# Exercise 3: Population-Based Single- and Multi-Objective Algorithm Analysis

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## Methods

**Representation & operators:** Solutions are binary vectors of length $n$ (nodes). We use bit–flip mutation with rate $1/n$ and no crossover. Initial populations are sampled uniformly at random.

**Uniform constraint:** For single-objective, we maximise $f(S)$ s.t. $|S| \leq B$ (uniform cardinality). We repair any over-budget offspring. Unless noted, we use greedy repair to iteratively drop the bit with smallest marginal loss until $|S| = B$.

**Single-objective selection & diversity:** Tournament selection (size $t$=3). We compare: Age (oldest non-elite removed first), Fitness sharing (Hamming-distance sharing; radius $\sigma$), Crowding (deterministic crowding replacement) and Niching (replace most similar; small niche list). All other operators are shared.

**Multi-objective:** We optimise $\langle f(S), -|S|\rangle$ with a simple Pareto EA: (1) choose a parent uniformly from the current *first* nondominated front; (2) bit–flip mutation ($1/n$); (3) pool + truncate to population size by recomputing the first front and applying *crowding distance* on that front. Thus dominated individuals are discarded and there is no secondary front nor external archive beyond the current population.

**Budgets & runs:** All plots shown here were generated with **30 independent runs** per configuration and a fixed per-run **10 000 evaluation budget**.[1] For the Single objective and Multi objective algorithms, we evaluate both **MaxCoverage** and **MaxInfluence** instances for populations $\{10, 20, 50\}$.

Table 1: Algorithm settings (kept constant unless mentioned)

| | |
|---|---|
| Mutation rate | $1/n$ |
| Single Objective tournament size | $t = 3$ |
| Single Objective elitism (Age) | 10% elites retained |
| Sharing radius $\sigma$ (Single Objective) | Hamming radius (problem-scaled) *(see code)* |
| Multi Objective parent choice | uniform over current first Pareto front |
| Multi Objective truncation | first front $\rightarrow$ crowding distance (NSGA-II style) |
| Populations | $\{10, 20, 50\}$ |
| Runs $\times$ budget | $30 \times 10{,}000$ evaluations |

[1]The Multi Objective driver supports other budgets (default 100k) but our figures were produced at 10000; see axes.

**Design process (Single-Objective):** The algorithm for exercise 3 was developed using Claude Sonnet 4.5. Initially a prompt was given to design and implement a single-objective evolutionary algorithm for monotone sub-modular problems with a uniform constraint. It was provided with the task sheet as reference material. This created an initial version of the code and the algorithm with four different diversity mechanisms: Fitness Sharing, Deterministic Crowding, Niching with Tournament Selection and Age-Based Diversity. Having these four different diversity mechanisms allowed me to experiment and find which worked optimally.

As such problems 2100 through 2103 were run with all four different diversity mechanisms, and Fitness Sharing and Age-Based Diversity were found to be the best performers by a significant margin. As such these two were carried into the runs of problems 2200 through 2203 which took significantly longer to run.

## Single-Objective: Population-Based GAs

We evaluate four population-based GA variants for monotone submodular maximisation with a uniform cardinality constraint. Variants share operators and differ only in their diversity/survivor mechanisms: *Age* (high turnover), *Fitness sharing* (niche penalties), *Crowding* (replace most similar), *Niching* (explicit niche maintenance). We test populations of size 10, 20 and 50 for 10,000 evaluations on the two instance families: *MaxCoverage* (step-wise gains), *MaxInfluence* (smoother gains; values logged negative—less negative is better).

### Fixed-Budget Behaviour (10000 evaluations)

**MaxCoverage (2100–2103):** With pop= 10, *Age* and *Fitness sharing* reach high values earliest; *Crowding/Niching* lag. At pop= 20, *Age* is clearly fastest and maintains the best endpoint, *Fitness sharing* close behind. At pop= 50, *Age* dominates—the extra turnover converts into steady gains—whereas *Fitness sharing* slows and *Crowding/Niching* often plateau. These trends are visible in Fig. 1, which compares mean progress across mechanisms for the MaxCoverage suite.

**MaxInfluence (2200–2203):** The ordering shifts: *Fitness sharing* achieves the best (least-negative) final medians across pops; *Age* is second. *Crowding/Niching* remain weaker and benefit least from pop= 50. See Fig. 2 for the corresponding MaxInfluence progress (note the inverted/negative scale where "less negative is better").

