Multidimensional Auctions of Contracts: An Empirical Analysis

Yunmi Kong

Rice University

Isabelle Perrigne

Rice University

Quang Vuong

New York University

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Motivation

- Multi-attribute auctions: Design-build auctions, scale auctions, auctions of contracts,...

  $\longrightarrow$ Bids contain several components

  $\longrightarrow$ Allocation rule

  $\longrightarrow$ Auctioning contracts: Trade-off between adverse selection (rent extraction) and moral hazard (incentives for effort)

  Laffont and Tirole (1986, 1987), McAfee and McMillan (1987)

  (single-dimensional private information models)

  $\longrightarrow$ Examples: Author-publisher, patent holder-licensee, landlord-sharecropper, etc

  $\longrightarrow$ Multidimensional private information: A richer framework to account for observed multidimensional multivariate bids

Paper Contributions

$\longrightarrow$ Develop a framework to analyze multivariate bids under a general allocation rule relying on a best-response approach

$\longrightarrow$ Analysis of oil/gas lease auctions in Louisiana

Bids = Cash payment and Royalty (sharing rule of expost revenues)

$\longrightarrow$ Modeling the value of the contract as an Option (accounts for future price uncertainty and volatility, duration of option, and probability of option exercise)

$\longrightarrow$ Adverse selection (principal's payoff function of bidder's private information) and moral hazard (sharing rule as incentive to exercise the option)

$\longrightarrow$ Rich set of counterfactuals: Comparison with fixed royalty auctions and scoring auctions, change of lease duration, exploiting oil price fluctuations

Related Literature

- Scoring Auctions
  - → Exogenous quality: Yoganarasimhan (2016), Krasnokutskaya, Song and Tang (2020), Laffont, Perrigne, Simioni and Vuong (2020)
  - → Endogenous quality: Che (1993), Asker and Cantillon (2008, 2010), Lewis and Bajari (2011), Takahashi (2018), Sant' Anna (2018), Hanazono, Hirose, Nakabayashi and Tsuruoka (2016),
- Auctions of real options
  - → Board (2007), Cong (2019), Bhattacharya, Ordin and Roberts (2021), Hernstadt, Kellogg and Lewis (2019)

Data

568 auctions of Louisiana onshore oil leases 1974-2003

Bids = (cash payment to be paid upfront, royalty rate on revenues contingent on oil extraction)

Lease duration 3 years (continues if production)

Cash Bid on average \$1,015 per acre, median at \$712

Royalty on average 23% (= median)

As comparison, 25% on private lands, 12.5% on public lands

Cash and royalty positively correlated (0.38)

How the winner is chosen?

Dominant Bid	64%
→ dominant bid wins	99%
→ dominant bid loses	1%
No Dominant Bid	36%
→ higher cash wins	68%
→ higher royalty wins	32%

No specific scoring rule but bidders are aware of past auctions → Choice probabilities

