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Adaptive Kalman Filtering for Vehicle Navigation

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Abstract. Kalman filters have been widely used for navigation and system integration. One of the key problems associated with Kalman filters is how to assign suitable statistical properties to both the dynamic and the observational models. For GPS navigation, the manoeuvre of the vehicle and the level of measurement noise are environmental dependent, and hardly to be predicted. Therefore to assign constant noise levels for such applications is not realistic.

In this paper, real-time adaptive algorithms are applied to GPS data processing. Two different adaptive algorithms are discussed in the paper. A number of tests have been carried out to compare the performance of the adaptive algorithms with a conventional Kalman filter for vehicle navigation. The test results demonstrate that the new adaptive algorithms are much robust to the sudden changes of vehicle motion and measurement errors.

Key words: GPS, adaptive filtering, vehicle navigation, real time positioning

1 Introduction

For some navigation applications, we need to know the precise position when a vehicle turns. In map matching processing, for example, the turning points of a vehicle have to be accurately determined in order to establish reliable map matching (Yu et al, 2002). In most cities, the local maps are based on local datum and they have to be transformed to WGS 84 coordinate system for the use of satellite navigation technology. One of the quick ways to establish the transformation parameters is to drive a car equipped with a GPS receiver in the different parts of the cities. Then the turning points of the trajectory are extracted and compared with the map data at the same locations to determine the transformation parameters (Hu et al, 2003). Currently Kalman filters have been widely used in different GPS receivers. However, a conventional Kalman filter is vulnerable for the determination of the turning points precisely.

The Kalman filtering is an optimal estimation method that has been widely applied in real-time dynamic data processing. A Kalman filter estimates the state of a dynamic system with two different models namely dynamic and observation models. The dynamic model describes the behaviour of state vector, while the observation model establishes the relationship between measurements and the state vector. Both models are associated with statistical properties to describe the accuracy of the models. For many applications, the model statistic noise levels are given before the filtering process and will maintain unchanged during the whole recursive process. Commonly, this a priori statistical information is determined by test analysis and certain knowledge about the observation type beforehand. If such a priori information is inadequate to represent the real statistic noise levels, Kalman estimation is not optimal and may cause to an unreliable results, sometimes even leads to filtering divergence (Mohamed and K.P. Schwarz 1999). For vehicle navigation, sudden acceleration or deceleration and sudden change of the directions are impossible to predict. Therefore it is difficult to design a system with constant noise variances that will satisfy all situations. One of the common problems with vehicle navigation using Kalman filter is so called 'over shooting' problem. That is the effect that the dynamic model keeps position estimation along with previous trend while a vehicle actually turns to another direction.

Adaptive filtering is trying to determine the statistic parameters of the dynamic system based on the behaviour of the system during data processing, and it has been paid much attention in Kalman filtering theory (Jia and Zhu, 1984, and Gustafsson, 2000). Different adaptive Kalman filtering algorithms have been studied for surveying and navigation applications. Chen (1992) and Mohamed and Schwarz (1999) applied adaptive Kalman filters for the integration of GPS and inertial navigation system (INS). Wang et al (1997) applied a simplified adaptive algorithm in kinematic GPS positioning. Chen et al (1999) uses adaptive filters to estimate the velocity of permanent GPS stations.

In this paper, we investigate the performance of two different adaptive Kalman filters for vehicle navigation using GPS, one based on the fading memory and one based on the variance estimation. Both algorithms make use of the predicted residuals. The fading memory approach tries to estimate a scale factor to increase the predicted variance components of the state vector. The variance estimation method, on the other hand, directly calculates the variance factor for the dynamic model. Both algorithms closely examine whether there are divergence in the filtering process. If there is no divergence, the conventional Kalman filtering is used. Otherwise, the adaptive algorithms are applied. Two examples of vehicle navigation with DGPS are given in the paper. It demonstrates that the positioning accuracy with the adaptive approaches is significantly better than the conventional Kalman filtering, especially when the vehicle turns around the corners.

