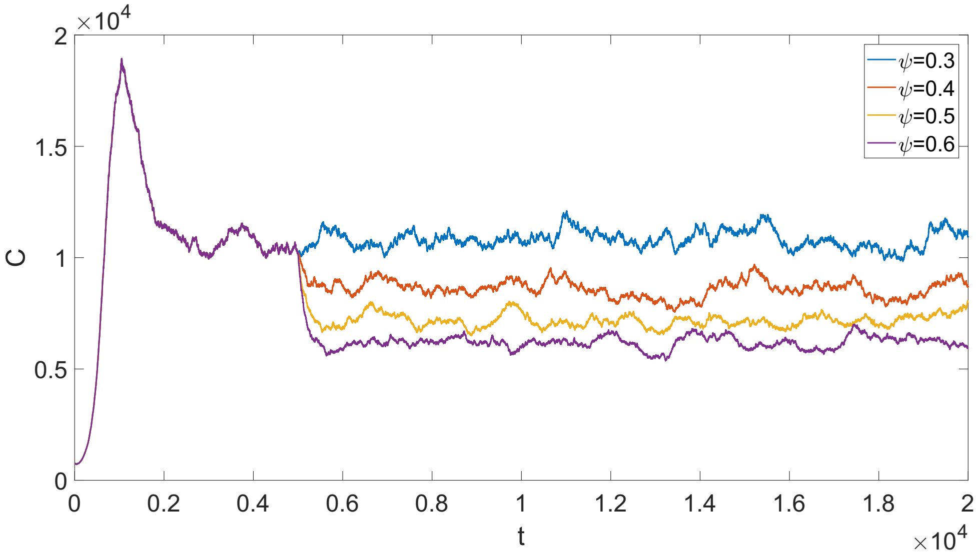

Fig. 2.2. The diagram tracks variation trends of carriers $C(t)$ with different ψ. With parameters: $x_0 = (75000, 1000, 750, 350, 1200, 800, 750)^T$, $\Pi = 10000$, $k = 1$, $\beta_1 = 0.8 \times 10^{-5}$, $\beta_2 = 0.6 \times 10^{-5}$, $\beta_3 = 0.4 \times 10^{-5}$, $\beta_4 = 0.7 \times 10^{-5}$, $\mu = 0.022$, $\sigma = 0.2$, $\rho_1 = 0.003$, $\gamma_1 = 0.2$, $\rho_2 = 0.09$, $\rho_3 = 0.4$, $\gamma_2 = 0.15$, $\rho_4 = 0.2$, $\gamma_3 = 0.3$, $\delta_1 = 0.33$, $\sigma_i = 0.0006$ $(i = 1, 3, 5, 6)$, $\sigma_2 = 0.0007$, $\sigma_i = 0.0008$ $(i = 4, 7)$.

