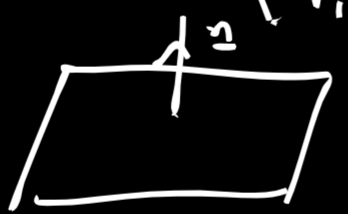
E4.6) $r_1 = (1,3,0)$

a) $r_2 = (3,1,0)$

$\vec{u} = (0,0,5)$. $P: x+y+z=0$. $\underset{\sim}{n} = (1,1,1)$



$\Pi_{\vec{n}}(r_1)$

= projection of line

$$= \frac{r_1 \cdot \underset{\sim}{n}}{|\underset{\sim}{n}|^2} n = \frac{\begin{pmatrix}1\\3\\0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac{1+3}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$= \frac{4}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}.$$

b) $\Pi_P(r_1)$ = projection of point r_1 onto plane P.

$\vec{AC} + \vec{CB} = \vec{AB}$

$\vec{CB} = \vec{AB} - \vec{AC}$

$$= \begin{pmatrix}1\\3\\0\end{pmatrix} - \frac{\begin{pmatrix}1\\3\\0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}{\left|\begin{pmatrix}1\\1\\1\end{pmatrix}\right|^2}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$= \begin{pmatrix}1\\3\\0\end{pmatrix} - \frac{4}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-\frac{1}{3}\\ \frac{5}{3}\\ -\frac{4}{3}\end{pmatrix}.$$

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c) $\pi_P(r_2)$ = projection of point r_2 onto P.

$$\vec{AB}+\vec{BC}=\vec{AC}$$

$$\vec{BC}=\vec{AC}-\vec{AB}$$

$$= \vec{r_2} - \frac{r_2\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}{\left|\begin{pmatrix}1\\1\\1\end{pmatrix}\right|^2}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}3\\1\\0\end{pmatrix} - \frac{\begin{pmatrix}3\\1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}{\left|\begin{pmatrix}1\\1\\1\end{pmatrix}\right|^2}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$= \begin{pmatrix}3\\1\\0\end{pmatrix} - \frac{3+1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

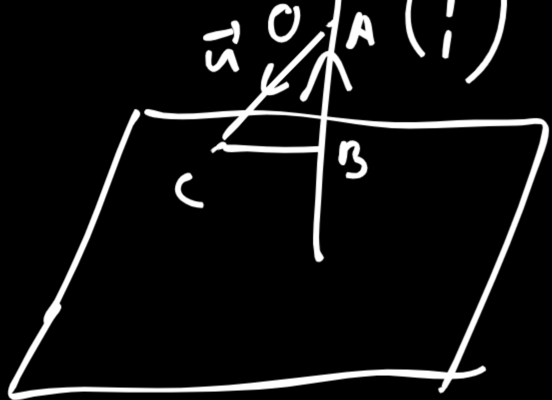
$$= \begin{pmatrix}\frac{5}{3}\\-\frac{1}{3}\\-\frac{4}{3}\end{pmatrix}.$$

d) $\pi_{\vec{n}}(\vec{u})$ = projection of vector $\vec{u}$ onto $\vec{n}$

$$\pi_{\vec{n}}(\vec{u}) = \frac{\begin{pmatrix}0\\0\\5\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}{\left|\begin{pmatrix}1\\1\\1\end{pmatrix}\right|^2}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac{5}{\sqrt{3}\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac{5}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$= \begin{pmatrix}\frac{5}{3}\\\frac{5}{3}\\\frac{5}{3}\end{pmatrix}.$$

e) $\vec{AB} + \vec{BC} = \vec{AC}$



$\pi_p(\vec{u}) = \vec{BC} = \vec{AC} - \vec{AB}$

$$= \begin{pmatrix}0\\0\\5\end{pmatrix} - \frac{\begin{pmatrix}0\\0\\5\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}}{\left\|\begin{pmatrix}1\\1\\1\end{pmatrix}\right\|^2}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

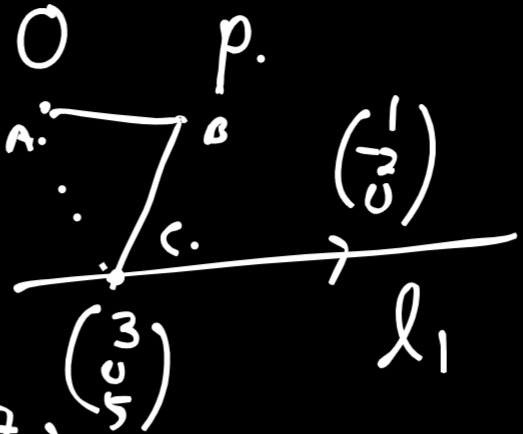
$$= \begin{pmatrix}0\\0\\5\end{pmatrix} - \frac{5}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix}-\frac{5}{3}\\-\frac{5}{3}\\\frac{10}{3}\end{pmatrix}}}$$

E4.7) $p=(10,10,10)$ $\quad l_1 : \left\{ \begin{pmatrix}3\\0\\5\end{pmatrix} + t\begin{pmatrix}1\\-2\\0\end{pmatrix}, t \in \mathbb{R} \right\}$.