### Robustness (End-of-Budget Distributions)

**MaxCoverage (2100–2103):** *Age* and *Fitness sharing* show higher medians for pop= 10/20; at pop= 50, *Age* has the best median and narrower spread. *Crowding/Niching* exhibit wider variance. Figure 3 summarises these end-of-budget distributions.

**MaxInfluence (2200–2203):** *Fitness sharing* yields the best medians and tight inter-quartile ranges across pops; *Age* is close but with larger variance. The distributional differences are shown in Fig. 4.

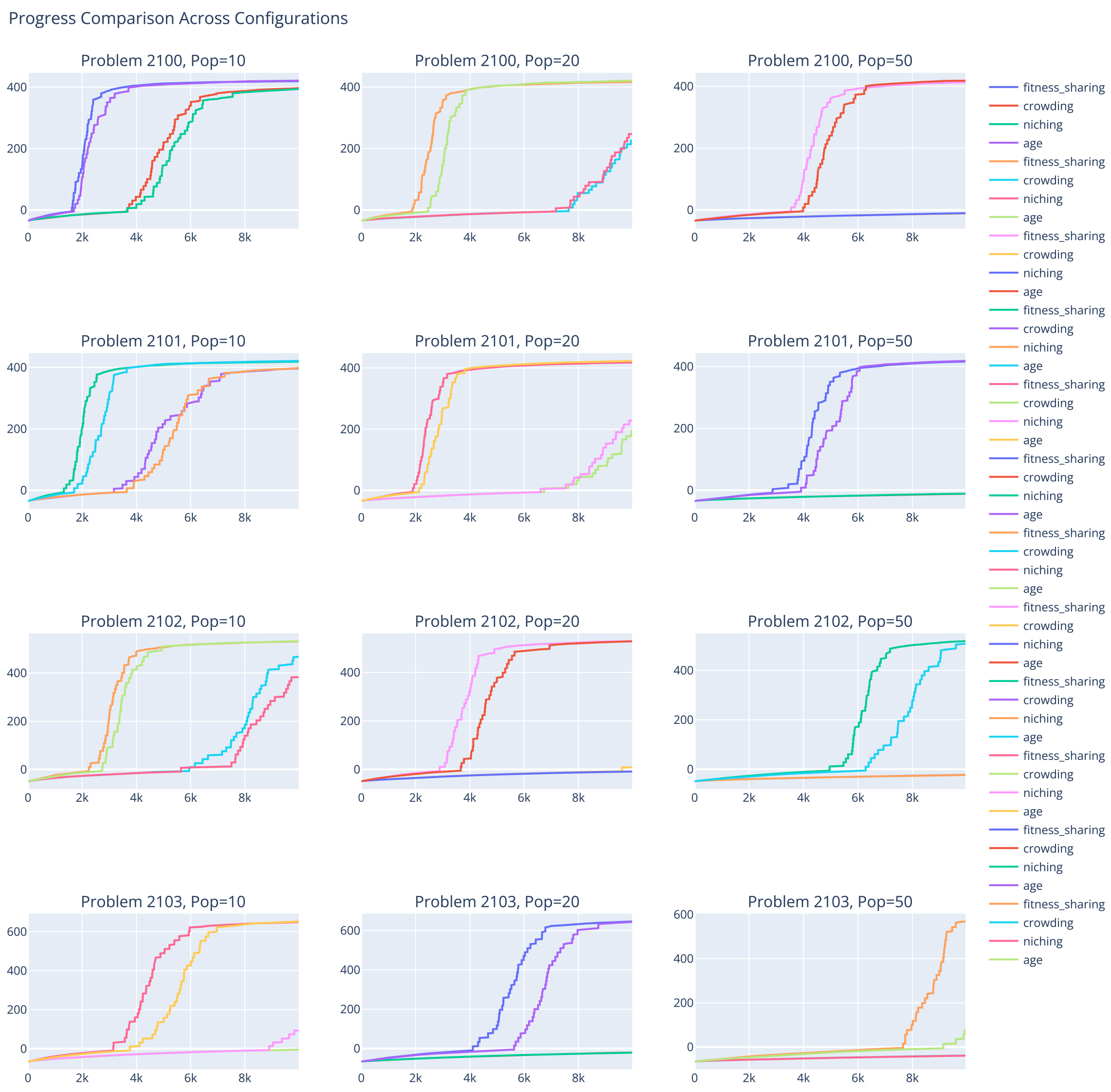


Figure 1: Single Objective fixed-budget progress for 10000 evaluations: **MaxCoverage** (2100–2103) across populations (Age, Fitness sharing, Crowding, Niching). Higher is better.

