DP

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2.1

The optimization problem is

$$\max_{c_t, p_t} \frac{1}{1-\nu}(c_t^\gamma(l-\theta_p p_t)^{1-\gamma})^{1-\nu} \tag{1}$$

s.t.

$$c_t = w_t p_t + (1-p_t)\mu_{ss} \tag{2}$$

$$\log w_t = \mu_w + \epsilon_t \tag{3}$$

$$p_t \in \{0, 1\} \tag{4}$$

2.2

Notice that p_t is discrete.

$$\begin{aligned} p_t^* &= \mathbb{1}\{\frac{1}{1-\nu}(w_t^\gamma(l-\theta_p)^{1-\gamma})^{1-\nu} \geq \frac{1}{1-\nu}(\mu_{ss}^\gamma l^{1-\gamma})^{1-\nu}\} \\ &= \mathbb{1}\{w_t^\gamma(l-\theta_p)^{1-\gamma} \geq \mu_{ss}^\gamma l^{1-\gamma}\} \end{aligned} \tag{5}$$

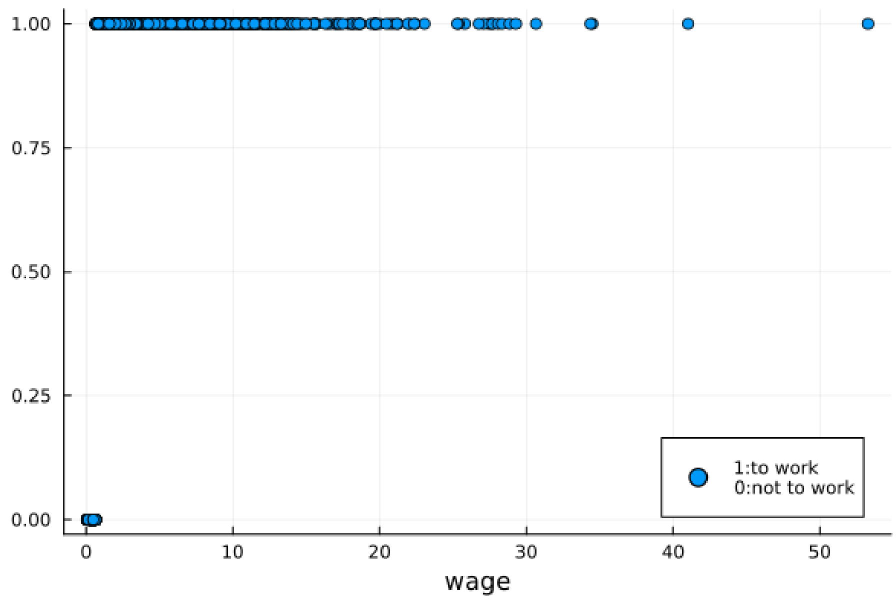
$$c_t^* = w_t p_t^* + (1-p_t^*)\mu_{ss} \tag{6}$$

We first solve p_t^* and then c_t^* is determined by (2).

2.3

As figure in 2.4, there are some people going to work and some not.

2.4



First, the analytical solution tells us that people choose to work when the wage is above the threshold, which is exactly what we see in the picture. Second, the density of wage is higher for the left side. This reflect the fact that the wage follows a lognormal distribution.

2.5

0.7922, it does not match since there is a selection bias.

2.6

Using MLE, the log-likelihood function is

$$logL(x) = \sum_{i=1}^{N} \mathbb{1}\{work\} logf(w_i \mid x) + (1 - \mathbb{1}\{work\}) logF(\mu_{ss}(\frac{l}{l-\theta_p})^{\frac{1-\gamma}{\gamma}}) \quad (7)$$

where f is the pdf of $Lognormal(x, \sigma)$ and F is the cdf. By MLE, we obtain that $\hat{\mu_w} = 0.4722$.

3.1

The linkage is the decision of retirement. A person would make the choice of retirement taking the future wage into account. Hence a dynamic model could help us to capture the characteristics.