2 The Adaptive Kalman Filtering Algorithms

Considering a general linear dynamic system:

$$X_{k+1} = \Phi_{k+1,k} X_k + \Gamma_{k+1,k} \Omega_k \quad (1)$$

$$L_{k+1} = B_{k+1} X_{k+1} + V_{k+1} \quad (2)$$

with

$$E(\Omega_k) = 0,$$

$$cov(\Omega_k, \Omega_j) = Q_{\Omega k} \delta_{kj},$$

$$E(V_k) = 0,$$

$$cov(V_k, V_j) = Q_{Lk} \delta_{kj},$$

$$cov(\Omega_k, V_j) = 0,$$

$$E(X_0) = \hat{X}_{0/0},$$

$$Q_{X_{0/0}} = var(X_0).$$

where k denotes epoch number; X_k is the state vector at epoch k; $\Phi_{k+1,k}$ is the state transition matrix; $\Gamma_{k+1,k}$ is vector of system disturbance; Ω_k is vector of dynamic model noise; L_{k+1} is vector of observation at epoch $k+1$; B_{k+1} represent design matrix for observation; V is observation noise; and δ_{kj} is the δ -function of Kronecker.

Kalman filtering estimation can be expressed as:

Prediction:

$$\hat{X}_{k+1/k} = \Phi_{k+1,k} \hat{X}_{k/k} \quad (3)$$

$$Q_{X(k+1/k)} = \Phi_{k+1,k} Q_{X(k/k)} \Phi^T_{k+1,k} + \Gamma_{k+1,k} Q_{\Omega k} \Gamma^T_{k+1,k} \quad (4)$$

Updating:

$$\hat{X}_{k+1/k+1} = \hat{X}_{k+1/k} + K_{k+1}(L_{k+1} - B_{k+1}\hat{X}_{k+1/k}) \quad (5)$$

$$Q_{X(k+1/k+1)} = (I - K_{k+1}B_{k+1}) Q_{X(k+1/k)} \quad (6)$$

$$K_{k+1} = Q_{X(k+1/k)} B^T_{k+1} (B_{k+1} Q_{X(k+1/k)} B^T_{k+!} + Q_{L(k+1)})^{-1} \quad (7)$$

where $\hat{X}_{k+1/k}$ is the predicted state vector; $Q_{X(k+1/k)}$ is the variance matrix for $\hat{X}_{k+1/k}$; K_{k+1} is the gain matrix; $\hat{X}_{k+1/k+1}$ is the estimation of filtering; and $Q_{X(k+1/k+1)}$ is its variance matrix.

The adaptive filtering with fading memory algorithm

The Kalman filtering estimation at epoch k can be considered as 'weighted' adjustment between the new measurements (observation model) and the predicted state vector based on the dynamic model and all previous measurements. If too much 'weight' were put to the dynamic model, the estimation would ignore the information from measurements and causes the divergence of the filtering process. The idea of fading memory is simple. By applying a factor S>1 to the predicted covariance matrix to deliberately increase the variance of the predicted state vector (Eq. 8), more 'weight' will be given to the measurements.

$$Q'_{X(k+1/k)} = S(\Phi_{k+1,k} Q_{X(k/k)} \Phi^T_{k+1,k} + \Gamma_{k+1,k} Q_{\Omega k} \Gamma^T_{k+1,k}) \quad (8)$$

The main difference between different fading memory algorithms is on how to calculate the scale factor S. One approach is to assign the scale factor as a constant, S =1

～1.4. When S =1, it becomes the conventional Kalman filtering. Obviously there are some drawbacks with a constant factor. For example, as the filtering proceeds, the precision of the filtering will decrease because the effects of old data will become less and less. The best way is to use a variant scale factor that will be determined base on the dynamic and observation model accuracy. In this paper, an algorithm is derived based on the size of the predicted residuals, which represent the difference between the measurements and the predicted state vector. The predicted residual vector is expressed as:

$$V_{k+1/k} = L_{k+1} - B_{k+1}(\hat{X}_{k+1/k}) \tag{9}$$

For a linear dynamic system, we have (Jia and Zhu, 1984)

$$\begin{aligned} &E(V_{k+1/k}V_{k+1/k}^T) \\ &= B_{k+1}Q_{X(k+1/k)}B_{k+1}{}^T + Q_{L(k+1)} \end{aligned} \tag{10}$$