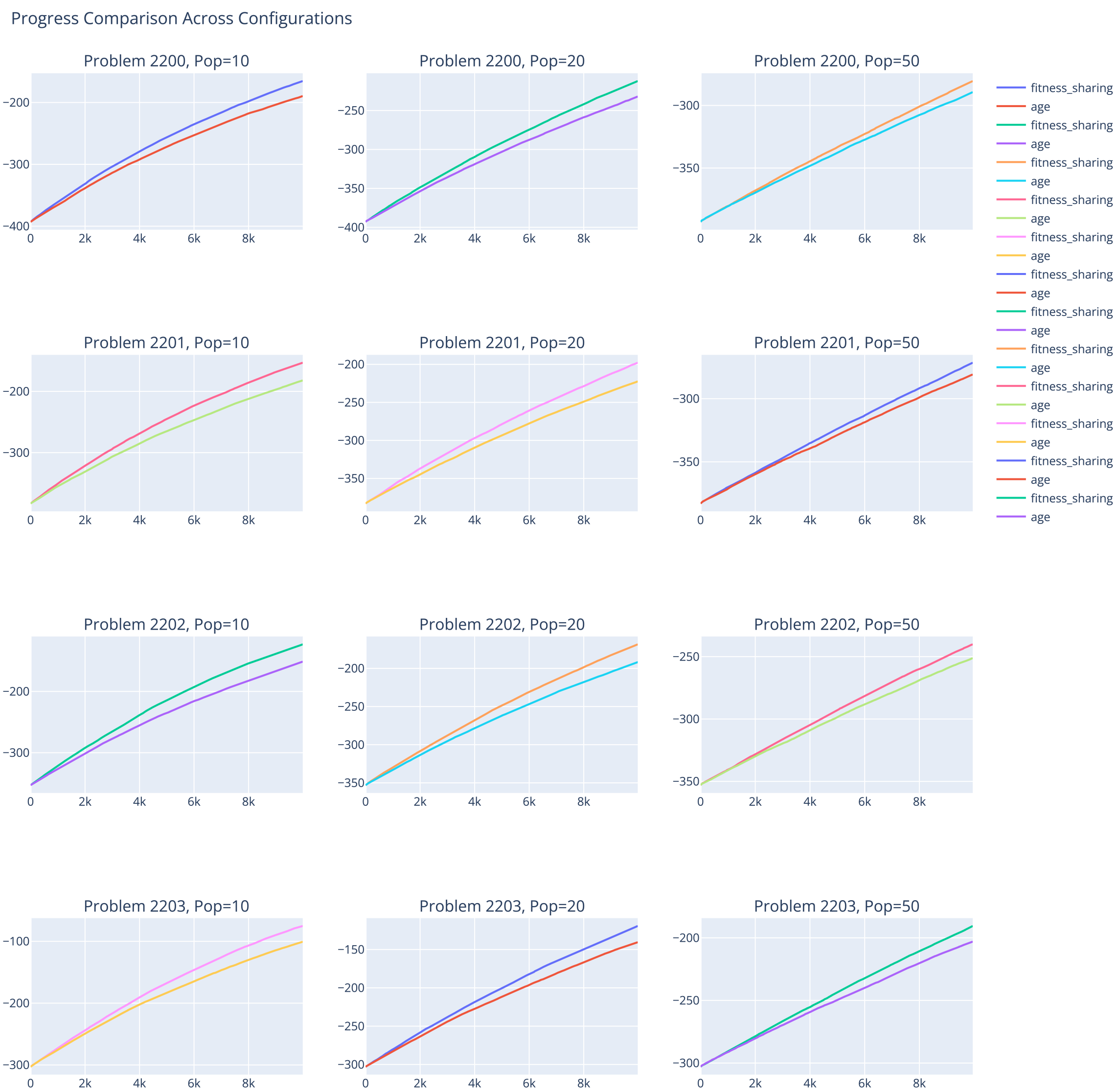


Figure 2: Single Objective fixed-budget progress for 10000 evaluations: **MaxInfluence** (2200–2203). Values are logged as negatives; less negative is better.

