CQF FitchLearning

SELECTED LECTURE SLIDES

Portfolio VaR and ES

Knowledge | Skills | Conduct

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Methods for Estimating VaR

First Method: Analytic variance-covariance approach (aka parametric VaR)

Under the analytic variance-covariance approach or 'delta normal' approach, we assume that the risk factors and the portfolio values are log-normally distributed or, equivalently, that their log returns (the log of the returns) are normally distributed.

This makes the calculation much simpler, since the normal distribution is completely characterised by its first two moments, and the analyst can derive the mean and the variance of the portfolio return distribution from:

a. The multivariate distribution of the risk factors

b. The composition of the portfolio

$$\boxed{\mathrm{VaR} = \underbrace{\mu}_{\text{drift}} + \underbrace{\sigma}_{\text{vol}}\underbrace{N^{-1}}_{\text{inverse normal std distn cumulative}}(X)}$$

② 

£1m

Analytic VaR

$$VaR = \overset{0}{\cancel{\mu}} + \sigma N^{-1}(x)$$

Problem 2: Analytic VAR: Compute (a) the 1-day VAR at 99% confidence, and (b) the 10-day VAR at 99% confidence, for a portfolio composed of a single asset whose value is £1 million, a volatility sigma_daily=1%

$\sigma_d = 1\%$

$VaR(T, X)$

(a) $T = \underline{\underline{1d}}$, $X = \underline{\underline{99\%}}$

(b) $T = \underline{\underline{10d}}$, $X = 99\%$

$\mu = 0$

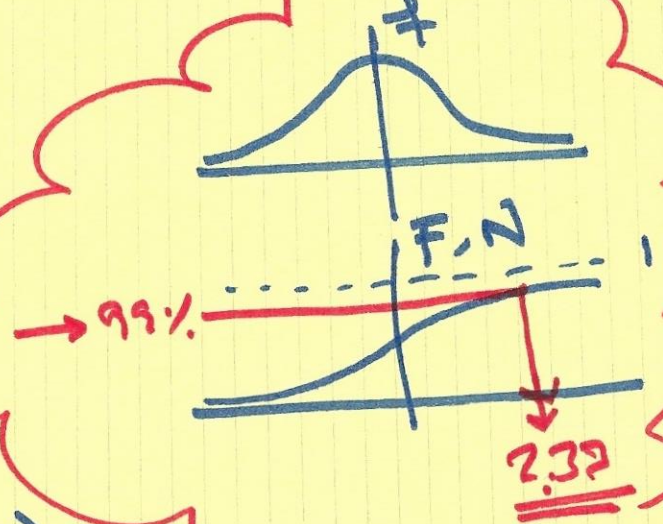
(a) $VaR(1d, 99\%) = (1\%)(\underline{\underline{2.33}}) = \underline{\underline{2.33\%}}$

or

$VaR(1d, 99\%) =$

$(2.33\%)(£1m)$

$= \underline{\underline{23{,}300\ £}}$



(b) $VaR(10d, 99\%)$

$= (23{,}300\ £)(\sqrt{10})$

$= \underline{\underline{73{,}681\ £}}$

$VaR_{10d} = \sqrt{10} \cdot VaR_{1d}$

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Example: Portfolio VaR and ES

$\pi =$ 

Example *We consider three stocks A, B and C. Their current prices are respectively 244, 135 and 315 dollars. We assume that their expected returns are equal to 50 bps, 30 bps and 20 bps on a daily basis, whereas their daily volatilities are 2%, 3% and 1%. The correlation of asset returns is given by the following matrix:*

μ

σ

CORR

$$\rho = \begin{pmatrix} 1.00 & 0.5 & 0.25 \\ 0.50 & 1.00 & 0.6 \\ 0.25 & 0.60 & 1.00 \end{pmatrix}$$

*Extract from Thierry Roncalli, Introduction to Risk Parity and Budgeting, CRC Press, 2014, pp. 75

Portfolio Risk Measures

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Let us assume that the asset returns are normally distributed: $R \sim \mathcal{N}(\mu, \Sigma)$. We have $\mu(x) = x^\top \mu$ and $\sigma(x) = \sqrt{x^\top \Sigma x}$. It follows that the standard deviation-based risk measure is:

$$\mathrm{SD}_c(x) = -x^\top \mu + c \cdot \sqrt{x^\top \Sigma x} \tag{2.1}$$

For the value-at-risk, we have $\Pr\{L(x) \leq \mathrm{VaR}_\alpha(x)\} = \alpha$. Because $L(x) = -R(x)$ and $\Pr\{R(x) \geq -\mathrm{VaR}_\alpha(x)\} = \alpha$, we deduce that:

$$\Pr\left\{\frac{R(x) - x^\top \mu}{\sqrt{x^\top \Sigma x}} \leq \frac{-\mathrm{VaR}_\alpha(x) - x^\top \mu}{\sqrt{x^\top \Sigma x}}\right\} = 1 - \alpha$$

It follows that:

$$\frac{-\mathrm{VaR}_\alpha(x) - x^\top \mu}{\sqrt{x^\top \Sigma x}} = \Phi^{-1}(1 - \alpha)$$

We finally obtain:

exp returns

α: significance level.

covariance matrix

$$\boxed{\mathrm{VaR}_\alpha(x) = -x^\top \mu + \Phi^{-1}(\alpha) \sqrt{x^\top \Sigma x}} \tag{2.2}$$

inv normal distribution

X: weights portfolio

Portfolio Risk Measures

$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}$

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port expected return

Σ : covariance matrix

The expected shortfall of portfolio x is then:

$$\boxed{\mathrm{ES}_\alpha(x) = -x^\top \mu + \frac{\sqrt{x^\top \Sigma x}}{(1-\alpha)} \phi\left(\Phi^{-1}(\alpha)\right)} \quad (2.3)$$

Like the value-at-risk, it is a standard deviation-based risk measure with $c = \phi\left(\Phi^{-1}(\alpha)\right) / (1-\alpha)$.

port weights vector

$\alpha = 99\%$

$(w_1\ w_2 \ldots w_3)$

③

938 $

	A	B	C	
$\mu \to$	50	30	20	(bps)
$\sigma \to$	2%	3%	1%	(%)

$$\rho = \begin{pmatrix} 1 & & \\ 0.5 & 1 & \\ 0.25 & 0.6 & 1 \end{pmatrix}$$

Calculate VaR, ES?