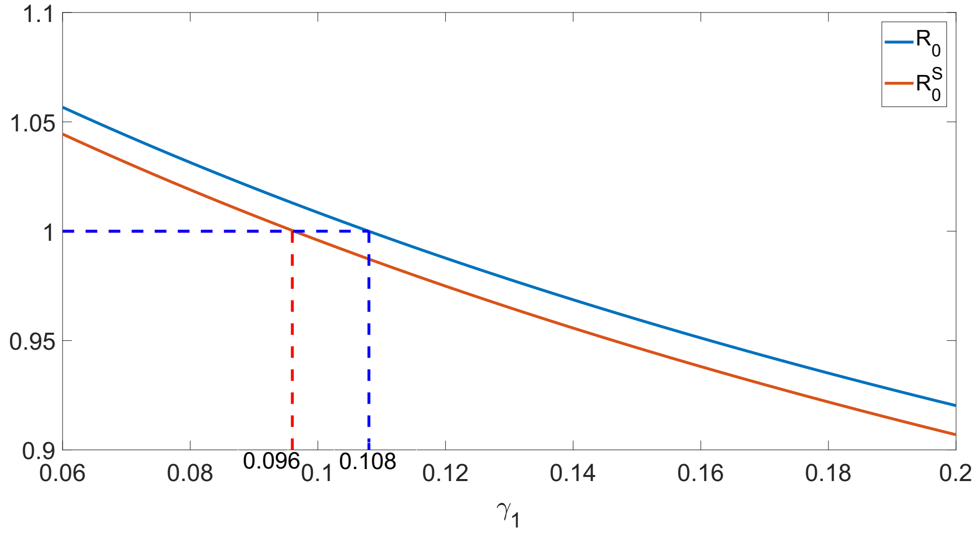

Fig. 3.1. The variation trends of R_0 and R_0^S with different γ_1. With parameters: $\Pi = 10000$, $k = 0.8$, $\beta_1 = 0.08 \times 10^{-5}$, $\beta_2 = 0.04 \times 10^{-5}$, $\beta_3 = 0.02 \times 10^{-5}$, $\beta_4 = 0.06 \times 10^{-5}$, $\mu = 0.025$, $\sigma = 0.2$, $\rho_1 = 0.1$, $\gamma_2 = 0.15$, $\rho_2 = 0.45$, $\rho_3 = 0.4$, $\rho_4 = 0.2$, $\gamma_3 = 0.3$, $\delta_1 = 0.33$, $\sigma_i = 0.02$ $(i = 1, 2)$, $\sigma_i = 0.1$ $(i = 3, 5)$, $\sigma_4 = 0.06$, $\sigma_6 = 0.02$, $\sigma_7 = 0.04$.

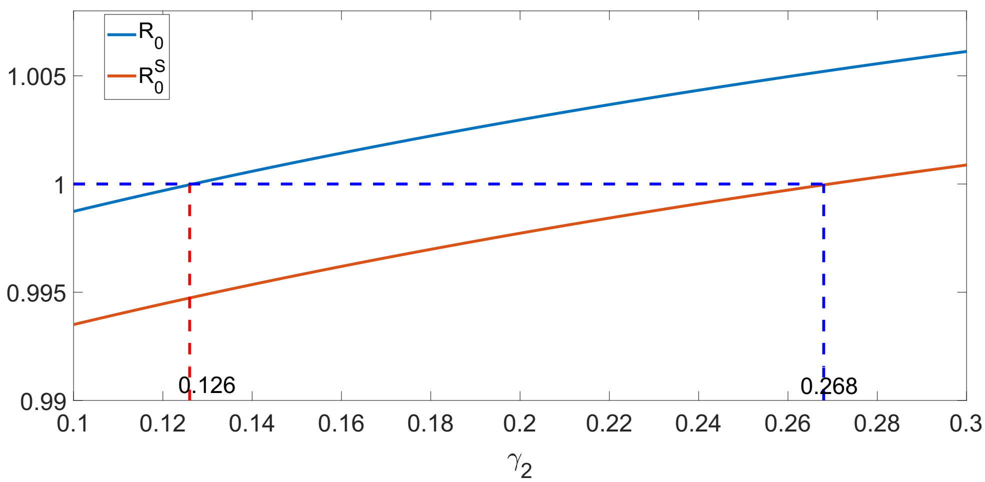

Fig. 3.2. The variation trends of R_0 and R_0^S with different γ_2. With parameters: $\Pi = 10000$, $k = 0.25$, $\beta_1 = 0.6 \times 10^{-5}$, $\beta_2 = 0.32 \times 10^{-5}$, $\beta_3 = 0.2 \times 10^{-5}$, $\beta_4 = 0.35 \times 10^{-5}$, $\mu = 0.04$, $\sigma = 0.2$, $\rho_1 = 0.25$, $\gamma_1 = 0.15$, $\rho_2 = 0.5$, $\rho_3 = 0.45$, $\rho_4 = 0.5$, $\gamma_3 = 0.3$, $\delta_1 = 0.33$, $\sigma_i = 0.02$ $(i = 1, 6)$, $\sigma_2 = 0.08$, $\sigma_i = 0.01$ $(i = 3, 5)$, $\sigma_4 = 0.06$, $\sigma_7 = 0.04$.