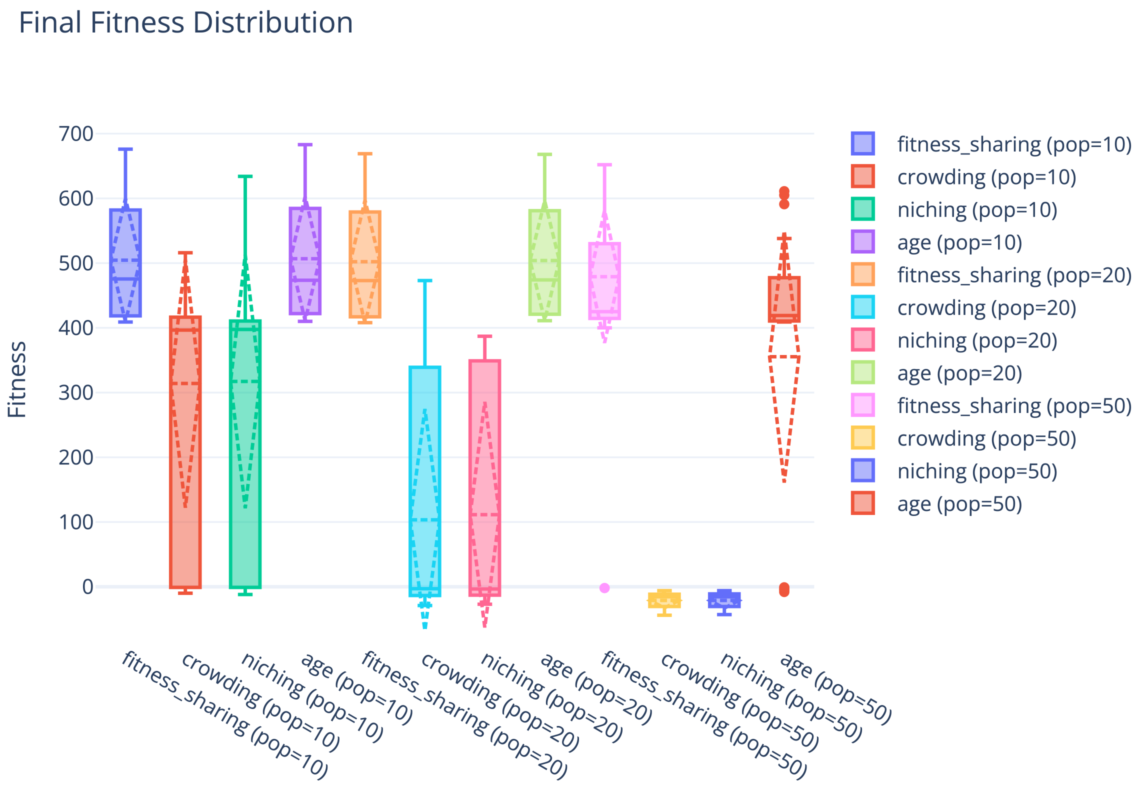


Figure 3: Single Objective final-fitness distributions for 10000 evaluations: **MaxCoverage** (2100–2103).

