3.21) $\vec{u}=(2,0,1)$ $\vec{v}=(1,-1,1)$ $\vec{w}=(0,2,3)$

$$\text{volume} = \det\left|\begin{matrix} 2 & 0 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & 3 \end{matrix}\right|$$

$$= |2(-3-2)-0+(2)|$$

$$= |-\overset{10}{\cancel{12}}+2| = \cancel{10}\ 8\,cm^3.$$

P3.22). $|M|=-2$. $|N|=7$.

a) $|M^T| = -2$.

b) $|3M| = 3^4|M| = \cancel{-6}\ 3^4(-2) = -162$

c) $|M^3| = (-2)^3 = -8$.

d) $|M\cdot N| = |M| \times |N| = -14$

e) $|M^{-1}NM| = 4 \times 7 = 28$.

P3.23 a) $$\begin{pmatrix} 3 & -2 & 0 & 1 \\ 0 & 1 & 3 & -1 \\ 5 & 0 & 1 & 4 \\ 0 & 3 & -4 & 2 \end{pmatrix} = 86.$$

b): -86.

c) -172.

3.24) $\det(A) = 6 - 6 = 0.$

since $\det(A) == 0$,

$\therefore$ rows are linearly dependent.

columns are linearly independent if and only if rows are linearly independent.

$\det(B) = 1(-1+2) - 4(-2-0) + 3(-4)$

$= 1 + 8 - 12 = -3.$

$\therefore$ it is linearly independent.

Columns are also linearly independent.

Since C has more rows than rank, it is linearly dependent. columns too.

$\det(D) = 0.$

$\therefore$ it is linearly dependent.

P3.25).

$x(r,\theta) = r\cos\theta \quad y(r,\theta) = r\sin\theta.$

$$\det(J) = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix}$$

$$= \left| \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} \right|$$

$$= r\cos^2\theta + r\sin^2\theta$$

$$= r(\cos^2\theta + \sin^2\theta)$$

$$= \underline{\underline{r}}.$$

P3.26) Spherical coordinates (ρ, θ, ϕ)

$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi$$

$$\text{abs}(\det(J_s)) = \text{abs}\left(\begin{vmatrix} \frac{\partial x}{\partial p} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial p} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial p} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{vmatrix}\right)$$

$$\frac{\partial x}{\partial p} = \sin\phi\cos\theta$$

$$\frac{\partial x}{\partial \theta} = -p\sin\phi\sin\theta$$

$$\frac{\partial x}{\partial \phi} = p\cos\phi\sin\theta$$

$$\frac{\partial y}{\partial p} = \sin\phi\sin\theta$$

$$\frac{\partial y}{\partial \theta} = p\sin\phi\cos\theta$$

$$\frac{\partial y}{\partial \phi} = p\cos\phi\sin\theta$$

$$\frac{\partial z}{\partial p} = \cos\phi$$

$$\frac{\partial z}{\partial \theta} = 0$$

$$\frac{\partial z}{\partial \phi} = -p\sin\phi$$

P3.27) a) Does not exist.

b) $$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right) \overset{R_2 \to R_2 - 2R_1}{\sim} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right)$$

$$\overset{R_1 \to R_1 - 2R_2}{\sim} \left(\begin{array}{cc|cc} 1 & 0 & 5 & -2 \\ 0 & 1 & -2 & 1 \end{array}\right).$$

$$\text{inv} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}.$$

c) $$\left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right)$$

$$\overset{R_1 \to \frac{1}{2}R_1}{\sim} \left(\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 2 & 4 & 0 & 1 \end{array}\right)$$

$$\overset{R_2 \to R_2 - 2R_1}{\sim} \left(\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -1 & 1 \end{array}\right)$$

$$\overset{R_1 \to R_1 - \frac{3}{2}R_2}{\sim} \left(\begin{array}{cc|cc} 1 & 0 & 2 & -\frac{3}{2} \\ 0 & 1 & -1 & 1 \end{array}\right).$$

3.28) $AB = C$

2×2. $\quad 2\times 2$.

$B = A^{-1}C$

Since $A = \begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} -7 & 4 \\ 2 & -1 \end{pmatrix}$

$B = \begin{pmatrix} -7 & 4 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & -4 \end{pmatrix}$

$= \begin{pmatrix} -21+4 & -14-16 \\ 6-1 & 4+4 \end{pmatrix} = \begin{pmatrix} -17 & -30 \\ 5 & 8 \end{pmatrix}$

3.29). $\left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 0 & -2 & -1 & 0 & 0 & 1 \end{array}\right)$

$R_2 \to R_2 - 2R_1$

$\sim \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & -7 & -5 & -2 & 1 & 0 \\ 0 & -2 & -1 & 0 & 0 & 1 \end{array}\right)$

$R_2 \to R_2 - 4R_3$

$\sim \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & -4 \\ 0 & -2 & -1 & 0 & 0 & 1 \end{array}\right)$

$R_3 \to R_3 + 2R_2$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & -4 \\ 0 & 0 & -3 & -4 & 2 & -7 \end{array}\right).$$

$R_3 \to -\frac{1}{3} R_3$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & -4 \\ 0 & 0 & 1 & \frac{4}{3} & -\frac{2}{3} & \frac{7}{3} \end{array}\right).$$

$R_1 \to R_1 - 4R_2$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 7 & 9 & 4 & 16 \\ 0 & 1 & -1 & -2 & 1 & -4 \\ 0 & 0 & 1 & \frac{4}{3} & -\frac{2}{3} & \frac{7}{3} \end{array}\right).$$

$R_1 \to R_1 - 7R_3$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & -1 & \\ 0 & 0 & 1 & \end{array}\right)$$

3.30) zero matrix has $\det(0) = 0$.

$\det(\text{zero matrix}^{-1}) = \frac{1}{\det(0)} = \frac{1}{0}$ = <u>undefined</u>.

P 3.33) $\begin{pmatrix} 1 & 3 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 4 & 0 \end{pmatrix}$

$LHS = \begin{pmatrix} a+3c & b+3d \\ -2a-c & -2b-d \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 4 & 0 \end{pmatrix}.$

$$\begin{aligned} a + \quad 3c &= 3 \\ b + 3d &= -5 \\ -2a \quad - c &= 4 \\ -2b \quad - d &= 0. \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 0 & 3 & -5 \\ -2 & 0 & -1 & 0 & 4 \\ 0 & -2 & 0 & -1 & 0 \end{array}\right).$$

$R_3 \to R_3 + 2R_1$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 0 & 3 & -5 \\ 0 & 0 & 5 & 0 & 10 \\ 0 & -2 & 0 & -1 & 0 \end{array}\right).$$

$R_4 \to R_4 + 2R_2$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 0 & 3 & -5 \\ 0 & 0 & 5 & 0 & 10 \\ 0 & 0 & 0 & 5 & -10 \end{array}\right).$$

$\therefore d = -2.$

$c = 2.$

$b + 3d = -5$

$b = -5 - 3(-2)$

$= 1.$

$a = 3 - 3c$

$= 3 - 6$

$= -3.$

3.34) $a=g$

$e=b=f$

$c=d=h.$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a & b \\ c & c \end{pmatrix}\begin{pmatrix} b & b \\ a & c \end{pmatrix}$$

$$= \begin{pmatrix} ab+ba & ab+bc \\ cb+ca & cb+cc \end{pmatrix}$$

$ab+ba = -2 \qquad ab = -1$

$ab+bc = -3 \qquad b(a+c) = -3$

$cb+ca = 0 \qquad c(b+a) = 0$

$cb+cc = 2. \qquad c(b+c) = 2.$