1 main derivatives

main derivatives to remember

$$(f \pm g)\prime = f\prime \pm g\prime$$

$$(f \times g)\prime = f\prime g + fg\prime$$

$$\left(\frac{f}{g}\right)\prime = \frac{f\prime g + fg\prime}{g^2}$$

$$\frac{df(u)}{dx} = \frac{df}{du} \times \frac{du}{dx}$$

$$\frac{d}{dx}|u| = \frac{u}{|u|} \times \frac{du}{dx}$$

$$[\log_a(x)]\prime = \frac{1}{x\ln(a)}$$

$$[\ln(x)]\prime = \frac{1}{x}$$

$$(a^x)\prime = a^x \ln(a)$$

$s(t) \ \sim \ position$

$s\prime(t) \ \sim \ velocity$

$s\prime\prime(t) \ \sim \ acceleration$

$s\prime(t) \ = \ 0 \ \sim \ stationary$

$f\prime(x) \ > \ 0 \ \sim \ incr.\ /\ right\ mov.$

$f\prime\prime(x) > 0 \ \sim \ conc.\ up\ /\ incr.$

$f\prime\prime(x) = 0 \ \sim \ point\ of\ inflection$

$f\prime(x) \ < \ 0 \ \sim \ decr.\ /\ left\ mov.$

$f\prime\prime(x) < 0 \ \sim \ conc.\ down\ /\ decr.$

2 tangent line

tangent line to function $f(x)\ at\ x \ = \ a$

$$y \ = \ mx \ + \ b$$

$$\begin{aligned} y &= f(a) \\ m &= f\prime(a) \\ x &= a \\ b &= f(a) \ - \ [f\prime(a) \ \times \ a] \end{aligned}$$

$Note\ -$

$$f\prime(a) \times g\prime(a) = -1 \ ; \ if \ g\prime(a) = -\frac{1}{f\prime(a)}$$

3 trig notes

important trig stuff to remember and trig inverse properties

$$\frac{\sin(x)}{\cos(x)} = \tan(x)$$

$$\begin{aligned} (\sin(x))\prime &= \cos(x) \\ (\cos(x))\prime &= -\sin(x) \end{aligned}$$

$$\sin(x) = a \ \Rightarrow x = \arcsin(a) + 2\pi n \ , \ x \ = \ \pi - \arcsin(a) + 2\pi n$$
$$\sin(x) = -a \ \Rightarrow x = \arcsin(-a) + 2\pi n \ , \ x = \pi + \arcsin(a) + 2\pi n$$

$$\cos(x) = a \ \Rightarrow x = \arccos(a) + 2\pi n \ , \ x \ = -\arccos(a) + 2\pi n$$
$$\cos(x) = -a \ \Rightarrow x = \arccos(-a) + 2\pi n \ , \ x = -\arccos(-a) + 2\pi n$$

$$\tan(x) = a \ \Rightarrow x = \arctan(a) + \pi n$$
$$\tan(x) = -a \ \Rightarrow x = \arctan(-a) + \pi n$$

4 main integrals

main integrals to remember

$$f(x) \;\Leftrightarrow F\prime(x)$$

$$\int f(x)dx = F(x) + c$$

$$\int_a^b f(x)dx = F(b) \; - F(a)$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_b^a f(x) = -\int_a^b f(x)dx$$

$$\int_a^b f(x)g\prime(x)dx = [f(x)g(x)]_a^b - \int_a^b f\prime(x)g(x)dx$$

$$f(x) = x^{\;a} \Rightarrow F(x) = \frac{x^{a+1}}{a+1}$$

$$f(x) = a^x \Rightarrow F(x) = \frac{a^x}{\ln(a)}$$

$$f(x) = e^x \Rightarrow F(x) = e^x$$

$$f(x) = \frac{1}{x} \Rightarrow F(x) = \ln(|x|)$$

$$f(x) = \sin(x) \Rightarrow F(x) = -\cos(x)$$

$$f(x) = \cos(x) \Rightarrow F(x) = \sin(x)$$

$$f(x) = \frac{1}{\cos^2(x)} \Rightarrow F(x) = \tan(x)$$

u - sub

$$\int_b^a f(g(x))g\prime(x)dx$$

$$u \;=\; g(x)$$

$$du \;=\; g\prime(x)dx$$

$$\therefore k\int_{g(b)}^{g(a)} f(u)du$$

definite integral of absolute value function

$$|x| = \begin{cases} x & if\ x \geqslant 0 \\ -x & if x < 0 \end{cases}$$

$$\therefore \int_a^c |x|dx = \int_a^b (-x)dx + \int_b^c xdx$$

eg.

