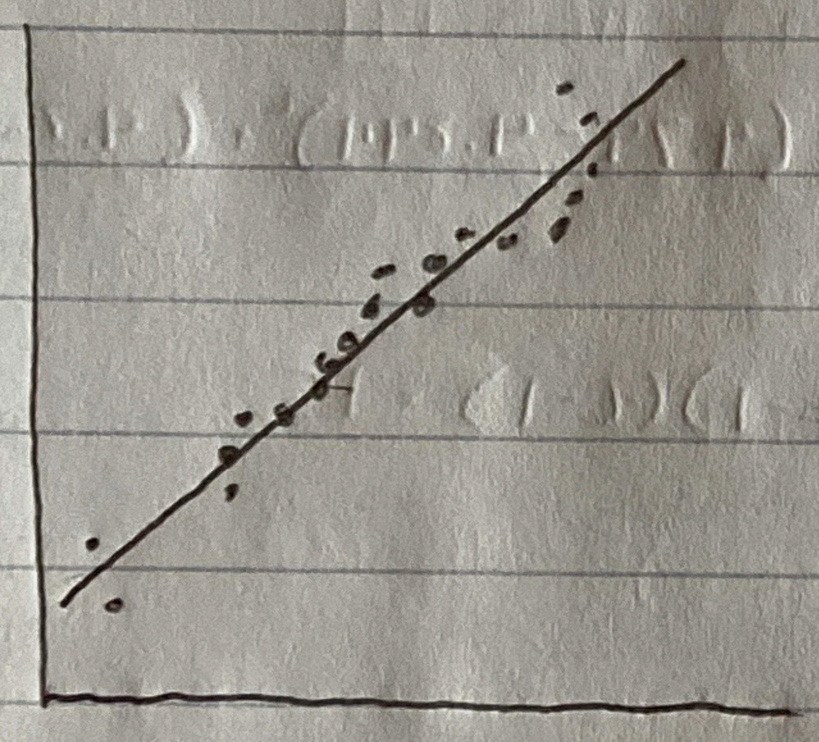
$S_R = 8.04 - 0.94 - 6.17 = \boxed{0.93} \rightarrow Df = 15$

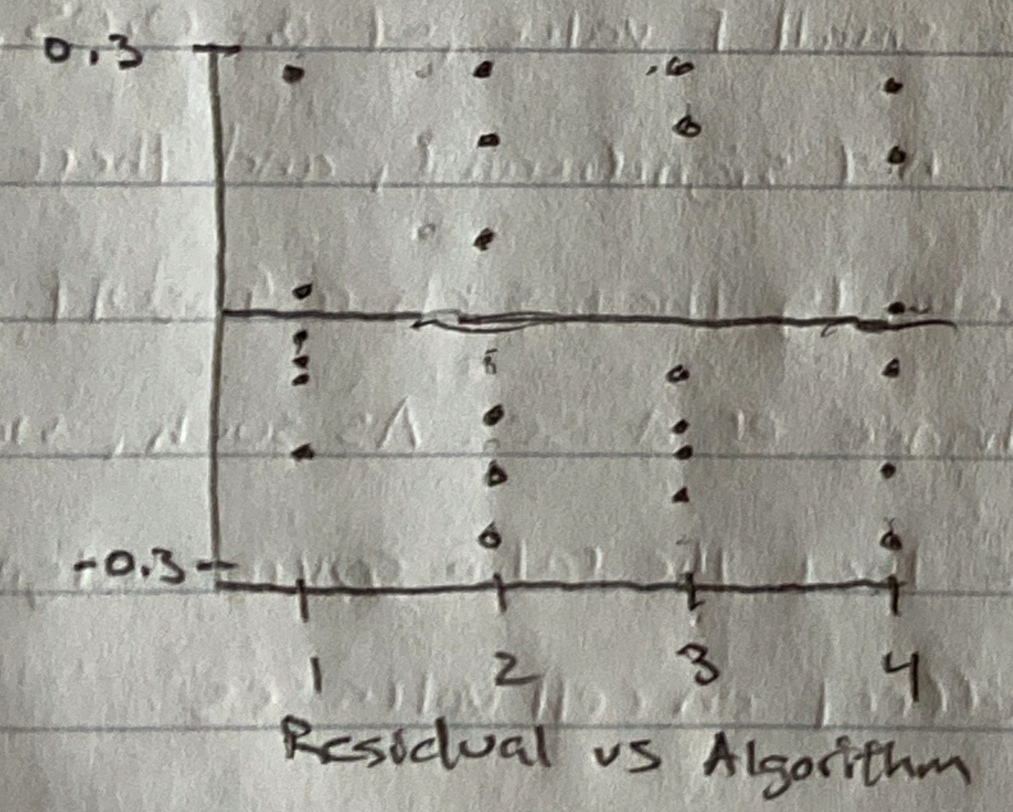
Source	Df	SS	MS	F
Treatment	3	6.17	2.06	33.26
Block	5	0.94	0.19	
Residual	15	0.93	0.062	
Total	23	8.04		

