University of Colorado
Boulder

ECEA 5715 Power Electronics Capstone Project
Milestone 2

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1. Approach

Take a screen shot of the LTspice averaged circuit model of your closed-loop converter. The screen shot should include only your averaged circuit model - it is not necessary to include the template header part provided in milestone2.asc (simulation commands and parameters, control signals, battery and bus models, measurement commands). Upload the screenshot followed by a brief description of how you approached the design of the control loop(s) around your converter, the type of control loop(s) employed (PWM voltage mode, peak current mode, or averaged current mode control), and the parameters: crossover frequency and phase margin for each control loop. Express each crossover frequency in kHz, and as a fraction of the corresponding converter switching frequency.

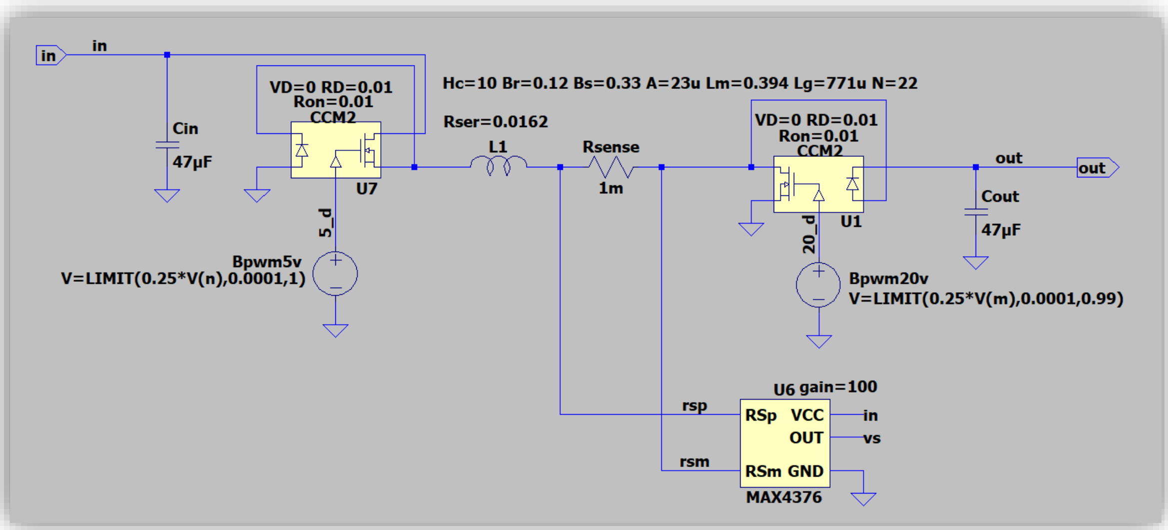

Figure 1. H-Bridge Synchronous Buck Boost Average Model

For Operating Point 2, the converter utilizes the Average Switch Model CCM2 U1 as a Boost converter, while CCM2 U7 would turn on 100% duty.
For Operating Point 1, the converter utilizes CCM2 U7 as a Buck converter, while CCM2 U1 would turn off with ~0% duty cycle.

Switching Frequency

$$f_{sw} = 1MHz$$

Operating Point 2

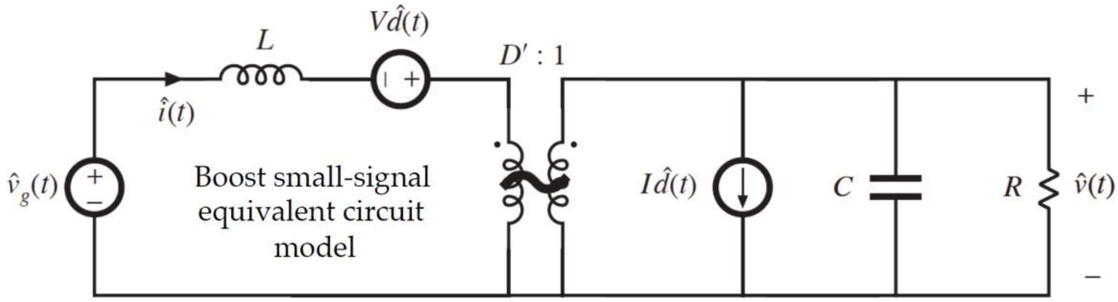

Figure 2. Boost small-signal equivalent circuit model

I chose the Averaged Current Mode Control for operating point 2, because Milestone 3 would require limit of current at operating point 2.