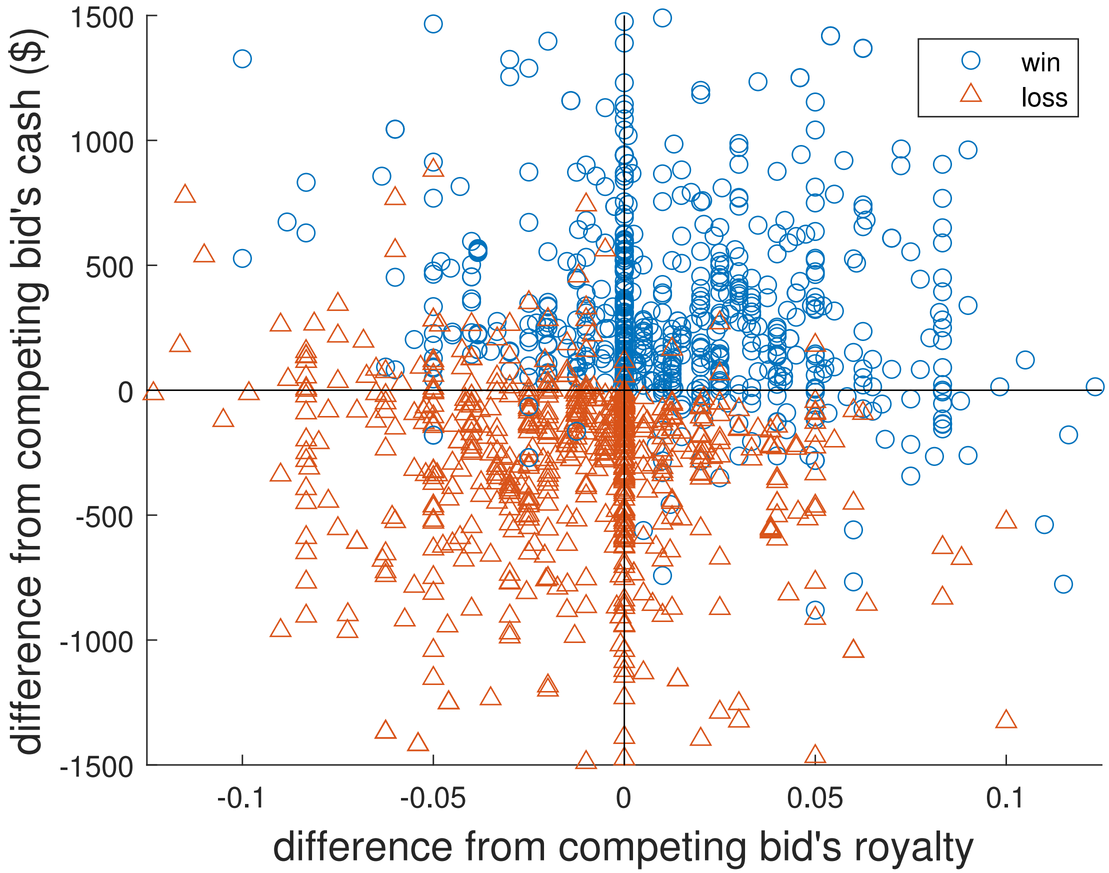
Probability of winning increasing in both cash and royalty

Contradiction with Laffont & Tirole (1987) and McAfee and McMillan (1987): Cash and royalty are negatively correlated, winner with the highest cash/lowest royalty

But one-dimensional types in their papers!

Are these auctions optimal in terms of generating revenue for Louisiana?

Figure: Choice Patterns



The Model

- A few facts to integrate in the model:

$\longrightarrow$ Auctioned Contract: Option

$\longrightarrow$ Royalty affects incentives to exercise the option and contract value

$\longrightarrow$ Multidimensional bids likely coming from multdimensional types in view of data scatter plot

- Notations

$(a, b) =$ (royalty rate, cash payment) — Firm's Bid

n firms bidding

$(\theta_1, \theta_2) =$ Firm's productivity (expected production volume) and cost — Firm's private information

$\sim F(\cdot, \cdot)$

t lease duration in years ($t = 3$)

r one year risk-free interest

p price at the time of the auction with volatility σ

Assumption: p follows a Geometric Brownian motion with volatility σ

Firm's Option Value

$$V(a, \theta_1, \theta_2) = e^{-rt}\left[p(1-a)\theta_1\Phi(x) - \theta_2\Phi(x - \sigma\sqrt{t})\right] \quad \text{with} \quad x \equiv \frac{\log[p(1-a)\theta_1/\theta_2] + \sigma^2 t/2}{\sigma\sqrt{t}},$$

where $\Phi(\cdot)$ cdf of Standard Normal

$p(1-a)\theta_1$: Firm's share of expected revenue

$\Phi(x - \sigma\sqrt{t})$: Ex ante probability of exercising the option

Black and Scholes (1973), Merton (1973), Black (1976)

	$V(a, \theta_1, \theta_2)$	Pr(exercise)
θ_1	+	+
θ_2	-	-
a	-	-
p	+	+
σ	+	?
t	+	?