→ After applying a ln() transformation on the data to make it additive, our F value is very large, thus resulting in a p value much smaller than $\alpha = 0.05$, indicating that the model is significant, and that ratio control algorithm does affect the pot noise

c)



Residuals lie on a relatively normal linear distribution, and thus fulfills the normality assumption.



Somewhat heteroscedastic, meaning caution is required when doing inference analysis

d) Because it was concluded that the ratio control algorithm has no effect on cell voltage, we must rely on which algorithm minimized pot noise (std dev of cell voltage), which on average could be seen in algorithm 2

2)

Control Algorithm (t)	1	2	3	4	5	6	mean
1	4.93 (0.05)	4.86 (0.07)	4.75 (0.05)	4.95 (0.06)	4.79 (0.03)	4.88 (0.05)	4.86 (0.04667)
2	4.85 (0.04)	4.91 (0.02)	4.79 (0.03)	4.85 (0.05)	4.75 (0.03)	4.85 (0.02)	4.833 (0.031667)
3	4.83 (0.09)	4.88 (0.13)	4.90 (0.11)	4.75 (0.15)	4.82 (0.08)	4.90 (0.12)	4.847 (0.113333)
4	4.89 (0.03)	4.77 (0.04)	4.94 (0.03)	4.86 (0.05)	4.79 (0.03)	4.76 (0.02)	4.835 (0.0416667)
mean	4.875 (0.0525)	4.855 (0.0575)	4.845 (0.055)	4.853 (0.0775)	4.788 (0.0475)	4.848 (0.0525)	$\boxed{4.844}$ (0.058334)

Time period (b)

0.05625

a) $S_t = b\sum_j (y_{\cdot j} - y_{\cdot\cdot})^2 = 6((4.86-4.844)^2 + (4.833+4.844)^2 + (4.847-4.844)^2 + (4.835-4.844)^2)$

$= \boxed{2.746E^{-3}} \rightarrow Df = t-1 = 3$

$S_b = t\sum_i (y_{i\cdot} - y_{\cdot\cdot})^2 = 4((4.875-4.844)^2 + (4.855-4.844)^2 + \ldots + (4.848-4.844)^2)$

$= \boxed{0.017} \rightarrow Df = b-1 = 5$

$S_D = \sum_{ij} (y_{ij} - y_{\cdot\cdot})^2 = (4.93-4.844)^2 + (4.86-4.844)^2 + \ldots + (4.79-4.844)^2 + (4.76-4.844)^2$

$= \boxed{0.092} \rightarrow Df = bt-1 = 23$

$S_R = 0.092 + 0.017 - 2.746E^{-3} = \boxed{0.072} \rightarrow Df = (b-1)(t-1) = 15$

Source	Df	SS	MS	F
Treatment	3	2.746E^{-3}	9.153E-4	0.19
Block	5	0.017	3.487E-3	
Residual	15	0.072	4.812E-3	
Total	23	0.092		

