ECE:5330 Graph Algorithms and Combinatorial Optimization

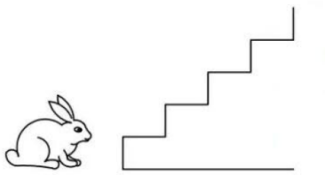
Spring 2024

Assignment 1

Due date: Feb. 02, 2024

1) Give an algorithm to find all nodes less than some value, X, in a binary heap. Your algorithm should run in O(K), where K is the number of nodes output. You need to give the main ideas of your algorithm and analyze the running time.
(15 points)

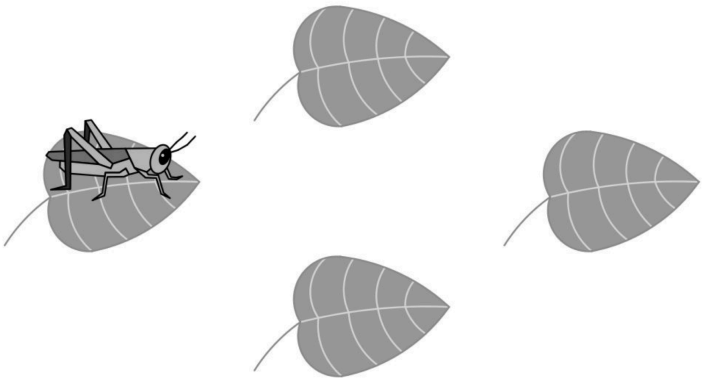
2) Buni Rabbit enjoys hopping up and down a set of stairs, one step at a time. In how many ways can Buni start on the ground, make a sequence of $2k$ hops, and end up back on the ground?



a) $k = 3$ **(10 points)**
b) $k = 5$ **(5 points)**

(Bonus Question) How to solve the problem when $k = n$ ($n > 0$) using a recursive algorithm? You need to give the main ideas of your algorithm and the recursive formula. **(20 points)**

3) A cricket randomly hops between 4 leaves, on each turn hopping to one of the other 3 leaves with equal probability. After $n \geq 0$ hops, what is the probability that the cricket has returned to the leaf where it started? Design a recursive algorithm to solve this problem. You need to give the main ideas of your algorithm and the recursive formula. **(10 points)**



4) Programming assignment. **(60 points)**

Using the programming language of your choice, implement the heap sort algorithm to sort a list of integers in non-decreasing order. You need to first implement the linear time `buildHeap` algorithm discussed in our class to construct a binary heap for the list of integers and then to perform the

`deleteMin` operations to output the sorted list of the integers. Your code should be able to take two file names as its input: the first one is for the text file used to keep the input integers and the second one is for the output text file used to store the sorted integers. Please also well document your code. So, our TA knows how to compile and run your code.