Let us introduce a scale factor S≥1 to the predicted covariance matrix

$$\begin{aligned} \dot{Q}_{X(k+1/k)} = S(&\Phi_{k+1,k}Q_{X(k/k)}Q_{k+1,k}^T \\ &+ \Gamma_{k+1}Q_{\Omega k}\Gamma_{k+1,k}^T) \end{aligned} \tag{11}$$

Then

$$V_{k+1/k}^T V_{k+1/k} \le Tr(SB_{k+1}Q_{X(k+1/k)}B_{k+1}^T + Q_{L(k+1)})$$

$$(s \ge 1) \tag{12}$$

As $s \ge 1$, we can further obtain that

$$V_{k+1/k}^T V_{k+1/k} \le STr(B_{k+1}Q_{X(k+1/k)}B_{k+1}^T + Q_{L(k+1)}) \tag{13}$$

When a filter is stable, we can estimate $E(V_{k+1/k}V_{k+1/k}^T)$ with the previous N (some time it is called the filtering window) epoch data:

$$E(V_{k+1/k}V_{k+1/k}^T) \approx \frac{1}{N}\sum_{j=0}^{N-1}(V_{k+1/k}V_{k+1/k}^T) \tag{14}$$

By combining Eqs 13 and 14 finally we can have

$$S \ge \frac{V_{k+1/k}^T V_{k+1/k}}{\frac{1}{N}(V_{k-N+j/k-N}^T V_{k-N+j/k-N})} \tag{15}$$

In practice, the number of measurements may vary from epoch to epoch, such as the GPS signal is blocked off by obstacles or the newly rising and down of satellite. Eq (15) can be modified as

$$S \ge \frac{\frac{1}{m_{k+1}}(V_{k+1/k}^T V_{k+1/k})}{\frac{1}{N}\sum_{j=0}^{N-1}\frac{1}{m_{k-N+j}}(V_{k-N+j/k-N}^T V_{k-N+j/k-N})} \tag{16}$$

where m_i is the number of measurements at epoch i.

When S≤1，it indicates that the filtering is in a steady state processing. When S>1, it indicates that the filtering may be in an unstable state or in a state of divergence. In practice, in order to avoid false alarm, a threshold S_0>1 is selected. When S>S_0, the adaptive algorithm is applied. Otherwise S =1 and the conventional Kalman filtering algorithm is used.

The adaptive filter with variance component estimation

This approach tries to estimate the variance factor of the dynamic model directly using the predicted residuals. Considering the prediction vector as a pseudo-observation, the Kalman filter equation can be expressed as a Gauss-Markov model:

$$V = A\hat{x}_{k/k} - L \tag{17}$$

with weight matrix P

where

$$V = \begin{bmatrix} v_{\hat{x}_{k/k-1}} \\ v_{L_k} \end{bmatrix}$$

$$A = \begin{bmatrix} I \\ B \end{bmatrix}$$

$$L = \begin{bmatrix} \hat{x}_{k/k-1} \\ L_k \end{bmatrix}, \quad P = \begin{bmatrix} Q_{X(k/k-1)}^{-1} & 0 \\ 0 & Q_{L(k)}^{-1} \end{bmatrix}$$

The predicted residual vector can be stated:

$$\tilde{V}_k = B_k\hat{X}_{k/k-1} - L_k \tag{18}$$

Because

$$cov(L_k) = \sigma_{L0}^2 P_k^{-1}$$

$$cov(X_{k/k-1}) = \sigma_{X0}^2 P_{X(k/k-1)}^{-1} = \sigma_{X0}^2 Q_{X(k/k-1)}$$

The covariance matrix of the predicted residuals can be expressed as:

$$cov(\tilde{V}_k) = \sigma_{X0}^2 B_k P_{X(k/k-1)}^{-1} B_k{}^T + \sigma_{L0}^2 P_{L_k}^{-1} \tag{19}$$

Therefore

$$
\begin{aligned}
&E(\widetilde{V}_k^T P_{L_k} \widetilde{V}_k) = tr(P_{L_k} D(\widetilde{V}_k)) = \\
&tr(P_{L_k}(\sigma^2_{X0} B_k P^{-1}_{X(k/k-1)} B_k^{\ T} + \sigma^2_{L0} P^{-1}_{L_k})) \\
&= \sigma^2_{X0} tr(P_{L_k} B_k P^{-1}_{X(k/k-1)} B_k^{\ T}) \\
&\quad + \sigma^2_{L0} m
\end{aligned}
\tag{20}
$$

where m is the number of measurements at the epoch.