→ Due to the small F value, at $\alpha = 0.05$, the model is not significant, and there is a large probability that such a result could be due to chance or noise. As such, we can determine that the ratio control algorithm has no effect on cell voltages

b) $\cancel{S_t = b\sum (y_{\cdot j} - y_{\cdot\cdot})^2 = 0.0249}$

$\cancel{S_b = t\sum (y_{i\cdot} - y_{\cdot\cdot})^2 = 1.1718}$

$\cancel{S_D = \sum_{ij} (y_{ij} + y_{\cdot\cdot})^2 =}$

→ based on data, there seems to be skews on residuals and qqplot so use log-transformed data, which would also yield easier values to work with

$S_t = b\sum_j (\ln(y_{\cdot j}) - \ln(y_{\cdot\cdot}))^2 = 6(\ln(0.04667) - \ln(0.058334))^2 + \ldots + (\ln(0.0416667) - \ln(0.058334))^2$

$= \boxed{6.17} \rightarrow Df = 3$

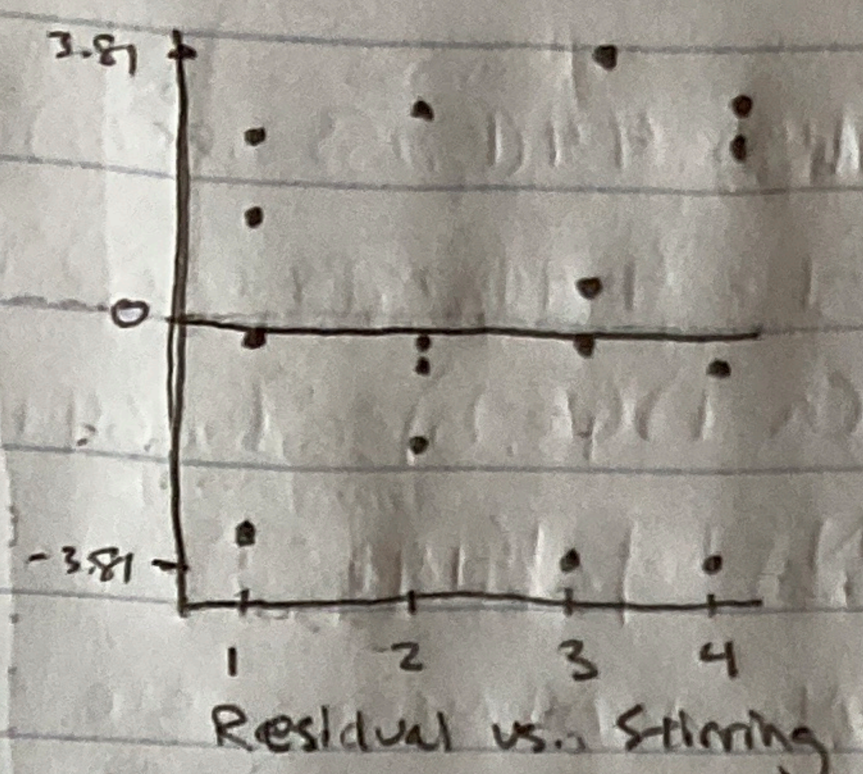
$S_b = t\sum (\ln(y_{i\cdot}) - \ln(y_{\cdot\cdot}))^2 = 4(\ln(0.0525) - \ln(0.058334))^2 + \ldots + (\ln(0.0525) - \ln(0.058334))^2$