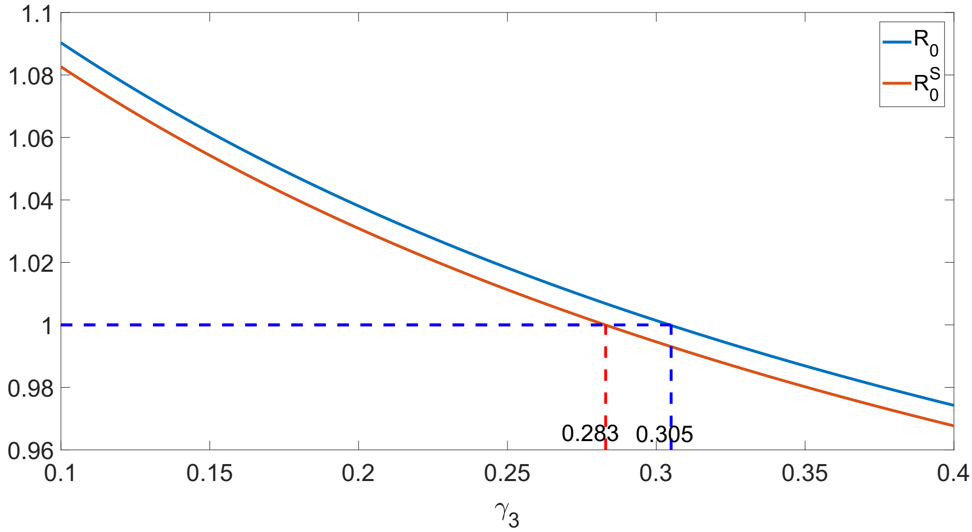

Fig. 3.3. The variation trends of R_0 and R_0^S with different γ_3. With parameters: $\Pi = 10000$, $k = 0.25$, $\beta_1 = 0.7 \times 10^{-5}$, $\beta_2 = 0.22 \times 10^{-5}$, $\beta_3 = 0.2 \times 10^{-5}$, $\beta_4 = 0.25 \times 10^{-5}$, $\mu = 0.04$, $\sigma = 0.2$, $\rho_1 = 0.25$, $\gamma_1 = 0.15$, $\rho_2 = 0.5$, $\rho_3 = 0.45$, $\rho_4 = 0.5$, $\gamma_2 = 0.3$, $\psi = 0.5$, $\delta_1 = 0.33$, $\sigma_i = 0.02$ $(i = 1, 6)$, $\sigma_2 = 0.008$. $\sigma_i = 0.1$ $(i = 3, 5)$, $\sigma_4 = 0.06$, $\sigma_7 = 0.04$.

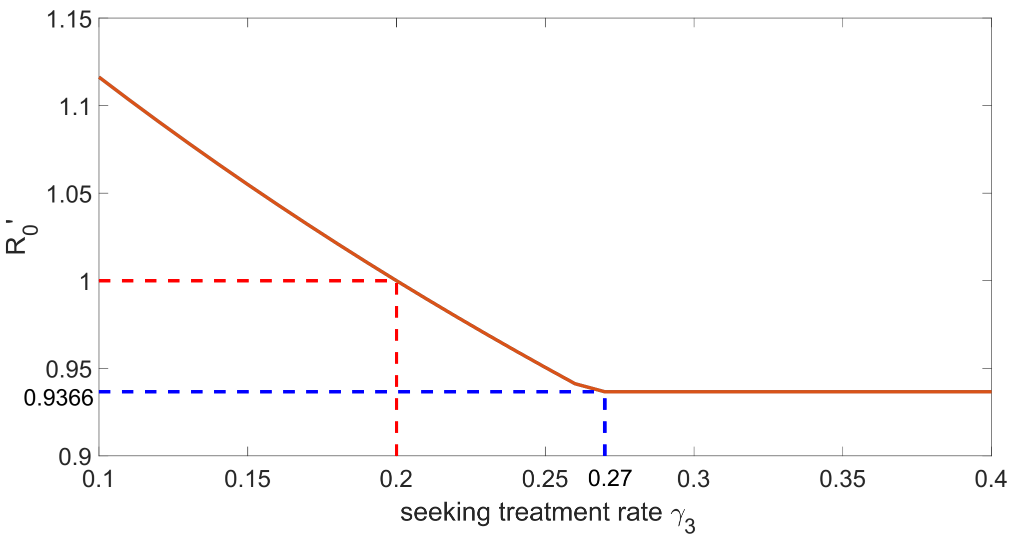

Fig. 3.4. The variation trend of $R_0^{'}$ with different γ_3. With parameters: $\Pi = 10000$, $k = 0.8$, $\beta_1 = 0.06 \times 10^{-5}$, $\beta_2 = 0.03 \times 10^{-5}$, $\beta_3 = 0.01 \times 10^{-5}$, $\beta_4 = 0.05 \times 10^{-5}$, $\mu = 0.025$, $\sigma = 0.2$, $\rho_1 = 0.1$, $\gamma_1 = 0.2$, $\gamma_2 = 0.15$, $\rho_2 = 0.45$, $\psi = 0.5$, $\rho_3 = 0.4$, $\rho_4 = 0.4$, $\delta_1 = 0.36$, $\sigma_i = 1.34$ $(i = 1, 2, 3, 4, 5, 7)$, $\sigma_6 = 0.74$.

