Project Proposal for SDS385 Statistical Models for Big Data

Applying Kalman Filter to Denoise GPS Data

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Goal

Nowsdays, GPS is widely used in vehicles, which can provide estimates of position of vehicles within a few meters. However, the GPS estimate can be noisey; readings "jump around" rapidly, though remaining within a few meters of the true position. Therefore, in order to get the accurate position of vehicles, filtering noise becomes very necessary. In this project, I will denoise GPS data using Kalman filter method.

Kalman filter is an optimal estimation method that has been widely used in smoothing noisy signals, generating non-observable states, and predicting future states. Kalman filter estimates the position in two stages: prediction and update. In the prediction stage, it produces a new position estimate based on the previous position and the dynamic model. And then in the update stage, a new measurement of the vehicle's position is taken from GPS, and the estimate is updated using a weighed average with more weight being given to estimates with higher certainty. Details about Kalman filter method are given in Method section.

Method

In Kalman filter, it is assumed that the evolution and measurement models are linear. That is,

$$x_t = F_t x_{t-1} + B_t u_t + \omega_t,$$

$$z_t = H_t x_t + v_t,$$

where x_t is the state vector containing the terms of interest (i.e. location, velocity) at time t; u_t is the vector containing any control inputs (i.e. steering angle); F_t is the state transition matrix applying the effect of each state parameter at time $t-1$ on the state at time t; B_t is the control input matrix which applies the effect of each control input in vector u_t on the state vector x_t; ω_t is the vector containing process noise for each variable in the state vector, and the noises are assumed to follow Gaussian distributions with known means 0 and covariances matrices Q_t; z_t is the vector of measurement; H_t is the transformation matrix that maps the state vector variables into the measurements; and v_t is the vector including the measurement noise for each observation, and it is also assumed that the noises are Gaussian with zero mean and covariances matrices R_t.

Kalman filter involves two steps: prediction step and update step. The prediction is based on the previous states and dynamic model. The equations are

$$\hat{x}_{t|t-1} = F_t \hat{x}_{t-1|t-1} + B_t u_t,$$

$$P_{t|t-1} = F_t P_{t-1|t-1} F_t^T + Q_t,$$

where $\hat{x}_{t|t-1}$, $P_{t|t-1}$ are priori state and covariance estimates, respectively. The update starts after a new measurement is available. The equations are given by

$$K_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R_t)^{-1},$$

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t (z_t - H_t \hat{x}_{t|t-1}),$$

$$P_{t|t} = P_{t|t-1}(I - K_t H_t),$$

where $\hat{x}_{t|t}$, $P_{t|t}$ are posteriori state and coveriance estimates, respectively. These two steps are repeated continuously to update the current state.