# Homework 2

Kai Aragaki

## Problem 1

### A

We can calculate the expectation value from the formula

$E[X] = \int_{-\infty}^{\infty} xf(x)dx$

We know that the area under the curve of a probability density function must equal 1. Additionally, we know that the uniform distribution is a rectangle with a length along the bottom equal to $b - a$. Therefore, the height must equal $\frac{1}{b-a}$. Since this is $f(x)$, our integral has simplified:

$E[X] = \frac{1}{b-a} \int_{-\infty}^{\infty} xdx = \frac{1}{b-a} |_{-\infty}^{\infty} x^2$

We also know that the uniform function is zero everywhere not between $a$ and $b$, simplifying this equation to

$E[X] = \frac{1}{b-a} |_a^b x^2 = \frac{1}{2}(a + b)$

So,

$E[X] = 7.5$

### B

The probability for any particular point for a probability density function is 0

### C

We know the cumulative distribution function is the integral of the probability density function from $-\infty$ to $t$. In part **A** we found

$f(x) = \frac{1}{b-a}$

Since we know the probability is 0 up to $a$, the CDF becomes

$|_a^t x \frac{1}{b-a} = \frac{t}{b-a} - \frac{a}{b-a} = \frac{t-a}{b-a}$

Therefore

$Pr(U > 6) = 1 - Pr(U \leq 6) = 1 - \frac{6-5}{10-5} = 1 - \frac{1}{5} = \frac{4}{5}$

### D

Using the above formula,

$Pr(7 < U < 9) = Pr(U < 9) - Pr(U < 7) = \frac{9-5}{10-5} - \frac{7-5}{10-5} = \frac{4}{5} - \frac{2}{5} = \frac{2}{5}$

# Problem 2

## A

```r
p2a <- pnorm(6, 5, 3)
p2a
```

```
## [1] 0.6305587
```

Roughly a 63% probability

## B

```r
p2b <- pnorm(0, 5, 3, lower.tail = FALSE)
p2b
```

```
## [1] 0.9522096
```

Roughly a 95% probability

## C

```r
p2c <- pnorm(5, 5, 3) - pnorm(0, 5, 3)
p2c
```

```
## [1] 0.4522096
```

Roughly a 45% probability

## D

```r
p2d <- pnorm(8, 5, 3) - pnorm(2, 5, 3)
p2d
```

```
## [1] 0.6826895
```

Roughly a 68% probability

## E

This essentially changes the distribution of $X$ to be $X \sim Normal(mean = 0,\ SD = 3)$

```r
p2e <- pnorm(2, 0, 3, lower.tail = FALSE)
p2e
```

```
## [1] 0.2524925
```

Roughly a 25% probability

# Problem 3

We know that if $Y = a + bX$ then $E(Y) = a + bE(X)$.

In this case, $Y = (X - 7) \times 100 = -700 + 100X$, making $a = -700$ and $b = 100$ so

$E(Y) = -700 + 100E(X) = -700 + 100 \times 7.373 = 37.3$

Additionally, we know that $SD(Y) = |b|SD(X)$. Using the information above:

$SD(Y) = |100| \times 0.129 = 12.9$

# Problem 4

## A

We know that when we draw from a population that is normally distributed with mean $\mu$ and standard deviation $\sigma$, the sampling distribution is $\bar{X} \sim Normal(\mu, \sigma/\sqrt{n})$ (provided the draws are iid). At 100 draws, we are told that what we see in real life should match up with the theoretical sampling distribution.

So we may ask, what is the chance of getting a number greater than 9.9 and less than 10.1 in the sampling distribution $Normal(10, 2.5/100)$?

```r
pnorm(10.1, 10, 0.025) - pnorm(9.9, 10, 0.025)
```

```
## [1] 0.9999367
```

It's almost certain that it will be within 0.1 units of the population mean.

## B

Using the same logic as above,

```r
pnorm(10.25, 10, 0.025, lower.tail = FALSE)
```

```
## [1] 7.619853e-24
```

As we may expect from the results seen in part A, there is a very slim chance that we will see a value that extreme.