QUANTUM COMPUTERS

The motivation behind Quantum Computing dates back to the year 1981, when the nobel laureate Richard Feynman questioned about the kind of computer which could possibly used to simulate events of physics. The answer is quantum! For the nature isn’t classical ,its measurement can be made possible only when it is made Quantum Mechanical. Hence sets of experiments are performed that deal with some fundamental and nobel aspects of Quantum Mechanics using Quantum Computers hosted by IBM QE.

The basic elements of information processing in quantum computers are Qubits that are analogous to the classical bits. The quantum computers are accessible through the cloud using Qiskit (open source software development for working with quantum computers). The Qiskit is based on the python programming language.

The quantum computers in IBM QE use a physical type of qubit called a superconducting transmon qubit, which is produced from superconducting materials such as niobium and aluminum, patterned on a silicon substrate.

QUBITS

The basic building blocks of information processing in quantum computers are called qubits. Qubits are the analogous version of bits of classical computers that are used as fundamental unit in information processing in quantum computing. A bit has generally two states **0** or **1** regarding the case of low or high voltage applied respectively from circuit theory. Similarly a qubit has also two states but these are quantum states which live in two-dimensional Hilbert space of the system. A general quantum state $|\psi\rangle$ is represented as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,\ \alpha \text{ and } \beta \text{ are complex scalars.}$$

where $|\alpha|^2$ and $|\beta|^2$ are the probability of finding the state along the direction of $|0\rangle$ and $|1\rangle$ respectively. Hence total probability is given as

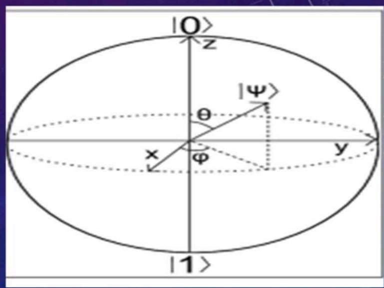
$$|\alpha|^2 + |\beta|^2 = 1$$

In quantum computers to get the probability of the states experiments are run number of times this is due to the variation in results every time when a new experiment is run.

$|0\rangle$ and $|1\rangle$ are standard basis, other basis used are $\{|+\rangle, |-\rangle\}$ named as Hadamard basis.

VISUALIZATION OF QUBIT

The visualization of qubit is an another type of representation of qubit called as The Bloch Sphere which is more versatile and help us to visualize in an easy way when it comes to the phase changes.



In figure ,the state vectors $|0\rangle$ and $|1\rangle$ are at north and south poles of the sphere respectively, where θ is the altitude angle and ϕ is azimuthal angle.

This is the representation of a single qubit system.

Considering the quantum state

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are complex scalars.

$$|\alpha|^2 + |\beta|^2 = 1$$

Which gives the total probability. { $|0\rangle = \binom{1}{0}$, $|1\rangle = \binom{0}{1}$ }

Now as for complex quantity we know

$$Z = x + iy = re^{i\theta}$$

So,

$$|\psi\rangle = r_\alpha e^{i\phi_\alpha}|0\rangle + r_\beta e^{i\phi_\beta}|1\rangle \text{ where } r_\alpha\,,\ r_\beta\,,\ \phi_\alpha\,,\ \phi_\beta \in R\,.$$

Which shows the dependence on 4 parameters and again,

$$e^{-i\phi_\alpha}\,|\psi\rangle = r_\alpha|0\rangle + r_\beta e^{i(\phi_\beta-\phi_\alpha)}|1\rangle$$

$$|\psi'\rangle = r_\alpha|0\rangle + r_\beta e^{i\phi}|1\rangle \text{ (reduced to 3 parameters)}$$

$$|\psi'\rangle = r_\alpha|0\rangle + (x+iy)|1\rangle$$

so the state vector defined by a point on the surface of unit 3D sphere fixed by θ, ϕ

Such that we can map points by defining

$$|\psi'\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$$

But in general we could have more than one qubit system. In that case we'll use the concept of composite system to work on.

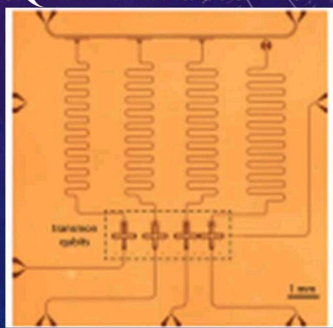
QUANTUM COMPUTERS AND QUANTUM GATES

Continuing with the theory of transmons manipulation of the transmon qubit state for initialization or other subsequent transformation of the qubit state so prepared has given rise to the field of circuit quantum electrodynamics cQED that uses the quantum dynamics of the electromagnetic field

in superconducting circuits.

