Derivatives Market Practice in Historical Perspective

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Option traders now and then

Myth 1:

People Did not Properly price options before Black-Scholes-Merton.

Myth 2:

Option traders today use the Black-Scholes-Merton formula.

Haug and Taleb: Option traders use (very) sophisticated heuristics, never the Black–Scholes–Merton formula

Put-Call Parity

Extremely Robust Arbitrage Principle

F-X=call-put

According to some modern text-books and famous Professors put-call parity invented 1969!

Joseph de la Vega 1688

“We say of those who buy means of a forward call contract and sell at fixed term or of those who sell by means of a put contract and buy at a fixed term that they shift the course of their speculation.”

Reprint in the book “Extraordinary Popular Delusions and the Madness of Crowds & Confusion de Confusiones” 1996 John Wiley & Sons, edited by Martin Fridson

Put-Call Parity early 1900

A) As pure arbitrage constrain.

B) To convert calls to puts, and puts and calls to straddles for hedging purpose. Options with options.

Option prices affected by supply and demand of options.

Higgins 1902/Nelson 1904

"It may be worthy of remark that 'calls' are more often dealt than 'puts' the reason probably being that the majority of 'punters' in stocks and shares are more inclined to look at the bright side of things, and therefore more often 'see' a rise than a fall in prices.

This special inclination to buy 'calls' and to leave the 'puts' severely alone dose not, however, tend to make 'calls' dear and 'puts' cheap, for it can be shown that the adroit dealer in options can convert a 'put' into a 'call,' a 'call' into a 'put', a 'call o' more' into a 'put- and-call,' in fact any option into another, by dealing against it in the stock. We may therefore assume, with tolerable accuracy, that the 'call' of a stock at any moment costs the same as the 'put' of that stock, and half as much as the Put-and-Call. "

Higgins 1902:

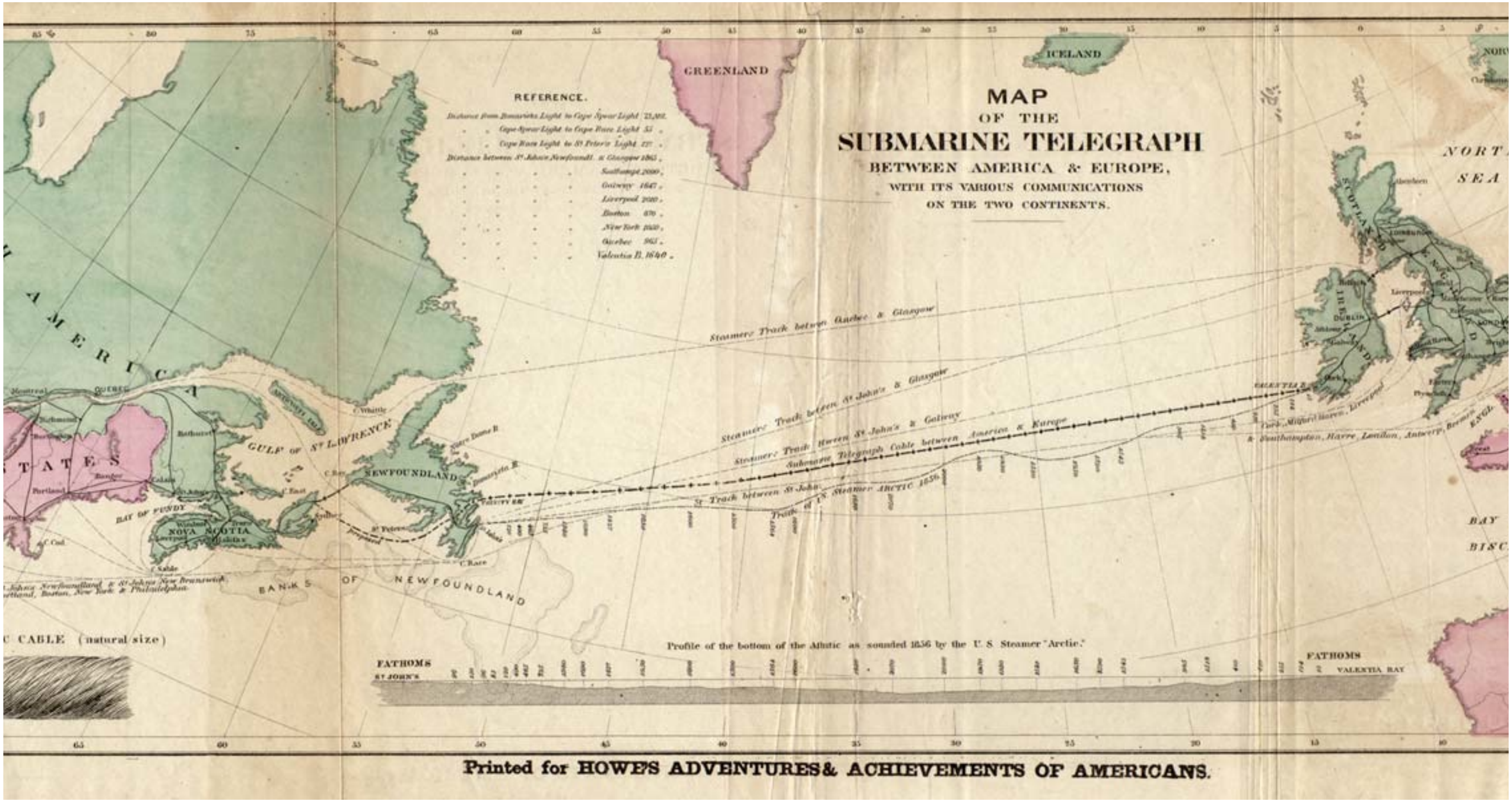
- 1. That a Call of a certain amount of stock can be converted into a Put-and- Call of half as much by selling one-half of the original amount.
- 2. That a Put of a certain amount of stock can be converted into a Put-and- Call of half as much by buying one-half of the original amount.
- 3. That a Call can be turned into a Put by selling all the stock.
- 4. That a Put can be turned into a Call by buying all the stock.
- 5. and 6. That a Put-and-Call of a certain amount of a stock can be turned into either a Put or twice as much by selling the whole amount, or into a Call of twice as much by buying the whole amount.