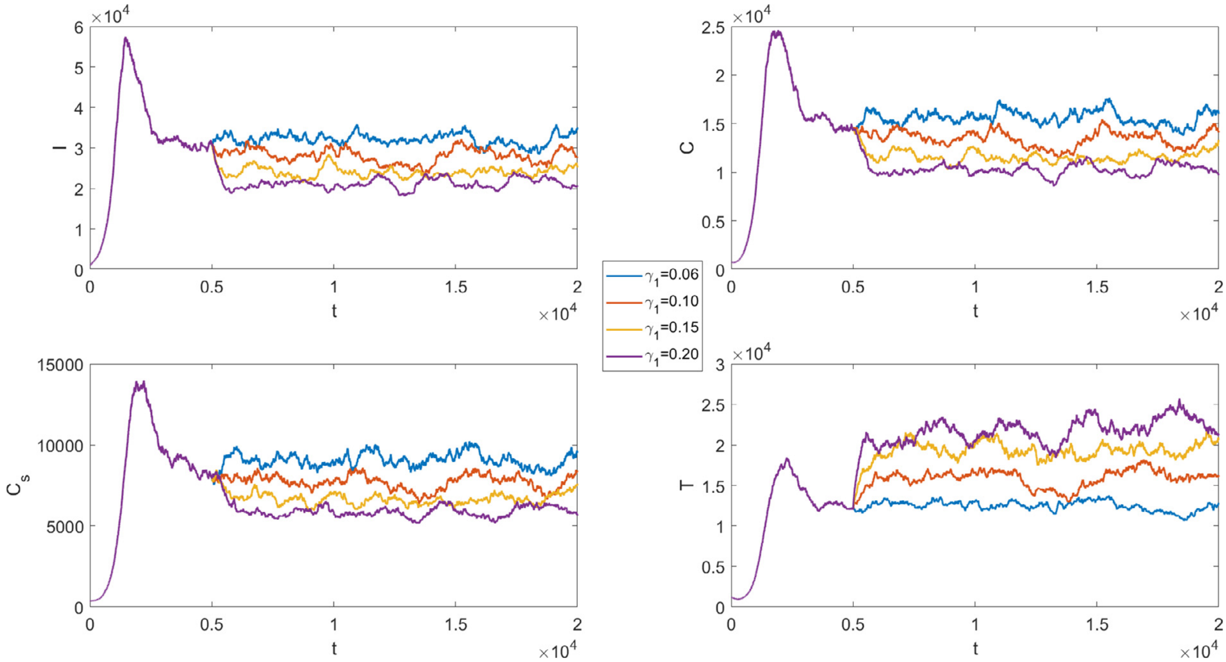

Fig. 3.5. The diagram tracks variation trends of carriers $I(t)$, $C(t)$, $C_s(t)$, and $T(t)$ with different γ_1. With parameters: $k = 0.5$, $\gamma_2 = \gamma_3 = 0.1$, $\psi = 0.3$. The other fixed parameter values are the same as in Fig 2.2.

