1) max $z = x_1 + 2x_2 + ax_3$

S.T. $x_1 + 2x_2 + 3x_3 \le 150$ (sunlight)

$2x_1 + 3x_2 + 2x_3 \le 150$ (chemical)

$x_1, x_2, x_3 \ge 0$

a is known constant

a. If first dual constraint has slack, then $\lambda_1 = 0$

By strong duality $p^* = d^* \rightarrow 120 = 150\lambda_1 + 150\lambda_2$

$\hookrightarrow 120 = 150\lambda_2$

$\lambda_2 = \frac{4}{5}$

min $150(0) + 150(\frac{4}{5})$

b) min $150\lambda_1 + 150\lambda_2$

S.T $\lambda_1 + 2\lambda_2 \ge 1$

$2\lambda_1 + 3\lambda_2 \ge 2$

$3\lambda_1 + 2\lambda_2 \ge 2$

$\hookrightarrow \lambda_1 = 0 \rightarrow 2\lambda_2 \ge 1 \rightarrow \lambda_2 \ge \frac{1}{2}$

$3\lambda_2 \ge 2 \rightarrow \lambda_2 \ge \frac{2}{3}$

$2\lambda_2 \ge 2 \rightarrow \lambda_2 \ge 1$

By $\hookrightarrow$ min $150\lambda_2 \rightarrow$ to minimize, need smallest value of λ_2 which is $\lambda_2 = 1$

$\boxed{\lambda_1 = 0, \lambda_2 = 1}$

c) New optimal area would not change, as line only intersects other hyperplanes at one point which does not change