CQF Exam One
Portfolio and Risk Techniques

January 2022 Cohort

Instructions

Answers to all questions **are required**. Requested mathematical and all computational workings must be provided. Portal, upload and logistical questions to Orinta.Juknaite@fitchlearning.com. Clarifying questions are welcome to Richard.Diamond@fitchlearning.com. Tutor is unable to re-explain calculation or confirm correct numerical answers. Please make a good use of lecture exercises/solutions.

- All workings, tables and plots **must** be presented in ONE PDF report:
  Q1 and Q2 require to provide mathematical formulae. Q1, Q2, and Q3 also need summary tables and plots. Q4 allows for handwritten workings, but can be coded/done in Excel.
  Copy final values and plots from Python/code output into Word/LaTeX. If you have handwritten workings – append pages to create ONE PDF file, named LASTNAME_REPORT_E1.
- Code computation in Python/other language but Exam One tasks can be implemented in Excel, particularly Q3 . All code/Excel to be uploaded as ONE zip file named LASTNAME_CODE.zip
- Submissions in Excel only / Python notebook only will receive a deduction in marks, particularly where there is unnecessary output and where output format does not match the type requested. Complicated submissions/multiple PDFs will result in a delay of your exam results.

Questions

Question 1. (24 marks) An investment universe of the following risky assets with a dependence structure (correlation) is given. Use the ready appropriate formulae from Portfolio Optimisation Lecture.

Asset	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$	$\boldsymbol{w}$
A	0.02	0.05	w_1
B	0.07	0.12	w_2
C	0.15	0.17	w_3
D	0.20	0.25	w_4

$$\boldsymbol{R} = \begin{pmatrix} 1 & 0.3 & 0.3 & 0.3 \\ 0.3 & 1 & 0.6 & 0.6 \\ 0.3 & 0.6 & 1 & 0.6 \\ 0.3 & 0.6 & 0.6 & 1 \end{pmatrix}$$

Consider minimum variance portfolio with a target return m.

$$\underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{2}\boldsymbol{w'\Sigma w} \qquad \text{s.t. } \boldsymbol{w'1} = 1, \qquad \mu_\Pi = \boldsymbol{w'\mu} = m$$

- Formulate the Lagrangian function and give its partial derivatives. No further derivation required.
- Compute the allocations $\boldsymbol{w}^*$ and portfolio risk $\sigma_\Pi = \sqrt{\boldsymbol{w'\Sigma w}}$, for $m = 4.5\%$.
  Now, stress the correlation matrix: multiply all correlations by $\times 1.25$ and $\times 1.5$, and compute the respective optimal allocations and portfolio risk (the same $m = 4.5\%$).

- Inverse optimisation: generate $> 2,000$ random allocations sets w' – these will not be optimal allocations. Plot the cloud of points of μ_Π vertically on σ_Π horizontally. Before computing μ_Π, σ_Π, you can standardise to satisfy $w'\mathbf{1} = 1$.

Question 2. (20 marks) Continue with the data from Question 1 and consider a tangency portfolio.

- Formulate the Lagrangian function and give its partial derivatives only.
- For the range of tangency portfolios given by $r_f = 50bps, 100bps, 150bps, 175bps$ optimal compute allocations (ready formula) and σ_Π. Present results in a table.
- Plot the true Efficient Frontier in the presence of risk-free earning asset for $r_f = 100bps, 175bps$ and specifically identify its shape.

Question 3. (42 marks) As a market risk analyst, each day you calculate VaR from the available prior data. Then, you wait ten days to compare your prediction value VaR_{t-10} to the realised return and check if the prediction about the worst loss was breached. You are given a dataset with *Closing Prices.*

- Implement VaR backtesting by computing 99%/10day Value at Risk using the rolling window of 21 returns to compute σ. (a) Report the percentage of VaR breaches and (b) number of consecutive breaches. (c) Provide a plot which clearly identifies breaches.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- For comparison, implement backtesting using variance forecast equation below (recompute on each day). Rolling window of 21 remains for σ_t^2 (past variance) computation. The equation is known as EWMA model, and you can check how variance forecast is done in the relevant lecture.

$$\sigma^2_{t+1\,|\,t} = \lambda\, \sigma^2_{t\,|\,t-1} + (1-\lambda)\, r_t^2$$

with $\lambda = 0.72$ value set to minimise out of sample forecasting error, and r_t refers to a return. Provide the same deliverables (a), (b), and (c).