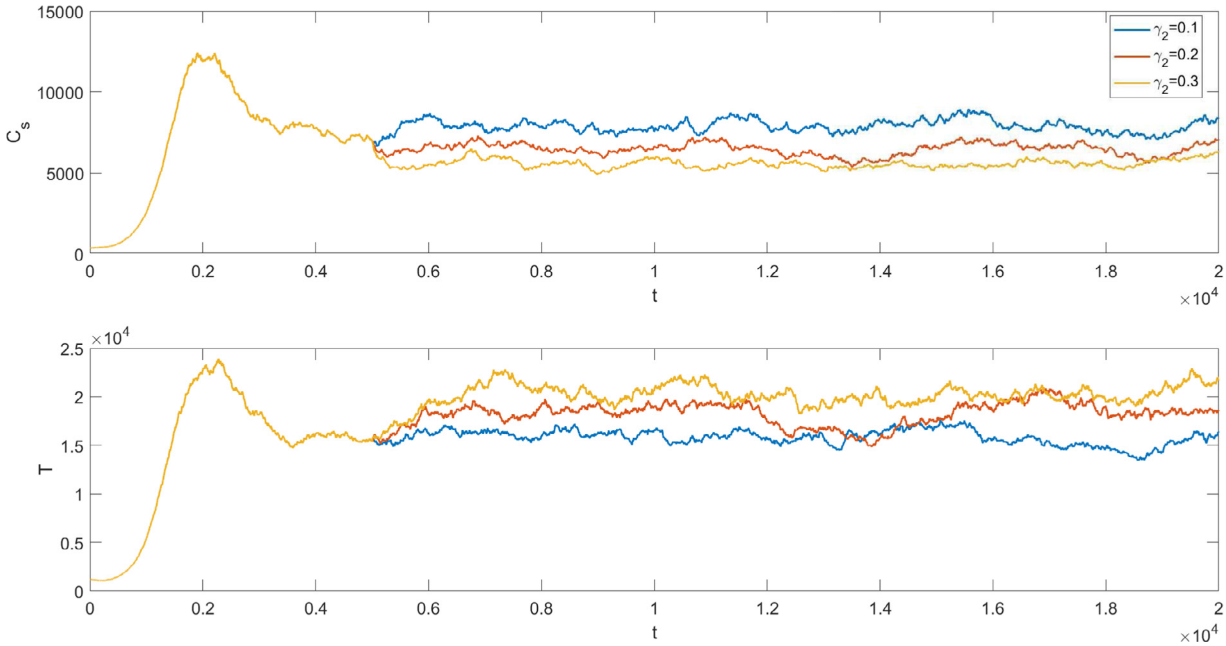

Fig. 3.6. The diagram tracks variation trends of carriers $C_s(t)$ and $T(t)$ with different γ_2. With parameters: $k = 0.5$, $\gamma_1 = \gamma_3 = 0.1$, $\psi = 0.3$. The other fixed parameter values are the same as in Fig 2.2.

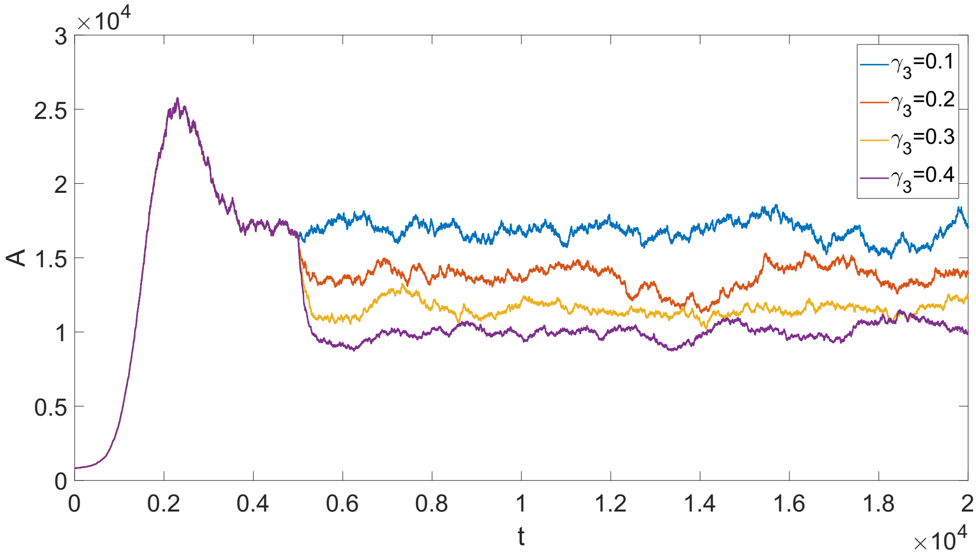

Fig. 3.7. The diagram tracks variation trends of carriers $A(t)$ with different γ_3. With parameters: $k = 0.5$, $\gamma_1 = \gamma_2 = 0.1$, $\psi = 0.3$. The other fixed parameter values are the same as Fig 2.2.

