

1 main derivatives

main derivatives to remember

$$(f \pm g)' = f' \pm g'$$

$$(f \times g)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g + fg'}{g^2}$$

$$\frac{df(u)}{dx} = \frac{df}{du} \times \frac{du}{dx}$$

$$\frac{d}{dx}|u| = \frac{u}{|u|} \times \frac{du}{dx}$$

$$[\log_a(x)]' = \frac{1}{x \ln(a)}$$

$$[\ln(x)]' = \frac{1}{x}$$

$$(a^x)' = a^x \ln(a)$$

$$s(t) \sim \text{position}$$

$$s'(t) \sim \text{velocity}$$

$$s''(t) \sim \text{acceleration}$$

$$s'(t) = 0 \sim \text{stationary}$$

$$f'(x) > 0 \sim \text{incr. / right mov.}$$

$$f''(x) > 0 \sim \text{conc. up / incr.}$$

$$f''(x) = 0 \sim \text{point of inflection}$$

$$f'(x) < 0 \sim \text{decr. / left mov.}$$

$$f''(x) < 0 \sim \text{conc. down / decr.}$$

2 tangent line

tangent line to function $f(x)$ at $x = a$

$$y = mx + b$$

$$y = f(a)$$

$$m = f'(a)$$

$$x = a$$

$$b = f(a) - [f'(a) \times a]$$

Note –

$$f'(a) \times g'(a) = -1 ; \text{ if } g'(a) = -\frac{1}{f'(a)}$$

3 trig notes

important trig stuff to remember and trig inverse properties

$$\frac{\sin(x)}{\cos(x)} = \tan(x)$$

$$(\sin(x))' = \cos(x)$$

$$(\cos(x))' = -\sin(x)$$

$$\sin(x) = a \Rightarrow x = \arcsin(a) + 2\pi n, x = \pi - \arcsin(a) + 2\pi n$$

$$\sin(x) = -a \Rightarrow x = \arcsin(-a) + 2\pi n, x = \pi + \arcsin(a) + 2\pi n$$

$$\cos(x) = a \Rightarrow x = \arccos(a) + 2\pi n, x = -\arccos(a) + 2\pi n$$

$$\cos(x) = -a \Rightarrow x = \arccos(-a) + 2\pi n, x = -\arccos(-a) + 2\pi n$$

$$\tan(x) = a \Rightarrow x = \arctan(a) + \pi n$$

$$\tan(x) = -a \Rightarrow x = \arctan(-a) + \pi n$$

4 main integrals

main integrals to remember

$$f(x) \Leftrightarrow F'(x)$$

$$\int f(x)dx = F(x) + c$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\int_a^b cf(x)dx = c \int_a^b f(x)dx$$

$$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_b^a f(x) = - \int_a^b f(x)dx$$

$$\int_a^b f(x)g'(x)dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx$$

$$f(x) = x^a \Rightarrow F(x) = \frac{x^{a+1}}{a+1}$$

$$f(x) = a^x \Rightarrow F(x) = \frac{a^x}{\ln(a)}$$

$$f(x) = e^x \Rightarrow F(x) = e^x$$

$$f(x) = \frac{1}{x} \Rightarrow F(x) = \ln(|x|)$$

$$f(x) = \sin(x) \Rightarrow F(x) = -\cos(x)$$

$$f(x) = \cos(x) \Rightarrow F(x) = \sin(x)$$

$$f(x) = \frac{1}{\cos^2(x)} \Rightarrow F(x) = \tan(x)$$

u - sub

$$\int_b^a f(g(x))g'(x)dx$$

$$u = g(x)$$

$$du = g'(x)dx$$

$$\therefore k \int_{g(b)}^{g(a)} f(u)du$$

definite integral of absolute value function

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\therefore \int_a^c |x|dx = \int_a^b (-x)dx + \int_b^c xdx$$

eg.

$$f(x) = |x+2| = \begin{cases} -(x+2) & \text{if } x < -2 \\ x+2 & \text{if } x \geq -2 \end{cases}$$

$$\therefore \int_{-4}^0 f(x)dx = \int_{-4}^{-2} -(x+2)dx + \int_{-2}^0 (x+2)dx$$