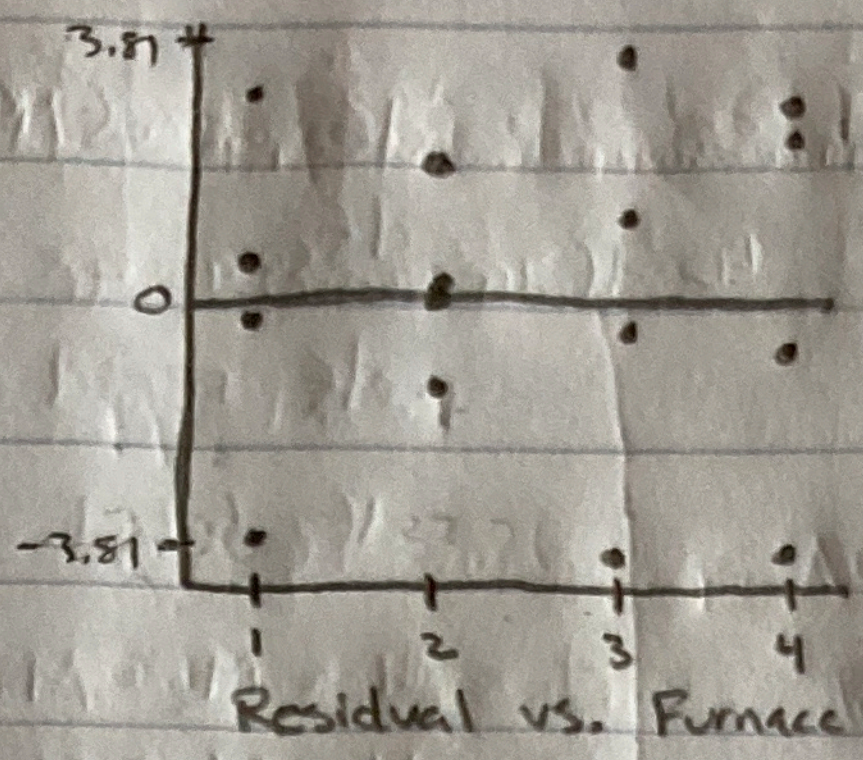
$= \boxed{0.94} \rightarrow Df = 5$

$S_D = \sum_{ij} (\ln(y_{ij}) - \ln(y_{\cdot\cdot}))^2 = (\ln(0.05) - \ln(0.058334))^2 + \ldots + (\ln(0.02) - \ln(0.058334))^2$

$= \boxed{8.04} \rightarrow Df = 23$

c)





Both are approximately homoscedastic, and thus allow us to safely infer based on F-test analysis

d) The model is $y_{jk} = \mu_{..} + \rho_j + \gamma_k + \varepsilon_{ij}$

e) Because there is no effect on grain size based on stirring rate, it should not matter what choice of combination of stirring rate and furnace is used to achieve a smaller grain size. Rather, some other variable should be tested to see if it is able to achieve smaller grains

f) Tukey multiplier $= q(4, 12, 0.05) = 4.199$

$\sqrt{\frac{MSE}{n}} = \sqrt{\frac{8.673}{4}} = 1.472$

Between stirring rate = 5 and 10: $8.5 - 5.75 \pm 4.199 \cdot 1.472 = 2.75 \pm 6.181$

$\rightarrow \boxed{(-3.431, 8.931)}$

5 and 15: $7.75 - 5.75 \pm 6.181 \rightarrow \boxed{(-4.181, 8.181)}$

5 and 20: $8.75 - 5.75 \pm 6.181 \rightarrow \boxed{(-3.181, 9.181)}$

10 and 15: $8.5 - 7.75 \pm 6.181 \rightarrow \boxed{(-5.431, 6.931)}$

10 and 20: $8.75 - 8.5 \pm 6.181 \rightarrow \boxed{(-5.931, 6.431)}$

15 and 20: $8.75 - 7.75 \pm 6.181 \rightarrow \boxed{(-5.181, 7.181)}$

HW6 DUE 11/15/2022

Stirring Rate (P)	Q: 1	2	3	4	
5	8	4	5	6	5.75
10	14	5	6	9	8.5
15	14	6	9	2	7.75
20	17	9	3	6	8.75
	13.25	6	5.75	5.75	

