P3.13

$$\underset{2\times 4}{\begin{pmatrix} 2 & 10 & -5 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}} \underset{4\times 2.}{\begin{pmatrix} 1 & 3 \\ 0 & 2 \\ 5 & 1 \\ -3 & -4 \end{pmatrix}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

$$= \begin{pmatrix} 2+0+(-25) & 6+20-5 \\ 5-9 & 1-12 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -23 & 21 \\ -4 & -11 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -2 \\ -15 & -15 \end{pmatrix}$$

could finish it out.

P3.14) $$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

$$= \begin{pmatrix} \frac{1}{2}-\frac{1}{2} & \frac{1}{2}+\frac{1}{2}. \\ \frac{1}{2}+\frac{1}{2} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \underline{\underline{X}}.$$

3.15) a)

$$L^2 = \begin{pmatrix} -1 & 1 & 3 \\ 3 & 0 & 3 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 & 3 \\ 3 & 0 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+3+9, & -1+6, & -3+3+3 \\ -3+9, & 3+6, & 9+3 \\ -3+6+3, & 3+0+2, & 9+6+1 \end{pmatrix}$$

$$= \begin{pmatrix} 13, & 5, & 3 \\ 6, & 9, & 12 \\ 6, & 5, & 16 \end{pmatrix}.$$

b)

$$LM = \underset{3\times 3.}{\begin{pmatrix} -1 & 1 & 3 \\ 3 & 0 & 3 \\ 3 & 2 & 1 \end{pmatrix}}\underset{3\times 4.}{\begin{pmatrix} -1 & 3 & 3 & 1 \\ -1 & 4 & -1 & 2 \\ -2 & 2 & 3 & -2 \end{pmatrix}}$$

$$= \begin{pmatrix} 1-1-6, & -3+4+6, & -3-1+9, & -1+2-6 \\ -3-6, & 9+6, & 9+9, & 3-6 \\ -3-2-2, & 9+8+2, & 9-2+3, & 3+4-2. \end{pmatrix}$$

$$= \begin{pmatrix} -6, & 7, & 5, & -5 \\ -9, & 15, & 18, & -3 \\ -7, & 19, & 10, & 5. \end{pmatrix}$$

c)

$$LN = \underset{3\times 3}{\begin{pmatrix} -1 & 1 & 3 \\ 3 & 0 & 3 \\ 3 & 2 & 1 \end{pmatrix}} \underset{3\times 2.}{\begin{pmatrix} 5 & 3 \\ 0 & 5 \\ 3 & -2 \end{pmatrix}}$$

$$= \begin{pmatrix} -5+9, & -3+5-6 \\ 15+9, & 9-6 \\ 15+3, & 17 \end{pmatrix} = \begin{pmatrix} 4, & -4 \\ 24, & 3 \\ 18, & 17 \end{pmatrix}.$$

$3\times 4 \to 4\times 3.$

d) $$M^T L = \underset{4\times 3}{\begin{pmatrix} -1 & -1 & -2 \\ 3 & 4 & 2 \\ 3 & -1 & 3 \\ 1 & 2 & -2 \end{pmatrix}} \begin{pmatrix} -1 & 1 & 3 \\ 3 & 0 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-3-6, & -1-4, & -3-3-2 \\ -3+12+6, & 3+4, & 9+12+2 \\ -3-3+9, & 3+6, & 9-3+3 \\ -1+6-6, & 1-4, & 3+6-2 \end{pmatrix}.$$

$$= \begin{pmatrix} -8, & -5, & -8 \\ 15, & 7, & 23 \\ 3, & 9, & 9. \\ -1, & -3, & 7 \end{pmatrix}.$$

e) $3\times 2 \to 2\times 3$ $\quad 2\times 3 \quad 3\times 3$

$$N^T L = \begin{pmatrix} 5 & 0 & 3 \\ 3 & 5 & -2 \end{pmatrix} \begin{pmatrix} -1 & 1 & 3 \\ 3 & 0 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