In our approach, we only try to estimate the variance factor of the dynamic model, as GPS measurement noise can be assigned to a reasonable level based on the type of GPS receiver used. Then the variance factor of the observation $cov(L_k) = \sigma^2_{L0} P^{-1}_{L_k}$ is assumed to be known. From Eq. 20, the estimation of the variance factor σ^2_{X0} can be formed as

$$
\begin{aligned}
\hat{\sigma}^2_{X0} = &\left[\ E(\widetilde{V}_k^T P_{L_k} \widetilde{V}_k) - \sigma^2_{L0} m\ \right] / \\
&tr(P_{L_k} B_k P^{-1}_{X(k/k-1)} B_k^{\ T})
\end{aligned}
\tag{21}
$$

Similarly we can derive the variance factor for the dynamic model noise:

$$
\begin{aligned}
\sigma^2_{\Omega 0} = &\left[\ \cdot(\widetilde{V}_k^T P_{L_k} \widetilde{V}_k) \right. \\
&- \sigma^2_{X0} tr(P_{L_k} B_k \Phi_{k,k-1} Q_{X(k-1/k)} \Phi^T_{k,k-1} B_k^{\ T}) \\
&\left. - \sigma^2_{L0} m)\ \right] /\ tr(P_{L_k} B_k \Gamma_{k,k-1} Q_{\Omega(k-1)} \Gamma^T_{k,k-1} B_k^{\ T})
\end{aligned}
\tag{22}
$$

3 Examples

In order to evaluate the performance of the adaptive algorithms on GPS positioning, a number of tests were carried out. In the first example, we used two TOPCON Java Legacy-E dual frequency GPS receivers. One receiver was set on the roof of the Tang Ping Yuan Building in the Hong Kong Polytechnic University as a reference station. Another GPS receiver was installed on roof of a car that was driven along the road, which is about 15 km from the university. Fig. 1 shows the trajectory of the car. Both dual frequency pseudorange and carrier phase measurements were collected during the test. The data collected were then post-processed using in-house GPS software developed by Hong Kong Polytechnic University. Firstly the reference trajectory was obtained with kinematic GPS positioning using carrier phase measurements. Then the L1 C/A code data were processed using different Kalman filters. A constant velocity model is adopted as the dynamic model for the Kalman filters. The state vector consists of geocentric coordinate (X, Y, Z) and velocity $(\dot{X}, \dot{Y}, \dot{Z})$. The accelerations $(\ddot{X}, \ddot{Y}, \ddot{Z})$ are considered as the dynamic model noise. The double difference pseudoranges were formed as the observation, and therefore, the receiver clock bias errors were not modeled in data processing.

The conventional Kalman filtering method was firstly used to process the DGPS data. In our test, two different dynamic noise levels, $\sigma_{\ddot{X}}$ =0.1m/s^2 and $\sigma_{\ddot{X}}$ =0.05m/s^2, had been chosen to analyze the dependence of the model noise on the positioning performance. Fig.s 2 and 3 show the positioning errors of the conventional Kalman filter with these two dynamic noise levels. It is clearly shown that the positioning errors are significantly different when different dynamic noise levels were selected. The peaks in Fig. 3 are larger than those in Fig. 2. The RMS errors are 2.18 m (Easting) and 1.85 m (Northing) and 4.02 m (Easting) and 2.80 m (Northing) for the noise levels of 0.1m/s^2 and 0.05m/s^2 respectively.