An diagram of a cQED with 4 transmons is given

in the figure.



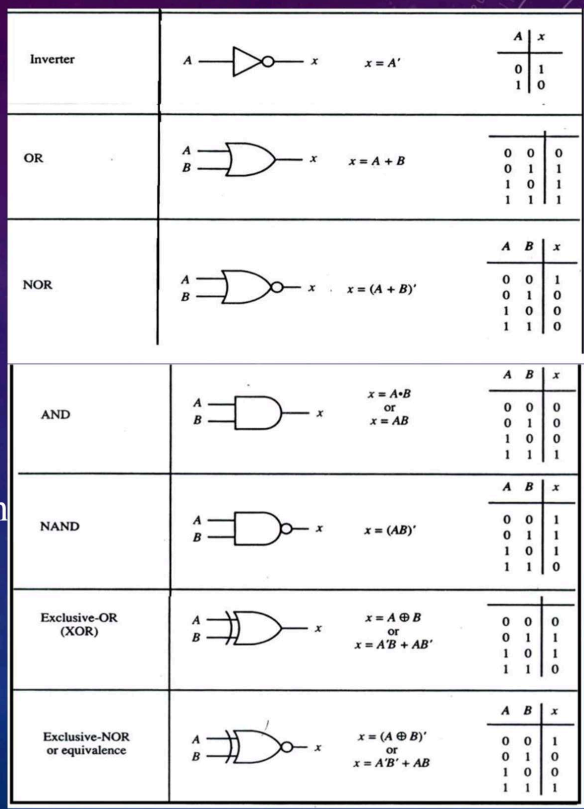
The IBMQ computers are based on a qubit that is essentially

a very small oscillator that is not harmonic but with a nonlinear tunable inductance to introduce a certain nonlinearity in the system so that two particular eigenstates could be selected to form a qubit that could be isolated from the other eigenstates for a given frequency of the electromagnetic radiation used to manipulate the two states.

Now moving onto the quantum gates that are analogous to classical gates in some way,

let's first deal with the classical gates.

The main purposes of computers are data inputting, manipulating, storing and outputting. some machinery is required to handle these bits. These are basically known as logic gates in classical computers and these are taken care of by the Boolean algebra.



		A	B	x
Inverter	$x = A'$	0		1
		1		0
OR	$x = A + B$	0	0	0
		0	1	1
		1	0	1
		1	1	1
NOR	$x = (A + B)'$	0	0	1
		0	1	0
		1	0	0
		1	1	0
AND	$x = A \bullet B$ or $x = AB$	0	0	0
		0	1	0
		1	0	0
		1	1	1
NAND	$x = (AB)'$	0	0	1
		0	1	1
		1	0	1
		1	1	0
Exclusive-OR (XOR)	$x = A \oplus B$ or $x = A'B + AB'$	0	0	0
		0	1	1
		1	0	1
		1	1	0
Exclusive-NOR or equivalence	$x = (A \oplus B)'$ or $x = A'B' + AB$	0	0	1
		0	1	0
		1	0	0
		1	1	1

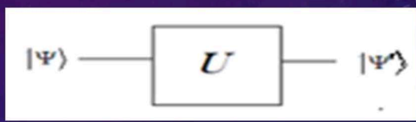
Like logic gates in classical computers which manipulates bits of information (0's and 1's), there is an analogous concept used in quantum computers called as quantum logic gates or quantum gates which manipulates quantum states.

Quantum gates are reversible(invertible)

The figure summarizes the various quantum gates acting on a qubit.