5 Area and (Volume of Revolution)

$f(x)$ about x axis with $x \in [a, b] \Rightarrow$

$$V = \pi \int_a^b [f(x)]^2 dx \vee V = 2\pi \int_a^b x[f(x)]dx$$

$f(x)$ and $g(x)$ about x axis where $x \in [a, b] \Rightarrow$

$$V = \pi \int_a^b [(f(x))^2 - (g(x))^2] dx \vee V = 2\pi \int_c^d y[g(y) - f(y)] ; g(y) \geq f(y) \text{ over } [c, d]$$

$f(x)$ and $g(x)$ about y axis where $y \in [c, d] \Rightarrow$

$$V = \pi \int_c^d [(g(y))^2 - (f(y))^2] dy \vee V = 2\pi \int_a^b x[f(x) - g(x)]dx ; f(x) \geq g(x) \text{ over } [a, b]$$

$f(x)$ and $g(x)$ about $x = h$ where $y \in [c, d] \Rightarrow$

$$V = \pi \int_c^d [(h - f(y))^2 - (h - g(y))^2] dy \vee V = \begin{cases} 2\pi \int_a^b (x - h)[f(x) - g(x)]dx & \text{if } h \leq a < b \\ 2\pi \int_a^b (h - x)[f(x) - g(x)]dx & \text{if } a < b \leq h \end{cases}$$

$f(x)$ and $g(x)$ about $y = k$ where $x \in [a, b] \Rightarrow$

$$V = \pi \int_a^b [k - f(x)]^2 - [k - g(x)]^2 dx \vee V = \begin{cases} 2\pi \int_c^d (y - k)[g(y) - f(y)]dx & \text{if } k \leq a < b \\ 2\pi \int_c^d (k - y)[g(y) - f(y)]dx & \text{if } a < b \leq k \end{cases}$$

Area of single and double regions

$f(x) \geq g(x)$ over $x \in [a, b]$

$$A = \int_a^b [f(x) - g(x)]dx$$

$f(x) = g(x)$ at $x = c$; $f(x) \geq g(x)$ over $x \in [a, b]$ and $g(x) \geq f(x)$ over $x \in [c, d]$

$$A = \int_a^b [f(x) - g(x)]dx + \int_c^d [g(x) - f(x)]dx$$

6 Algebra rules

important algebra rules

trig rules

$$\cos^2(a) + \sin^2(a) = 1$$

$$\therefore \sin^2(a) = 1 - \cos^2(a) \wedge \cos^2(a) = 1 - \sin^2(a)$$

$$\cos(a \pm b) = \cos(a) \cos(b) \mp \sin(a) \sin(b)$$

$$\cos(2a)$$

$$= \cos^2(a) - \sin^2(a)$$

$$= 1 - 2\sin^2(a)$$

$$= 2\cos^2(a) - 1$$

$$\sin(a \pm b) = \sin(a) \cos(b) \pm \cos(a) \sin(b)$$

$$\sin(2a) = 2\sin(a) \cos(a)$$

$$\sin(-a) = -\sin(a)$$

$$\cos(-a) = +\cos(a)$$

$$\tan(-a) = -\tan(a)$$

triangle rules

$$Area = \frac{1}{2}|AB| \times |BC|$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

exponent rules

$$1, 1^a = 1$$

$$2, a^0 = 1$$

$$3, (ab)^n = a^n b^n$$

$$4, (a^b)^c = a^{bc}$$

$$5, a^{b+c} = a^b a^c$$

$$6, 0^a = 0$$

$$7, \frac{a^m}{a^n} = a^{m-n}$$

$$8, \left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

fraction rules

$$1, \frac{0}{a} = 0$$

$$2, \frac{a}{a} = 1$$

$$3, \left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^c$$

$$4, a^{-b} = \frac{1}{a^b}$$

$$5, \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{(a \times d)}{(b \times c)}$$

$$6, \frac{-a}{-b} = \frac{a}{b}$$

log rules

$$a^x = b \Leftrightarrow \log_a(b) = x$$

$$\log_a(x^b) = b \log_a(x)$$

$$\log_a\left(\frac{1}{x}\right) = \log_a(x^{-1}) = -\log_a(x)$$

$$\log_a(b) = \frac{\ln(a)}{\ln(b)}$$

$$\log_x(x) = 1$$

$$\log_x\left(\left(\frac{1}{x}\right)^n\right) = \log_x(x^{-n}) = -n$$

$$\log_{a^b}(x) = \frac{1}{b} \log_a(x)$$

$$a^{\log_a(b)} = b$$

radical rules

$$1, \sqrt{1} = 1$$

$$2, \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$3, \sqrt{a} \sqrt{a} = a$$

$$4, \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$5, \sqrt{0} = 0$$

$$6, \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$7, \frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

