Effects of Temperature Dependent Molecular Diffusivities on Heat and Momentum Transport in Rapidly Rotating Rayleigh-Bénard Convection

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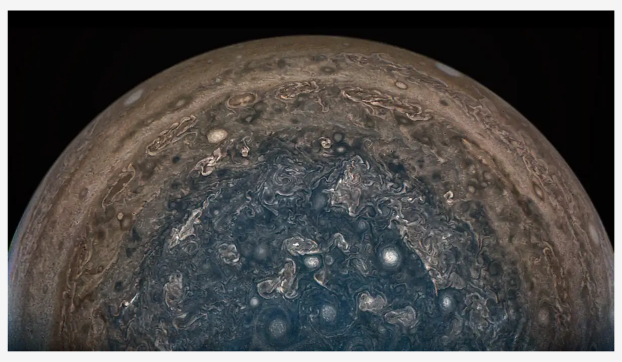
Motivation

Most numerical and experimental studies into Rayleigh-Bénard Convection (RBC) assume that momentum and heat transport are done by turbulent mixing and advection, and that molecular diffusivities can be neglected. This allows RBC to be studied with the the classical Oberbeck-Boussinesq (OB) assumption, where fluid material properties are constant everywhere except in the buoyant force term. [1]



The interior of gas giants comprises of multiple layers of rapidly rotating fluids with height and temperature dependent material properties. Using Jovian-inspired material properties with non-Oberbeck-Boussinesq (NOB) Rayleigh-Bénard equations, we want to investigate the effect of temperature-dependent viscosity, ν and thermal diffusivity, κ on the flow morphology of rapidly-rotating RBC.

Governing Equations

- DNS code: 4^{th} order accurate finite volume method solver, `goldfish` [2]
- Governing equations [3]:

$$\nabla \cdot \mathbf{u} = 0$$

$$D_t\mathbf{u} = -\nabla p + \Gamma^{-\frac{3}{2}}Pr^{-\frac{1}{2}}Ra^{\frac{1}{2}}\nabla \cdot \left[\nu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathbf{T}}\right)\right] + T\hat{z} + Ro^{-1}\Gamma^{\frac{1}{2}}\mathbf{u}\times\hat{z}$$

$$D_tT = \Gamma^{\frac{3}{2}}Ra^{-\frac{1}{2}}Pr^{\frac{1}{2}}\nabla\cdot[\kappa\nabla T]$$

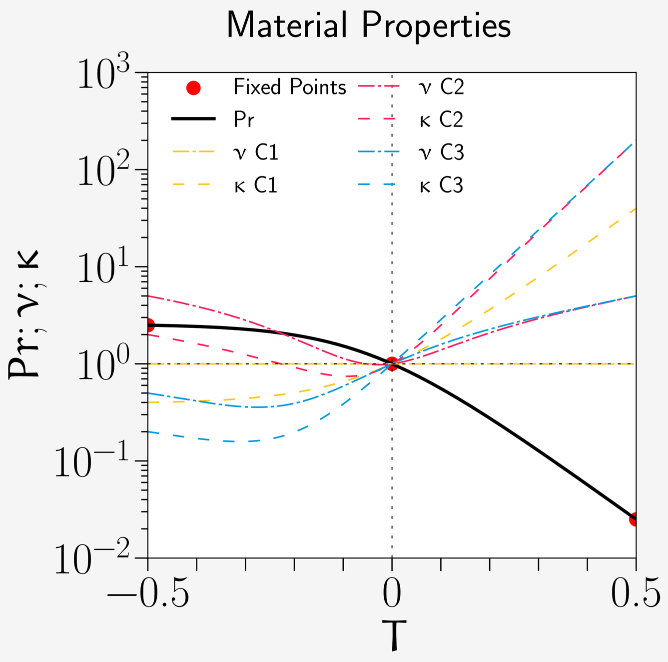
- where

$$Ra = \frac{g\alpha\Delta H^3}{\kappa\nu}, \quad Pr = \frac{\nu}{\kappa}, \quad Ro = \sqrt{\frac{\alpha g H\Delta}{2\Omega H}},$$

$$\Gamma = \frac{R}{H}, \quad Ek = \sqrt{\frac{Ro^2Pr}{Ra}} = \frac{\nu}{2\Omega H^2}$$

- Boundary conditions:
  - Top & bottom plate: No-slip, isothermal
  - Sidewall: Free-slip, adiabatic