3.2

$$\max_{c_1,p_1,c_2,p_2} \frac{1}{1-\nu}(c_1^{\gamma}(l-\theta_p p_1)^{1-\gamma})^{1-\nu} + \beta\mathbb{E}(\frac{1}{1-\nu}(c_2^{\gamma}(l-\theta_p p_2)^{1-\gamma})^{1-\nu}) \tag{8}$$

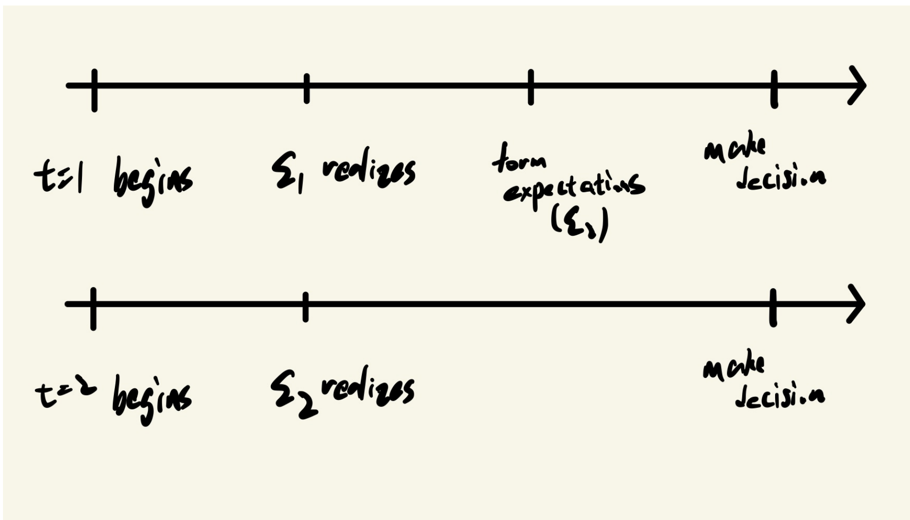
s.t.

$$\begin{array}{ll} p_t \in \{0\} & if\ p_{t-1} = 0 \\ p_t \in \{0,1\} & if\ p_{t-1} = 1 \end{array} \qquad t = 1, 2.\text{[1]} \tag{9}$$

$$c_t = w_t p_t + (1-p_t)\mu_{ss} \qquad t = 1, 2. \tag{10}$$

$$log w_t = \mu_w + \epsilon_t \qquad \epsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2) \tag{11}$$

3.3



3.4

ν is now relevant. In the static model, the CRRA form can be taken off due to the fact that monotonic transformation does not affect the preference. However, in the dynamic one, ν is the key parameter that affect the intertemporal substitution. (e.g. $\nu = 0$ then the two periods are perfect substitutes while $\nu = 1$ means that the two periods are Cobb-Douglas.)

3.5

The state s_t consists of p_{t-1}, w_t(or ϵ_t) and actually t itself since if $t = 2$ then there is no next period.

[1] We assume that $p_0 = 1$ otherwise the person retires from the beginning.

3.6

$$V_t(p_{t-1}, w_t) = \max_{c_t, p_t} \frac{1}{1-\nu}(c_1^\gamma (l - \theta_p p_t)^{1-\gamma})^{1-\nu} + \beta \mathbb{E} V_{t+1}(p_t, w_{t+1}) \tag{12}$$

where $V_3 = 0$.

3.7

The choice set of an individual is $\{(0,0),(1,0),(1,1)\}$ where (p_1, p_2) represents the decision of work and hence retirement. Then the choice is decided by comparing the outcomes. Let I, II, III be the lifetime value respectively.

$$I = (1+\beta)\frac{1}{1-\nu}(\mu_{ss}^\gamma l^{1-\gamma})^{1-\nu} \tag{13}$$

$$II = \frac{1}{1-\nu}(w_t^\gamma (l-\theta_p)^{1-\gamma})^{1-\nu} + \beta \frac{1}{1-\nu}(\mu_{ss}^\gamma l^{1-\gamma})^{1-\nu} \tag{14}$$

$$III = \frac{1}{1-\nu}(w_t^\gamma (l-\theta_p)^{1-\gamma})^{1-\nu} + \beta \frac{1}{1-\nu}(l-\theta_p)^{(1-\gamma)(1-\nu)} \mathbb{E}[w_{t+1}^{\gamma(1-\nu)}] \tag{15}$$

Notice that $\mu_w + \epsilon_{t+1}$ follow normal distribution and hence by the moment generating function of normal distribution, we have

$$\begin{aligned} \mathbb{E}[w_{t+1}^{\gamma(1-\nu)}] &= \mathbb{E}[e^{(\mu_w + \epsilon_{t+1})\gamma(1-\nu)}] \\ &= e^{\mu_w \gamma(1-\nu) + \frac{1}{2}\gamma^2(1-\nu)^2\sigma^2} \end{aligned} \tag{16}$$

