# Lecture 2 – Geometry of Linear Programs

Module 4 – Linear Programming Basics
CS/ISyE/ECE 524



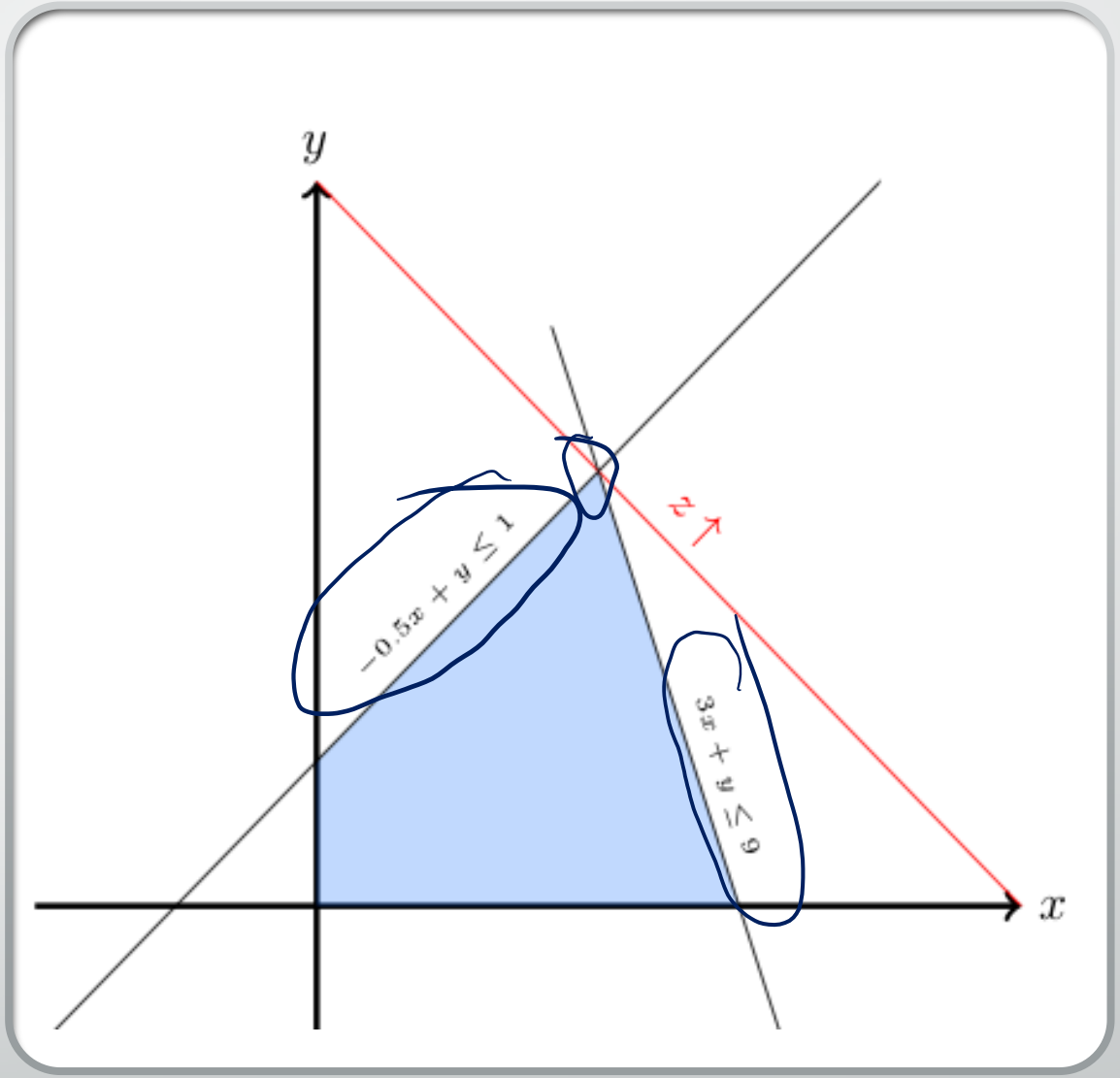
# Solving Linear Programs

We've seen the graphical method. We'll see other algorithms later, but let's start building an intuition for what we might encounter....

There are exactly four possible cases for an LP….

1. There is *exactly one* optimal solution, and it lies at one of the **extreme points** of the polyhedron. We say the LP **is feasible and has an optimal solution.**

An **extreme point** or **vertex** is the intersection of *n* linearly independent hyperplanes.