Rationalizing denominators

$$\frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

$$\frac{a}{\sqrt{b} \pm c} = \frac{a(\sqrt{b} \mp c)}{b \mp c^2}$$

7 Solving Inequalities

linear

1, *rewrite equation in terms of x*

quadratic

- 1, *write polynomial in correct form \sim eg. $ax^2 + bx + c > 0$*
- 2, *find critical values AKA set equation $= 0$*
- 3, *plug in values above / below critical values and see if they satisfy the inequality*
- 4, *create the intervals that satisfy the inequality*

rational

1, *write equation in correct form \sim*

$$\frac{f(x)}{g(x)} < 0$$

2, *find critical values \sim*

$$f(x) = 0 \Rightarrow x = a$$

$$g(x) = 0 \Rightarrow x = b$$

3, *plug in values above / below critical values and see if they satisfy the inequality*

4, *create the intervals that satisfy the inequality*

absolute value

Given $|u| > a$ with $a \in \mathbb{R}^+$

$$\Rightarrow u < -a \vee u > a$$

Given $|u| < a$ with $a \in \mathbb{R}^+$

$$\Rightarrow -a < u < a$$

logarithmic

for $a > 1$

$$1, \log_a(f(x)) \leq \log_a(g(x)) \Leftrightarrow f(x) \leq g(x), f(x) > 0$$

$$2, \log_a(f(x)) \geq \log_a(g(x)) \Leftrightarrow f(x) \geq g(x), g(x) > 0$$

for $a < 1$

$$1, \log_a(f(x)) \leq \log_a(g(x)) \Leftrightarrow f(x) \geq g(x), f(x) > 0$$

$$2, \log_a(f(x)) \geq \log_a(g(x)) \Leftrightarrow f(x) \leq g(x), g(x) > 0$$

square root

$$\sqrt{f(x)} \leq g(x) \Rightarrow f(x) > 0 \wedge g(x) \geq 0$$

$$\sqrt{f(x)} \geq g(x) \Rightarrow f(x) > 0 \wedge g(x) \geq 0 \wedge g(x) < 0$$

$$\text{Note} - u^n > a \Rightarrow u < -\sqrt[n]{a} \wedge u > \sqrt[n]{a}$$

Note - multiplying or dividing by a negative number flips the sign

8 Parametric Calculus and Vectors

Tangent line

$$x(t) = x, \ y(t) = y \text{ at } t = a \text{ or } (x_t, y_t)$$

tangent in point slope form

$$y - y_0 = m(x - x_0)$$

$$y_0 = y(a)$$

$$x_0 = x(a)$$

$$m = \frac{y'(a)}{x'(a)}$$

$$\text{vertical at } x'(t) = 0 \wedge y'(t) \neq 0$$

$$\text{horizontal at } y'(t) = 0 \wedge x'(t) \neq 0$$

Area and Length (Distance)

Given

$$x(t) = x$$

$$y(t) = y$$

$$t \in [a, b]$$

$$A = \int_a^b y(t)x'(t)dt$$

$$L = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$$

Velocity, Speed, Acceleration

velocity

$$v(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} ; \ v_x(t) = \begin{pmatrix} x'(t) \\ 0 \end{pmatrix} ; \ v_y(t) = \begin{pmatrix} 0 \\ y'(t) \end{pmatrix}$$

speed

$$||\vec{v}(t)|| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$$

extreme

$$||\vec{v}'(t)|| = 0 \Rightarrow t = n$$

$$||v(n)|| = a ; \ a = \text{max/min speed}$$

acceleration

$$a(t) = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix}$$

Note –

second derivative

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left[\frac{dy}{dx} \right]}{\frac{dx}{dt}} = \frac{\left(\frac{y''(t)}{x'(t)} \right)}{\frac{dx}{dt}}$$

Angle between tangent lines/vectors

$$\tan a = \frac{|m_1 - m_2|}{|1 + m_1 m_2|}$$

$$\therefore a = \arctan \left(\frac{|m_1 - m_2|}{|1 + m_1 m_2|} \right)$$

vectors,

$$\cos a = \frac{\vec{a} \times \vec{b}}{||\vec{a}|| \times ||\vec{b}||}$$

$$\therefore a = \arccos = \left(\frac{\vec{a} \cdot \vec{b}}{||\vec{a}|| \times ||\vec{b}||} \right)$$