VaR(X,T)
ES(X,T)

$$VaR_\alpha(x) = -x^T\mu + \Phi^{-1}(\alpha)\sqrt{x^T \cdot \Sigma \cdot x}$$

$$x = \begin{pmatrix} \frac{244}{938} \\ \frac{135}{938} \\ \frac{315}{938} \end{pmatrix} = \begin{pmatrix} 52\% \\ 14.4\% \\ 33.6\% \end{pmatrix}$$

$$x^T = \begin{pmatrix} 52\% & 14.4\% & 33.6\% \end{pmatrix}$$

$$\mu = \begin{pmatrix} 50 \\ 30 \\ 20 \end{pmatrix} \times 10^{-4}$$

$$\Rightarrow = -\begin{pmatrix} 52\% & 14.4\% & 33.6\% \end{pmatrix} \begin{pmatrix} 50 \\ 30 \\ 20 \end{pmatrix} \times 10^{-4}$$

$=$

$1 \times 3 \quad 3 \times 1$

scalar

$$\mathrm{VaR}(x) = -x^T\mu + \underline{\Phi^{-1}(\alpha)\sqrt{x^T \Sigma x}}$$

$$\Sigma = \begin{array}{cccc} \sigma_1 & \sigma_2 & \sigma_3 & \\ \left[\begin{array}{ccc} \sim & \sim & \sim \\ \sim & \sim & \sim \\ \sim & \sim & \sim \end{array}\right] & & & \begin{array}{c}\sigma_1\\ \sigma_2\\ \sigma_3\end{array} \end{array}$$

correlation matrix

$$\Sigma = \begin{array}{cc} \begin{array}{ccc} 2\% & 3\% & 1\% \end{array} & \\ \left[\begin{array}{ccc} 1 & & \\ 0.5 & 1 & \\ 0.25 & 0.6 & 1 \end{array}\right]_{3\times3} & \begin{array}{c} 2\% \\ 3\% \\ 1\% \end{array} \end{array}$$

$$\Sigma = \left[\begin{array}{c} (2\%)(2\%)(1) \\ (2\%)(3\%)(0.5) \\ \\ \\ \end{array}\right]_{3\times3}$$

$$\underbrace{N^{-1}(x)}_{2.33} \qquad x = 99\%$$

$$\sqrt{(\sim\ \sim\ \sim)\left[\ \Sigma\ \right]\begin{pmatrix}\sim\\ \sim\\ \sim\end{pmatrix}}$$

$$\underbrace{1\times3\ \checkmark\ 3\times3\ \checkmark\ 3\times1}_{1\times1 \therefore \text{Scalar}}$$

$$(2.33)\sqrt{(52\%\ 14\%\ 33.5\%)\begin{pmatrix}\sim & \sim & \sim\\ \sim & \sim & \sim\\ \sim & \sim & \sim\end{pmatrix}\begin{pmatrix}52\%\\ 14\%\\ 33\%\end{pmatrix}}$$

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Example: Portfolio VaR and ES

We consider Portfolio #1 composed of two stocks A, one stock B and one stock C. The value of the portfolio is then equal to 938 dollars. We deduce that the weights x are $(52.03\%, 14.39\%, 33.58\%)$. Computation gives $\mu(x) = 37.0$ bps and $\sigma(x) = 1.476\%$. In Table 2.1, we report the values taken by the value-at-risk and the expected shortfall for different confidence levels α. Because these risk measures are computed using Formulas (2.2) and (2.3) with the portfolio weights, the values are expressed as a percentage. For example, the value-at-risk of the portfolio for $\alpha = 99\%$ is equal to[2]:

$$\mathrm{VaR}_{99\%}(x) = -0.370\% + 2.326 \times 1.476\% = 3.06\%$$

For the expected shortfall, we obtain[3]:

$$\mathrm{ES}_{99\%}(x) = -0.370\% + 2.667 \times 1.476\% = 3.56\%$$

```
%
% LAB: VAR and ES portfolio
% Thierry Roncalli, Introduction to Risk Parity and Budgeting, CRC Press, 2014
% Example page 75
%
% Fitch Learning UK
% Jan 2018
%

close all
clear all

% STEP 1: inputs
% three stocks A,B,C
p=[244 135 315]';
x=[0.5203 0.1439 0.3358]';
mu=[50 30 20]'./10000;
vol=[2 3 1]./100;
rho=[1 0.5 0.25;0.5 1 0.6;0.25 0.6 1];

Sigma=[4.0000e-004  3.0000e-004  5.0000e-005;
  3.0000e-004  9.0000e-004  1.8000e-004;
  5.0000e-005  1.8000e-004  1.0000e-004]

alpha=0.99;


% STEP 2: calculations

% compute VAR
VAR_x = -x'*mu + norminv(alpha)*sqrt(x'*Sigma*x);

% compute ES
ES_x = -x'*mu + (sqrt(x'*Sigma*x))/(1-alpha)*normpdf(norminv(alpha));


% STEP 3: output
VAR_x
ES_x
```