References

[1] P.K. Roy, S. Saha, F.A. Basir, Effect of awareness programs in controlling the disease HIV/AIDS: an optimal control theoretic approach, Adv. Differ. Equ. 2015 (1) (2015) 217.
[2] B. Tomas, G. Vytautas, et al., Consistent prediction of mutation effect on drug binding in HIV-1 protease using alchemical calculations, J Chem. Theory Comput. 14 (7) (2018) 3397–3408.
[3] P.K. Roy, S. Chowdhury, A.N. Chatterjee, et al., A mathematical model on CTL mediated control of HIV infection in a long term drug therapy, J. Biol. Syst. 21 (3) (2013) 1350019.
[4] R. Liu, J. Wu, H. Zhu, Media/psychological impact on multiple outbreaks of emerging infectious diseases, Comput. Math. Methods. M. 8 (3) (2007) 153–164.
[5] P.K. Roy, A.N. Chatterjee, Anti-viral drug treatment along with immune activator IL-2: a control-based mathematical approach for HIV infection, Int. J. Control. 85 (2) (2012) 220–237.
[6] S.D. Hove-Musekwa, F. Nyabadza, The dynamics of an HIV/AIDS model with screened disease carriers, Comput. Math. Methods. M. 10 (4) (2015) 287–305.
[7] R. Arrington-Sanders, N.J. Wheeler, P. Matson, et al., A system-level approach to improve HIV screening in an urban pediatric primary care setting, PEDIATRICS (2018).
[8] A.M. Geretti, H. Austin, G. Villa, et al., Point-of-care screening for a current hepatitis c virus infection: Influence on uptake of a concomitant offer of HIV screening, Sci. Rep. 8 (1) (2018).
[9] L. Raynell, S. Stuart, F. Jackie, et al., HIV infection after prenatal screening: an open window leading to perinatal infection, AIDS Care. 31 (3) (2019) 306–309.
[10] A. Tripathi, R. Naresh, D. Sharma, Modelling the effect of screening of unaware infectives on the spread of HIV infection, Appl. Math. Comput. 184 (2007) 1053–1068.
[11] F. Nyabadza, A mathematical model for combating HIV/AIDS in southern africa, J. Biol. Syst. 14 (2006) 357–372.
[12] M.A. Khan, M. Rahman, et al., Awareness of sexually transmitted disease among women and service providers in rural bangladesh, Int. J. Std. Aids. 8 (11) (1997) 688–696.
[13] M.S. Rahman, M.L. Rahman, Media and education play a tremendous role in mounting AIDS awareness among married couples in bangladesh, AIDS. Res. Ther. 4 (1) (2007) 10.
[14] X.R. Mao, E. Renshaw, G. Marion, Environmental brownian noise suppresses explosions in population dynamics, Stoch. Proc. Appl. 97 (1) (2002) 95–110.
[15] N. Dalal, D. Greenhalgh, X.R. Mao, A stochastic model for internal HIV dynamics, J. Math. Anal. Appl. 341 (2) (2008) 1084–1101.
[16] L. Wang, D.Q. Jiang, A note on the stationary distribution of the stochastic chemostat model with general response functions, Appl. Math. Lett. 73 (2017).
[17] L. Wang, D.Q. Jiang, Ergodic property of the chemostat: A stochastic model under regime switching and with general response function, Nonlinear Anal. Hybird. Syst. 27 (2018) 341–352.
[18] P.J. Witbooi, Stability of an SEIR epidemic model with independent stochastic perturbations, Phys. A. 392 (2013) 4928–4936.
[19] S. Peng, X. Zhu, Necessary and sufficient condition for comparison theorem of 1-dimensional stochastic differential equations, Stoch. Proc. Appl. 116 (2006) 370–380.
[20] X.R. Mao, G. Marion, E. Renshaw, Environmental noise suppresses explosion in population dynamics, Stoch. Proc. Appl. 97 (2002) 95–110.
[21] H. Liu, Q. Yang, D.Q. Jiang, The asymptotic behavior of stochastically perturbed DI SIR epidemic models with saturated incidences, Automatica 48 (5) (2012) 820–825.
[22] Q. Luo, X.R. Mao, Stochastic population dynamics under regime switching, J. Math. Anal. Appl. 334 (1) (2009) 69–84.
[23] Q. Liu, D.Q. Jiang, N.Z. Shi, Threshold behavior in a stochastic SIQR epidemic model with standard incidence and regime switching, Appl. Math. Comput. 316 (2018) 310–325.
[24] E. Yves, B. Danhree, K. Rodoumta, Mathematical analysis of HIV/AIDS stochastic dynamic models, Appl. Math. Model. 40 (2016) 9131–9151.
[25] Y. Wang, D.Q. Jiang, A. Alsaedi, et al., Modeling a stochastic HIV model with logistic target cell growth and nonlinear immune response function, Physica. A. 501 (1) (2018) 276–292.
[26] M.U. Nsuami, P.J. Witbooi, A model of HIV/AIDS population dynamics including ARV treatment and pre-exposure prophylaxis, Adv. Differ. Equ. 2018 (1) (2018) 11.

