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數位影像處理 DIP Homework Chapter 4_1 (100 pts)

1. If a discrete sequence (*M=4*) is {*f(0), f(1), f(2), f(3)* }={4, 2, 1, 5}, please find its DFT *F(u)*? and count how many multiplications and additions in the *M=4* DFT ? (30)

   F(u)={12, 3+3j, -2, 3-3j} 64 mult, 56 add

2. If a discrete sequence (*M=8*) is {*f(0), f(1), f(2), f(3), f(4), f(5), f(6), f(7)*}={ 4, 2, 1, 5, 4, 2, 1, 5}, please find its DFT *F(u)*? and count how many multiplications and additions in the *M=8* DFT ? (30)

   4×8^2 mult, $4\times 8^2 - 2\times 8$ add

3. If a discrete sequence (*M=8*) is {*f(0), f(1), f(2), f(3), f(4), f(5), f(6), f(7)*}={ 4, -2, 1, -5, 4, -2, 1, -5}, please find its DFT *F(u)*? and count how many multiplications and additions in the *M=8* DFT ? (30)

   4×8^2 mult, $4\times 8^2 - 2\times 8$ add

4. Please state the relation between the question (1), (2), and (3) ? (10)

   (1) (2) 的關係 → 頻域做 up sample → 時域 pattern 重覆

   (2) (3) 的關係 → 時域 $\times (e^{\frac{j2\pi k}{4}})$ = 頻域 ciraular shift

   4 #

① DFT

$e^{-j\frac{2\pi}{N}kn}$

$\Rightarrow X[k]=\sum_{n=0}^{3}x[n]\,W_4^{kn}$, $k=0,1,2,3$

$\Rightarrow F(0)=4+2+1+5=12$

$F(1)=4+2e^{-j\frac{2\pi}{4}}+1e^{-j\frac{2\pi}{4}\times 2}+5e^{-j\frac{2\pi}{4}\times 3}$

$=3+3j$

$F(2)=4+2e^{-j\frac{2\pi}{4}\times 2}+1e^{-j\frac{2\pi}{4}\times 4}+5e^{-j\frac{2\pi}{4}\times 6}$

$=-2$

$F(3)=4+2e^{-j\frac{2\pi}{4}\times 3}+1e^{-j\frac{2\pi}{4}\times 6}+5e^{-j\frac{2\pi}{4}\times 9}$

$=4+2j-1-5j$

$=3-3j$

$4N$ mult $\times N$

$(4N-2)$ add $\times N$

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② DFT

$F(0)=4+2+1+5+4+2+1+5=24$

$F(1)=4+2\cdot e^{-j\frac{2\pi}{8}}+1\cdot e^{-j\frac{2\pi}{8}\times 2}+\cdots+5\cdot e^{-j\frac{2\pi}{8}\cdot 7}$

$=0$

$F(2)=4+2\cdot e^{-j\frac{2\pi}{8}\times 2}+\cdots+5\cdot e^{-j\frac{2\pi}{8}14}$

$=6+6j$

$F(3)=4+2\cdot e^{-j\frac{2\pi}{8}\times 3}+\cdots+5\cdot e^{-j\frac{2\pi}{8}21}$

$= 0$

$F(4) = 4 + 2 \cdot e^{-j\frac{2\pi}{8}\times 4} + \cdots + 5 \cdot e^{-j\frac{2\pi}{8}\times 8}$

$= -4$

$F(5) = 4 + 2 \cdot e^{-j\frac{2\pi}{8}\times 5} + \cdots + 5 \cdot e^{-j\frac{2\pi}{8}35}$

$= 0$

$F(6) = 6 - 6j$

$F(7) = 0$

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③

$F(0) = 4 - 2 + 1 - 5 + 4 - 2 + 1 - 5 = -4$

$F(1) = 4 - 2 \cdot e^{-j\frac{2\pi}{8}} + 1 \cdot e^{-j\frac{2\pi}{8}\times 2} + \cdots - 5 \cdot e^{-j\frac{2\pi}{8}\cdot 7}$

$= 0$

$F(2) = 4 - 2 \cdot e^{-j\frac{2\pi}{8}\times 2} + \cdots - 5 \cdot e^{-j\frac{2\pi}{8}14}$

$= 6 - 6j$

$F(3) = 4 - 2 \cdot e^{-j\frac{2\pi}{8}\times 3} + \cdots - 5 \cdot e^{-j\frac{2\pi}{8}21}$

$= 0$

$F(4) = 4 - 2 \cdot e^{-j\frac{2\pi}{8}\times 4} + \cdots - 5 \cdot e^{-j\frac{2\pi}{8}\times 8}$

$= 24$

$F(5) = 4 - 2 \cdot e^{-j\frac{2\pi}{8}\times 5} + \cdots - 5 \cdot e^{-j\frac{2\pi}{8}35}$

$= 0$

$F(6) = 6 + 6j$

$F(7) = 0$
