ECE:5330 Graph Algorithms and Combinatorial Optimization

Spring 2024

Assignment 3

Due date: March 01, 2024

1. Using the programming language of your choice, implement the *Huffman's* algorithm from Section 4.8 for the optimal prefix code problem. You need to use binary heap in your implementation. Your program should be able to take two file names as its input of the command line execution: the first one is for the text file used to keep the frequency of each letter, and the second one is for the output text file used to store the prefix code of each letter. Please also well document your code. So, our TA knows how to compile and run your code.

(60 points)

In this problem the input file format is:

[number_of_letters]
[letter #1 : frequency of letter #1]
[letter #2 : frequency of letter #2]
...

and the output file format is:

[letter #1 : prefix code of letter #1]
[letter #2 : prefix code of letter #2]
...

Below is an example input file:

5
a : 0.32
b : 0.25
c : 0.20
d : 0.18
e : 0.05

and the corresponding output file is:

a : 11
b : 10
c : 01
d : 001
e : 000

2. Page 189: Problem 2 **(8 points)**

3. Page 199: Problem 20 **(8 points)**

4. Properties of Dijkstra's Algorithm **(8+8 points)**

We assume that $G = (V, E)$ is a weighted directed graph, and $s \in V$ is a designated source node. Denote by $\delta(u)$ the minimum distance from s to u in the graph (measured in terms of the sum of edge weights).

At any point in an execution of Dijkstra's algorithm between iterations of the main loop, we define S to be the subset of V consisting of the nodes that have already been processed, and $Q = V - S$. For any node u, $d[u]$ is the shortest path estimate of u, that is, the minimum weight of a path from s to u in which all nodes, except possibly the final node u, are in S. Prove the following two properties.

(a) After any number of iterations of the main loop, if u is any node in V, then $d[u] \geq \delta(u)$.

(b) After any number of iterations of the main loop, if $u \in S$ and $v \in Q$, then $\delta(u) \leq \delta(v)$.

5. Consider the problem of making change for n cents using the fewest number of coins. Assume that each coin's value is an integer. Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Prove that your algorithm yields an optimal solution. **(8 points)**

Bonus Problem: **(20 points)**

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