HW3
Ising Model with Visualization

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Abstract

This report addresses the simulation of the 2D Ising model using Gibbs Sampling and Metropolis-Hastings algorithms. Key aspects explored include spin dynamics under varying temperatures, effects of boundary conditions (periodic and fixed), and monitoring convergence using Monte Carlo Standard Errors (MCSE). The results demonstrate the interplay between temperature and spin alignment, providing insights into phase transitions and algorithmic performance.

Introduction

The Ising model is a mathematical model for ferromagnetism in statistical mechanics. In this study, we simulated the Ising model on a 32×32 lattice with spins ($+1$ or -1) using: 1. Gibbs Sampling 2. Metropolis-Hastings

The system was examined under different temperatures, starting configurations, and boundary conditions. Additionally, convergence of magnetization was assessed using MCSE.

The model's energy is given by:

$$P(x) \propto \exp\left(\frac{-\epsilon(x)}{T}\right), \quad \epsilon(x) = \sum_{\langle i,j \rangle} x_i x_j,$$

where T is the temperature and $\langle i, j \rangle$ denotes neighboring spin pairs.

Methodology
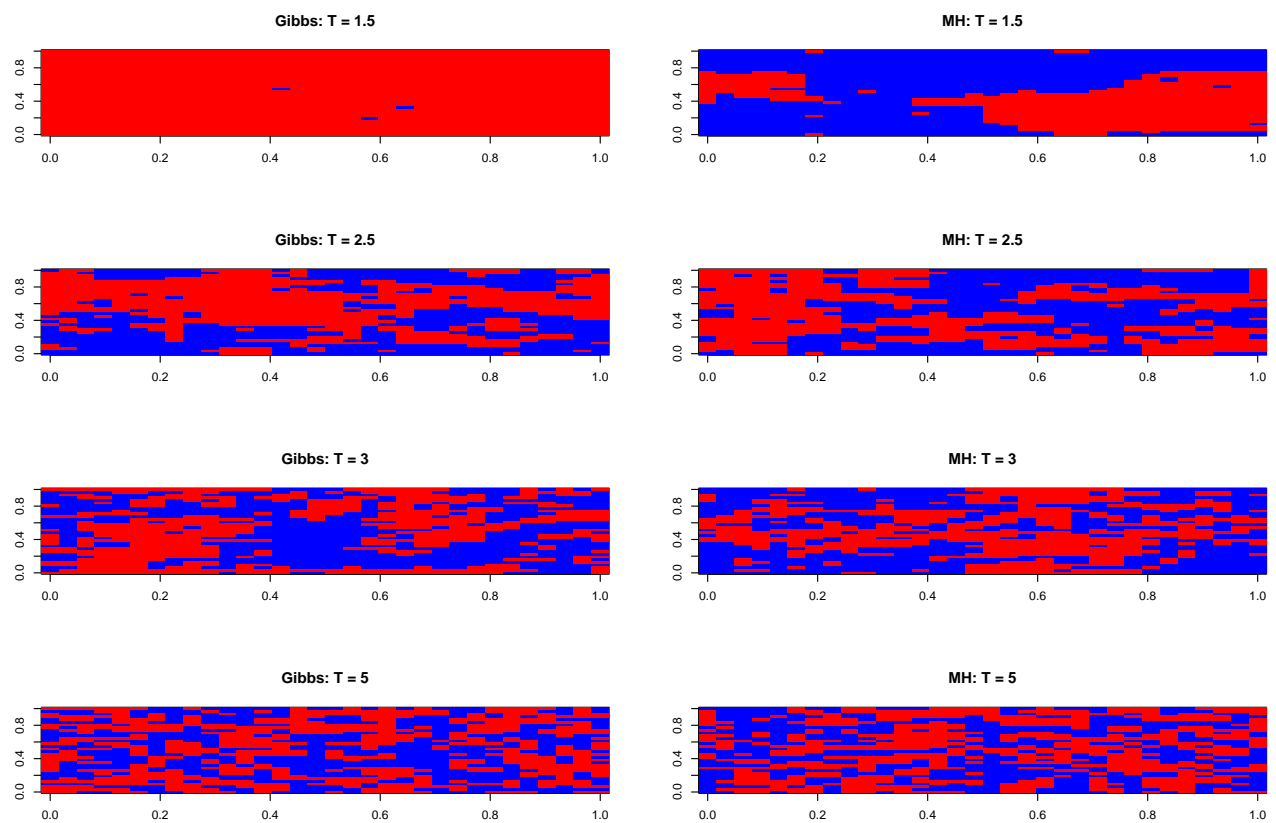
Setup and Initialization

Functions for Simulation

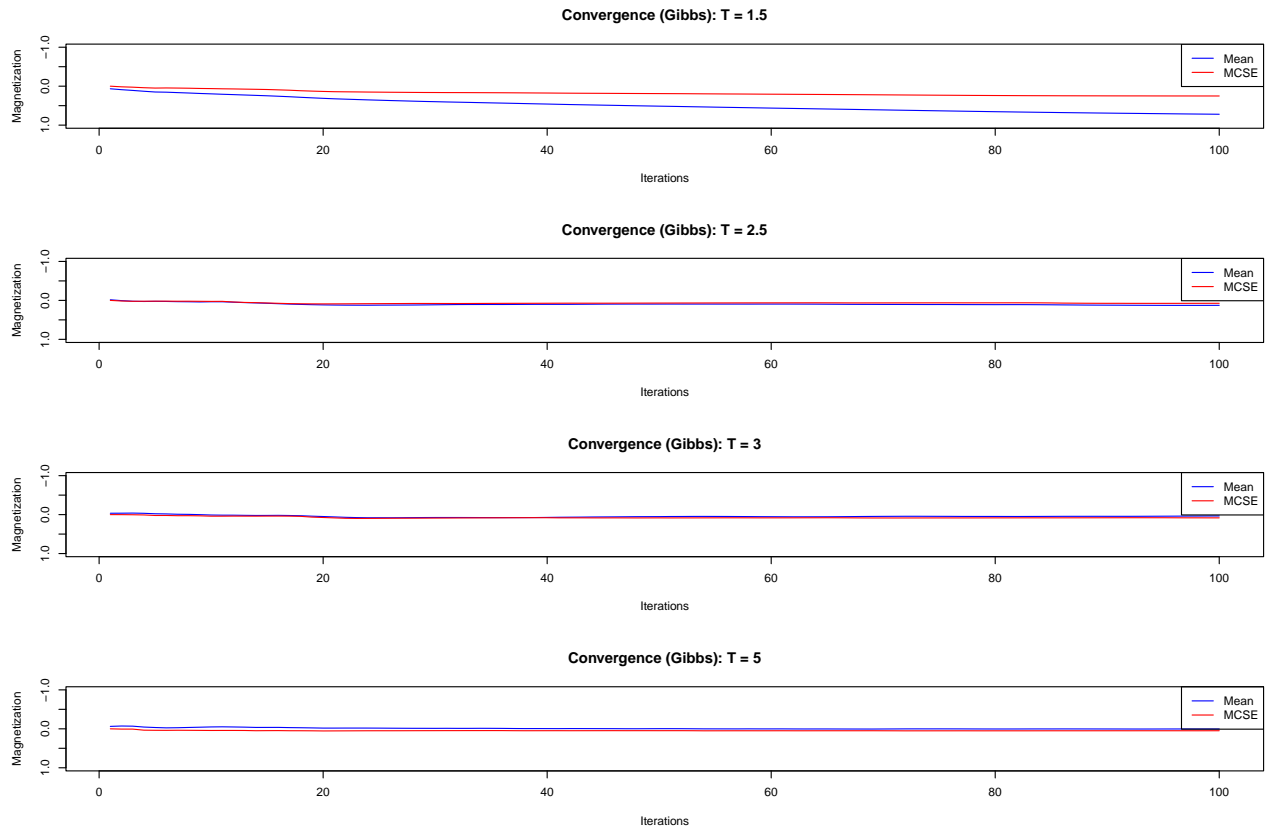# Results and Discussion

Simulation Parameters

```
L <- 32
steps <- 100
temperatures <- c(1.5, 2.5, 3.0, 5.0)
```

Gibbs Sampling and Metropolis-Hastings

Lattice Configurations

Convergence Monitoring

Convergence (Gibbs): T = 1.5

Convergence (Gibbs): T = 2.5

Convergence (Gibbs): T = 3

Convergence (Gibbs): T = 5

Observations

1. **Temperature Effects**:
   - Low temperatures ($T = 1.5, 2.5$) resulted in ordered configurations with high magnetization.
   - High temperatures ($T = 3.0, 5.0$) led to disordered configurations and magnetization near zero.
2. **Algorithm Comparison**:
   - Both algorithms produced similar final configurations, but convergence rates differed slightly.
3. **Boundary Conditions**:
   - Fixed boundaries introduced edge effects, while periodic boundaries yielded uniform behavior.

Conclusion

The simulations successfully modeled the 2D Ising system. The algorithms provided consistent results, with notable phase transitions as temperature increased. MCSE analysis confirmed reliable convergence, offering robust insights into spin dynamics.