Lecture 4 – Network Flow Problems: The Transportation Problem

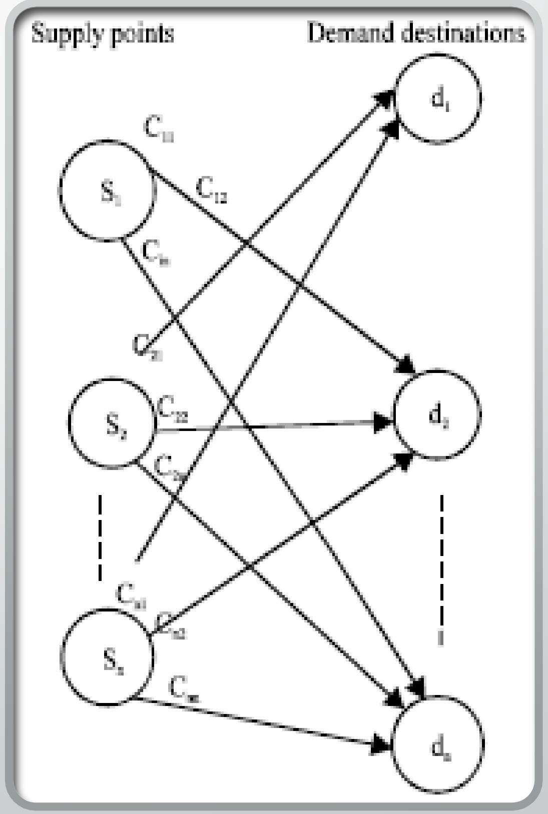
Module 5 – Special Cases of Linear Programs
CS/ISyE/ECE 524

Transportation problems

The assignment (matching) problem is a type of **transportation problem**

Objective of transportation problem is to transport a commodity (sandwich) from several possible sources (kinds of sandwich) to several possible destinations (children) at minimal cost

- Sources have known supply limits
- Destinations have known demands
- Edges may have capacity limits
- Each edge has an associated cost



A more complicated transportation example

The story:

- Han Solo has three big crime lords he's trying to keep happy and is planning three new storage bases for his smuggled goods
- Each crime lord requires a certain number of deliveries each day and each storage site can make deliveries a specified number of times per day
- It costs Han 4 credits/parsec/unit to haul deliveries
- Distances to each crime lord are given below:

The question:

How can Han meet the crimelords' demands at minimal cost?

	Distance from storage site to crime lord (parsecs)			Max deliveries/day from site
Storage site	Jabba the Hutt	Pyke Syndicate	Sollima	
1	50	25	84	24
2	12	75	18	12
3	64	39	70	9
Demand	16	20	9	= 45 total

The general transportation model

Data

- $I = \{1, 2, 3\}$ – the set of storage sites
- $J = \{1, 2, 3\}$ – the set of crime lords
- d_{ij} – Distance from site i to crime lord j
- c_{ij} – cost of traveling on path (i, j) $(= 4 \, * d_{ij})$
- m_j – demand of crime lord j
- s_i – supply from site i

Variables

- Assign deliveries to routes:

x_{ij} = # deliveries Han sends from site i to crime lord j (x nonnegative)

Objective

- Minimize costs:

$$\min_{x} \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$$

Constraints

- Meet demand:

$$\sum_{i \in I} x_{ij} = m_j \; \forall j \in J$$

- Not exceed supply:

$$\sum_{j \in J} x_{ij} = s_i \; \forall i \in I$$

Question: Why can we use equalities in the constraints?

In Julia: Transportation Example