$$f(x) = |x+2| = \begin{cases} -(x+2) & if\ x < -2 \\ x+2 & if x \geqslant -2 \end{cases}$$

$$\therefore \int_{-4}^0 f(x)dx = \int_{-4}^{-2} -(x+2)dx + \int_{-2}^0 (x+2)dx$$

5 Area and (Volume of Revolution)

$f(x)\ about\ x\ axis\ with\ x \in\ [a,\ b]\ \Rightarrow$

$$V = \pi \int_a^b [f(x)]^2 dx \vee V = 2\pi \int_a^b x[f(x)]dx$$

$f(x)\ and\ g(x)\ about\ x\ axis\ where\ x \in\ [a,\ b]\ \ \Rightarrow$

$$V = \pi \int_a^b \left[(f(x))^2 - (g(x))^2\right] dx \vee V = 2\pi \int_c^d y[g(y) - f(y)]\ ;\ g(y) \geqslant f(y)\ over\ [c, d]$$

$f(x)\ and\ g(x)\ about\ y\ axis\ where\ y \in [c,\ d]\ \Rightarrow$

$$V = \pi \int_c^d \left[(g(y))^2 - (f(y))^2\right] dy \vee V = 2\pi \int_a^b x[f(x) - g(x)]dx\ ;\ f(x) \geqslant g(x)\ over\ [a,\ b]$$

$f(x)\ and\ g(x)\ about\ x = h\ where\ y \in\ [c,\ d] \Rightarrow$

$$V = \pi \int_c^d \left[(h - f(y))^2 - (h - g(y))^2\right] dy\ \ \vee V = \begin{cases} 2\pi \displaystyle\int_a^b (x-h)[f(x) - g(x)]dx & if\ h \leqslant a < b \\ 2\pi \displaystyle\int_a^b (h-x)[f(x) - g(x)]dx & if\ a < b \leqslant h \end{cases}$$

$f(x)\ and\ g(x)\ about\ y = k\ where\ x \in\ [a,\ b]\ \Rightarrow$

$$V = \pi \int_a^b [k - f(x))^2 - (k - g(x))^2]dx\ \ \vee V = \begin{cases} 2\pi \displaystyle\int_c^d (y-k)[g(y) - f(y)]dx & if\ k \leqslant a < b \\ 2\pi \displaystyle\int_c^d (k-y)[g(y) - f(y)]dx & if\ a < b \leqslant k \end{cases}$$

$Area\ of\ single\ and\ double\ regions$

$f(x)\ \geqslant g(x)\ over\ x \in [a,\ b]$

$$A = \int_a^b [f(x) - g(x)]dx$$

$f(x) = g(x)\ at\ x = c;\ f(x) \geqslant g(x)\ over\ x \in [a,\ b]\ and\ g(x) \geqslant f(x)\ over\ x \in [c,\ d]$

$$A = \int_a^b [f(x) - g(x)]dx + \int_c^d [g(x) - f(x)]dx$$

6 Algebra rules

important algebra rules

trig rules

$\cos^2(a) + \sin^2(a) = 1$
$\therefore \sin^2(a) = 1 - \cos^2(a) \wedge \cos^2(a) = 1 - \sin^2(a)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$
$\cos(2a)$
$= \cos^2(a) - \sin^2(a)$
$= 1 - 2\sin^2(a)$
$= 2\cos^2(a) - 1$
$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$
$\sin(2a) = 2\sin(a)\cos(a)$

