P3.5a). $\left(\begin{array}{ccc|c} 1 & -1 & -2 & 1 \\ -2 & 2 & 4 & -2 \\ 3 & -3 & -6 & 3 \end{array}\right)$.

$R_2 \to R_2 + 2R_1$

$\sim \left(\begin{array}{ccc|c} 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & -3 & -6 & 3 \end{array}\right)$.

$\sim \left(\begin{array}{ccc|c} 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$.

$x_1 - x_2 - 2x_3 = 1$.

free: x_2, x_3.

basic: x_1.

$x_1 = x_2 + 2x_3 + 1$.

$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \right.$

$\left. \forall s, \forall t \in \mathbb{R} \right\}$.

3.5b).

$$\left(\begin{array}{cccc|c} 2 & -2 & 3 & 2 & 2 \\ 0 & 0 & 5 & 3 & 3 \\ 6 & -6 & -1 & 0 & 0 \\ 6 & -6 & 9 & 6 & 6 \end{array}\right).$$

$R_4 \to R_4 - R_3$

$$\sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & 2 & 2 \\ 0 & 0 & 5 & 3 & 3 \\ 6 & -6 & -1 & 0 & 6 \\ 0 & 0 & 10 & 6 & 6 \end{array}\right).$$

$$\sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & 2 & 2 \\ 0 & 0 & 5 & 3 & 3 \\ 6 & -6 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

$R_3 \to R_3 - 3R_1$

$$\sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & 2 & 2 \\ 0 & 0 & 5 & 3 & 3 \\ 0 & 0 & -6 & -6 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

$$\sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & 2 & 2 \\ 0 & 0 & 5 & 3 & 3 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

$$\sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & 2 & 2 \\ 0 & 0 & 0 & -2 & -2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

$$\sim \left(\begin{array}{cccc|c} 2 & -2 & 3 & 2 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

$$\sim \left(\begin{array}{cccc|c} 1 & -1 & \frac{3}{2} & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Careful...!

-

Let $x_4 = s$.

$$\therefore \left\{ \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \forall s \in \mathbb{R} \right\}.$$

P3.(a).

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 4 & 2 & -2 & 0 \end{array}\right)$$

$R_3 \to R_3 - 2R_1$ $\sim$

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

basis: x_1, x_2.

free: x_3.

$$2x_1 + x_2 - x_3 = 0$$

$$x_2 + x_3 = 0. \quad x_3 = s$$

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \forall s \in \mathbb{R} \right\}$$

b). $\left(\begin{array}{ccc|c} 2 & 0 & 1 & 5 \\ 1 & 4 & 2 & 2 \\ 0 & 2 & 1 & 1 \end{array}\right)$.

$R_1 \to \frac{1}{2}R_1$ $\sim$ $\left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 2.5 \\ 1 & 4 & 2 & 2 \\ 0 & 2 & 1 & 1 \end{array}\right)$.

$R_2 \to R_2 - R_1$ $\sim$ $\left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{5}{2} \\ 0 & 4 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 2 & 1 & 1 \end{array}\right)$.

$R_3 \to R_3 - \frac{1}{2}R_2$ $\sim$ $\left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{5}{2} \\ 0 & 4 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{5}{4} \end{array}\right)$.

$$x_1 + \frac{1}{2}x_3 = \frac{5}{2}.$$

$$4x_2 + \frac{3}{2}x_3 = -\frac{1}{2}.$$

$$\frac{1}{4}x_3 = \frac{5}{4}.$$

$x_2 = -2$.

$x_3 = 5$.

$$x_1 = \frac{5}{2} - \frac{1}{2}x_3$$

$$= \frac{5}{2} - \frac{5}{2}.$$

$$= 0.$$

$$4x_2 = -\frac{1}{2} - \frac{3}{2}(5)$$

$$= -8.$$

$$\left\{ \begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix} \right\}.$$

.

3.6(c). $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 4 & -2 & -4 & 5 \\ 3 & -3 & -6 & 3 \end{array}\right)$.

$R_2 \to R_2 - 4R_1$ $\sim$ $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -6 & -12 & -3 \\ 3 & -3 & -6 & 3 \end{array}\right)$.

$R_3 \to R_3 - 3R_1$ $\sim$ $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -6 & -12 & -3 \\ 0 & -6 & -12 & -3 \end{array}\right)$.

$\sim$ $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -6 & -12 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$.

$x_1 + x_2 + 2x_3 = 2$.

$-6x_2 = -3 + 12x_3$.

Let $x_3 = s$.

$$x_2 = \frac{1}{2} - 2s.$$

$$x_1 = 2 - 2x_3 - x_2$$

$$= 2 - 2s - \left(\frac{1}{2} - 2s\right)$$

$$= \frac{3}{2}$$

$$\therefore \left\{ \begin{pmatrix} \frac{3}{2} \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} : s \in \mathbb{R} \right\}$$

P3.7) $A \in \mathbb{R}^{5\times 5}$.

$A\vec{x} = \vec{b}$ has infinite solutions.

rank of 4.

P3.8). $ABCD = AD$.

$CD = B^{-1}A^{-1}AD$

$= B^{-1}D$.

$C = B^{-1}DD^{-1}$

$= B^{-1}$