$l_2 : \left\{ \begin{pmatrix}6\\0\\0\end{pmatrix} + t\begin{pmatrix}1\\-2\\0\end{pmatrix}, t \in \mathbb{R} \right\}$.

a) $\pi_{l_1}(p)$ = projection of p onto l_1.



$\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{BC} = \vec{AC} - \vec{AB}$

$= \begin{pmatrix}3\\0\\5\end{pmatrix} - \begin{pmatrix}10\\10\\10\end{pmatrix} = \begin{pmatrix}-7\\-10\\-5\end{pmatrix}$

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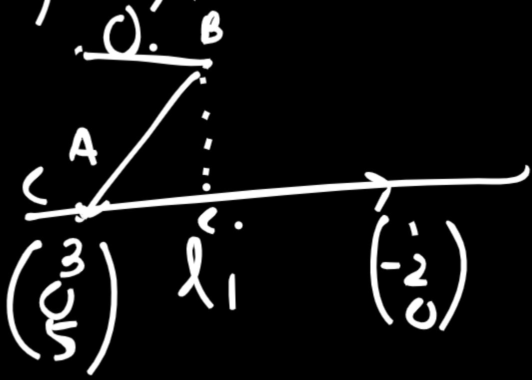
$$\pi_{l_1}(p) = \frac{\begin{pmatrix}-7\\-10\\-5\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\0\end{pmatrix}}{\left\|\begin{pmatrix}1\\-2\\0\end{pmatrix}\right\|^2}\begin{pmatrix}1\\-2\\0\end{pmatrix} = \frac{-7+20}{5}\begin{pmatrix}1\\-2\\0\end{pmatrix} = \frac{13}{5}\begin{pmatrix}1\\-2\\0\end{pmatrix}$$

$$= \begin{pmatrix}\frac{13}{5}\\-\frac{26}{5}\\0\end{pmatrix}.$$

b) $d(p, l_1)$ = distance between p and l_1.



$$\overrightarrow{CB} = \begin{pmatrix}10\\10\\10\end{pmatrix} - \begin{pmatrix}3\\0\\5\end{pmatrix} = \begin{pmatrix}7\\10\\5\end{pmatrix}.$$

$$d(p, l_1) = \left\| \begin{pmatrix}7\\10\\5\end{pmatrix} - \frac{\begin{pmatrix}7\\10\\5\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\0\end{pmatrix}}{\left\|\begin{pmatrix}1\\-2\\0\end{pmatrix}\right\|^2}\begin{pmatrix}1\\-2\\0\end{pmatrix} \right\|$$

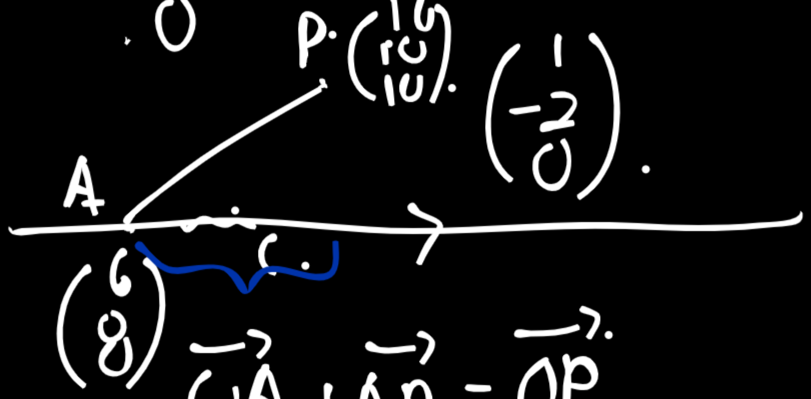
$\overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}.$

$\overrightarrow{AC} = \overrightarrow{AB} - \overrightarrow{CB}.$

$$= \left\| \begin{pmatrix}7\\10\\5\end{pmatrix} - -\frac{13}{5}\begin{pmatrix}1\\-2\\0\end{pmatrix} \right\|$$

$$= \left\| \begin{pmatrix}\frac{48}{5}\\\frac{24}{5}\\5\end{pmatrix} \right\| \approx 11.84 \text{ units (2d.p).}$$

