Lab4: Back-Propagation Through Time (BPTT)

Lab Objective:

In this project, you are going to implement RNN from scratch. The optimization method is BPTT (Back-Propagation Through Time), which is mentioned on page 9, chapter 10. You may also need to implement every necessary function by using Numpy or pure Python.

Important Date:

1. Experiment Report Submission Deadline: 4/25 (Thu) 12:00
2. Demo date: 4/25 (Thu)

Turn in:

1. Experiment Report (.pdf)
2. Source code

Notice: zip all files in one file and name it like「DLP_LAB4_your studentID_name.zip」, ex: 「DLP_LAB4_0756172_鍾嘉峻.zip」

Lab Description:

- Understand RNN model
  - Basic Structure
  - Forward Propagation
- Understand BPTT
  - Gradient vanish & explosion problem
  - Compare with Back-Propagation

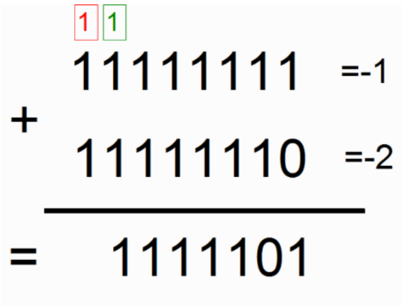
Requirements:

- Implement RNN network
  - Construct the neural network
  - Forward Propagation
  - Back-Propagation Through Time
  - Only Numpy or pure Python

- Apply your RNN model into Binary Addition task
  - Do the Binary Addition task with your RNN model

Task – Binary Addition:

- Introduction: The goal is trying to predict the result of two binary numbers addition
- Binary Number:
  - Each Number is less than 256/2 (total eight digits)
  - You can use Numpy to create numbers
- Actions:
  - A + B = C
  - Given A and B (each of them will less than 256/2), predict the correct answer C



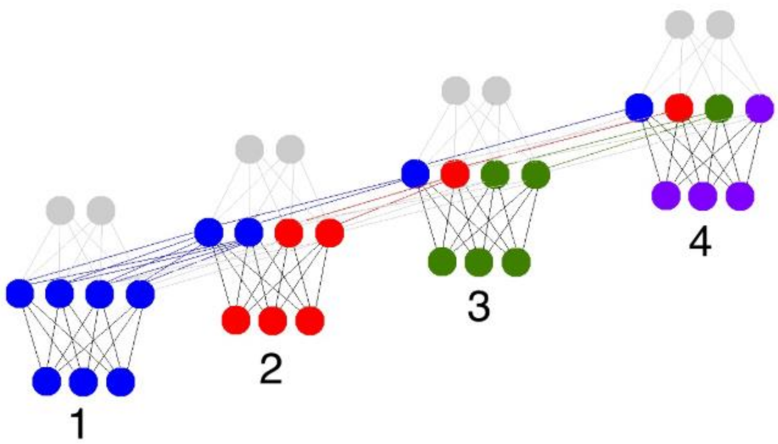
Picture 1. Example of binary addition

- Error:
  - Simply count how many digits are different between the ground truth and your result
- Accuracy:
  - Count how many correct answers in the last 1000 iterations

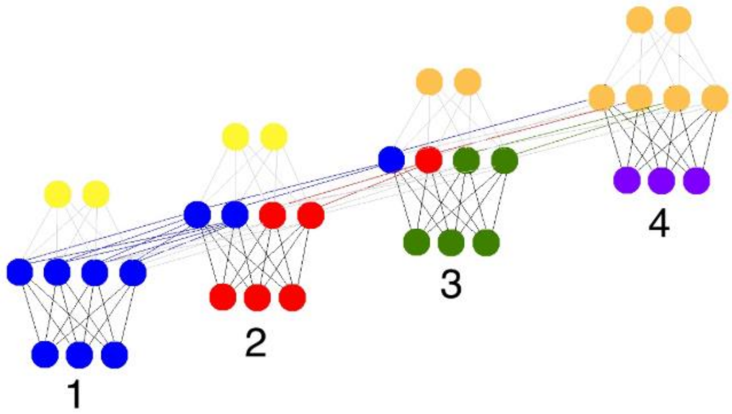
Implementation Details:

Network Architecture

- RNN model
  - Binary Dimension: 8
  - Input Dimension: 2
  - Hidden Dimension: 16
  - Output Dimension: 1
- Training Parameters:
  - Iteration: 20000
  - Alpha: 0.1
  - Spending Time: in a few minutes
- Forward Propagation

