Report of Written assignment

In section 3.1 Univariate Polynomials, the author discusses how a tanh neural network can approximate univariate polynomials.

Lemma 3.1 first shows that any odd-degree univariate polynomial can be approximated by a shallow tanh neural network.

The specific method is as follows: by employing the pth order central difference operator and expressing the central difference operator as a combination of tanh functions, it can then be regarded as a neural network.

We introduce the pth order central finite difference operator δ_h^p for any $f \in C^{p+2}([a,b])$ for some $p \in \mathbb{N}$ by

$$\delta_h^p[f](x) = \sum_{i=0}^{p} (-1)^i \binom{p}{i} f\left(x + \left(\frac{p}{2} - i\right)h\right). \tag{15}$$

Lemma 3.2 shows that a shallow tanh neural network can approximate any univariate polynomial, including both odd-degree and even-degree cases.