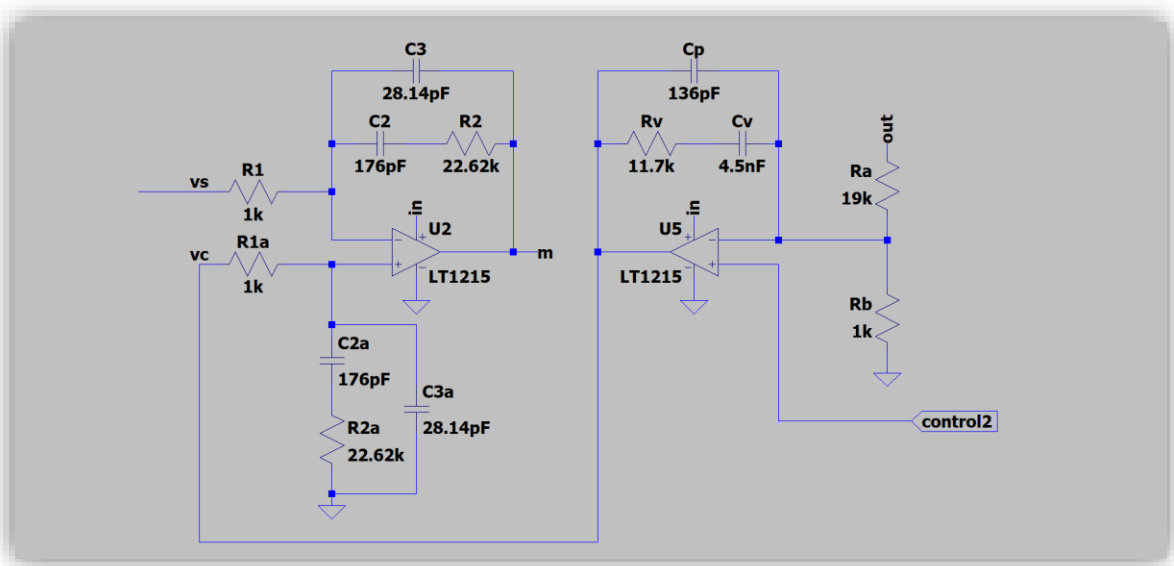

Figure 3. Averaged Current Mode Control Compensator

First calculate the approximate lossless required duty cycle for the operating point.

$$D = 1 - \frac{V_g}{V_{out}} = 1 - \frac{9.6}{20} = 0.52$$

Actual Duty Cycle with losses:

$$D_{real} = 0.5488$$

$$G_{id}(s) = \frac{\hat{\imath}_L}{\hat{d}}\bigg|_{\hat{v}_g=0} = G_{id0}\frac{1+\frac{s}{\omega_{zi}}}{1+\frac{1}{Q}\frac{s}{\omega_o}+\frac{s^2}{\omega_o^2}}$$

$$G_{id0} = \frac{2V_{out}}{D'^2R} = \frac{2\cdot 20}{(1-0.52)^2\cdot 6.67} = 26.029A$$

$$f_o = \frac{D'}{2\pi\sqrt{LC}} = \frac{(1-0.52)}{2\pi\sqrt{18uH\cdot 47\mu F}} = 2.627kHz$$

$$f_{zi} = \frac{1}{\pi RC} = \frac{1}{\pi\cdot 6.67\cdot 47\mu F} = 1.015kHz$$

$$Q = D'R\sqrt{\frac{C}{L}} = (1-0.52)6.67\sqrt{\frac{47}{18}} = 5.171$$

$$T_{iu0} = R_f\cdot\frac{1}{V_M}\cdot G_{id0} = 0.1\cdot\frac{1}{4}\cdot 26 = 0.650716 = -3.73dB$$

For Asymptotic behavior of $T_{iu}(s)$ around f_{ci}

$$T_{iu}(s) = T_{iu0}\frac{1+\frac{s}{\omega_{zi}}}{1+\frac{1}{Q}\frac{s}{\omega_o}+\frac{s^2}{\omega_o^2}} \to T_{iu0}\frac{\frac{s}{\omega_{zi}}}{\frac{s^2}{\omega_o^2}} \to T_{iu0}\frac{\omega_o^2/\omega_{zi}}{s}$$

$$T_{iu0}\frac{\omega_o^2}{\omega_{zi}} = \frac{R_f}{V_M}\cdot\frac{2V_{out}}{D'^2R}\cdot\frac{D'}{LC}\cdot\frac{RC}{2} = \frac{R_f}{V_M}\cdot\frac{V_{out}}{L}$$

$$T_{iu}(s) = \frac{R_f}{V_M}\cdot\frac{V_{out}}{sL}$$

We want

$$T_u(s) = \frac{R_f}{V_M}\cdot\frac{V_{out}}{sL}G_{cm} = 1$$

By choice, choosing 100kHz as my crossover frequency of my Current Loop.