Therefore,

$$III = \frac{1}{1-\nu}(w_t^\gamma (l-\theta_p)^{1-\gamma})^{1-\nu} + \beta \frac{1}{1-\nu}(l-\theta_p)^{(1-\gamma)(1-\nu)} e^{\mu_w \gamma(1-\nu) + \frac{1}{2}\gamma^2(1-\nu)^2\sigma^2} \tag{17}$$

Thus the value function at $t = 1$ is given by

$$V_1 = \max\{I, \max\{II, III\}\} \tag{18}$$

and V_2 is similar to the one in problem **2**. Hence the policy functions are

$$p_1^* = \mathbb{1}\{\max\{II, III\} \geq I\} \tag{19}$$

$$p_2^* = \mathbb{1}\{w_2^\gamma (l-\theta_p)^{1-\gamma} \geq \mu_{ss}^\gamma l^{1-\gamma}\} \tag{20}$$

and c_t^* can be obtained by plugging (19) and (20) into (10).

5.1

We discuss these parameters in **5.5**.

5.2

Table 1: Descriptive Statistics

Variable	Mean	Std.dev	Median
con_1	2.7486	3.6853	1.6242
con_2	2.7157	3.4265	1.6438

5.3/4

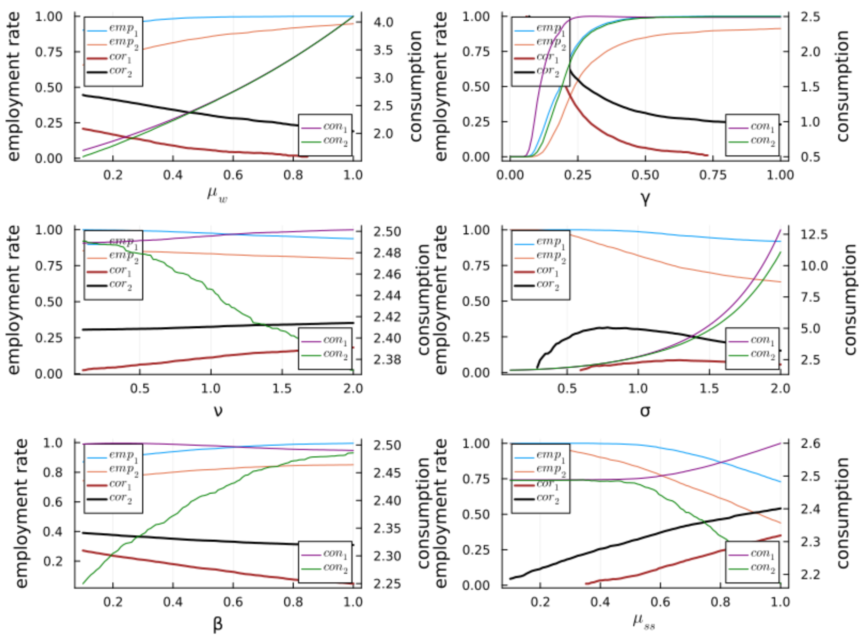


Figure 1: Comparative statics.[2]

5.5

- μ_w controls the mean of wage so when μ_w increases, the employment rate also increases.
- γ represents the contemporal proportion of the expenditure on each item (consumption and leisure). Although we are here considering a discrete

[2]Parameters' defaults are $\mu_w = \mu_{ss} = \gamma = \nu = 0.5$, $\sigma = \beta = 0.9$.

choice case, γ still controls the threshold of working. As what the intuition predicts, the employment rate and hence the consumption grow with γ, which reflect the relative importance in people's mind.

- ν reflects the reciprocal of the elasticity of intertemporal substitution and also the tendency of risk averse. The effects of a higher ν here are somewhat complicated. First, since the agent is now more sensitive to the risk of having a low wage in the future, the incentive to keep on working deminishes. Second, he is less sensitive to the discount factor which is 0.9 as a default. This prompts him to retire later and care more about the future.
- σ also has a positive wealth effect since lognormal is a right-skewed distribution. We may see that the consumption of period 1 increases due to this wealth effect. Besides, the σ also controls the variance and hence one may retire in the first period due to the risk of w_2.
- β is the time discount factor. When β increases, one would weight the future more and hence the consumptions in each period are closer to the case of consumption smoothing.
- μ_{ss} the government pension would lead to decrease in labor and also have a wealth effect hence we can observe that the employment rate decreases and the consumption increases.

Code: `https://github.com/KaiJyunWang/labor_econ`