CQF Exercises Binomial Method

The calculations in this problem set centre on using the delta hedged portfolio

1. (Ice-breaker) A share price is currently £80. At the end of three months, it will be either £84 or £76. Ignoring interest rates, calculate the value of a three-month European call option with exercise price £79.

2. A share price is currently £92. At the end of one year, it will be either £86 or £98. Calculate the value of a one-year European call option with exercise price £90 using a single-step binomial tree. The risk-free interest rate is 2% p.a. with continuous compounding.

3. A share price is currently £15. At the end of three months, it will be either £13 or £17. Interest rates are zero. Calculate the value of a three-month European style 'power' option with payoff $\max(S^2 - 159, 0)$ where S is the share price at the end of three months.

4. A share price is currently £75. At the end of three months, it will be either £59 or £92. What are the risk-neutral probabilities that the share price rises or falls? The risk-free interest rate is zero.

5. A binary call option (also called digital option) $B_C(S,t)$ has a payoff of one if at expiry it is ITM, i.e. $S(T) > E$, and zero otherwise

$$B_C(S,T) = \begin{cases} 1 & S(T) > E \\ 0 & \text{otherwise.} \end{cases}$$

   A share price is currently £80. At the end of three months, it will be either £84 or £76. Ignoring interest rates, calculate the value of a three-month **binary** call option with strike price £79. **Note this is similar to question 1, but with the option now being a digital**.

6. Implement the multi-step binomial method as described in the Binomial Method lecture with the following variables and parameters: stock $S = 100$, interest rate $r = 0.05$ (continuously compounded) for a call option with strike $E = 100$, $T = 1$. Use four time steps. Calculate the value of the option for a range of volatilities and plot the results.
   Now with volatility $\sigma = 0.2$, plot the value of the call option as the number of time steps increases.

7. A share price is currently £63. At the end of each three-month period, it will change by going up £3 or going down £3. Calculate the value of a **six-month** European **put** option with strike price £61. The risk-free interest rate is 4% per annum with continuous compounding.

8. An asset S with value α today follows the two-step Binomial tree given by

$$\begin{array}{cccc} & & & \alpha+20 \\ & & \alpha+10 & \\ & \alpha & & \alpha \\ & & \alpha-10 & \\ & & & \alpha-20 \\ \text{Time} & 0 & T_1 & T \end{array}$$

The risk-free interest rate $r = 0$. Construct the corresponding option pricing tree

$$\begin{array}{cccc} & & & V_2 \\ & & V_1 & \\ & V & & V_0 \\ & & V_{-1} & \\ & & & V_{-2} \\ \text{Time} & 0 & T_1 & T \end{array}$$

for a European call option with payoff $V(S,T) = \max(S - \alpha - 5, 0)$, to show

$$\begin{array}{cccc} & & & 15 \\ & & 15/2 & \\ & V = 15/4 & & 0 \\ & & 0 & \\ & & & 0 \\ \text{Time} & 0 & T_1 & T \end{array}$$