$P(a, b|n)$: Firm's probability of winning when bidding royalty a and cash payment b with n bidders

Indirect approach (best response) *a la* GPV (2000)

Firm's Maximization Problem:

$$\max_{a,b} \left[V(a, \theta_1, \theta_2) - b \right] P(a, b|n)$$

Leading to FOC:

$$V_a(a, \theta_1, \theta_2) = -\frac{P_a(a, b|n)}{P_b(a, b|n)}$$

$$V(a, \theta_1, \theta_2) = b + \frac{P(a, b|n)}{P_b(a, b|n)}$$

Remark: Bidders exploit their private information to choose (a, b) reducing their payment without compromising their winning probability $P \rightarrow$ Adverse Selection

Moral Hazard arising from poor incentives to exercise option with large a

Identification and Estimation

Observables: $(a_{i\ell}, b_{i\ell}, W_{i\ell}, Z_\ell), i = 1, \ldots, n_\ell, \ell = 1, \ldots, L$

Remark: Z_ℓ includes tract characteristics as well as interest r_ℓ, oil price p_ℓ
and volatility σ_ℓ, $W_{i\ell}$ win/loss dummy

Model Primitive: $F(\theta_1, \theta_2)|Z, n)$

PROPOSITION: *The joint distribution $F(\cdot, \cdot|\cdot, n)$ is identified.*

Comment: The two FOCs identifies $(\theta_{1i\ell}, \theta_{2i\ell})$ from each submitted bid $(a_{i\ell}, b_{i\ell})$ since $P(\cdot, \cdot|\cdot, n)$ is observed and $V(\cdot, \cdot, \cdot)$ is a known function

Estimation of $P(\cdot, \cdot|n)$ as a nonparametric (kernel) regression of win/loss dummies $W_{i\ell}$ on $(a_{i\ell}, b_{i\ell}, Z_\ell)$ given n

Estimation Results

(θ_1, θ_2) affiliated across bidders, $n = 2$

$\longrightarrow F(\cdot, \ldots, \cdot|n)$, $P(a, b|a, b, n)$

Estimation of winning probability

$$\hat{P}(\cdot, \cdot|a, b, n) = \int \hat{C}(\cdot - a_-, \cdot - b_-|n)\hat{g}_{a_-,b_-|a,b}(a_-, b_-|a, b, n)da_-b_-,$$

with $g_{a_-,b_-|a,b,n}(\cdot, \cdot|\cdot, \cdot, n)$ is the joint conditional distribution of (a_-, b_-) (estimated semiparametrically with Gaussian copula) and $C(\cdot, \cdot|n$ pairwise choice probability (estimated via sieve approximation of $\mathrm{E}[W = 1|a, b, a_-, b_-, n]$)

Auction Heterogeneity: $P(\cdot, \cdot|a, b, z, n)$

Predict the mean of $a_{i\ell}$ and $\log b_{i\ell}$ conditional on Z_ℓ using ‘leave-one-out’ regressions

$\longrightarrow$ Normalized bids $\tilde{a}_{i\ell} = a_{i\ell} - \mathrm{E}(a_{-\ell}|Z_{-\ell})$, $\tilde{b}_{i\ell} = \log b_{i\ell} - \mathrm{E}(\log b_{-\ell}|Z_{-\ell})$ to estimate the joint bid density

Z_ℓ: oil price p, interest, acreage, acreage^2, royalty recipient, township production index, heatmap cash index (geological and geographic information)

Estimation of $F(\cdot, \cdot | z)$:

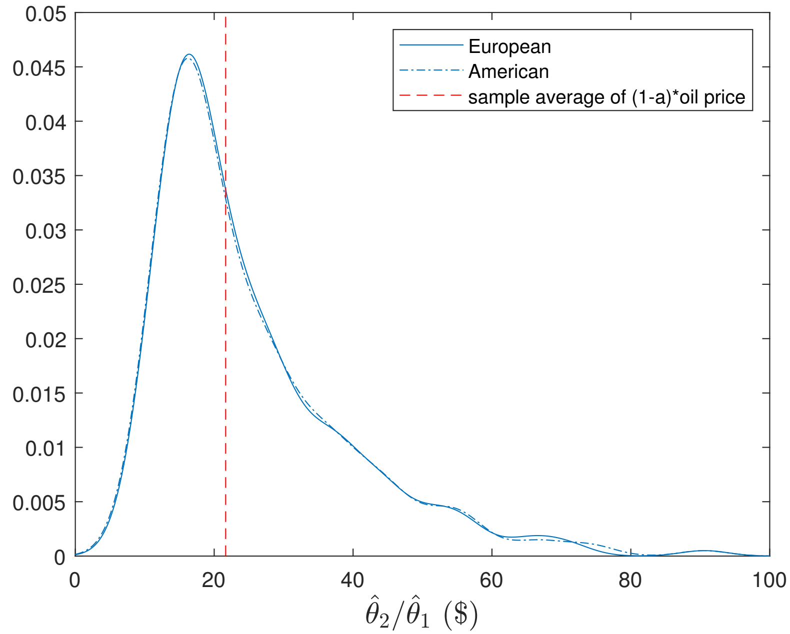
(1) Estimate $\hat{\theta}_{1i\ell}, \hat{\theta}_{2i\ell}$ solving the FOCs