$\sin(-a) = -\sin(a)$
$\cos(-a) = +cos(a)$
$\tan(-a) = -\tan(a)$

triangle rules

$Area = \frac{1}{2}|AB| \times |BC|$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

log rules

$a^x = b \Leftrightarrow \log_a(b) = x$

$\log_a\left(x^b\right) = b\log_a(x)$

$\log_a\left(\frac{1}{x}\right) = \log_a\left(x^{-1}\right) = -\log_a(x)$

$\log_a(b) = \frac{\ln(a)}{\ln(b)}$

$\log_x(x) = 1$

$\log_x\left(\left(\frac{1}{x}\right)^n\right) = \log_x\left(x^{-n}\right) = -n$

$\log_{a^b}(x) = \frac{1}{b}\log_a(x)$

$a^{\log_a(b)} = b$

exponent rules

1, $1^a = 1$

2, $a^0 = 1$

3, $(ab)^n = a^n b^n$

4, $\left(a^b\right)^c = a^{bc}$

5, $a^{b+c} = a^b a^c$

6, $0^a = 0$

7, $\frac{a^m}{a^n} = a^{m-n}$

8, $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

fraction rules

1, $\frac{0}{a} = 0$

2, $\frac{a}{a} = 1$

3, $\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^c$

4, $a^{-b} = \frac{1}{a^b}$

5, $\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{(a \times d)}{(b \times c)}$

6, $\frac{-a}{-b} = \frac{a}{b}$

radical rules

1, $\sqrt{1} = 1$

2, $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

3, $\sqrt{a}\sqrt{a} = a$

4, $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

5, $\sqrt{0} = 0$

6, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

7, $\frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

Rationalizing denominators

$$\frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$
$$\frac{a}{\sqrt{b} \pm c} = \frac{a\left(\sqrt{b} \mp c\right)}{b \mp c^2}$$

7 Solving Inequalities

linear
1, $rewrite\ equation\ in\ terms\ of\ x$

quadratic
1, $write\ polynomial\ in\ correct\ form\ \sim\ eg.\ ax^2 + bx + c\ > 0$
2, $find\ critical\ values\ AKA\ set\ equation\ =\ 0$
3, $plug\ in\ values\ above\ /\ below\ critical\ values\ and\ see\ if\ they\ satisfy\ the\ inequality$
4, $create\ the\ intervals\ that\ satify\ the\ inequality$

rational
1, $write\ equation\ in\ correct\ from\ \sim$
$\frac{f(x)}{g(x)} < 0$

2, $find\ critical\ values\ \sim$
$f(x) = 0 \Rightarrow x = a$
$g(x) = 0 \Rightarrow x = b$

3, $plug\ in\ values\ above\ /\ below\ critical\ values\ and\ see\ if\ they\ satisfy\ the\ inequality$

4, $create\ the\ intervals\ that\ satify\ the\ inequality$

absolute value
$Given\ |u| > a\ with\ a \in \mathbb{R}^+$

$\Rightarrow u < -a\ \vee u > a$

$Given\ |u| < a\ with\ a \in \mathbb{R}^+$

$\Rightarrow -a < u < a$
logarithmic
for a > 1
1, $\log_a(f(x)) \leqslant \log_a(g(x))\ \Leftrightarrow f(x) \leqslant g(x),\ f(x) > 0$
2, $\log_a(f(x)) \geqslant \log_a(g(x))\ \Leftrightarrow f(x) \geqslant g(x),\ g(x) > 0$

for a < 1
1, $\log_a(f(x)) \leqslant \log_a(g(x))\ \Leftrightarrow f(x) \geqslant g(x),\ f(x) > 0$
2, $\log_a(f(x)) \geqslant \log_a(g(x))\ \Leftrightarrow f(x) \leqslant g(x),\ g(x) > 0$

square root
$\sqrt{f(x)} \leqslant g(x)\ \Rightarrow f(x) > 0 \wedge g(x) \geqslant 0$

$\sqrt{f(x)} \geqslant g(x) \Rightarrow f(x) > 0 \wedge g(x) \geqslant 0\ \wedge g(x) < 0$

$critical\ values\ at\ -\ \sqrt{f(x)} = g(x) \wedge f(x) = 0 \wedge g(x) = 0$

> $Note\ - u^n > a \Rightarrow u < -\sqrt[n]{a} \wedge u > \sqrt[n]{a}$
> $Note\ -\ multiplying\ or\ dividing\ by\ a\ negative\ number\ flips\ the\ sign$