Reinach 1961

“Although I have no figures to substantiate my claim, I estimate that over 60 per cent of all Calls are made possible by the existence of Converters.”

Reinach understands put-call parity do not hold for American options, mention how converters try to arbitrage on it.

Five attempts 1857, two in 1858, 1865, 1866 lasting connection



International Arbitrage Trading

Telegraph

- 1858 0.1 Words per minute
- 1866 8 words per minute
- 1900 >120 words per minute

Horrible with such high frequency trading!
Is it not?

Options Arbitrage Between London and New York (Nelson 1904)

Up to 500 messages per hour and typically 2,000 to 3,000 messages per day where sent between the London and the New York market through the cable companies. Each message flashed over the wire system in less than a minute.

Empirical Research pre-1973 markets

Options priced much as today

There was a volatility skew

Empirical regularities in implied vol etc similar.

Mixon, S. (2008): “Option Markets and Implied Volatility: Past Versus Present

Kairys, J. P. and N. Valerio (1997) The Market for Equity Options in the 1870s, Journal of Finance, Vol LII, NO. 4., Pp 1707−1723.

Delta Hedging

Delta Hedging invented in 1960's to 1970's ?

Dynamic Delta Hedging Invented ?

Continuous Time Dynamic Delta Hedging invented in 1970's Black-Scholes-Merton

Nelson 1904

"Sellers of options in London as a result of long experience, if they sell a Call, straightway buy half the stock against which the Call is sold; or if a Put is sold; they sell half the stock immediately."

Standard options at that time in London always issued at-the-money forward and European style

Nelson 1904

“The regular London option is always either a Put or a Call, or both, at the market price of the stock at the time the bargain is made, to which is immediately added the cost of carrying or borrowing the stock until the maturity of the option.”

Put-Call Parity 1902/1904

Static delta hedging 1902/1904 for at-the-money options. Nelson/Higgins

Static delta hedging any option 1967, Thorp

Idea of dynamic delta hedging 1904? 1969 Thorp

Continuous-time dynamic delta hedging 1973 Merton

Arbitrage early 1900

1. Postage
2. Interest expenses
3. Insurance
4. Transportation
5. Transportation arbitrage
6. Credit risk

Option Pricing Formulas Before Black-Scholes-Merton

Bachelier (1900) Normal distribution. Says little about how to hedge out risk in options.

Bronzin (1908) based on put-call parity, several types of distributions.

Sprenkle (1960), Book of Cootner

Ayres (1963), Book of Cootner

Boness (1964),

Samuelson (1965)

Thorp (1969)

Professor Bronzin 1908, Option Pricing:

$$F(x) = \int_x^{\omega} f(x)\,dx \text{ resp. } F_1(x) = \int_x^{\omega_1} f_1(x)\,dx \tag{5}$$

die Gesamtprobabilitäten dar, daß die Schwankungen über resp. unter B am Liquidationstermin die Größe x übersteigen; wir werden bald erfahren, welche bedeutende Rolle gerade diese Funktionen in den späteren Betrachtungen spielen werden.

