Expansion detected by cycles

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Abstract

1 On the hitting number of sets

Definition 1.1. *Let S be a set and $\mathcal{F} \subseteq 2^S$ a family of subsets. Then*

$$\tau(\mathcal{F}) := \min\{|P| : P \subseteq S, P \cap F \neq \emptyset, \text{for all } F \in \mathcal{F}\}$$

*is called the **hitting number** of $\mathcal{F}$.*

Definition 1.2. *Let S be a set and $\mathcal{F} \subseteq 2^S$ a family of subsets. Then*

$$\Delta_{\mathcal{F}} := \left\{ S \subseteq \mathcal{F} : \bigcap_{F \in S} F \neq \emptyset \right\}$$

*is called the **cut complex** of $\mathcal{F}$.*

Definition 1.3. *Let X be a simplicial complex on the vertex set V and $S \subseteq X$ a set of simplices of X. Then S is called a **covering** of X if for every vertex $v \in V$ there exists a simplex $\sigma \in S$, such that $v \in \sigma$.*

Proposition 1.1. *Let S be a set and $\mathcal{F} \subseteq 2^S$ a family of subsets. Then we have:*

$$\tau(\mathcal{F}) = \min\{|S| : S \text{ is a covering of } \Delta_{\mathcal{F}}\}$$

Proof. □

References