$$G_{cm} = \frac{L\omega_c}{V_{out}} \cdot \frac{V_M}{R_f} = \frac{18\mu H(2\pi(100kHz))}{20} \cdot \frac{4}{0.1} = 22.62$$

By choice, for f_z and f_p

$$f_z = \frac{f_c}{2.5} = 40kHz$$
$$f_p = 2.5f_c = 250kHz$$

Approximate Phase Margin

$$\phi_m = 90° - \tan^{-1}\frac{f_z}{f_p} - \tan^{-1}\frac{f_p}{f_z} = 46°$$

Required Resistors and Capacitors value

$$R_1 = 1k\Omega$$

$$R_2 = G_{cm}R_1 = 22.62 \times 1k = 22.62k\Omega$$

$$C_2 = \frac{1}{2\pi R_2 f_z} = \frac{1}{2\pi \cdot 22.62k \cdot 40k} = 176pF$$

$$C_3 = \frac{1}{2\pi R_2 f_p} = \frac{1}{2\pi \cdot 22.62k \cdot 250k} = 28.14pF$$

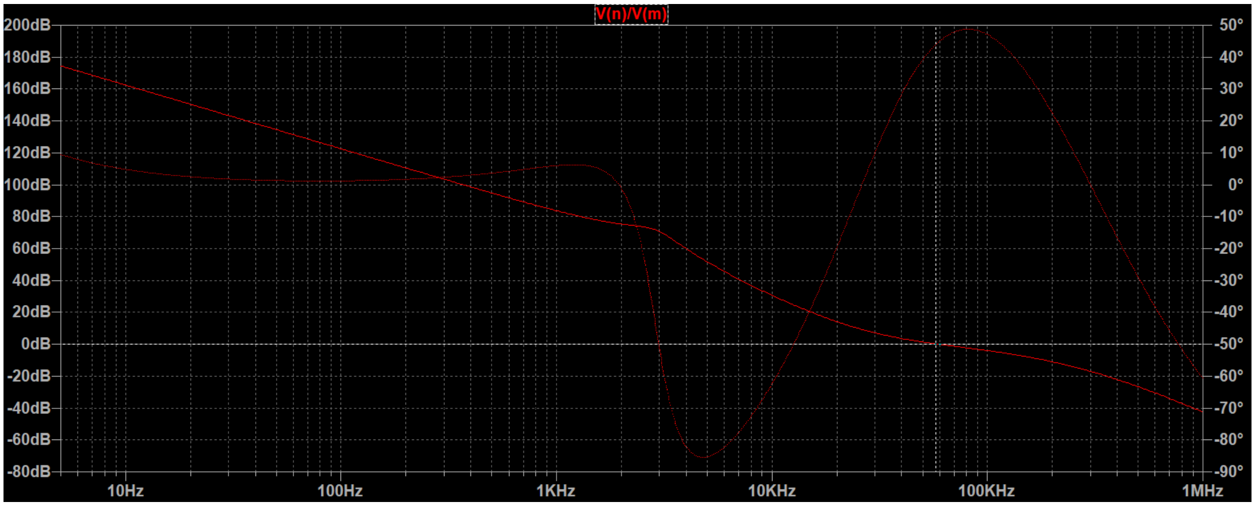

Figure 4. Inner Current Loop Gain

The crossover frequency happened at 57kHz with 44º Phase Margin, 0.057 of f_{sw}

Outer Voltage Design of ACM

Desired voltage loop crossover frequency

$$f_{cv} \leq \frac{1}{10} f_{ci} = 10kHz$$

By choice

$$V_{ref} = 1V$$

$$H = \frac{1}{20}$$

Voltage loop gain with compensator for frequencies well below the cross-over frequency of the current control loop.

$$T_v(s) = H \cdot G_{vc} \cdot G_{cv} = H \cdot \frac{1}{R_f} \cdot \frac{G_{vd}}{G_{id}} \cdot G_{cv}$$