Vector summary notes

1, addition and subtraction

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \pm \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \end{pmatrix}$$

2, magnitude (length)

$$||\vec{a}|| = \sqrt{[a_1]^2 + [a_2]^2}$$

3, scalar × vector

$$n \times \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} na_1 \\ na_2 \end{pmatrix}$$

4, vector × vector

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = (a_1 \times b_1) + (a_2 \times b_2)$$

5, perpendicular vectors

$$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \times \vec{b} = 0$$

6, vector equation of line

$$y = mx + b \Leftrightarrow r = \begin{pmatrix} x \\ y \end{pmatrix} + t \begin{pmatrix} \text{run} \\ \text{rise} \end{pmatrix} x, \ y \text{ are points on } y$$

7, vector to general form linear eq.

$$\vec{v} = \begin{pmatrix} \pm a \\ \pm b \end{pmatrix} \Leftrightarrow [\mp bx] + [\pm ay] = c$$

$$(x, y) \in \{\text{all points on } \vec{v}\}$$

8, angles between to vectors

$$A(x_a, y_a) \Leftrightarrow \begin{pmatrix} x_a \\ y_a \end{pmatrix}, \ B(x_b, y_b) \Leftrightarrow \begin{pmatrix} x_b \\ y_b \end{pmatrix}, \ C(x_c, y_c) \Leftrightarrow \begin{pmatrix} x_c \\ y_c \end{pmatrix}$$

$$\angle ABC = \text{angle between } \vec{BA} \text{ and } \vec{BC}$$

Note – vector from any point A to any point B given by

$$\vec{AB} = \vec{b} - \vec{a} = \begin{pmatrix} x_b - x_a \\ y_b - y_a \end{pmatrix}$$

9 Coordinate Geometry and Circle Equations

Circle equations

general form. $\sim x^2 + y^2 + 2gx + 2fy + c = 0$

center = $(-g, -f) \wedge$ radius = $\sqrt{g^2 + f^2 - c}$

center - radius form
 $(x - h)^2 + (y - k)^2 = r^2$

center = $(h, k) \wedge$ radius = r

Given

$A(x_1, y_1), C(x_3, y_3), B(x_2, y_2)$

Algebraic approach to find eq.

- 1, sub A, B, C into general form to create 3 eq.
- 2, solve equations for missing f, g, c

Geometric approach to find eq.

- 1, create perpendicular bisector for AB and BC
- 2, $PB_1 = PB_2$ then solve for x
- 3, $PB_1(x) = y$
- 4, \therefore center $(x, y) = (h, k)$
- 5, $r = d(\text{center}, A/B/C)$

Tangent line at point

Given, Center = $C(x_c, y_c)$, Point = $P(x_p, y_p)$

tangent eq. at P \sim

$$l : (x_c - x_p)(x - x_p) + (y_c - y_p)(y - y_p) = 0$$

Line equations

general form. $\sim ax + by + c = 0$

Given,

$A(x_1, y_1)$

$B(x_2, y_2)$

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

\therefore line equation

$$y - y_1 = m(x - x_1)$$

Given,

$x - \text{int.} = a$

$y - \text{int.} = b$

\therefore line equation

$$\frac{1}{a}x + \frac{1}{b}y = 1$$

Intersections

two lines

$$y = ax + b$$

$$y = cx + d$$

1, $ax + b = cx + d$ [make equations equal]

2, $x = \left(\frac{d - b}{a - c}\right)$ [solve for x]

3, $y = m\left(\frac{d - b}{a - c}\right) + b$ [sub x into line eq.]

Note - $f'(x) = g'(x)$ when touching

line and circle

$$y_l = mx + b$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

1, $x^2 + y_l^2 + 2gx + 2fy_l + c = 0$ [sub line eq.]

2, $x_i = n$ [solve for x_i]

3, $y_i = m(n) + b$ [sub x_i into line eq.]

circle and circle

$$a, x^2 + y^2 + 2gx + 2fy + c = 0$$

$$b, x^2 + y^2 + 2gx + 2fy + c = 0$$

1, $a - (b)$ [subtract eq. b from a]

2, $y = mx + b$ [rewrite into line eq.]

3, $x_i = a(mx + b)$ [sub line eq. into a / b]

4, $y_i = x_i m + b$ [sub x - int. into line eq.]