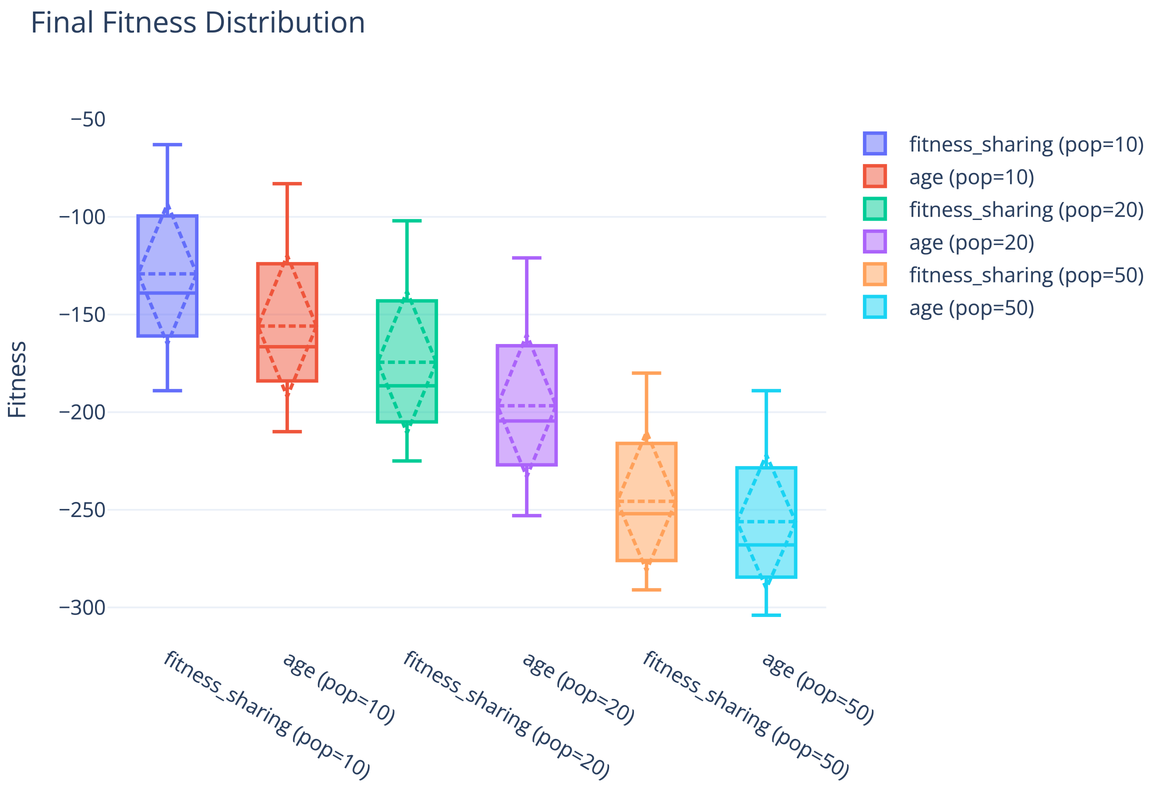


Figure 4: Single Objective final-fitness distributions for 10000 evaluations: **MaxInfluence** (2200–2203). Less negative is better.

## Multi-objective: Pareto EA

We jointly optimise value and cost $\langle f(S), -|S|\rangle$ under the same 10000 evaluations budget and populations $\{10, 20, 50\}$.

### Multi Objective Setup & Metrics:

We report: (i) **first-run** Pareto fronts at the budget (to visualise attained trade-offs), (ii) **anytime** best-objective envelopes across 30 runs (best–worst shading). *Note:* our progress metric follows the primary objective $f$; it is not a hypervolume/epsilon indicator.

### Pareto Quality at the Budget:

Across **MaxCoverage**, fronts are smooth and concave with visible knees; across **MaxInfluence** they are near-linear (small marginal gains) yet separate clearly by cost. **Effect of population size:** pop$=$ 10 recovers a coarse front but misses mid–high-cost knees; pop$=$ 20 densifies the front and strengthens extremes; pop$=$ 50 thickens coverage further but offers only small gains over pop$=$ 20 at 10000. These patterns can be read across Figs. 5–7.


Figure 5: Multi Objective fixed-budget Pareto fronts (first run) (pop $= 10$): **MaxCoverage** (top) concave with knees; **MaxInfluence** (bottom) near-linear trade-offs.

### Progress Plots (Best–Worst Across Runs)

**MaxCoverage:** Rapid early gains followed by jumps around 2–4k evaluations as larger cardinalities are discovered; variance bands narrow after the jump. Pop$=$ 20 reaches the plateau earlier than pop$=$ 10; pop$=$ 50 yields only slight extra acceleration. See Fig. 8 (pop$=$ 10) and Fig. 9 (pop$=$ 20), with the pop$=$ 50 envelope in Fig. 10.

**MaxInfluence:** Smooth, incremental progress with steady variance reduction. Pop$=$ 20 improves both speed and final envelopes over pop$=$ 10; pop$=$ 50 shows diminishing returns at 10000. The three populations are compared in Figs. 8–10.