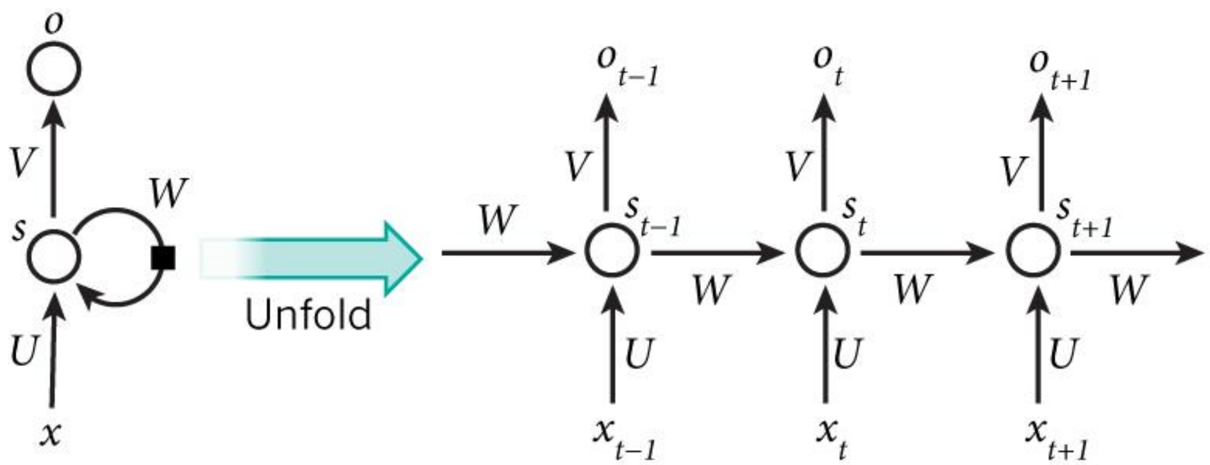


- BPTT



Methodology:

- **Algorithm – RNN model**



The above diagram shows an RNN being unrolled (or unfolded) into a full network. By unrolling, we mean that we write out the network for the complete sequence. For example, if the sequence we care about is a sentence of 5 words, the network would be unrolled into a 5-layer neural network, one layer for each word. The formulas that govern the computation happening in an RNN are as follows:

- x_t is the input at time step t. For example, x_1 could be a one-hot vector corresponding to the second word of a sentence.
- s_t is the hidden state at time step t. It's the "memory" of the network. s_t is calculated based on the previous hidden state and the input at the current step: $s_t = f(Ux_t + Ws_{t-1})$. The function f usually is a nonlinearity such as tanh or ReLU. s_{-1}, which is required to calculate the first hidden state, is typically initialized to all zeroes.
- o_t is the output at step t. For example, if we wanted to predict the next word in a sentence, it would be a vector of probabilities across our vocabulary. $o_t = \mathrm{softmax}(Vs_t)$.

- **Forward Propagation**

$$
\begin{aligned}
\boldsymbol{a}^{(t)} &= \boldsymbol{b} + \boldsymbol{W}\boldsymbol{h}^{(t-1)} + \boldsymbol{U}\boldsymbol{x}^{(t)}, \\
\boldsymbol{h}^{(t)} &= \tanh(\boldsymbol{a}^{(t)}), \\
\boldsymbol{o}^{(t)} &= \boldsymbol{c} + \boldsymbol{V}\boldsymbol{h}^{(t)},
\end{aligned}
$$

where $\boldsymbol{U}, \boldsymbol{W}, \boldsymbol{V}$ correspond respectively to connections for

– Input-to-hidden units ($\boldsymbol{U}$)

– Hidden-to-hidden units ($\boldsymbol{W}$)

– Hidden-to-output units ($\boldsymbol{V}$)

and $\boldsymbol{b}, \boldsymbol{c}$ are biase

- **BPTT**

To compute $\nabla_{\boldsymbol{W}} L$, we observe that:

- The immediate child nodes of W are all h(t)'s, and
- The chain rule for tensorsa can be applied to arrive at

$$
\nabla_{\boldsymbol{W}} L = \sum_t \sum_i \left(\frac{\partial L}{\partial h_i^{(t)}} \right) (\nabla_{\boldsymbol{W}} h_i^{(t)})
$$

$$
\boldsymbol{X}_{m \times n} \xrightarrow{g(\boldsymbol{X})} \boldsymbol{Y}_{s \times k} \xrightarrow{f(\boldsymbol{Y})} z_{1 \times 1}
$$

$$
\nabla_{\boldsymbol{X}} z = \sum_j \left(\frac{\partial z}{\partial Y_j} \right) \nabla_{\boldsymbol{X}} Y_j,
$$