8 Parametric Calculus and Vectors

Tangent line

$x(t) = x,\ y(t) = y\ at\ t = a\ or\ (x_t, y_t)$

tangent in point slope form

$y - y_0 = m(x - x_0)$

$y_0 = y(a)$
$x_0 = x(a)$

$$m = \frac{y\prime(a)}{x\prime(a)}$$

$vertical\ at\ x\prime(t) = 0 \wedge y\prime(t) \neq 0$

$horizontal\ at\ y\prime(t) = 0 \wedge x\prime(t) \neq 0$

Angle between tangent lines/vectors

lines,

$$\tan a = \frac{|m_1 - m_2|}{|1 + m_1 m_2|}$$

$$\therefore a = \arctan\left(\frac{|m_1 - m_2|}{|1 + m_1 m_2|}\right)$$

vectors,

$$\cos a = \frac{\vec{a} \times \vec{b}}{||\vec{a}|| \times ||\vec{b}||}$$

$$\therefore a = \arccos = \left(\frac{\vec{a} \cdot \vec{b}}{||\vec{a}|| \times ||\vec{b}||}\right)$$

Area and Length (Distance)

Given
$x(t) = x$
$y(t) = y$

$t \in [a,\ b]$

$$A = \int_a^b y(t)x\prime(t)dt$$

$$L = \int_a^b \sqrt{[x\prime(t)]^2 + [y\prime(t)]^2}dt$$

Velocity, Speed, Acceleration

velocity

$$v(t) = \begin{pmatrix} x\prime(t) \\ y\prime(t) \end{pmatrix}\ ;\ v_x(t) = \begin{pmatrix} x\prime(t) \\ 0 \end{pmatrix}\ ;\ v_y(t) = \begin{pmatrix} 0 \\ y\prime(t) \end{pmatrix}$$

speed

$$||\overrightarrow{v(t)}|| = \sqrt{[x\prime(t)]^2 + [y\prime(t)]^2}$$

extreme

$||\overrightarrow{v\prime(t)}|| = 0\ \Rightarrow t = n$

$||v(n)|| = a\ ;\ a = max/min\ speed$

acceleration

$$a(t) = \begin{pmatrix} x\prime\prime(t) \\ y\prime\prime(t) \end{pmatrix}$$

Note −
second derivative

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}} = \frac{\left(\frac{y\prime\prime(t)}{x\prime\prime(t)}\right)}{x\prime\prime(t)}$$

Vector summary notes

1, addition and subtraction

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \pm \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 \pm b_1 \\ a_2 \pm b_2 \end{pmatrix}$$

2, magnitude (length)

$$||\vec{a}|| = \sqrt{[a_1]^2 + [a_2]^2}$$

$$\begin{matrix} & a_1 \\ a_2 & \begin{pmatrix} na_1 \\ na_2 \end{pmatrix} \end{matrix}$$

4, vector×vector

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = (a_1 \times b_1) + (a_2 \times b_2)$$

5, perpendicular vectors

$$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \times \vec{b} = 0$$

6, vector equation of line

$$y = mx + b \Leftrightarrow r = \begin{pmatrix} x \\ y \end{pmatrix} + t\begin{pmatrix} run \\ rise \end{pmatrix} x,\ y\ are\ points\ on\ y$$

7, vector to general form linear eq.

$$\vec{v} = \begin{pmatrix} \pm a \\ \pm b \end{pmatrix} \Leftrightarrow [\mp bx] + [\pm ay] = c$$

$$(x,\ y) \in \{all\ points\ \ on\ \vec{v}\}$$

8, angles between to vectors

$$A(x_a,\ y_a) \Leftrightarrow \begin{pmatrix} x_a \\ y_a \end{pmatrix},\ B(x_b,\ y_b) \Leftrightarrow \begin{pmatrix} x_b \\ y_b \end{pmatrix},\ C(x_c,\ y_c) \Leftrightarrow \begin{pmatrix} x_c \\ y_c \end{pmatrix}$$

$$\angle ABC\ = angle\ between\ \overrightarrow{BA}\ and\ \overrightarrow{BC}$$

$Note\ -\ vector\ from\ any\ point\ A\ to\ any\ point\ B\ given\ by$

$$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} x_b - x_a \\ y_b -\ y_a \end{pmatrix}$$