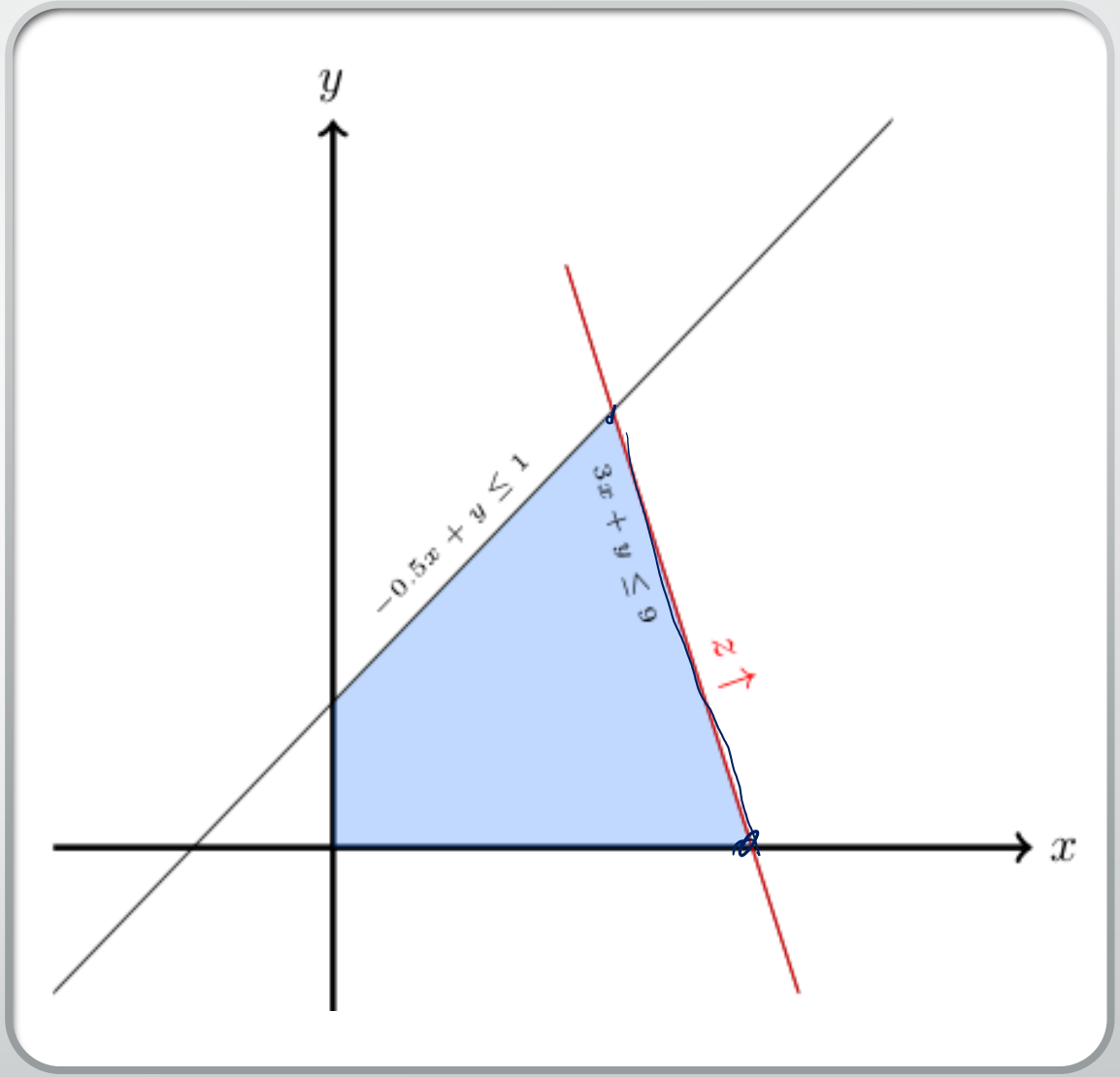


# There are exactly four possible cases for an LP….

**2**. There are *infinitely many* solutions, and they lie along a **face** of the polyhedron. We say the LP **is feasible and has an optimal solution.**