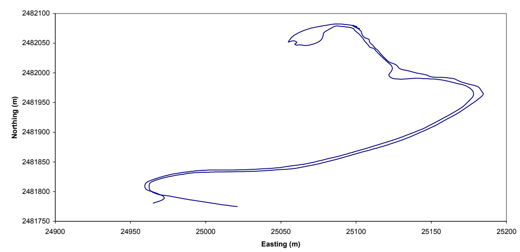


Fig. 1 Trajectory of the Car

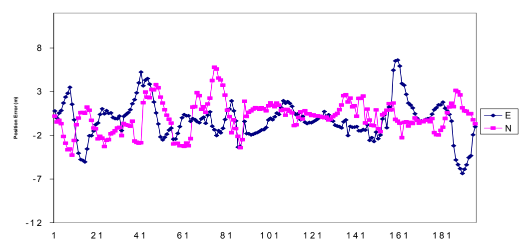


Fig.2 Positioning Error with conventional Kalman Filter ($\sigma_{\ddot{X}}$ =0.1 m/s^2)

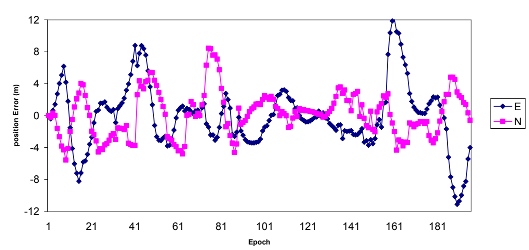


Fig. 3 Positioning Error with conventional Kalman Filter ($\sigma_{\ddot{X}}$ =0.05 m/s^2)

It is much clearer to compare the estimated trajectories with differential pseudorange with the 'true' one estimated by carrier phase measurements. Fig. 4 and 5 show the 'true' trajectory estimated with carrier phase measurements and the estimated trajectory using C/A code pseudorange, using conventional Kalman filter. On the straight lines, the positioning errors are similar with different noise levels. However, with tight constraint on the dynamic noise level (0.05 m/s^2), the positioning errors are significantly larger when the car turns.

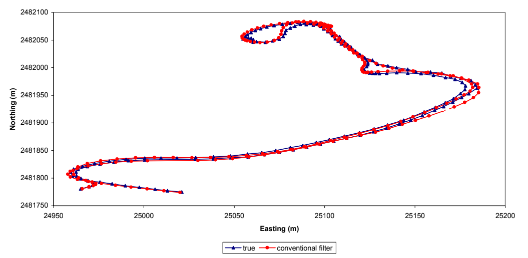


Fig. 4 Estimated trajectory with conventional Kalman filter ($\sigma_{\ddot{X}}$ =0.1 m/s^2)

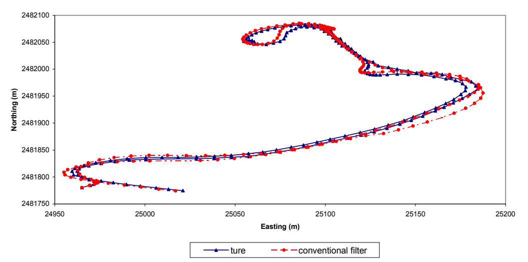


Fig. 5 Estimated trajectory with conventional Kalman filter ($\sigma_{\ddot{X}}$ =0.05 m/s^2)

Then the same data set was processed using the adaptive filters discussed in section 2. Fig. 6 and 7 show the positioning errors with the two adaptive Kalman filters respectively. Comparing Fig. 5 and 6 with Fig. 1 and 2, it is clearly shown that the positioning errors with the adaptive filters are significantly smaller than the conventional filter. For fading memory filter, the positioning errors are reduced to 0.91 m and 1.34 m RMS for easting and northing respectively. Better results can be achieved with the variance estimation method, with the RMS error of 0.72 m and 1.21 m for easting and northing respectively. Also, the errors in Fig. 6 and 7 are uniformly distributed and this means the positioning errors are not associated with the sudden manoeuvre changes of the car. Tab. 1 summarizes the RMS error of positioning error using different filtering algorithms. For the adaptive filters, the initial noise levels are chosen as the same as the conventional Kalman filter. The dynamic noise level strongly affects the performance of the conventional Kalman filter. For fading memory filter, the positioning errors are significantly reduced. However, the positioning accuracy is still affected by the selection of initial dynamic noise level. The variance estimation filter has better performance than the fading memory filter and it is not affected by the selection of the initial dynamic noise level. A number of other tests have also been carried out and the results confirm the above conclusions.