9 Coordinate Geometry and Circle Equations

Circle equations

$general\ form.\ \sim\ x^2 + y^2 + 2gx + 2fy + c = 0$

$center\ =\ (-g, -f) \wedge radius = \sqrt{g^2 + f^2 - c}$

$center - radius\ form$
$(x-h)^2 + (y-k)^2 = r^2$

$center = (h,\ k)\ \wedge\ radius = r$

$Given$
$A(x_1,\ y_1),\ C(x_3,\ y_3),\ B(x_2,\ y_2)$

Algebraic approach to find eq.
1, sub A, B, C into general form to create 3 eq.
2, solve equations for missing f, g, c

Geometric approach to find eq.
1, create perpendicular bisector for AB and BC
2, $PB_1 = PB_2\ then\ solve\ for\ x$
3, $PB_1(x) = y$
4, $\therefore center(x,\ y) = (h,\ k)$
5, $r = d(center,\ A/B/C)$

Tangent line at point
$Given,\ Center = C(x_c,\ y_c),\ Point = P(x_p,\ y_p)$

$tangent\ eq.\ at\ P\ \sim$
$l : (x_c - x_p)(x - x_p) + (y_c - y_p)(y - y_p) = 0$

Intersections
two lines

$y = ax + b$
$y = cx + d$

1, $ax + b = cx + d\ [make\ equations\ equal]$
2, $x = \left(\dfrac{d-b}{a-c}\right)\ [solve\ for\ x]$
3, $y = m\left(\dfrac{d-b}{a-c}\right) + b\ [sub\ x\ into\ line\ eq.]$

$Note\ -\ f\prime(x) = g\prime(x)\ when\ touching$

line and circle

$y_l = mx + b$
$x^2 + y^2 + 2gx + 2fy + c = 0$

1, $x^2 + {y_l}^2 + 2gx + 2fy_l + c = 0\ [sub\ line\ eq.]$
2, $x_i = n\ [solve\ for\ x_i]$
3, $y_i = m(n) + b\ [sub\ x_i\ into\ line\ eq.]$

circle and circle

$a,\ x^2 + y^2 + 2gx + 2fy + c = 0$
$b,\ x^2 + y^2 + 2gx + 2fy + c = 0$

1, $a - (b)\ [subtract\ eq.\ b\ from\ a]$
2, $y = mx + b\ [reqwrite\ into\ line\ eq.]$
3, $x_i = a(mx + b)\ [sub\ line\ eq.\ into\ a\ /\ b]$
4, $y_i = x_i m + b\ [sub\ x - int.\ into\ line\ eq.]$

Line equations

$general\ form.\ \sim\ ax + by + c = 0$

Given,
$A(x_1,\ y_1)$
$B(x_2,\ y_2)$

$slope\ = m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore line\ equation$
$y - y_1 = m(x - x_1)$

Given,
$x - int. = a$
$y - int. = b$

$\therefore line\ equation$
$\dfrac{1}{a}x + \dfrac{1}{b}y = 1$

Line and Point formulae

$A(x_1,\ y_1)$
$B(x_2,\ y_2)$
$line\ :\ Ax + By + C = 0$

distance A to B

$$d(A,\ B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

midpoint A to B

$$M = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$$

distance A to line

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

10 properties of functions

Asymptotes of rational functions
$HA\ defined\ as\ \lim_{|x|\to\infty} f(x)$

method
$f(x) = \frac{x^2}{x^2} \Rightarrow HA:\ at\ y = \frac{a}{b}$

$f(x) = \frac{x^3}{x^2} \Rightarrow HA\ (slant):\ at\ y = mx + b$

$f(x) = \frac{ax^2}{bx^3} \Rightarrow HA:\ at\ y = 0$

$f(x) = \frac{x^4}{x^2} \Rightarrow HA:\ no\ asymptote$

$$VA\ defined\ as\ \begin{cases} \lim_{|x|\to a^-} f(x) = \pm\infty \\ \lim_{|x|\to a^+} f(x) = \pm\infty \end{cases}$$

method
$Find\ where\ function\ becomes\ undef.$

Long division

$eg.\ y = \frac{x^3 - 2x}{x^2 - 5}$

$1,\ divide\ leading\ coefficients$
$\therefore \frac{x^3}{x^2} = x$

$2,\ multiply\ bottom\ by\ 1,$
$\therefore x \times \left(x^2 - 5\right) = x^3 - 5x$