Line and Point formulae

$A(x_1, y_1)$

$B(x_2, y_2)$

$$\text{line} : Ax + By + C = 0$$

distance A to B

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

midpoint A to B

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

distance A to line

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

10 properties of functions

Asymptotes of rational functions

HA defined as $\lim_{|x| \rightarrow \infty} f(x)$

method

$$f(x) = \frac{x^2}{x^2} \Rightarrow HA : \text{at } y = \frac{a}{b}$$

$$f(x) = \frac{x^3}{x^2} \Rightarrow HA \text{ (slant)} : \text{at } y = mx + b$$

$$f(x) = \frac{ax^2}{bx^3} \Rightarrow HA : \text{at } y = 0$$

$$f(x) = \frac{x^4}{x^2} \Rightarrow HA : \text{no asymptote}$$

$$VA \text{ defined as } \begin{cases} \lim_{|x| \rightarrow a^-} f(x) = \pm\infty \\ \lim_{|x| \rightarrow a^+} f(x) = \pm\infty \end{cases}$$

method

Find where function becomes undef.

Function transformation

- 1, $f(x+c) \Rightarrow (y, x+c)$; $c < 0$ right, $c > 0$ left
- 2, $f(x)+c \Rightarrow (y+c, x)$; $c < 0$ down, $c > 0$ up
- 3, $cf(x) \Rightarrow (cy, x)$; $0 < c < 1$ compress, $c > 1$ stretch
- 4, $f(cx) \Rightarrow (y, cx)$; $0 < c < 1$ stretch, $c > 1$ compress
- 5, $-f(x) \Rightarrow (-y, x)$; reflect along x -axis
- 6, $f(-x) \Rightarrow (y, -x)$; reflect along y -axis

Long division

$$\text{eg. } y = \frac{x^3 - 2x}{x^2 - 5}$$

1, divide leading coefficients

$$\therefore \frac{x^3}{x^2} = x$$

2, multiply bottom by 1,

$$\therefore x \times (x^2 - 5) = x^3 - 5x$$

3, subtract top from 2,

$$\therefore x^3 - 2x - (x^3 - 5x) = 3x$$

4, answer

$$1, + \frac{3}{\text{denominator}}$$

Note - repeat 1, to 3, if 3, degree > 1

Undefined

$$1, 0^0 = \text{und.}$$

$$2, \frac{x}{0} = \text{und.}$$

$$3, \log_a(b) = \text{und. if } a \leq 0 \vee b \leq 0$$

$$4, \log_1(a) = \text{und.}$$

Trig. transformations

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$

$$-\sin(x) = \sin(-x) = \sin(x \pm \pi)$$

$$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

$$-\cos(x) = \cos(x \pm \pi)$$

11 Misc Notes

Absolute Value Equations

Given $|f(x)| = g(x)$

1, find the intervals of $|f(x)|$

2, solve for $-f(x) = g(x) \wedge f(x) = g(x)$

3, validate solutions for the intervals

eg.

$$|x^3 - 3x^2| = 2x$$

$$1, |x^3 - 3x^2| = \begin{cases} (x^3 - 3x^2) & f(x) > 0 \text{ for } x > 3 \\ -(x^3 - 3x^2) & f(x) \leq 0 \text{ for } x < 0 \vee 0 \leq x < 3 \end{cases}$$

2, for $f(x) \leq 0$ solve, $-(x^3 - 3x^2) = 2x \Rightarrow x = 0, x_1, x_2 = \pm n$
 \therefore Note - $[x_2 \text{ is not valid}]$

3, for $f(x) > 0$ solve, $(x^3 - 3x^2) = 2x \Rightarrow x_1, x_2, x_3 = 0, 1, 2$
 \therefore Note - $[x_1 \text{ is not valid}]$

Trig Equations

$$\sin x = \sin a \Leftrightarrow x = a + 2\pi n \vee x = \pi - a + 2\pi n$$

$$\cos x = \cos a \Leftrightarrow x = a + 2\pi n \vee x = -a + 2\pi n$$

$$\tan x = \tan a \Leftrightarrow x = a + \pi n$$

| | $\frac{\pi}{3}$ | $\frac{\pi}{4}$ | $\frac{\pi}{6}$ |
|----------|----------------------|----------------------|----------------------|
| $\sin x$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ |
| $\cos x$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\tan x$ | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ |

Optimization \sim distances

$f(x) \geq g(x)$ over $x \in [a, b]$

$$D(x) = f(x) - g(x)$$

$$(f(x) - g(x))' = 0 \Rightarrow x = n$$

$$D(n) = y - \text{distance}$$

Note - if $(f(x) - g(x))' = 0 \Rightarrow x = n \wedge x = z$ with $z < r$
 $z = \min$ and $r = \max$ over $x \in [a, b]$

Trigonometric inequalities

1, $\sin x \sim$ Note - $< x <$

$\geq a$

$$\arcsin(a) + 2\pi n \leq x \leq \pi - \arcsin(a) + 2\pi n$$

$\leq a$

$$-\pi - \arcsin(a) + 2\pi n \leq x \leq \arcsin(a) + 2\pi n$$

2, $\cos x \sim$ Note - $< x <$

$\geq a$

$$-\arccos(a) + 2\pi n \leq x \leq \arccos(a) + 2\pi n$$

$\leq a$

$$-\arccos(a) + 2\pi n \leq x \leq 2\pi - \arccos(a) + 2\pi n$$

3, $\tan x \sim$ Note - $< x <$

$\geq a$

$$\arctan(a) + \pi n \leq x < \frac{\pi}{2} + \pi n$$

$\leq a$

$$-\frac{\pi}{2} + \pi n < x \leq -\arctan(a) + \pi n$$

Factoring Cubic Polynomials

By grouping

$$f(x) = x^3 - 4x^2 + 3x - 12$$

$$= (x^3 - 4x^2) + (3x - 12)$$

$$= x^2(x - 4) + 3(x - 4)$$

$$= (x^2 + 3)(x - 4)$$

Rational Root Theorem

$$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0 \text{ assuming } \{a_1, a_2, a_3, a_0\} \in \mathbb{Q}$$

$$1, \text{ roots} = \pm \frac{\text{factors of } a_0}{\text{factors of } a_3} = \pm \{r_1, \dots, r_n\}$$

2, check all possible zeros via sub.of r into $p(x)$

$$p(r) = 0$$

$$\therefore \frac{p(x)}{x \pm r} = ax^2 + bx + c$$

$$\therefore p(x) = (x \pm r)(ax^2 + bx + c)$$

$$\therefore p(x) = (x \pm r)(p \pm x)(q \pm x) \text{ [factor standard quadratic]}$$

12 Misc Notes 2

Uneven segment formula

$P(x, y)$ divides line $A(x_a, y_a)$ to $B(x_b, y_b)$ by ratio $m : n$

$$P(x, y) = \left(\frac{mx_b + nx_a}{m+n}, \frac{my_b + ny_a}{m+n} \right)$$

eg. $A(2, 3) ; B(4, -8)$ ratio $= \frac{1}{3} : \frac{2}{3}$

$$P(x, y) = \left(\frac{\frac{1}{3}(4) + \frac{2}{3}(2)}{\frac{1}{3} + \frac{2}{3}}, \frac{\frac{1}{3}(-8) + \frac{2}{3}(3)}{\frac{1}{3} + \frac{2}{3}} \right)$$

$$\therefore P(x, y) = \left(\frac{8}{3}, -\frac{2}{3} \right)$$

simple formulas

Circle

$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

Cylinder

$$\text{Volume} = \pi r^2 h$$

Problem solving approaches general

1, remember to write out what you are doing a little

calculus

- 1, visually understand the problem as much as you can
- 2, remember to apply everything as accurately as possible
- 3, remember most problems you will face require substitution

geometry

- 1, if you get confused remember to draw triangles
- 2, always draw the problem to get a better understanding of it
- 3, remember to apply circle theorems when possible

vectors

- 1, vectors need to be constructed differently depending on use
- 2, vector addition can be used to find points (in certain circumstances)

trigonometry

- 1, if you are asked to find all solutions in an interval, find ALL
- 2, remember that triangles also often come into play here

Rules to only apply in specific circumstances

Notes – If the integrand is a trig. func. with a power > 1 then you have to rewrite

$$\int \cos^2(x) dx = \int \frac{1}{2} + \frac{1}{2} \cos(2x) dx = \frac{1}{2} \int 1 + \cos(2x) = \frac{1}{2} \left(1 + \frac{1}{2} \sin(2x) \right)$$

using the relationship \sim

$$\cos(2x) = 2\cos^2(x) - 1$$

Notes – If the denominator power $\neq 1$ then do not use $\frac{1}{x} \Rightarrow \ln(|x|)$

$$\int \frac{1}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = -2x^{\frac{1}{2}}$$