Banking Competiotion Research Record

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1. Literature

1.1. Bank Run and Banking Competition

- Diamond and Dybvig (1983).
- Goldstein and Pauzner (2005).
- M. Egan, Hortaçsu, and Matvos (2017).
- Gertler and Kiyotaki (2015).
- Wang, Whited, Wu, and Xiao (2022).
- M. L. Egan, Hortaçsu, Kaplan, Sunderam, and Yao (2025).

2. Monopolistic Bank with Liquidity Motive

2.1. Household

Time is continuous, starting from $t = 0$ and going on forever. Following Moreira and Savov (2017), we model the households with liquidity needs. There is a unit measure of identical households solving the following probelm

$$V_t = \max \mathrm{E}_t \left[\int_t^\infty e^{-\rho t} \log(W_t(\psi dC_t + dc_t)) \right]. \tag{1}$$

where C_t is the consumption fraction in liquidity event, $\psi > 1$ is the marginal utility of liquidity event consumption and c_t is the consumption fraction outside of the liquidity event.

A liquidity event comes with rate h with size $d\overline{C}_t$. That is, $dC_t \leq d\overline{C}_t$. Conditional on the occurence of a liquidity event, the size follows $\mathrm{Exp}(\eta)$.

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There are two type of securities issued by the bank, money m and equity e. The money can be used to finance both consumptions while the equity can only finance c_t. The constraints are

$$\frac{dW_t}{W_t} = m_t dr_t^m + e_t dr_t^e - dC_t - dc_t \tag{2}$$

$$dC_t \leq \min\left\{m_t, d\overline{C}_t\right\} \tag{3}$$

$$m_t + e_t = 1. \tag{4}$$

Recursify equation (1),

$$\rho V_t(W_t)dt = \log(W_t) + \max_{m_t, e_t, dC_t, dc_t} \mathrm{E}_t\left[\log(\psi dC_t + dc_t)\right] + \mathrm{E}_t\left[dV_t\right]. \tag{5}$$

where

$$dV_t = V_t dt + V_W dW_t + \frac{1}{2} V_{WW}(dW_t)^2. \tag{6}$$

Given m_t, e_t, conditional on the liquidity event, the household decides (dC_t, dc_t):

$$\begin{aligned} &\max_{dC_t, dc_t} \log(\psi dC_t + dc_t) + \theta_t(\min\left\{m_t, d\overline{C}_t\right\} - dC_t) \\ &\quad + V_t dt + V_W \mathrm{E}\left[m_t dr_t^m + e_t dr_t^e - dC_t - dc_t\right] + \frac{1}{2} V_{WW} \mathrm{E}\left[W_t^2 (m_t dr_t^m + e_t dr_t^e - dC_t - dc_t)^2\right] \end{aligned} \tag{7}$$

The first order condition with complementary slackness is

$$\frac{\psi}{\psi dC_t + dc_t} - \theta_t - V_W W_t - V_{WW} \mathrm{E}\left[W_t^2 (m_t dr_t^m + e_t dr_t^e - dC_t - dc_t)\right] = 0 \tag{8}$$

$$\frac{1}{\psi dC_t + dc_t} - V_W W_t - V_{WW} \mathrm{E}\left[W_t^2 (m_t dr_t^m + e_t dr_t^e - dC_t - dc_t)\right] = 0 \tag{9}$$

$$\theta_t(\min\left\{m_t, d\overline{C}_t\right\} - dC_t) = 0. \tag{10}$$

Thus

$$\theta_t = \frac{\psi - 1}{\psi dC_t + dc_t} > 0$$

and the liquidity constraint always binds.

$$dC_t = \min\left\{m_t, d\overline{C}_t\right\}.$$

Now,

$$dc_t$$

3. Monopolistic Bank with Information Frictions

The bank can choose to keep operation or exit and taking E_0. In the continuation region, the bank solves the HJB

$$\begin{aligned}\rho_B E(k,a) = \max_r -rD(r,a) + E_a rD(r,a) + \phi(\alpha k + E((1-\alpha)k,a) - E(k,a)) \\ + E_k \mu k_t + \eta \int \max\left\{E((1-\kappa)k,a) - E(k,a), E_0 - E(k,a)\right\} dF_\kappa \\ + \lambda \int \max_{k \le k' \le \min\left\{k+\bar{I},D\right\}} E(k',a) - E(k,a) dF_I(\bar{I}).\end{aligned} \tag{11}$$

$E_k > 0$ so $k' = \min\left\{k + \bar{I}, D\right\}$.

$$\begin{aligned}(\rho_B + \phi + \lambda + \eta)E(k,a) = \max_r -rD(r,a) + E_a rD(r,a) + \phi(\alpha k + E((1-\alpha)k,a)) \\ + E_k \mu k_t + \eta \int \max\left\{E((1-\kappa)k,a), E_0\right\} dF_\kappa \\ + \lambda((1-F_I(D-k))E(D,a) + \int_0^{D-k} E(k+\bar{I},a) dF_I(\bar{I})).\end{aligned} \tag{12}$$

Given household asset holdings a, the unit measure of households solve

$$\max_{\theta_t \in [0,1]} \mathrm{E}_t\left[\frac{da_t}{a_t}\right] - \frac{\gamma}{2}\mathrm{Var}_t\left[\frac{da_t}{a_t}\right] \tag{13}$$

subject to $da_t = \theta_t a_t d\tilde{r}_t$. The solution is given by

$$\theta_t = \frac{\mathrm{E}_t\left[d\tilde{r}_t\right]}{\gamma \mathrm{Var}_t\left[d\tilde{r}_t\right]}, \quad \Rightarrow \quad D(r,a) = \frac{\mathrm{E}\left[d\tilde{r}_t\right]}{\gamma \mathrm{Var}\left[d\tilde{r}_t\right]} a. \tag{14}$$

Let $k^*(a)$ be the continuation boundary. We have $E(k^*(a),a) = E_0$. Suppose that the when the bank default, the risky asset is illiquid and all the deposit disappears. Hence

$$da_t = \theta_t a_t r dt - \theta_{t^-} a_{t^-} dN_t \tag{15}$$

where the rate of N_t is $\eta P((1-\kappa)k \le k^*(a)) = \eta(1 - F_\kappa(1 - k^*/k)) = \eta\delta$.

$$\theta_t = \begin{cases} 1, & \frac{1}{\gamma}\left[\frac{r}{\eta\delta} - 1\right] > 1 \\ \frac{1}{\gamma}\left[\frac{r}{\eta\delta} - 1\right], & \text{o.w.} \\ 0, & \frac{1}{\gamma}\left[\frac{r}{\eta\delta} - 1\right] < 0. \end{cases} \tag{16}$$

Take the parametric assumption that F_I is $Exp(\beta)$ and F_κ is $U(0,1)$. The optimal deposit rate solves

$$\begin{aligned}(E_a - 1)(D + rD_r) - \lambda\beta \exp(-\beta(D-k))D_r E(D,a) \\ + \lambda \exp(-\beta(D-k))E_k(D,a)D_r + E(D,a)\beta\exp(-\beta(D-k)) = 0.\end{aligned} \tag{17}$$

We solve the interior case and truncate when needed.

References

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