(2) Estimate semiparametrically $F(\cdot, \ldots, \cdot | z)$ from estimated types using a Gaussian copula

$\hat{\theta}_2/\hat{\theta}_1$: Estimated cost per barrel of oil

Remark: All results robust to American option rather than European one

Figure: Estimated density of unit cost



$\longrightarrow$ A priori large proportion of tracts not profitable to exercise option (oil extraction)

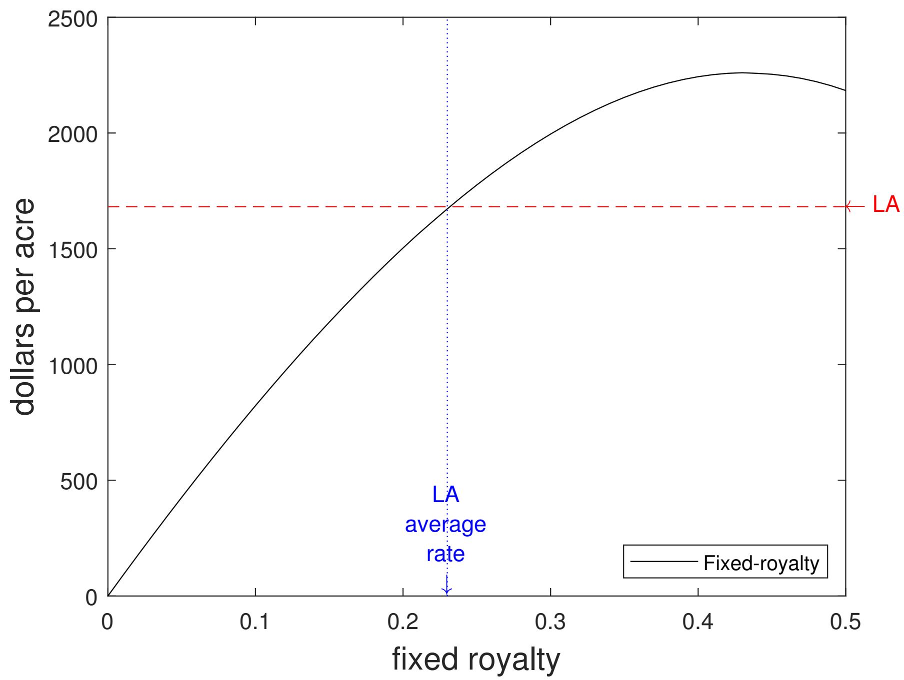
$\longrightarrow$ Estimates of production and cost in line with observations at the township and industry

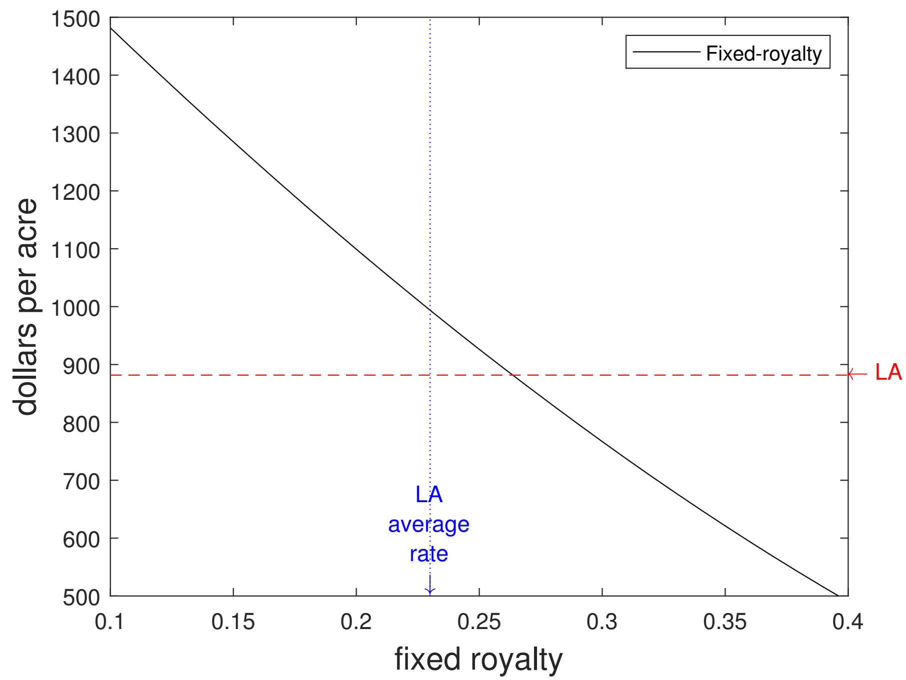
$\longrightarrow$ $\text{corr}(\theta_1, \theta_2) = 0.86$, correlation of prodction and cost across bidders at 0.81 and 0.9