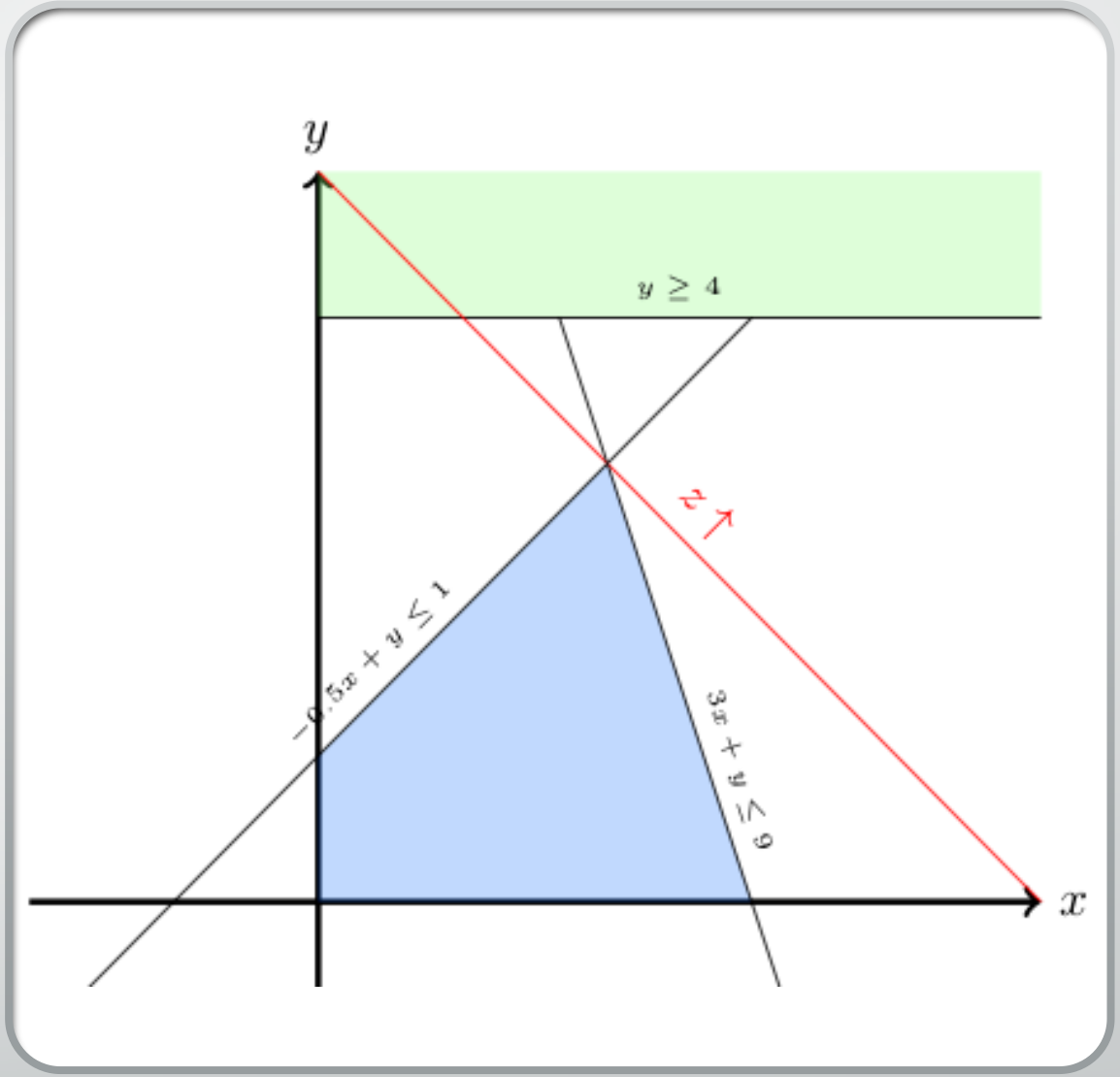
A **face** is a set $\{x \in P \mid a^T x = b\}$ where $a^T x \leq b$ is a **valid inequality** of the polyhedron $P$.

$a^T x \leq b$ is a **valid inequality** for $P$ if $a^T x \leq b$ for all $x \in P$.



There are exactly four possible cases for an LP….