Where

$$G_{vd}(s) = \frac{V_{out}}{D'} \cdot \frac{1 - s\frac{L}{D'^2 R}}{1 + s\frac{L}{D'^2 R} + s^2 \frac{LC}{D'^2}}$$

$$G_{id}(s) = \frac{2V}{D'^2 R} \cdot \frac{1 + s\frac{RC}{2}}{1 + s\frac{L}{D'^2 R} + s^2 \frac{LC}{D'^2}}$$

For Asymptotic behavior of $T_{vu}(s)$ around f_{cv}

$$T_{vu} = H \cdot \frac{D'R}{2R_f} \cdot \frac{1 - s\frac{L}{D'^2 R}}{1 + s\frac{RC}{2}} \to H \cdot \frac{D'R}{2R_f} \cdot \frac{1}{1 + s\frac{RC}{2}} = H \cdot \frac{D'}{R_f sC}$$

We would only need a simple PI compensator

$$G_{cv}(s) = G_{vm}(1 + \frac{\omega_{zv}}{s})$$

We want

$$G_{vm} \cdot H \cdot \frac{D'}{R_f sC} = 1$$

$$G_{vm} = \frac{2\pi f_{cv} C R_f}{D'H} = \frac{2\pi \cdot 10kHz \cdot 47uF \cdot 0.1}{(1 - 0.52)\frac{1}{20}} = 12.3$$

Choose the voltage divider for H, also these two resistors can be seen as a series resistor by Thevenin Theorem.

$$R_a = 19k\Omega$$
$$R_b = 1k\Omega$$

Required Resistors and Capacitors value

$$R_v = \frac{R_a \cdot R_b}{R_a + R_b} \cdot G_{vm} = \frac{19k \cdot 1k}{20k} \cdot 12.3 = 950 \cdot 12.3 = 11.7k\Omega$$

By choice set f_{zv} well below f_{cv}, f_{pv} much higher than f_{cv}

$$f_{zv} = 3kHz$$
$$f_{pv} = 100kHz$$

Approximate Phase margin:

$$\varphi_m = 90 - \tan^{-1}\frac{f_{zv}}{f_{cv}} = 90 - \tan^{-1}\left(\frac{3}{100}\right) = 73.3°$$

$$C_v = \frac{1}{2\pi R_v f_{zv}} = \frac{1}{2\pi \cdot 11.7k \cdot 3k} = 4.5nF$$

$$C_p = \frac{1}{2\pi R_v f_{pv}} = \frac{1}{2\pi \cdot 11.7k \cdot 100k} = 136pF$$

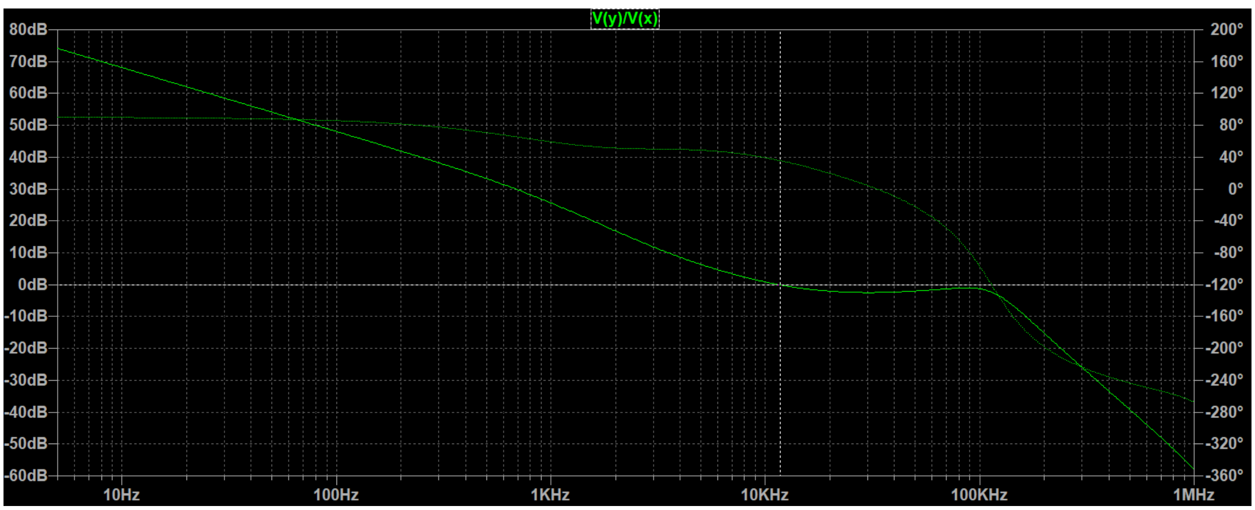

Figure 5. Outer Voltage Loop gain

The crossover frequency happened at 11.5kHz with 36º Phase Margin, 0.0115 of f_{sw}