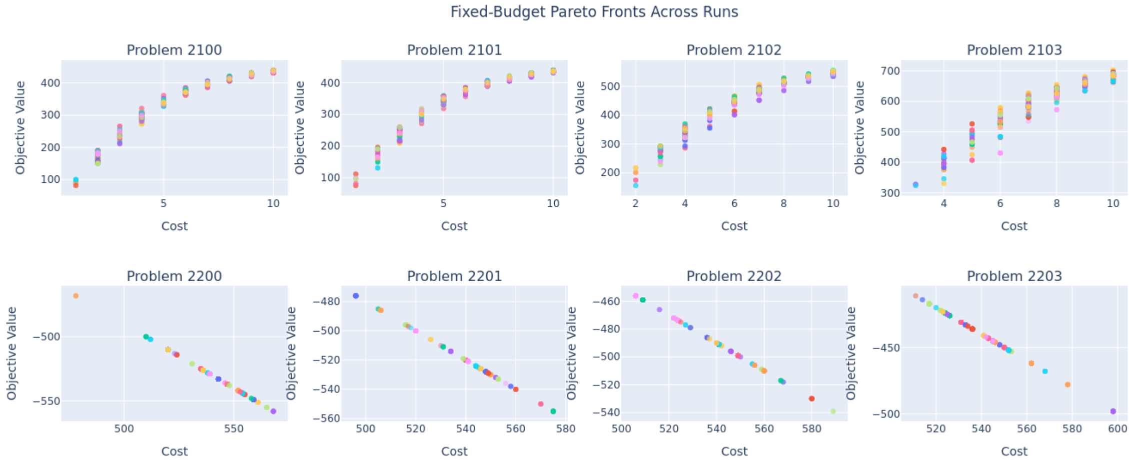


Figure 6: Multi Objective fronts (pop = 20): denser coverage and stronger extremes (low and high cost).

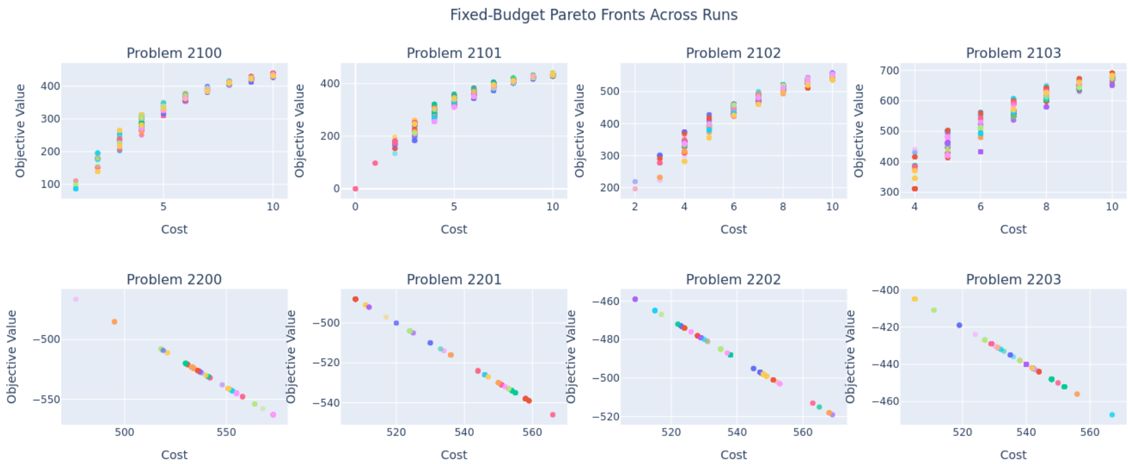


Figure 7: Multi Objective fronts (pop = 50): thick coverage, but only marginal gains over pop = 20 at 10000.


Figure 8: Multi Objective progress plots (pop = 10): bands shrink after key discoveries (notably on MaxCoverage).


Figure 9: Multi Objective progress plots (pop = 20): earlier plateaus on MaxCoverage and narrower final bands across suites.


Figure 10: Multi Objective progress plots (pop = 50): small gains over pop = 20 within the 10000 budget.

### Findings

Multi Objective search must cover a frontier. Population size of 20 balances the diversity ensuring both the extremes and knees are kept and the turnover to refine incumbents; pop= 50 spends more budget maintaining diversity, thus leaving us with fewer effective generations per individual at fixed budget, so the returns are diminished. The effect is stronger on **MaxCoverage** where knee discovery gives large jumps; on **MaxInfluence**, marginal gains are small, so extra population helps less. At 10000 evaluations, a **population of 20** offers the best trade-off between front coverage, convergence speed, and run-to-run stability across both algorithms.

## Conclusion

At 10000 evaluations, **Age** with a larger population excels on coverage-like Single Objective instances via accelerated exploitation, whereas **Fitness sharing** is most robust on influence-like Single Objective instances. In Multi Objective, a population of size 20 best balances coverage and convergence.