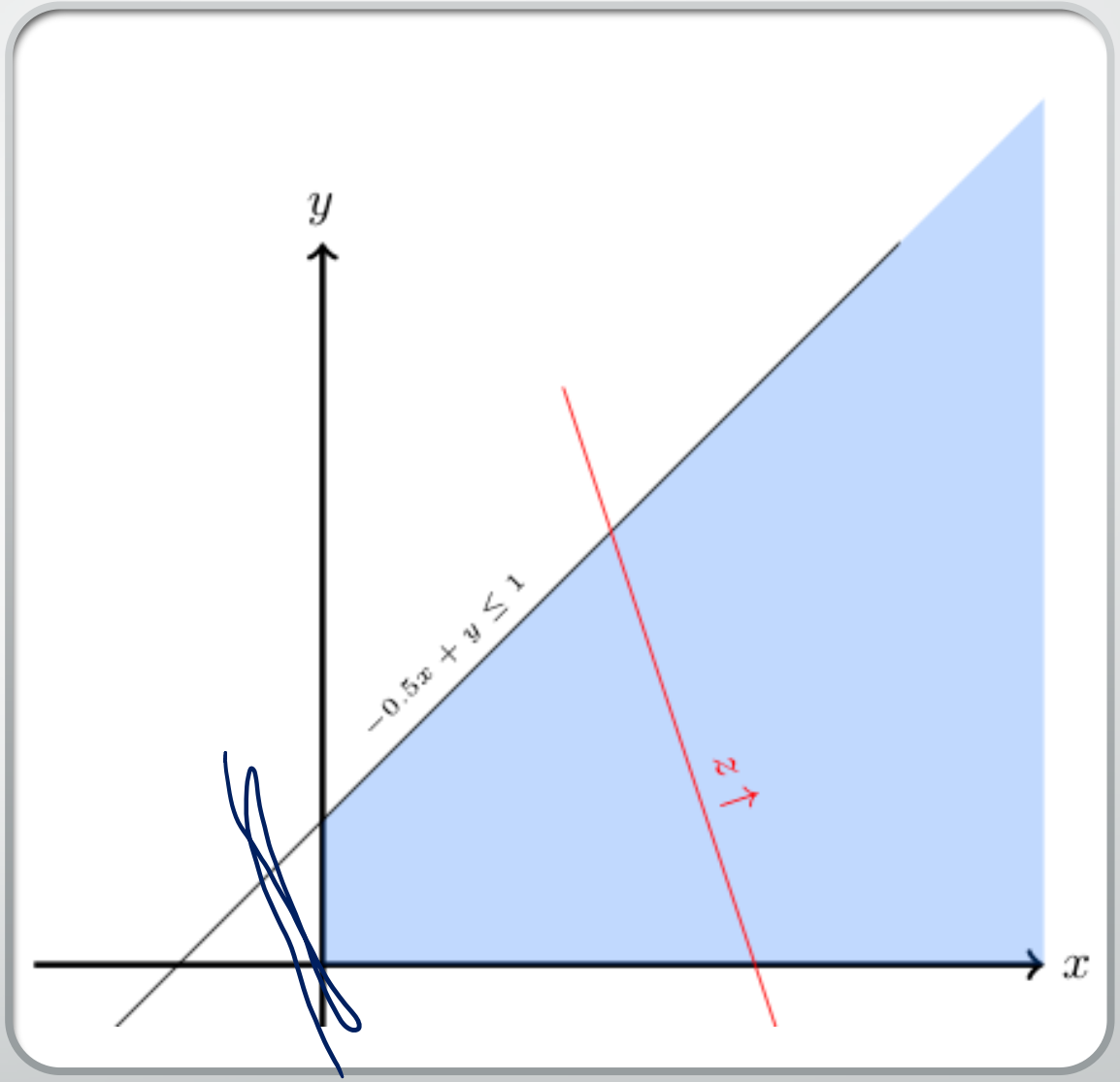
**3.** The feasible region is *empty* (no points lie in the intersection of the halfspaces). We say the LP is **infeasible.**



# There are exactly four possible cases for an LP….

**4.** The feasible region is *unbounded,* and we can make the objective function arbitrarily large. We say the LP is **unbounded**.

*Beware: An unbounded feasible region does **not necessarily** imply an unbounded LP!*



After modeling your LP and solving it, the solver unexpectedly returns "infeasible." Which of the following modeling errors could have caused this?

A. Incorrect objective function

B. Forgot to include a constraint

C. Added an extra constraint by mistake

D. Any of A, B, or C could have caused it

E. Either A or B could have caused it

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Suppose you have a constrained minimization problem. You then add an extra linear constraint to the problem. What might the optimal objective value do?

A. increase

B. stay the same

C. decrease

D. any of A, B, or C could happen

E. either A or B could happen

# Linear Programming Basics Module Learning Outcomes

Now, you should be able to ….

- Give a formal definition of a linear program (LP)
- Transform any LP into standard form
- Graphically deduce whether an LP
  - has exactly one optimal solution
  - has infinitely many optimal solutions,
  - is infeasible, or
  - is unbounded.
- Begin to gain an intuition for the properties of LPs