Operating Point 1

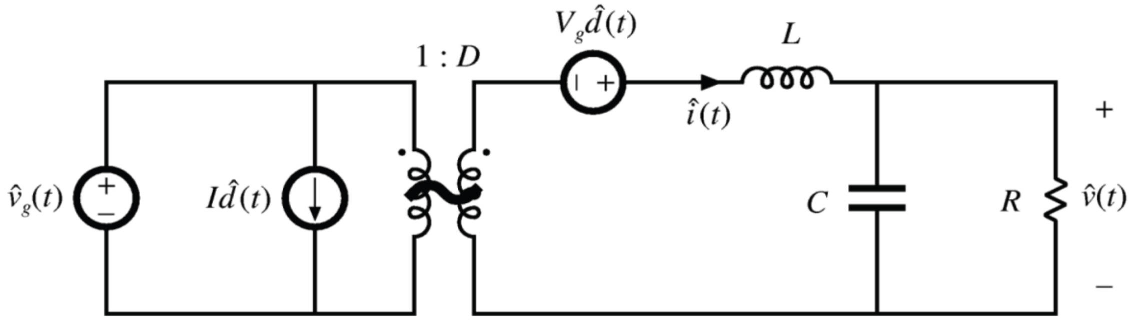

Figure 6. Buck small-signal equivalent circuit model

I chose Voltage Mode for operating point 1 for a simpler and less component design.

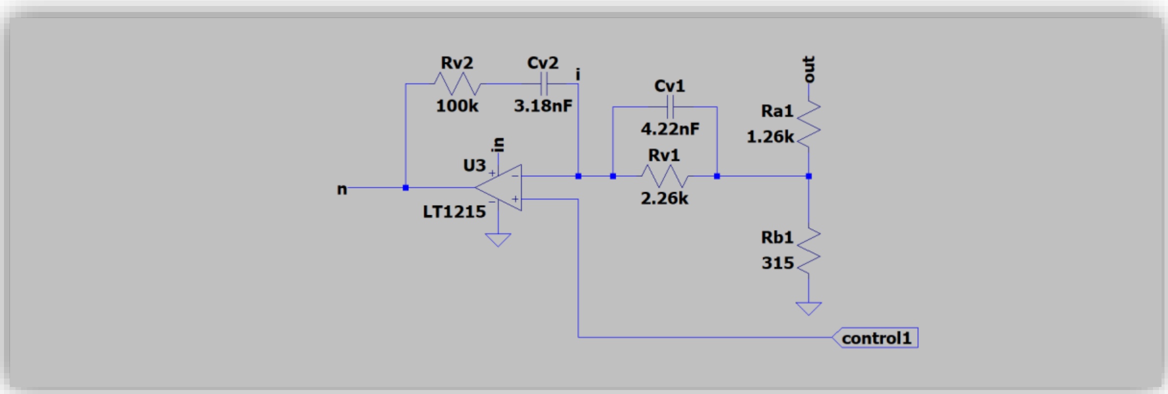

Figure 7. Voltage Mode Control Compensator

First calculate the approximate lossless required duty cycle for the operating point.

$$D = \frac{V_{out}}{V_g} = \frac{5}{12.6} = 0.396825$$

Actual Duty Cycle with losses:

$$D_{real} = 0.4032$$

$$G_{vd}(s) = \frac{V_{out}}{D} \cdot \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

$$G_{d0} = V_g = \frac{V_{out}}{D} = \frac{5}{0.396825} = 12.6V$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{18uH \cdot 47\mu F}} = 5.47kHz$$

$$Q = R\sqrt{\frac{C}{L}} = 2.5\sqrt{\frac{47}{18}} = 4.04$$

$$T = G_c \cdot \frac{1}{V_M} \cdot G_{vd} \cdot H$$

By choice

$$V_{ref} = 1V$$
$$H = \frac{1}{5}$$

Uncompensated Loop DC Gain

$$T_{u0} = H \cdot \frac{1}{V_M} \cdot V_g = \frac{1}{5} \cdot \frac{1}{4} \cdot 12.6 = 0.63 = -4.01dB$$

