STAT/ME 424 Homework 6

Due Tue Nov 15, 2022

1. An aluminum master alloy manufacturer produces grain refiners in ingot form. The company produces the product in four furnaces. Each furnace is known to have its own unique operating characteristics, so any experiment run in the foundry that involves more than one furnace will consider furnaces as a block variable. The process engineers suspect that stirring rate impacts the grain size of the product. Each furnace can be run at four different stirring rates. A randomized block design is run for a particular refiner and the resulting grain size data is shown in Table 1.

Table 1: Stirring rate data

Stirring	Furnace			
rate	1	2	3	4
5	8	4	5	6
10	14	5	6	9
15	14	6	9	2
20	17	9	3	6

   (a) Is there any evidence that stirring rate impacts grain size? Test the appropriate hypothesis at level $\alpha = 0.05$.

   (b) Make a normal quantile plot of the residuals from the experiment. Interpret the plot.

   (c) Plot the residuals versus furnace number and versus stirring rate. Do the plots convey any useful information?

   (d) State the model you use to answer the above questions and carry out a test to check its validity.

   (e) What should the process engineers recommend concerning the choice of stirring rate and furnace for this grain refiner if small grain size is desirable?

   (f) Estimate the pairwise differences between mean values of the stirring rates with 95% simultaneous Tukey confidence intervals.

2. Aluminum is produced by combining alumina with other ingredients in a reaction cell and applying heat by passing electric current through the cell. Alumina is added continuously to the cell to maintain the proper ratio of alumina to other ingredients. Four different ratio control algorithms were investigated in an experiment. The response variables studied were related to cell voltage. Specifically, a sensor scans the cell voltage several times each second, producing thousands of voltage measurements during each run of the experiment. The process engineers decided to use the average voltage and the standard deviation of cell voltage over the run as the response variables. The average voltage is important because it impacts cell temperature, and the standard deviation of voltage (called "pot noise") is important because it impacts the overall cell efficiency.

   The experiment was conducted as a randomized block design, where six time periods were selected as the blocks, and all four ratio control algorithms were tested in each time period. The average cell voltage and the standard deviation of voltage (shown in parentheses) for each cell are given in Table 2.

   Table 2: Voltage data

Control	Time period					
Algorithm	1	2	3	4	5	6
1	4.93 (0.05)	4.86 (0.04)	4.75 (0.05)	4.95 (0.06)	4.79 (0.03)	4.88 (0.05)
2	4.85 (0.04)	4.91 (0.02)	4.79 (0.03)	4.85 (0.05)	4.75 (0.03)	4.85 (0.02)
3	4.83 (0.09)	4.88 (0.13)	4.90 (0.11)	4.75 (0.15)	4.82 (0.08)	4.90 (0.12)
4	4.89 (0.03)	4.77 (0.04)	4.94 (0.05)	4.86 (0.05)	4.79 (0.03)	4.76 (0.02)

   (a) Analyze the average cell voltage data. (Use $\alpha = 0.05$.) Does the choice of ratio control algorithm affect the average cell voltage?

   (b) Perform an appropriate analysis on the standard deviation of voltage. Does the choice of ratio control algorithm affect the pot noise?

   (c) Conduct any residual analyses you deem appropriate.

   (d) Which ratio control algorithm would you select if your objective is to reduce both the average cell voltage and the pot noise?