$3,\ subtract\ top\ from\ 2,$
$\therefore x^3 - 2x - \left(x^3 - 5x\right) = 3x$

$4,\ answer$
$1,\ + \frac{3,}{denominator}$

$Note\ -\ repeat\ 1,\ to\ 3,\ if\ 3,\ degree\ >\ 1$

Function transformation
$1,\ f(x+c) \Rightarrow (y,\ x+c)\ ;\ c < 0\ right,\ c > 0\ left$

$2,\ f(x) + c \Rightarrow (y+c,\ x);\ c < 0\ down\ ,\ c > up$

$3,\ cf(x) \Rightarrow (cy,\ x);\ 0 < c < 1\ compress,\ c > 1\ strech$

$4,\ f(cx) \Rightarrow (y,\ cx);\ 0 < c < 1\ strech,\ c > 1\ compress$

$5,\ -f(x) \Rightarrow (-y,\ x);\ reflext\ along\ x - axis$

$6,\ f(-x) \Rightarrow (y,\ -x);\ reflect\ along\ y - axis$

Undefined
$1,\ 0^0 = und.$

$2,\ \frac{x}{0} = und.$

$3,\ log_a(b) = und.\ if\ a \leqslant 0\ \vee b \leqslant 0$

$4,\ log_1(a) =\ und.$

Trig. transformations
$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$
$-\sin(x) = \sin(-x) = \sin(x \pm \pi)$

$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$
$-\cos(x) = \cos(x \pm \pi)$

11 Misc Notes

Absolute Value Equations

Given $|f(x)| = g(x)$

$1, find\ the\ intervals\ of\ |f(x)|$

$2,\ solve\ for\ -f(x) = g(x) \wedge f(x) = g(x)$

$3,\ validate\ solutions\ for\ the\ intervals$

eg.
$|x^3 - 3x^2| = 2x$

$$1,\ |x^3 - 3x^2| = \begin{cases} \left(x^3 - 3x^2\right) & f(x) > 0\ for\ x > 3 \\ -\left(x^3 - 3x^2\right) & f(x) \leqslant 0\ for\ x < 0 \vee 0 \leqslant x < 3 \end{cases}$$

$2,\ for\ f(x) \leqslant 0\ solve,\ -\left(x^3 - 3x^2\right) = 2x\ \Rightarrow x = 0,\ x_{1,\ 2} = \pm n$
$\therefore Note\ -\ [x_2\ is\ not\ valid]$

$3,\ for\ f(x)\ > 0\ solve,\ \left(x^3 - 3x^2\right) = 2x\ \Rightarrow x_{1,\ 2,\ 3} = 0,\ 1,\ 2$
$\therefore Note\ -\ [x_1\ is\ not\ valid]$

Trig Equations

$\sin x = \sin a \Leftrightarrow x = a + 2\pi n \vee x = \pi - a + 2\pi n$

$\cos x = \cos a \Leftrightarrow x = a + 2\pi n \vee x = -a + 2\pi n$

$\tan x = \tan a \Leftrightarrow x = a + \pi n$

	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
$\sin x$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\cos x$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\tan x$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

Optimization $\sim$ *distances*
$f(x) \geqslant g(x)\ over\ x \in [a,\ b]$

$D(x)\ = f(x) - g(x)$

$(f(x) - g(x))\prime = 0 \Rightarrow x = n$

$D(n) = y - distance$

$Note\ -\ if\ (f(x) - g(x))\prime = 0 \Rightarrow x = n \wedge x = z\ with\ z < r$
$z = min\ and\ r = max\ over\ x \in [a,\ b]$

Trigonometric inequalities

$1,\ \sin \mathbf{x}\ \sim\ Note -\ <\ x\ <$
$\geqslant \mathbf{a}$
$\arcsin(a) + 2\pi n \leqslant x \leqslant \pi - \arcsin(a) + 2\pi n$

$\leqslant \mathbf{a}$
$-\pi - \arcsin(a) + 2\pi n \leqslant x \leqslant \arcsin(a) + 2\pi n$