By choice, choosing 50kHz as my crossover frequency f_c, and for f_z and f_p with desirable 53º Phase Margin

$$f_z = f_c\sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}} = 16.7kHz$$

$$f_p = f_c\sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}} = 150kHz$$

For Asymptotic behavior of $T(s)$ around f_c

$$T_{uo} \cdot (\frac{f_o}{f_c})^2 \cdot G_{co} \cdot \sqrt{\frac{f_p}{f_z}} = 1$$

$$G_{co} = \frac{1}{T_{uo}} \cdot (\frac{f_c}{f_o})^2 \cdot \sqrt{\frac{f_z}{f_p}} = \frac{1}{0.63} \cdot (\frac{50}{5.47})^2 \cdot \sqrt{\frac{16.7}{150}} = 44.25 = 33dB$$

By choice, $10f_L < f_c$,

$$f_L = 500Hz$$

Required Resistors and Capacitors value

$$G_{co} = \frac{R_{v2}}{R_{v1}}$$
$$R_{v2} = \frac{1}{2\pi f_L C_{v2}}$$
$$R_{v1} = \frac{1}{2\pi f_z C_{v1}}$$
$$R_3 = \frac{1}{2\pi f_p C_{v1}}$$

By choice, $10f_L < f_c$, and default resistance of R_{v2}

$$f_L = 500Hz$$
$$R_{v2} = 100k\Omega$$

$$R_{v1} = \frac{R_{v2}}{G_{co}} = \frac{100k\Omega}{44.25} = 2.26k\Omega$$

$$C_{v2} = \frac{1}{2\pi f_L R_{v2}} = \frac{1}{2\pi \cdot 500Hz \cdot 100k\Omega} = 3.18nF$$

$$C_{v1} = \frac{1}{2\pi f_z R_{v1}} = \frac{1}{2\pi \cdot 16.7kHz \cdot 2.26k\Omega} = 4.22nF$$

$$R_3 = \frac{1}{2\pi f_p C_{v1}} = \frac{1}{2\pi \cdot 150kHz \cdot 4.22nF} = 252\Omega$$

Choose the voltage divider for H, also these two resistors can be seen as a series resistor by Thevenin Theorem to replace R_3

$$H = \frac{R_{b1}}{R_{a1} + R_{b1}} = \frac{1}{5}$$

$$R_3 = \frac{R_{a1} \cdot R_{b1}}{R_{a1} + R_{b1}} = H \cdot R_{a1}$$

$$R_{a1} = \frac{R_3}{H} = \frac{252}{0.2} = 1.26k\Omega$$

$$R_{b1} = R_{a1} \frac{H}{1 - H} = 1.26k\Omega \cdot \frac{0.2}{0.8} = 315\Omega$$

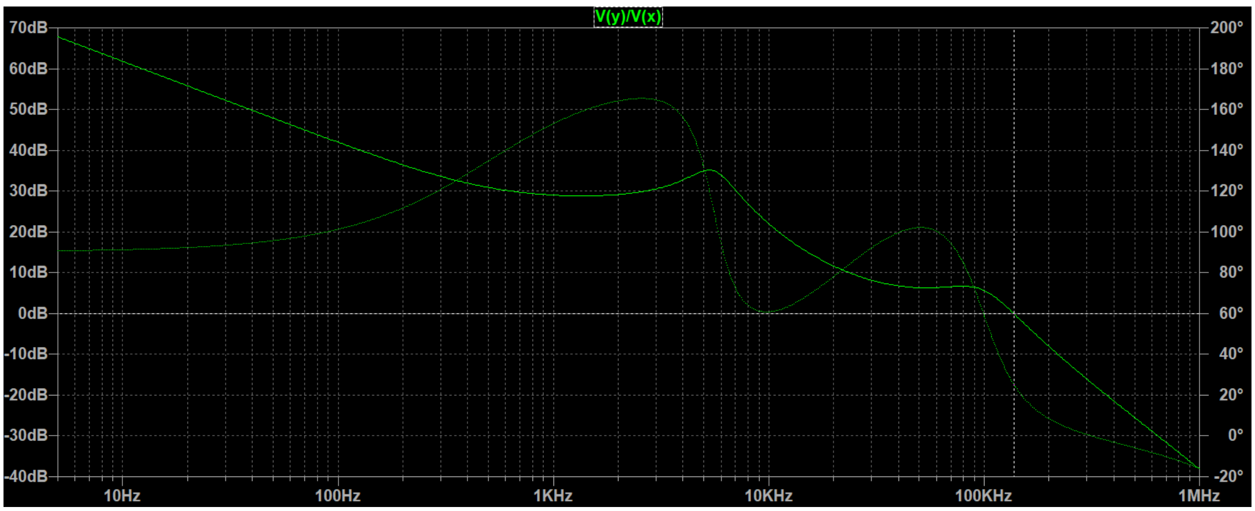

Figure 8. Operating point 1 Voltage Mode Loop Gain

The crossover frequency happened at 137kHz with 26⁰ Phase Margin, 0.137 of f_{sw}