	Gate	Operator symbol	Operator	Matrix
σ_x operator	$\boxed{\mathrm{X}}$	σ_x	$\|0\rangle\langle 1\| + \|1\rangle\langle 0\|$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
σ_y operator	$\boxed{\mathrm{Y}}$	σ_y	$-i(\|0\rangle\langle 1\| - \|1\rangle\langle 0\|)$	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
σ_z operator	$\boxed{\mathrm{Z}}$	σ_z	$\|0\rangle\langle 0\| - \|1\rangle\langle 1\|$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
NOT	$\oplus$	NOT, σ_x	$\|0\rangle\langle 1\| + \|1\rangle\langle 0\|$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\sqrt{\mathrm{NOT}}$	$\boxed{\sqrt{\mathrm{NOT}}}$	$\sqrt{\mathrm{NOT}}$	$e^{i\frac{\pi}{4}\sigma_x}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$
x rotation	$\boxed{\mathrm{X}(\phi)}$	$R_x(\phi)$	$e^{-i\frac{\phi}{2}\sigma_x}$	$\begin{pmatrix} \cos\frac{\phi}{2} & -i\sin\frac{\phi}{2} \\ -i\sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{pmatrix}$
y rotation	$\boxed{\mathrm{Y}(\phi)}$	$R_y(\phi)$	$e^{-i\frac{\phi}{2}\sigma_y}$	$\begin{pmatrix} \cos\frac{\phi}{2} & -\sin\frac{\phi}{2} \\ \sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{pmatrix}$
z rotation	$\boxed{\mathrm{Z}(\phi)}$	$R_z(\phi)$	$e^{-i\frac{\phi}{2}\sigma_z}$	$\begin{pmatrix} e^{-i\frac{\phi}{2}} & 0 \\ 0 & e^{i\frac{\phi}{2}} \end{pmatrix}$
Phase	$\boxed{\phi(\alpha)}$	$\phi(\alpha)$	$\|0\rangle\langle 0\| + e^{i\alpha}\|1\rangle\langle 1\|$	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$
Hadamard	$\boxed{\mathrm{H}}$	H	$\frac{1}{\sqrt{2}}(\sigma_x + \sigma_z) = e^{-i\frac{\pi}{4}\sigma_y}\sigma_z$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Consider a qubit in an arbitrary state $|\psi\rangle$ and a quantum gate U acting on the state vector $|\psi\rangle$ as follows



$$U|\psi\rangle = |\psi'\rangle$$

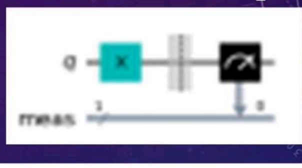
From the fact that all quantum evolution preserves the norm of the state vector we have

$$\langle\psi|\psi\rangle = \langle\psi'|\psi'\rangle$$

Or, $U^{\dagger}U = I$

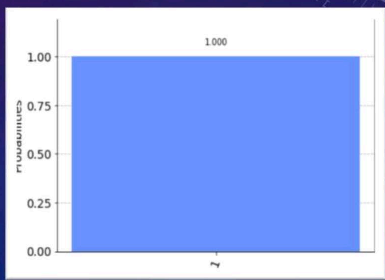
Therefore U is a unitary.

Introducing examples of quantum gates that are run on IBM QE as an experiment by using the programming language python.



- ❑ **Pauli X Gate**

When it acts on $|0\rangle$: $X|0\rangle = |1\rangle$.



- ❑ **Controlled- Not Gate**

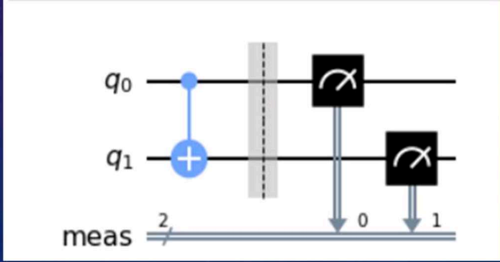
The controlled-NOT gate or **CNOT** gate is an useful gate that operates on quantum registers having two qubits. It flips the second qubit of the two qubit system if and only if the first qubit is in state $|1\rangle$. CNOT is used to generate entangled state.

The matrix representation of **CNOT** is given as

$$C_{NOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

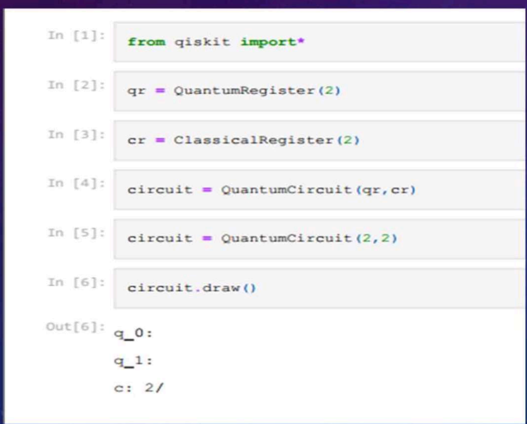
The quantum circuit of **CNOT** gate is shown below

```
cir = QuantumCircuit(2)
cir.cx(0,1)
cir.measure_all()
%matplotlib inline
cir.draw(output='mpl')
```