Material Properties



Fixed Points	Material	Pr
$T = -0.5$	Water	2.5
$T = 0.0$	-	1.0
$T = 0.5$	Liquid metal	0.025

Case	κ	ν
C1	$\frac{39}{64}\exp\left(4\,\text{atanh}\left(\frac{32}{33}\right)T\right) + \frac{25}{64}$	1
C2	$\exp\left(-2\ln\left(\frac{1}{10}\left(1001-\sqrt{1001901}\right)\right)\right)T + \left(\frac{999}{5} - \frac{\sqrt{1001901}}{5}\right)T$	$\frac{\exp\left(-2\ln\left(\frac{1}{10}\left(1001-\sqrt{1001901}\right)\right)\right)T + \left(\frac{999}{5}-\frac{\sqrt{1001901}}{5}\right)T}{\frac{39}{64}\exp\left(4\,\text{atanh}\left(\frac{32}{33}\right)T\right)+\frac{25}{64}}$
C3	$\exp\left(-2\ln\left(101-10\sqrt{102}\right)T\right) + \left(198-20\sqrt{102}\right)T$	$\frac{\exp\left(-2\ln\left(101-10\sqrt{102}\right)T\right)+\left(198-20\sqrt{102}\right)T}{\frac{39}{64}\exp\left(4\,\text{atanh}\left(\frac{32}{33}\right)T\right)+\frac{25}{64}}$

References

[1] Gary A. Glatzmaier. *Introduction to Modeling Convection in Planets and Stars: Magnetic Field, Density Stratification, Rotation*. Student ed. Princeton University Press, 2014. ISBN: 9780691141732. (Visited on 02/19/2025).

[2] Olga Shishkina et al. "A fourth order accurate finite volume scheme for numerical simulations of turbulent Rayleigh–Bénard convection in cylindrical containers". In: *Comptes Rendus Mécanique* (Jan. 2005).

[3] Ronald L. Panton. *Incompressible Flow*. Wiley, July 2013. ISBN: 9781118713075.

[4] S. Chandrasekhar. *Hydrodynamic and Hydromagnetic Stability*. Dover Books on Physics Series. Dover Publications, 1981. ISBN: 9780486640716.

[5] Susanne Horn et al. "Prograde, retrograde, and oscillatory modes in rotating Rayleigh–Bénard convection". In: *Journal of Fluid Mechanics* (Oct. 2017).

[6] J. Herrmann et al. "Asymptotic theory of wall-attached convection in a rotating fluid layer". In: *Journal of Fluid Mechanics* (Oct. 1993).

[7] Susanne Horn et al. "Rotating non-Oberbeck–Boussinesq Rayleigh–Bénard convection in water". In: *Physics of Fluids* (May 2014).

Convection Onset Predictions

Stationary bulk convection [4]