2. Simulation

Place your Milestone 2 LTspice files into a dedicated folder. Make sure all simulations run correctly using the files in that folder. Remove all .raw files, but include all files necessary to run the required simulations (all .sch. .asy, .lib files). Create a .zip file of the folder and upload this .zip file. The folder must include a single schematic (.sch) file that contains the template header from milestone2.sch and the averaged circuit model of your closed-loop converter. The template tests your converter averaged model at two operating points, in this order

1. USB output 20V at 3A, Vbatt = 9.6V (operating point 2)
2. USB output 5V at 2A, Vbatt = 12.6V (operating point 1)

In the template header you should adjust the parameters **t1**, and **Tend** as necessary so that the template measurements of the two operating points are made in steady state. Your simulation should use the control signal parameters **Vref1 5V**, **Vref1 20V**, (and **Vref2 5V**, **Vref2 20V** if the second control signal source is used), to set the references for your control loop(s) as necessary to reach the two operating points defined above, and you should adjust these parameters as well.
Also define the switching period parameter **Ts** so that the template runs ac simulation over an appropriate range of frequencies for your design.

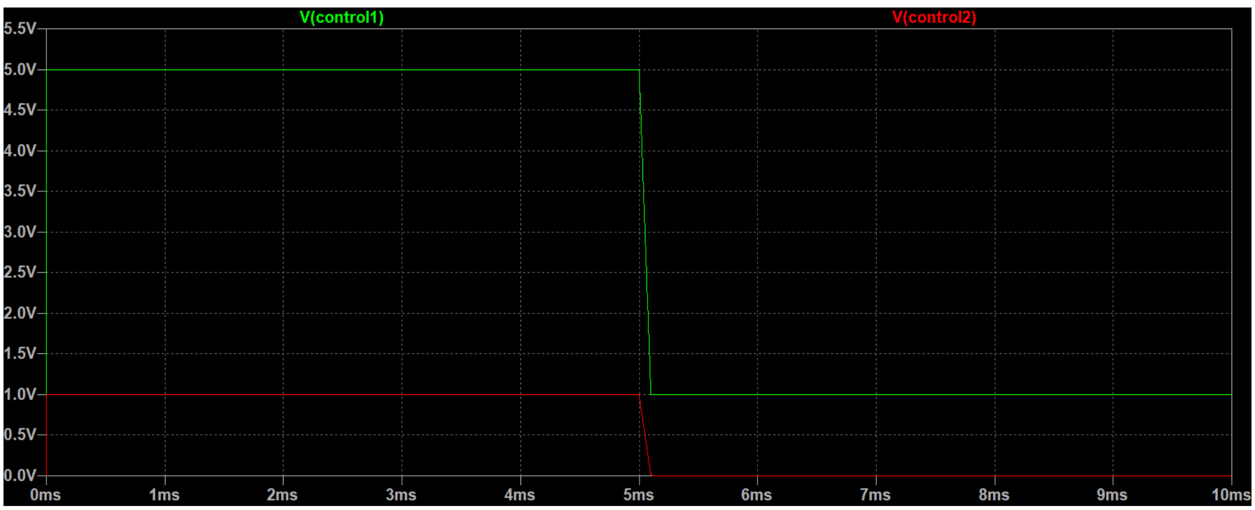

Figure 9. V(control1) and V(control2)