- To complete the evaluation, we need to know further $\nabla_{\boldsymbol{h}^{(t)}} L$, which can be evaluated using the same chain rule as

$$
\begin{aligned}
\nabla_{\boldsymbol{h}^{(t)}} L &= \left(\frac{\partial \boldsymbol{h}^{(t+1)}}{\partial \boldsymbol{h}^{(t)}} \right)^T (\nabla_{\boldsymbol{h}^{(t+1)}} L) + \left(\frac{\partial \boldsymbol{o}^{(t)}}{\partial \boldsymbol{h}^{(t)}} \right)^T (\nabla_{\boldsymbol{o}^{(t)}} L) \\
&= \boldsymbol{W}^T \boldsymbol{H}^{(t+1)} (\nabla_{\boldsymbol{h}^{(t+1)}} L) + \boldsymbol{V}^T (\nabla_{\boldsymbol{o}^{(t)}} L)
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{H}^{(t+1)} &= \left(\frac{\partial \boldsymbol{h}^{(t+1)}}{\partial \boldsymbol{a}^{(t+1)}} \right)^T \\
&= \begin{bmatrix}
1 - (h_1^{(t+1)})^2 & 0 & \dots & 0 \\
0 & 1 - (h_2^{(t+1)})^2 & \dots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \dots & 1 - (h_n^{(t+1)})^2
\end{bmatrix}
\end{aligned}
$$

$$
\nabla_{\boldsymbol{o}^{(t)}} L = \hat{\boldsymbol{y}}^{(t)} - \boldsymbol{y}^{(t)}
$$

and

$$\nabla_{\boldsymbol{h}^{(\tau)}} L = \boldsymbol{V}^T (\nabla_{\boldsymbol{o}^{(\tau)}} L) = \boldsymbol{V}^T (\hat{\boldsymbol{y}}^{(\tau)} - \boldsymbol{y}^{(\tau)})$$

- In matrix form, $\nabla_{\boldsymbol{W}} L$ is given as

$$\nabla_{\boldsymbol{W}} L = \sum_t \boldsymbol{H}^{(t)} (\nabla_{\boldsymbol{h}^{(t)}} L) \boldsymbol{h}^{(t-1)T}$$

- The gradient on the remaining parameters can be obtained similarly

$$\nabla_{\boldsymbol{U}} L = \sum_t \boldsymbol{H}^{(t)} (\nabla_{\boldsymbol{h}^{(t)}} L) \boldsymbol{x}^{(t)T}$$

$$\nabla_{\boldsymbol{V}} L = \sum_t (\nabla_{\boldsymbol{o}^{(t)}} L) \boldsymbol{h}^{(t)T}$$

$$\nabla_{\boldsymbol{b}} L = \sum_t \boldsymbol{H}^{(t)} (\nabla_{\boldsymbol{h}^{(t)}} L)$$

$$\nabla_{\boldsymbol{c}} L = \sum_t \nabla_{\boldsymbol{o}^{(t)}} L$$

Rule of Thumb:

- The error should decrease very fast if you write the correct code.
- Don't set training iteration too large, the error should be close to 0 after 10000 iterations.

Scoring Criteria:

- Report (70%)
  - A plot shows episode rewards of at least 10000 training episodes (10%)
  - Describe how to generate data? (10%)
  - Explain the mechanism of forward propagation (20%)
  - Explain the mechanism of BPTT (20%)
  - Describe how the code work (the whole code) (10%)
  - More you want to say

- Performance(30%)
  - Accuracy * 100

References:

[1] M.P. Cuéllar and M. Delgado and M.C. Pegalajar (2006). An Application of Non-linear Programming to Train Recurrent Neural Networks in Time Series Prediction Problems. Enterprise Information Systems VII. Springer Netherlands. pp. 95–102.

[2] Pangolulu "Rnn From Scratch." Retrieved from Github: https://github.com/pangolulu/rnn-from-scratch

[3] Kjw0612. "Awesome Recurrent Neural Networks" Retrieved from Github: https://github.com/kjw0612/awesome-rnn