c) $\pi_{\ell_2}(p)$



$$\vec{OA}+\vec{AP}=\vec{OP}$$

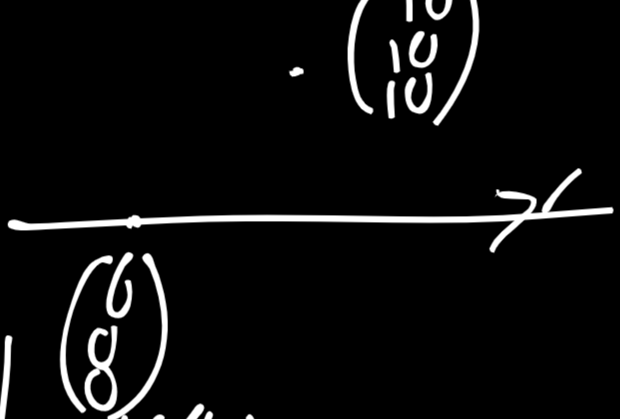
$$\vec{AP}=\vec{OP}-\vec{OA}$$

$$\pi_{\ell_2}(p)=\frac{\begin{pmatrix}4\\10\\10\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\0\end{pmatrix}}{\left\|\begin{pmatrix}1\\-2\\0\end{pmatrix}\right\|^2}\begin{pmatrix}1\\-2\\0\end{pmatrix} \qquad =\begin{pmatrix}10\\10\\10\end{pmatrix}-\begin{pmatrix}6\\0\\0\end{pmatrix}=\begin{pmatrix}4\\10\\10\end{pmatrix}.$$

$$=\frac{4-20}{5}\begin{pmatrix}1\\-2\\0\end{pmatrix}=-\frac{16}{5}\begin{pmatrix}1\\-2\\0\end{pmatrix}+\begin{pmatrix}6\\0\\0\end{pmatrix}.$$

$$=\begin{pmatrix}\frac{14}{5}\\ \frac{32}{5}\\ 0\end{pmatrix}.$$

d) $d(p,\ell_2)$ $\qquad p=\begin{pmatrix}10\\10\\10\end{pmatrix}$



$$\left\|\begin{pmatrix}4\\10\\10\end{pmatrix}-\frac{\begin{pmatrix}4\\10\\10\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\0\end{pmatrix}}{\left\|\begin{pmatrix}1\\-2\\0\end{pmatrix}\right\|^2}\begin{pmatrix}1\\-2\\0\end{pmatrix}\right\|=\left\|\begin{pmatrix}4\\10\\10\end{pmatrix}-\frac{-16}{5}\begin{pmatrix}1\\-2\\0\end{pmatrix}\right\|$$

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$$= \left\| \begin{pmatrix} \frac{36}{5} \\ \frac{18}{5} \\ 10 \end{pmatrix} \right\|$$

$$\approx 12.84 \text{ units.}$$

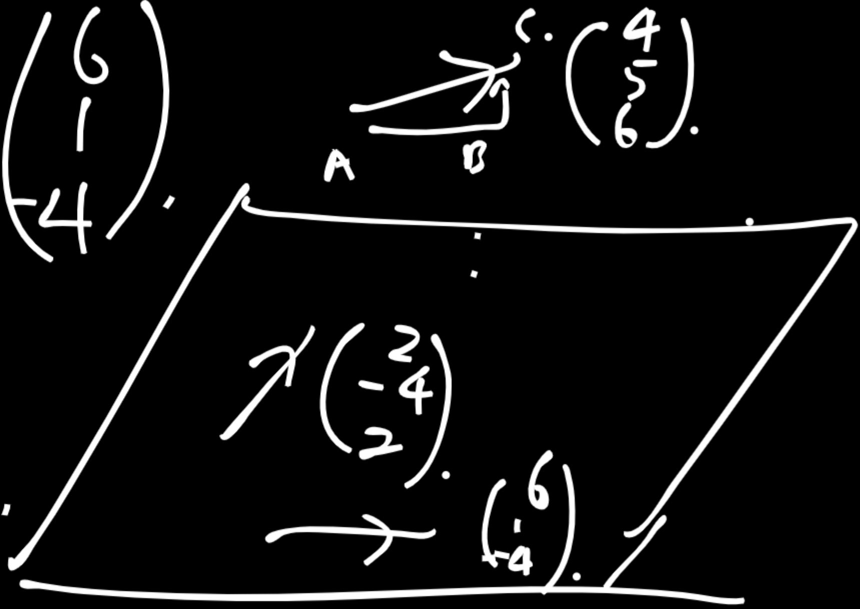
e) $d(l_1, l_2)$. since l_1 & l_2 are $\parallel$,

$$\vec{\ } = \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 9 \\ 0 \end{pmatrix}$$

3

$=$

E4.8). $$\hat{n} = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} \times \begin{pmatrix} 6 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 14 \\ 20 \\ 26 \end{pmatrix}$$



$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\vec{BC} = \vec{AC} - \vec{AB}$$

$$\pi_{\underline{n}\perp}(\vec{v}) = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} - \frac{\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 14 \\ 20 \\ 26 \end{pmatrix}}{\left\| \begin{pmatrix} 14 \\ 20 \\ 26 \end{pmatrix} \right\|^2} \begin{pmatrix} 14 \\ 20 \\ 26 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} - \frac{13}{53} \begin{pmatrix} 14 \\ 20 \\ 26 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{30}{53} \\ \frac{5}{53} \\ -\frac{20}{53} \end{pmatrix}$$