Research Record for the Project "Large vs Small Banks"

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1. Model

1.1. Household

The time is continuous, $t \in [0, \infty)$. There is a unit mass of households. Each household solves the following problem:

$$\rho V dt = \max_{c,\theta,e} u(c)dt + \mathrm{E}_t [dV] \tag{1}$$

subject to

$$da = \left[a \left(\sum_{n=1}^{N} \theta_n dr_n + e_n dr_n^e \right) \right] - cdt \tag{2}$$

$$a \geq 0 \tag{3}$$

where θ_0 is the asset proportion holding on hand, θ_n is the asset proportion as deposits and e_n is the proportion as equity.

Derive deposit and equity demand.

$\chi = (\chi_1, \ldots, \chi_N)$ are the believes about banks' risky asset holding proportion. At rate λ, a bank's portfolio is going to be revealed. Let $\tilde{\chi}_t$ be the underlying portfolio,

$$\chi_t = \mathrm{E} [\tilde{\chi}_t | \mathcal{F}_t] .$$

Now let $d\zeta_t$ be the new signal.

$$\begin{aligned} d\chi_t &= \mathrm{E} [\tilde{\chi}_{t+dt} | \mathcal{F}_t, d\zeta_t] - \mathrm{E} [\tilde{\chi}_t | \mathcal{F}_t] = \mathrm{E} [\tilde{\chi}_t + d\tilde{\chi}_t | \mathcal{F}_t, d\zeta_t] - \mathrm{E} [\tilde{\chi}_t | \mathcal{F}_t] \\ &= \mathrm{E} [d\tilde{\chi}_t] \end{aligned}$$

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Think about the information shocks.

1.2. Bank

The banks have two type of assets. One is a risk-free bond B_t with fixed return rate r^f. The others are risky assets k_t that require costs to build. A risky asset cannot be held by different banks so the single bank can only hold $k_t \in \{0, 1, \ldots\}$. The risky assets arrive at rate λ_k, requiring cost C, and pays off R at rate λ_R. After R arrives, the risky asset is destructed. Total asset value is

$$A_t = q_{B,t} B_t + q_{k,t} k_t. \tag{4}$$

d and e are the proportion of asset holding financed by deposit and equity.

$$\begin{aligned}
\mu_e E dt = \max_{d,e,B,k} & r^f B dt + d(q_B B) + d(q_k k) - \mathrm{E}\left[A d \cdot dr + (Ae - E) dr^e\right] \\
& + \lambda_k \int \mathbf{1}\left\{x \leq q_B B\right\} \max\left\{q_k(k+1) + q_B B - C - E, 0\right\} dF_C(x) \\
& + \lambda_R k \int q_k(k-1) + q_B B + x - E dF_R(x)
\end{aligned} \tag{5}$$

There are N banks, competing in the security market in monopolistic competition manner. The bank n can issue two types of security, deposit d and equity e. The returns satisfying that

$$dr^d = \mu^d dt \quad \text{and} \quad dr^e = \mu^e dt + dZ_t^n,$$

where dZ_t^n is a compensated Poisson process with rate λ_n. Since the banks are risk neutral, they will fund their investments through issuing as many deposits as they can.

References