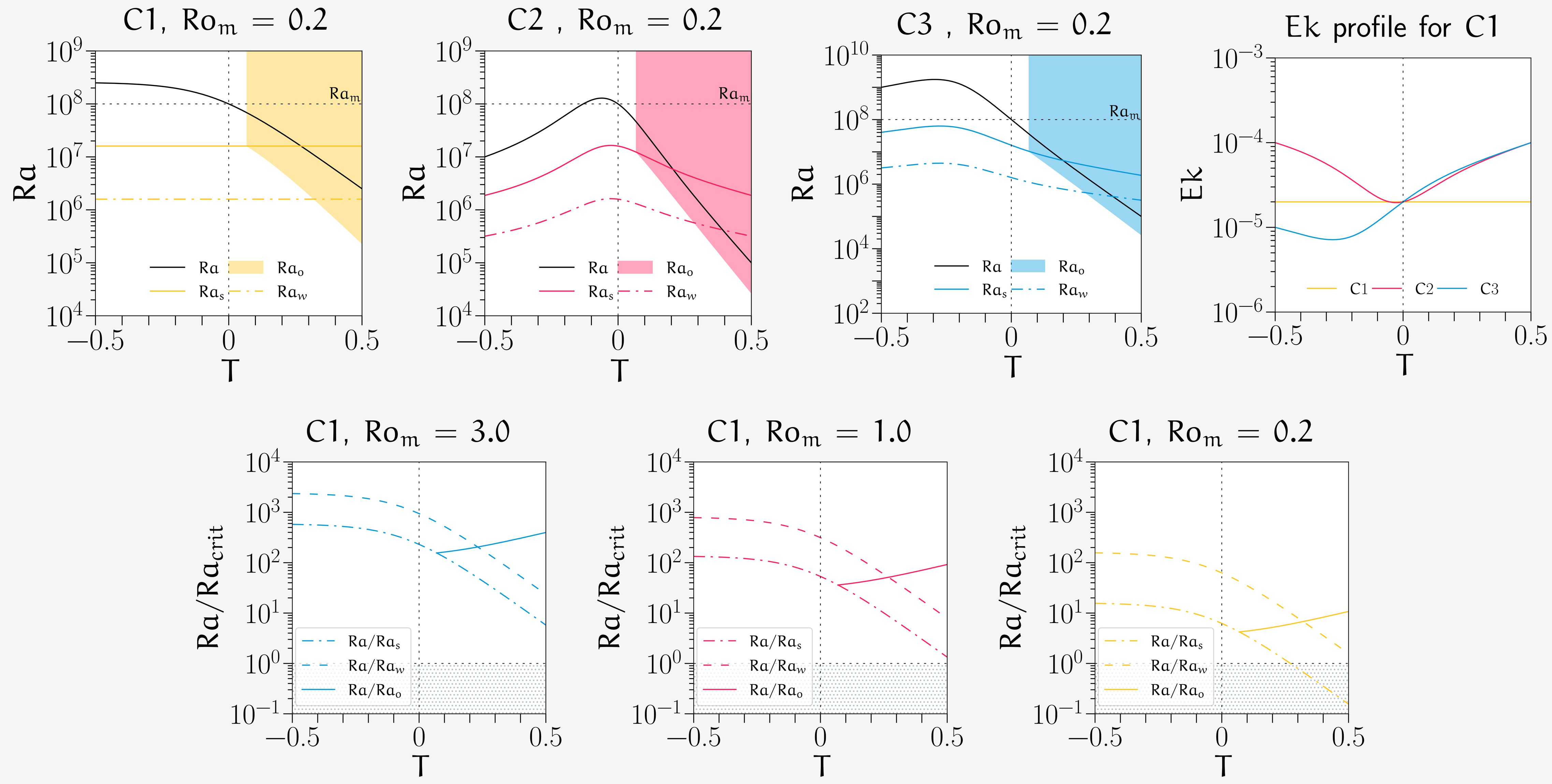
$$Ra_s = \frac{3}{2}\left(2\pi^4\right)^{\frac{1}{3}}Ek^{-\frac{4}{3}}$$

Oscillatory convection ($Pr > 0.68$) [5]

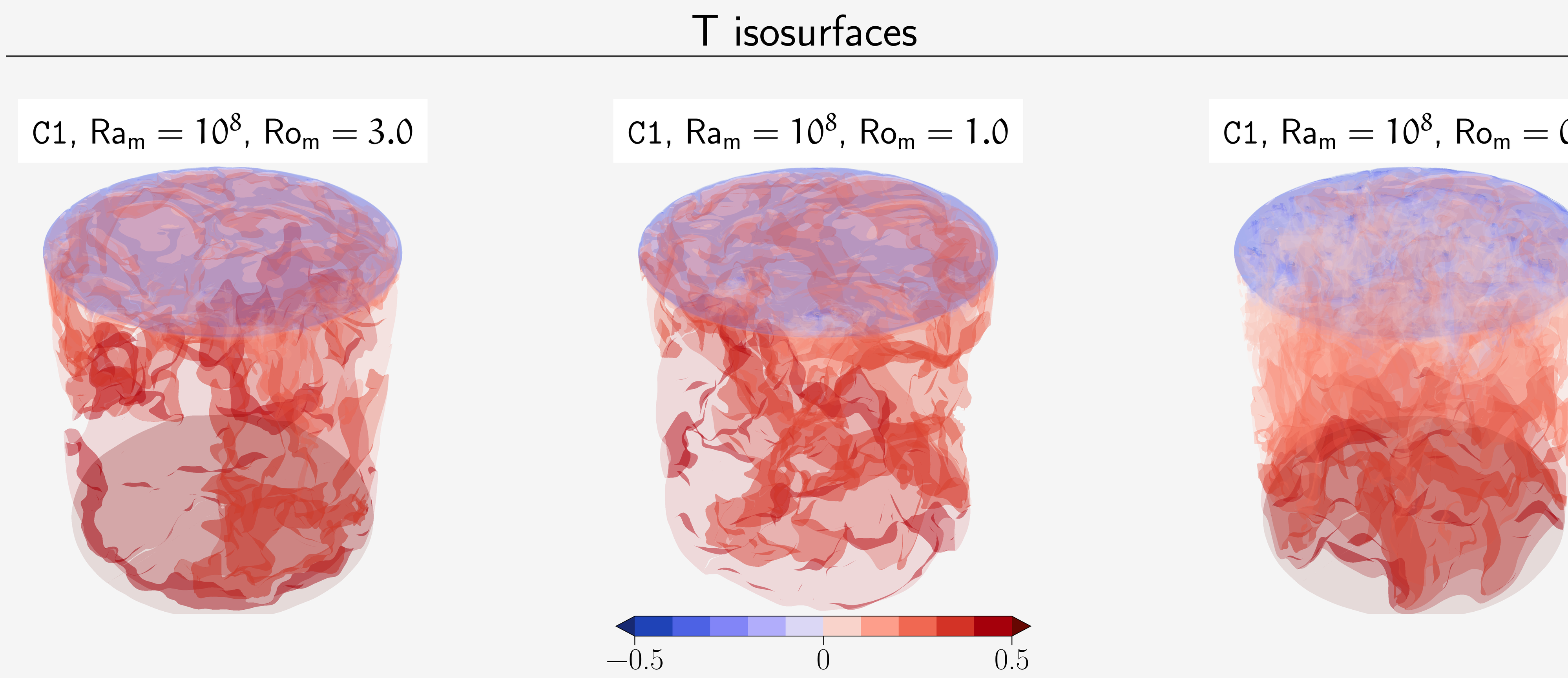
$$Ra_o = \frac{3}{2}\left(2\pi\,Pr\right)^{\frac{4}{3}}\left(1+Pr\right)^{-\frac{1}{3}}Ek^{-\frac{4}{3}}$$

