Homework III

1. Implement Gibbs sampling algorithm and Metroplis-Hasting algorithm for simulating from an Ising model defined on a 32x32 grids. You may try different temperatures, T, different staring points and different boundary conditions. Here we set $\pi(x) \propto \exp(-\varepsilon(x)/T)$, where $\varepsilon(x) = \sum_{i \sim j} x_i x_j$ and $x_i \in \{-1,+1\}$. Note that the convergence of the MCMC should be assessed to ensure that the number of iterations is sufficient. For example, the Monte Carlo Standard Error (MCSE) can serve as one potential diagnostic metric. Another commonly used method is the Gelman-Rubin Statistic.

   A reference website: https://rajeshrinet.github.io/blog/2014/ising-model/
   A reference for MCMC convergence detection:
   https://bookdown.org/rdpeng/advstatcomp/monitoring-convergence.html

2. Generate 100 random numbers, $U_{0,k}, k = 1, \ldots, 10$ and $U_{i,j}, i \neq j, i, j = 1, \ldots, 10$. Now, consider a travelling salesman problem in which the salesman starts from city 0 and must travel in turn to each of the 10 cities, 1, …, 10, according to some permutation of 1, …, 10. Let $U_{i,j}$ be the reward earned by the salesman when he/she goes directly from city i to city j. Use Simulated Annealing (SA) algorithm to find the maximal possible return for the salesman.

   A reference website: https://medium.com/@francis.allanah/travelling-salesman-problem-using-simulated-annealing-f547a71ab3c6

   https://medium.com/hunter-cheng/python-%E6%A8%A1%E6%93%AC%E9%80%80%E7%81%AB%E6%BC%94%E7%AE%97%E6%B3%95-simulated-annealing-sa-%E6%B1%82%E8%A7%A3%E6%97%85%E8%A1%8C%E6%8E%A8%E9%8A%B7%E5%93%A1%E5%95%8F%E9%A1%8C-traveling-salesman-problem-tsp-9971c9dc23f6