↳ Total = 123, $\mu_{..} = 7.6875$

$df_{stir} = a-1 = 4-1 = 3$

$df_{furnace} = b-1 = 4-1 = 3$

$df_{resid} = (a-1)(b-1) = 9$

$df_{total} = N-1 = 16-1 = 15$

$H_0 = \mu_1 = \mu_2 = \mu_3 = \mu_4$ → All means are same

$H_A = \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$ → At least one mean is different

a) $S_p = qr \sum_j (y_{j\cdot} - y_{..})^2 = 4((5.75-7.6875)^2 + (8.5-7.6875)^2 + (7.75-7.6875)^2 + (8.75-7.6875)^2$

$= 4(3.7539 + 0.6602 + 0.00391 + 1.12891) = \boxed{22.19}$

$S_q = pr \sum_k (y_{\cdot k} - y_{..})^2 = 4((13.25-7.6875)^2 + (6-7.6875)^2 + (5.75-7.6875)^2 + (5.75-7.6875)^2$

$= 4(30.9414 + 2.8477 + 3.7539 + 3.7539) = \boxed{165.19}$

$S_D = \sum_j \sum_k (y_{jk} - y_{..})^2 = (8-7.6875)^2 + (4-7.6875)^2 + \ldots + (3-7.6875)^2 + (6-7.6875)^2$

$= \boxed{265.44}$

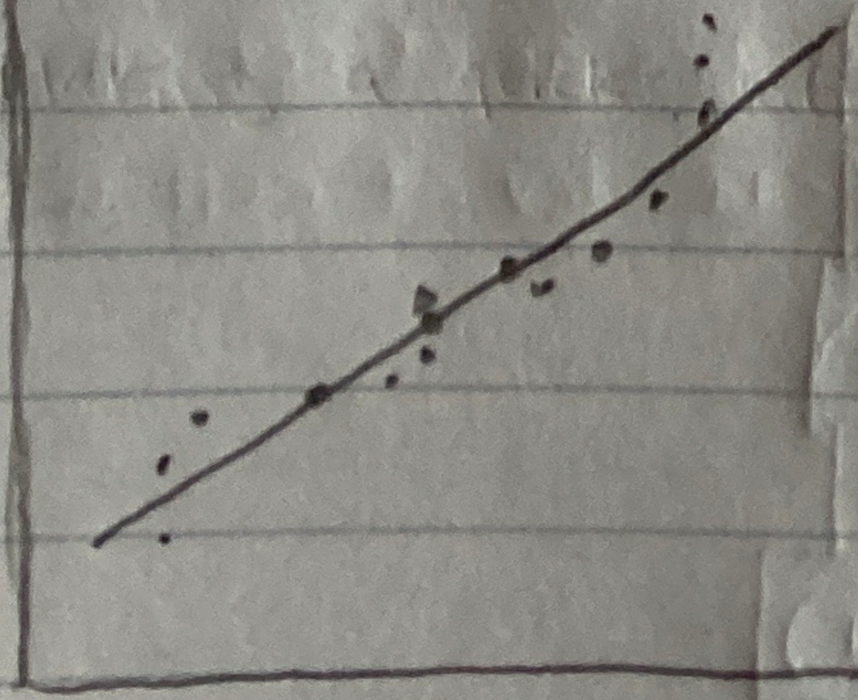
$S_R = 265.44 - 165.19 - 22.19 = \boxed{78.06}$

↳

Source	Df	SS	MS	F
Treatment	3	22.19	7.397	0.853
Block	3	165.19	55.063	6.349
Residual	9	78.06	8.673	
Total	15	265.44		

→ Based on the relatively small F value of the treatment, it is much smaller than the F critical value at $\alpha = 0.05$, thus failing to have significance. Thus, we fail to reject the null that all means are the same, and that stirring rates has an effect on mean grain size

b)



↳ Rough sketch from R studio

We see that the residuals lie on a linear distribution, and satisfies the assumption that the residuals are normal (normality assumption)