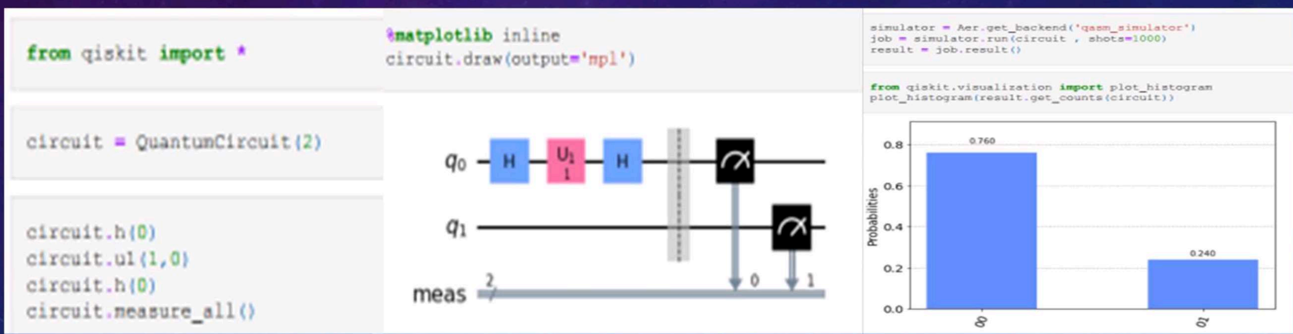
PROGRAMING IN QUANTUM COMPUTERS

Programming in Quantum computers using IBM QE. We now move further with various experiments run by python codes in IBM QE using jupyter notebook interface illustrated below.



➢ MACH-ZEHDNER INTERFEROMETER (MZI) EXPERIMENT

The interferometer demonstrates the principle of superposition of electromagnetic waves to create interference. In the quantum version of Mach-Zehnder interferometer interference phenomenon of one quantum state $|\psi\rangle$ is observed. Shown below is the code setting up the quantum circuit with the resulting probability amplitudes .



The quantum circuit for the MZI is set up by taking the Hadamard gates and phase shift gates into consideration. The unitary transformation at the beam splitter is dealt by the Hadamard gate and phase shifter gate shifts the phase of the qubit.

So the following process takes place

$$|0\rangle \rightarrow (\boldsymbol{H}) \rightarrow \frac{1}{\sqrt{2}}(\,|0\rangle + \,|1\rangle)$$

$$\frac{1}{\sqrt{2}}(\,|0\rangle + \,|1\rangle) \rightarrow (\boldsymbol{\phi}) \rightarrow \frac{1}{\sqrt{2}}(\,|0\rangle + \,e^{i\phi}|1\rangle)$$

$$\frac{1}{\sqrt{2}}(\,|0\rangle + \,e^{i\phi}|1\rangle) \;\rightarrow (\boldsymbol{H}) \rightarrow \; e^{i\phi}[\,\cos(\frac{\phi}{2})|0\rangle + \,\sin(\frac{\phi}{2})|1\rangle]$$

Where '***H***' is the Hadamard gate and '$\boldsymbol{\phi}$' is the phase shifter gate.

We can obtain results by running the experiments on IBM QE for various phase shifters acting in the setup of MZI by changing the value of $\boldsymbol{\phi}$.

➢ ENTANGLEMENT AND BELL STATES EXPERIMENTS

When two or more quantum system interact to generate a composite state it may so happen that the composite quantum state is not separable as tensor product of the component states. Such states are called entangled states which have no classical analogs. Bell states are a set of entangled states generated from two qubits.

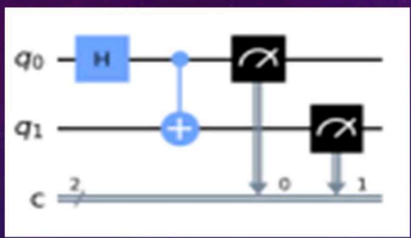
The four bell states are:

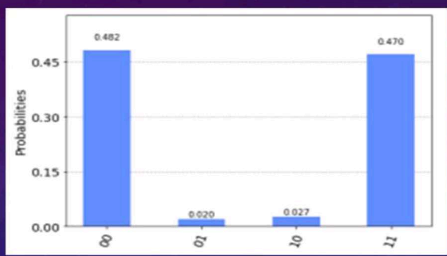
$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(\,|01> - |10>)\ ,\quad |\psi^+\rangle = \frac{1}{\sqrt{2}}(\,|01> + |10>)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(\,|00> - |11>)\ ,\quad |\phi^+\rangle = \frac{1}{\sqrt{2}}(\,|00> + |11>)$$

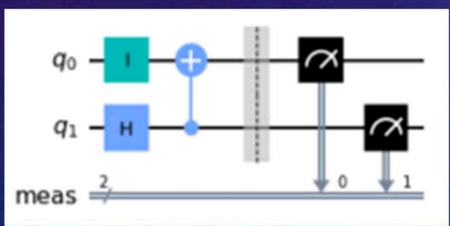
entanglement and bell states are demonstrated in the following experiment in IBM QE .