2×3

$$= \begin{pmatrix} -5+9, & 5+6, & 15+3 \\ -3+15-6, & 3-4, & 9+15-2 \end{pmatrix}$$

$$= \begin{pmatrix} 4, & 11, & 18 \\ 6, & -1, & 22 \end{pmatrix}.$$

P3.16) a) A^2+B^2

$$= \begin{pmatrix} \cos\alpha & 1 \\ -1 & -\sin\alpha \end{pmatrix} \begin{pmatrix} \cos\alpha & 1 \\ -1 & -\sin\alpha \end{pmatrix} + \begin{pmatrix} \sin\alpha & 0 \\ 0 & -\sin\alpha \end{pmatrix} \begin{pmatrix} \sin\alpha & 0 \\ 0 & -\sin\alpha \end{pmatrix}$$

$$= \begin{pmatrix} (\cos\alpha)^2-1, & \cos\alpha-\sin\alpha \\ -\cos\alpha+\sin\alpha, & -1+(\sin\alpha)^2 \end{pmatrix} + \begin{pmatrix} (\sin\alpha)^2 & 0 \\ 0 & (\sin\alpha)^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0, & \cos\alpha-\sin\alpha. \\ -\cos\alpha+\sin\alpha & 2(\sin\alpha)^2-1 \end{pmatrix} = \begin{pmatrix} 0, & \cos\alpha-\sin\alpha \\ \sin\alpha-\cos\alpha & -\cos 2\alpha \end{pmatrix}$$

b) $A^2 + ($

$$= \begin{pmatrix} \cos^2\alpha - 1, & \cos\alpha - \sin\alpha \\ \sin\alpha - \cos a, & \sin^2\alpha - 1 \end{pmatrix} + \begin{pmatrix} 1 & -\cos\alpha \\ \sin\alpha & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\alpha, & -\sin\alpha \\ 2\sin\alpha - \cos a, & \sin^2\alpha. \end{pmatrix}$$

3.17) $\det\begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix} = 2\times0 - 3\times1 = -3.$

b)

$$\det\begin{pmatrix} 0 & 5 & 3 \\ 0 & 1 & \cdot 1 \\ 0 & 1 & 0 \end{pmatrix} = 0\times(-1) - 5(0) + 0$$
$$= 0$$

c)

$$\det\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \\ 4 & -2 & 0 \end{pmatrix} = 1\times(2) - 2(-4)$$
$$= 2 + 8 = 10$$

3.19) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$.

- A system of linear equations is linearly independent if there exists c_1, c_2, c_3 such that $c_i \neq 0$.

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right).$$

$R_2 \to R_2 - R_1$
$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right).$$

$R_3 \to R_3 + R_2$
$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right).$$

It is linearly independent.

b) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \right\}$.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right).$$

$$\overset{R_2 \to R_2 - R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right).$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

$\therefore$ 3 vars with 2 eqns.

$\therefore$ <u>dependent</u>.

c) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}.$

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 0 \\ 2 & -2 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 \end{array}\right)$$

$R_2 \to R_2 - 2R_1$

$$\sim \left(\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 0 \\ 0 & 4 & 0 & -2 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 \end{array}\right).$$

$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 4R_1$

$$\sim \left(\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 0 \\ 0 & 4 & 0 & -2 & 0 \\ 0 & 4 & 1 & -3 & 0 \\ 0 & 4 & 1 & -3 & 0 \end{array}\right).$$

duplicate.
linearly dependent.

3.20). $\vec{v} = (3, -5)$.

$\vec{w} = (1, -1)$

$$\text{Area} = \left| \begin{matrix} \vec{v} \\ \vec{w} \end{matrix} \right|$$

$$= \left| \begin{matrix} 3 & -5 \\ 1, & -1 \end{matrix} \right| = -3 - (-5)$$

$$= 2 \text{ cm}^2.$$

3.21)