Wall-attached convection [6]

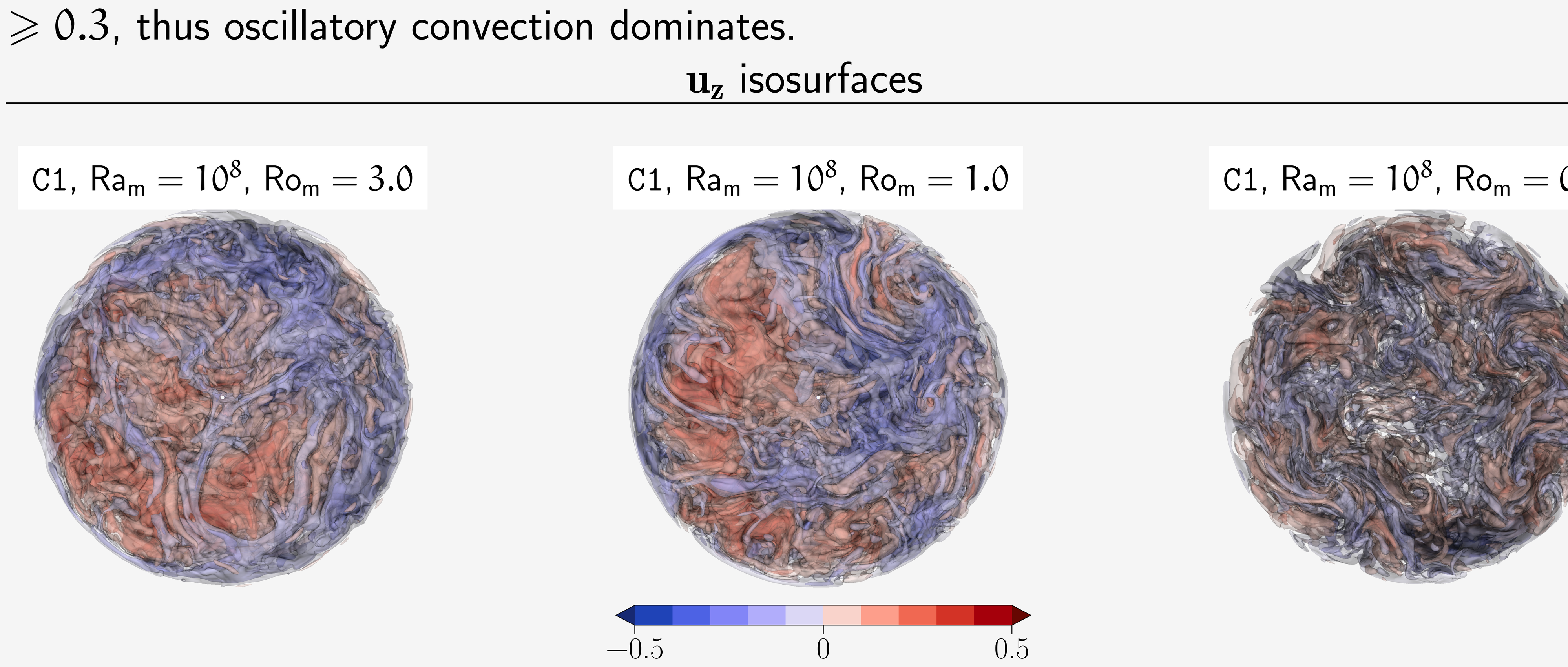
$$Ra_w = \pi^2\left(6\sqrt{3}\right)^{\frac{1}{2}}Ek^{-1}$$