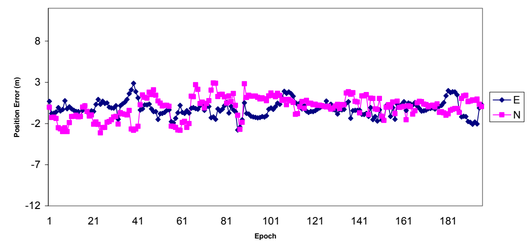


Fig. 6 Positioning Error with Fading Memory Filter

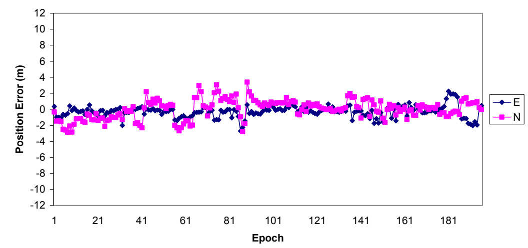
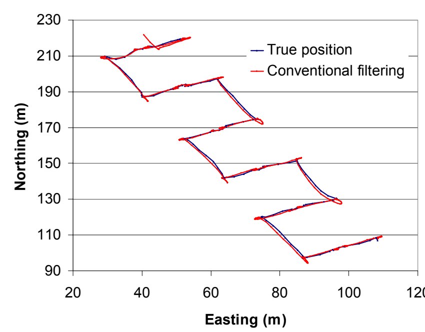
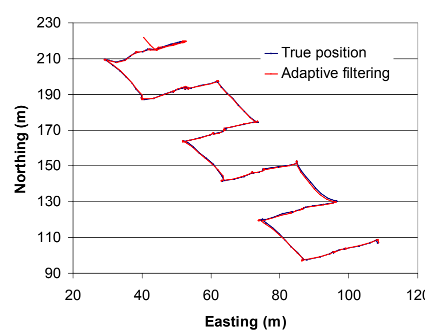


Fig.7 Positioning Error with Variance Estimation Filter



(a)Conventional Kalman filter



(b)Adaptive Kalman filter

Fig.8 Estimated trajectories with different filtering types for example 2

Tab. 1 Positioning Errors with Different Filters (Example 1)

Filter type	Conventional KF		Fading Memory		Variance Estimation	
Noise Level (m/s^2)	0.1	0.05	0.1	0.05	0.1	0.05
Easting (m)	2.18	4.02	0.91	1.28	0.72	0.72
Northing (m)	1.85	2.80	1.34	1.55	1.21	1.21

The second example comes from cross section surveying of highway. Leica SR229 GPS receivers were used to survey the cross section profile (as shown in Fig. 8). The position of the profile is determined by GPS RTK technique with the accuracy of centimeter level. Then the conventional Kalman filters and the two adaptive filters discussed in this paper are used to process differential psuedorange data. The positioning errors with these three methods are plotted in Fig. 8. It is clear to see in Fig. 8 that the position estimation accuracy with the conventional Kalman filter is much worse than that with the adaptive filters. The peak errors in Fig. 8 associated with the turns of the trajectory. Tab. 2 shows the RMS error of the three filters.

Tab. 2 Positioning Errors with Different Filters (m) (Example 2)

Filter type	Convent-ional KF	Fading Memory	Variance Estimation
Easting	1.25	0.92	0.88
Northing	0.93	0.53	0.41

4. Conclusion

Conventional Kalman filter is very sensitive to the selection of the dynamic model noise level. One of the approaches tries to use adaptive filters to adjust the dynamic model noise based on the divergence of the dynamic and the observation models. In this paper, two adaptive Kalman filtering algorithms have been derived, one is simply applying a scale factor to the predicted covariance matrix (fading memory) and another to estimate the variance factor directly. The test demonstrates that both adaptive algorithms are better than the conventional Kalman filter, especially when the car changes its manoeuvre along the road. The computation for the variance estimation filter is slightly more complex than the fading memory filter. However, the positioning accuracy with variance estimation filter is better.

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