Question 4. (14 marks) Liquidity-Adjusted VaR (LVaR) is effectively VaR itself plus VaR of the bid/ask spread. The latter is our liquidity adjustment. It has not been introduced in Market Risk lecture, however to compute LVaR simply use the formula:

$$\begin{aligned} \text{LVaR} &= \text{VaR} + \Delta_{Liquidity} \\ &= \text{Portfolio Value} \times \left[-\mu + \text{Factor} \times \sigma + \frac{1}{2}(\mu_{Spread} + \text{Factor} \times \sigma_{Spread}) \right] \end{aligned}$$

Use the positive value of the Standard Normal Factor for the correct percentile. For the following cases, report (a) the proportion attributed to VaR and (b) the proportion attributed to liquidity adjustment.

(a) Consider a portfolio of USD 16 million composed of shares in a technology company. Daily mean and volatility of its returns are 1% and 3%, respectively. Bid-ask spread also varies with time, its daily mean and volatility are 35 bps and 150 bps. Compute 99%/1D LVaR and attributions to it,

(b) Now consider GBP 40 million invested in UK gilts. Take the daily volatility of portfolio returns as 3% and bid-ask spread is 15 bps (no spread volatility). Compute 99%/1D LVaR and attributions. What would happen if the bid-ask spread increases to 125 bps?

Further Instructions

Clarifying only questions are welcome to Richard.Diamond@fitchlearning.com, but not re-explanation. Please make good use of lecture exercises particularly, Market Risk Lecture and problem-solving sessions. The tutor is unable to confirm numerical answers and methods of calculation/spreadsheets.

To compute the 99%/10day Value at Risk for an investment in the market index on the rolling basis. We drop the expected return (mean) from the VaR formula

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- Appropriate Factor value to be used (Standard Normal Percentile), the tutor will not confirm the numerical value. It is also your task to identify the eligible number of observations for which VaR is available and can be backtested: N_{obs} will not be confirmed.
- Compute a column of rolling standard deviation over log-returns for observations $1-21, 2-22, \ldots$. Compute VaR for each day t, after the initial period. This is your worst loss prediction.
- **Regardless** of how many observations there are in a sample (10, 21, 100, etc.), variance is *an average of squared daily differences* $\frac{\sum (r_t-\mu)^2}{(N-1)}$ and so, timescale remains 'daily'.
- VaR is fixed at time t and compared to the return realised from t to $t+10$. A breach occurs when that forward realised 10-day return $\ln(S_{t+10}/S_t)$ is below the VaR_t quantity.

  $$r_{10D,t+10} \; < \; \text{VaR}_{10D,t} \qquad \text{means breach, given both numbers are negative.}$$

  In Excel, you will have a column for VaR_t series, a column of $r_{10D,t+10}$ series, and indicator column $\{0,1\}$ for a breach using $IF()$ function.
- To obtain the conditional probability of breach $N_{conseq}/N_{breaches}$, identify consecutive breaches. For example, the sequence $1, 1, 1$ means two consecutive breaches occurred.
- As an extra (**not a requirement**), you can apply statistical tests to the issue of independence of breaches in VaR (eg, conditional coverage, Christoffersen's exceedance independence).

  Alternatively, simply provide Q-Q plots for 1D returns and conclude if Normally distributed returns was a reasonable assumption. Lecture One (Module One) solutions give instruction on Q-Q plots.

EWMA scheme for variance forecasting is a practitioner's balance between naive sample std dev and GARCH models. The main disadvantage of GARCH(1,1) in the context of 10-day VaR is that the full GARCH model is too biased towards the long-term average variance.

With the same rolling window, eg 21 or 42 observations (depends on exam task). EWMA estimated for each next period as follows:

$$\sigma^2_{t+1\,|\,t} = \lambda\, \sigma^2_{t\,|\,t-1} + (1-\lambda)\, r_t^2$$

we recommend $\lambda = 0.72$, which is smaller than the original RiskMetrics method, but minimises out of sample forecasting errors.

VaR practitioners/risk managers also test the issue of independence of breaches in VaR quantitatively. The applicable statistical test is: Christoffersen's exceedances independence.

END OF EXAM