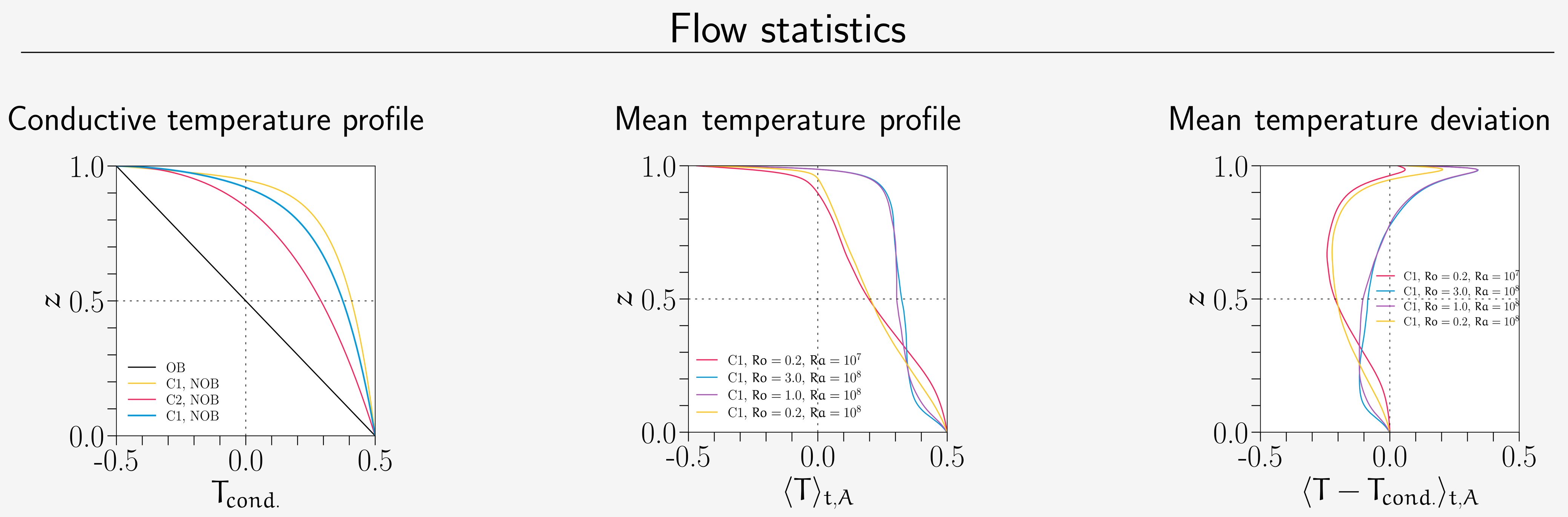


Results



- As rotation rate increases (i.e. Ro_m decreases), the supercriticalities decreases.
- Case C1 at $Ro_m = 1.0$, 3.0 and $Ra_m = 10^8$ still permits all three types of convection to occur.
- For case C1 at $Ro_m = 0.2$ and $Ra_m = 10^8$, stationary convection is prohibited at approximately $T \geqslant 0.3$, thus oscillatory convection dominates.





- Flow results show strong NOB characteristics as the fluid has a much higher $T_c - T_m$.
- Rotation is observed to suppress NOB effects, as the rapidly-rotating cases ($Ro_m = 0.2$) show a much lower $T_c - T_m$ that the slowly-rotating cases ($Ro_m = 1.0, 3.0$).
- These results were also observed in [7].