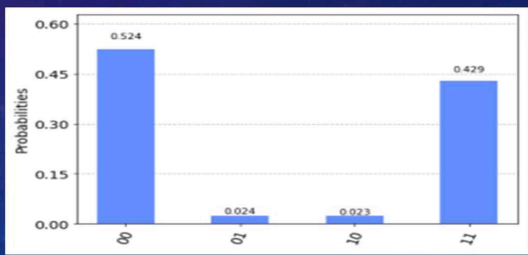
Quantum Circuit and probability amplitudes result for entanglement are below.





Quantum Circuit and probability amplitudes result for Bell state $|\phi^+\rangle$ are below.





➢ QUANTUM TELEPORTATION EXPERIMENT

Teleportation is application of quantum entanglement that allows one (Alice) to send a quantum state $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$ to another person (Bob). The novelty of this procedure is that the quantum state itself is not sent using any tangible process. The only piece of information sent in an usual sense is the result of some measurement made by Alice that is communicated by her to Bob. The procedure can be discussed in the following steps (→)

$$\rightarrow \quad |\beta_{00}> = \frac{1}{\sqrt{2}}(|00> + |11>)$$

$$\rightarrow \quad |\psi\rangle = |\chi\rangle \otimes |\beta_{00}\rangle$$

$$\rightarrow \quad |\psi'\rangle = U_{CNOT}|\psi\rangle$$

$$\rightarrow \quad |\psi' > = \frac{1}{\sqrt{2}}(\alpha|000> + \alpha|011> + \beta|110> + \beta|101>)$$

$$\rightarrow \quad |\psi''> = H\,|\psi'>$$

$$\rightarrow |\psi''> = \frac{1}{2}[\alpha(|000> + |100> + |011> + |111>) + \beta(|010> - |110> + |001> - |101>)]$$

So, for Bob to have the state $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$ he must know the result Alice obtained when she measured her part of the state.

For Alice measuring |00> Bob gets directly | χ>|

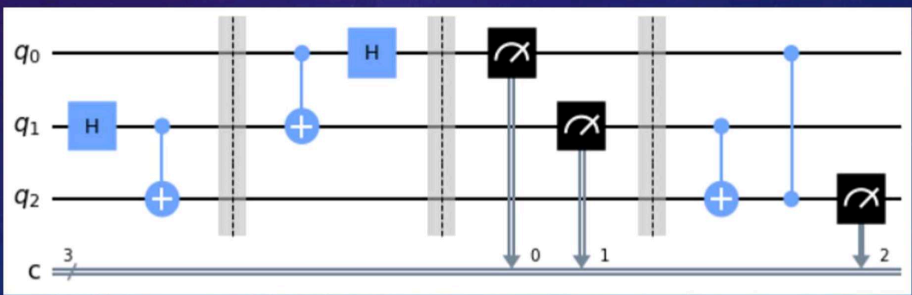
For Alice measuring |01> Bob applies X gate: X(α|1> + β|0>) = |χ>

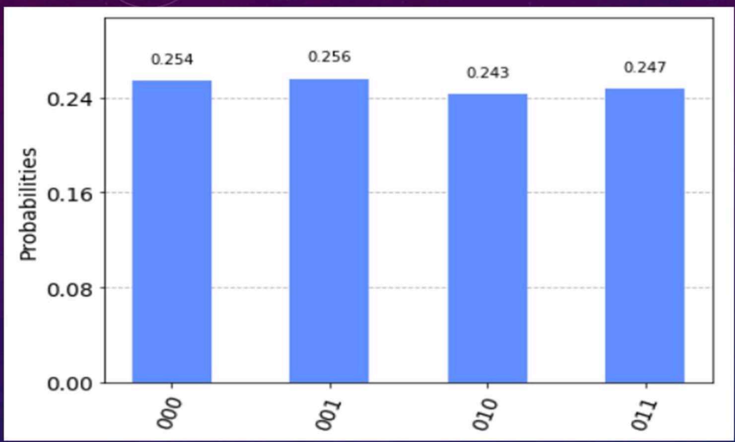
For Alice measuring |10> Bob applies Z gate: Z(α|0> - β|1>) = |χ>

For Alice measuring |11> Bob applies, ZX(α|1> - β|0>) = |χ>

Where X and Z are Pauli's gates.

This procedure of quantum teleportation is obtained by performing the experiment in the quantum computer IBM QE. Hence we got the below mentioned results that is the circuit setup and the probability amplitude of the entangled states.





This confirms the fact that the results we have got in the above probability amplitude matches the theory we have discussed earlier in this section. This completes the teleportation as in the starting two people Alice and Bob shared a pair of qubits in the entangled state.

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