.param Vref1_5V=1 Vref1_20V=5
.param Vref2_5V=0 Vref2_20V=1

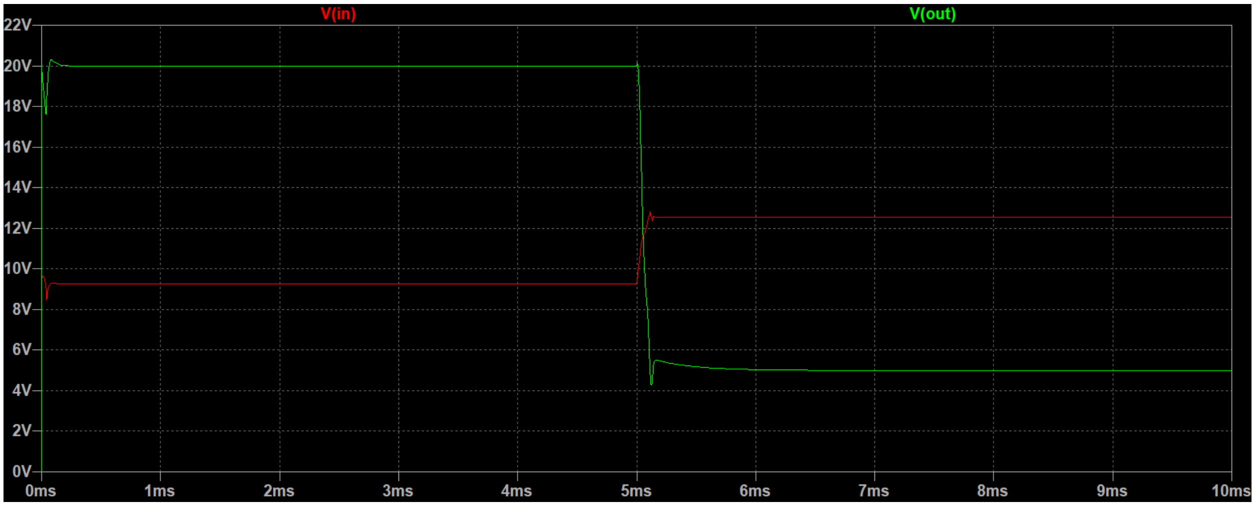

Figure 10. V(in) and V(out)

```
start_20v: 0.00499=0.00499
end_20v: 0.005=0.005
start_5v: 0.009989=0.009989
end_5v: 0.009999=0.009999
vout_20v: AVG(v(out))=19.9926 FROM 0.00499 TO 0.005
vout_5v: AVG(v(out))=4.99464 FROM 0.009989 TO 0.009999
```

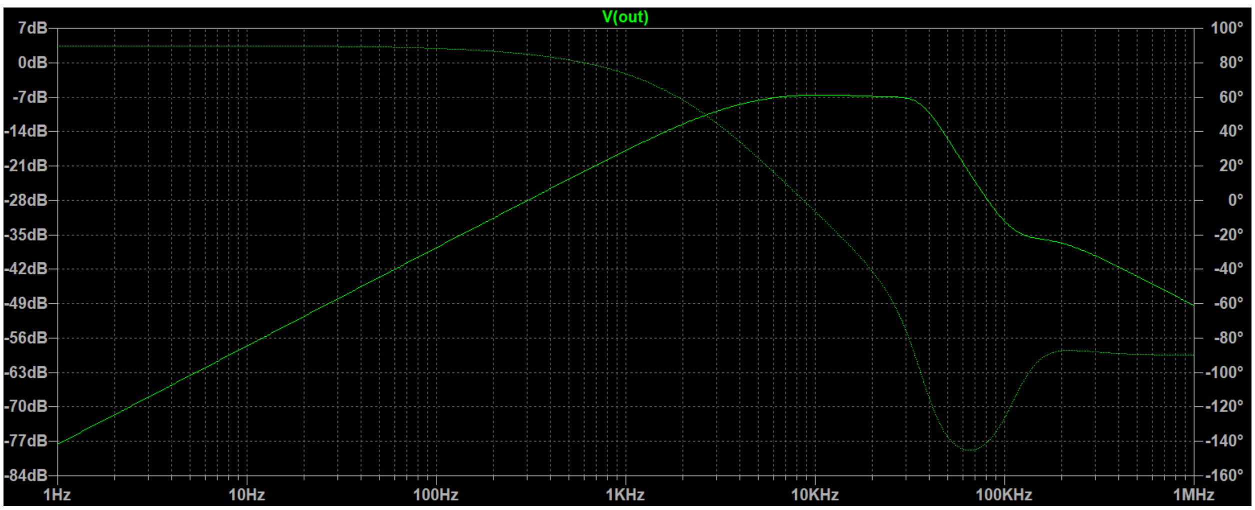

Figure 11. V(out) as Output Impedance plot

```
zout_20v_db: MAX(mag(v(out)))=(-6.53305dB,0°) FROM 1 TO 1e+006
zout_20v_ohms: 10^(zout_20v_db/20)=(0.471354dB,0°)
```

The Output Impedance is measured as 0.47Ω around 8.9kHz