$\longrightarrow$ Exante option exercise probability estimated at 0.44 compared to exercise probability at 0.42

Counterfactual: Cash-Royalty vs Fixed-Royalty Auctions

Figures: Royalty and Cash Payment Revenues





No benefit of having a cash-royalty auction, it even reduces the cash revenue by 11% compared to a 23% fixed-royalty auction

Government revenue: Louisiana auctions outperform fixed-royalty auctions if royalty less than 18% or larger than 48%

Exercise probability: No benefit either

Overall, adverse selection of cash-royalty bidding (leaving too much rents to firms) dominates the benefits of royalty flexbility

Counterfactual: Scoring vs Fixed-Royalty Auctions

Score $S(a, b) = b - wS/a^{\rho}$ with w optimal weight on royalty, $\rho \in [1, 10]$ curvature parameter

Table: Scoring vs Fixed-Royalty Auctions

	Fixed-royalty auction	Scoring auction, $\rho = 1$
Mean royalty	30%	37%
Median royalty	30%	20%
Total government revenue	\$2,889	\$2,743
Royalty revenue	\$1,995	\$1,196
Cash revenue	\$893	\$1,547
Firm information rents	\$944	\$1,300
Pr(option exercise)	0.41	0.45
Social surplus	\$3,832	\$4,042

Contrast with Asker and Cantillon (2008), again adverse selection is the main driving force

Problem of optimal mechanism under multidimensional private information

Counterfactual: Lease Duration and Timing in Fixed-Royalty Auctions

Lease duration $t = 6$ instead of 3

Overall decreases exercise probability but increases option value and cash bids. At 23% fixed royalty, 0.46 to 0.40 and total revenue would increase by 16%

Exploiting oil price fluctuations, a 20% increase in p

Overall increases exercise probability and government revenue

Extension

Our methodology extends to a general framework of multi-attribute auctions such as scoring and scale auctions

$(\theta_1, \ldots, \theta_{K+1}) \sim F(\cdot, \ldots, \cdot|n)$

$(b_1, \ldots, b_{K+1})$ bid components with b_{K+1} cash component

$V(b_1, \ldots, b_K, \theta_1, \ldots, \theta_{K+1})$

$[V(b_1, \ldots, b_K, \theta_1, \ldots, \theta_{K+1}) - b_{K+1}]P(b_1, \ldots, b_{K+1}|n)$

$$\begin{aligned} V_k(b_1, \ldots, b_K, \theta_1, \ldots, \theta_{K+1}) &= -\frac{P_k(b_1, \ldots, b_{K+1}|n)}{P_{K+1}(b_1, \ldots, b_{K+1}|n)}, k = 1, \ldots, K \\ V(b_1, \ldots, b_K, \theta_1, \ldots, \theta_{K+1}) &= b_{K+1} + \frac{P(b_1, \ldots, b_{K+1}|n)}{P_{K+1}(b_1, \ldots, b_{K+1}|n)}, \end{aligned}$$

The probability can be defined using the score $S(b_1, \cdots, b_{K+1})$

Concluding Remarks

Royalty bidding exacerbates adverse selection and induces too much moral hazard

Fixed-royalty auctions generate more government revenue and dominate scoring auctions

More research needed on optimal mechanism under muldimensionsl private information