[27] D. Jasmina, C.J. Silva, D.F.T. Delfim, A stochastic SICA epidemic model for HIV transmission, Appl. Math. Lett. 84 (2018) 168–175.
[28] F. Rodrigues, C.J. Silva, D.F.M. Torres, et al., Optimal control of a delayed HIV model, Discrete. Cont. Dyn-B. 23 (1) (2017).
[29] J. Jia, G. Qin, Stability analysis of HIV/AIDS epidemic model with nonlinear incidence and treatment, Adv. Differ. Equ. 2017 (1) (2017).
[30] L. Cai, S. Guo, S. Wang, Analysis of an extended HIV/AIDS epidemic model with treatment, Appl. Math. Comput. 236 (2014) 621–627.
[31] L. Imhof, S. Walcher, Exclusion and persistence in deterministic and stochastic chemostat models, J. Differ. Equ. 217 (2005) 26–53.
[32] Q. Liu, D.Q. Jiang, N.Z. Shi, T. Hayat, A. Alsaedi, Stationarity and periodicity of positive solutions to stochastic SEIR epidemic models with distributed delay, Discrete. Cont. Dyn-B. 22 (6) (2017) 2479–2500.
[33] F. Li, S. Zhang, X. Meng, Dynamics analysis and numerical simulations of a delayed stochastic epidemic model subject to a gene, Comp. Appl. Math. 38 (2019) 95.
[34] X. Yu, S. Yuan, T. Zhang, Survival and ergodicity of a stochastic phytoplankton-zooplankton model with toxin-producing phytoplankton in an impulsive polluted environment, Appl. Math. Comput. 347 (15) (2019) 249–264.
[35] Y. Zhao, D.Q. Jiang, The threshold of a stochastic SIS epidemic model with vaccination, Elsevier Science Inc., 2014.
[36] L. Zu, et al., Ergodic property of a lotka-volterra predator-prey model with white noise higher order perturbation under regime switching, Appl. Math. Comput. 330 (2018) 93–102.
[37] Q. Liu, D.Q. Jiang, T. Hayat, et al., Stationary distribution and extinction of a stochastic predatorcprey model with additional food and nonlinear perturbation, Appl. Math. Comput. 320 (2018) 226–239.
[38] R. Khasminskii, Stochastic Stability of Differential Equations, Springer, Berlin, Heidelberg Publishing, 1980.
[39] T.C. Gard, Introduction To Stochastic Differential Equations, Marcel Dekker INC, New York, 1988.
[40] G. Strang, Linear Algebra and Its Applications, Thomson Learning INC, London, 1988.
[41] C. Zhu, G. Yin, Asymptotic properties of hybrid diffusion systems, SIAM J. Control. Optim. 46 (2007) 1155–1179.
[42] K. Qi, D.Q. Jiang, Threshold behavior in a stochastic HTLV-i infection model with CTL immune response and regime switching, Math. Meth. Appl. Sci. 41 (2018) 6866–6882.
[43] D.J. Higham, An algorithmic introduction to numerical simulation of stochastic differential equations, SIAM Rev. 43 (3) (2001) 525–546.
[44] X.R. Mao, Stochastic differential equations and applications, Elsevier, 2007.
[45] M.M. Gao, D.Q. Jiang, T. Hayat, The threshold of a chemostat model with single-species growth on two nutrients under telegraph noise, Commun. Nonlinear. Sci. 75 (2019) 160–173.
[46] Q. Liu, D.Q. Jiang, Stationary distribution and extinction of a stochastic predator-prey model with distributed delay, Appl. Math. Lett. 78 (2018) 79–87.
[47] R. Durrett, Stochastic Calculus, CRC Press, 1996.