$2,\ \cos \mathbf{x}\ \sim\ Note -\ <\ x\ <$
$\geqslant \mathbf{a}$
$-\arccos(a) + 2\pi n \leqslant x \leqslant \arccos(a) + 2\pi n$

$\leqslant \mathbf{a}$
$-\arccos(a) + 2\pi n \leqslant x \leqslant 2\pi - \arccos(a) + 2\pi n$

$3,\ \tan \mathbf{x} \sim\ Note\ -\ <\ x\ <$
$\geqslant \mathbf{a}$
$\arctan(a) + \pi n \leqslant x < \frac{\pi}{2} + \pi n$

$\leqslant \mathbf{a}$
$-\frac{\pi}{2} + \pi n < x \leqslant -\arctan(a) + \pi n$

Factoring Cubic Polynomials
By grouping

$f(x) = x^3 - 4x^2 + 3x - 12$

$= \left(x^3 - 4x^2\right) + (3x - 12)$
$= x^2(x - 4) + 3(x - 4)$
$= \left(x^2 + 3\right)(x - 4)$

Rational Root Theorem

$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0\ assuming\ \{a_{1,\ 2,\ 3,\ 0}\} \in Q$

$1,\ roots = \pm\frac{factors\ of\ a_0}{factors\ of\ a_3} = \pm\{r_1, ..., r_n\}$

$2,\ check\ all\ possible\ zeros\ via\ sub.of\ r\ into\ p(x)$

$p(r) = 0$

$\therefore \frac{p(x)}{x \pm r} = ax^2 + bx + c$
$\therefore p(x) = (x \pm r)\left(ax^2 + bx + c\right)$
$\therefore p(x) = (x \pm r)(p \pm x)(q \pm x)\ [factor\ standard\ quadratic]$

12 Misc Notes 2

Uneven segment formula

$P(x,\ y)$ *devides line* $A(x_a,\ y_b)$ *to* $B(x_b,\ y_b)$ *by ratio* $m\ :\ n$

$$P(x,\ y)\ =\ \left(\frac{mx_b+nx_a}{m+n},\ \frac{my_b+ny_a}{m+n}\right)$$

$$eg.\ A(2,\ 3)\ ;\ B(4,\ -8)\ ratio\ =\ \frac{1}{3}\ :\ \frac{2}{3}$$

$$P(x,\ y)\ =\ \left(\frac{\frac{1}{3}(4)+\frac{2}{3}(2)}{\frac{1}{3}+\frac{2}{3}},\ \frac{\frac{1}{3}(-8)+\frac{2}{3}(3)}{\frac{1}{3}+\frac{2}{3}}\right)$$

$$\therefore P(x,\ y)=\left(\frac{8}{3},\ -\frac{2}{3}\right)$$

simple formulas

Circle
$Circumference\ =\ 2\pi r$
$Area\ =\ \pi r^2$

Cylinder
$Volume\ =\ \pi r^2 h$

Problem solving approaches general
1, remember to write out what you are doing a little

calculus
1, visually understand the problem as much as you can
2, remember to apply everything as accuratly as possible
3, remember most problems you will face require substitution

geometry
1, if you get confused remember to draw triangles
2, always draw the problem to get a better understanding of it
3, remember to apply circle theorems when possible

vectors
1, vectors need to be constructed differently depending on use
2, vector addition can be used to find points (in certain circumstances)

trigonometry
1, if you are asked to find all solutions in an interval, find ALL
2, remember that triangles also often come into play here

Rules to only apply in specific circumstances

Notes – If the integrand is a trig. func. with a power > 1 then you have to rewrite

$$\int \cos^2(x)dx = \int \frac{1}{2}+\frac{1}{2}\cos(2x)dx = \frac{1}{2}\int 1+\cos(2x) = \frac{1}{2}\left(1+\frac{1}{2}\sin(2x)\right)$$

using the relationship $\sim$

$$cos(2x) = 2cos^2(x)-1$$

Notes – If the denominator power $\neq 1$ then do not use $\mathbf{\frac{1}{x}} \Rightarrow \ln(|\mathbf{x}|)$

$$\int \frac{1}{\sqrt{x}}dx = \int x^{-\frac{1}{2}}dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = -2x^{\frac{1}{2}}$$