Tragen wir auf einer horizontalen Geraden rechts von einem Punkte 0 die Marktschwankungen über B, links davon hingegen die Schwankungen unter B auf und errichten wir in den jeweiligen Endpunkten Senkrechte, welche die entsprechenden Funktionswerte $f(x)$ bezw. $f_1(x)$ darstellen sollen, so entstehen zwei kontinuierliche Kurven C und C_1, die wir füglich Schwankungswahrscheinlichkeitskurven nennen werden (siehe Fig. 23); die zwischen irgend zwei Ordinaten

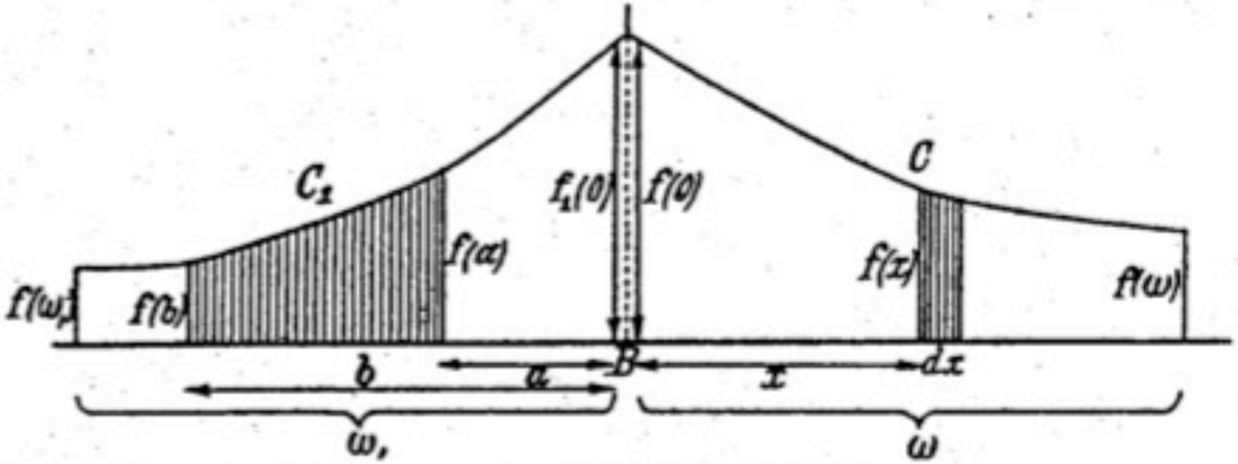
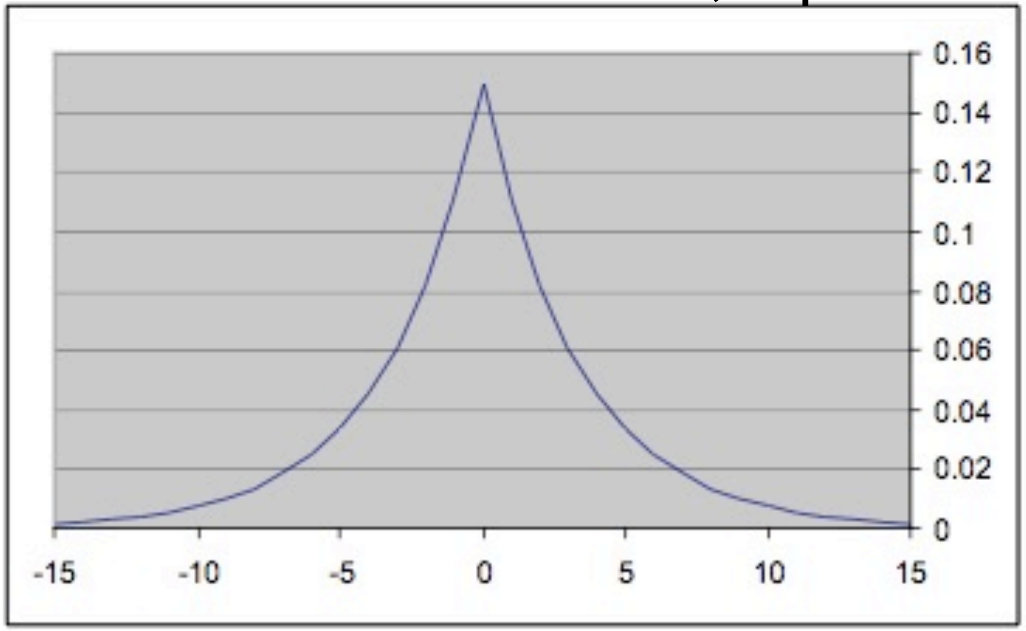
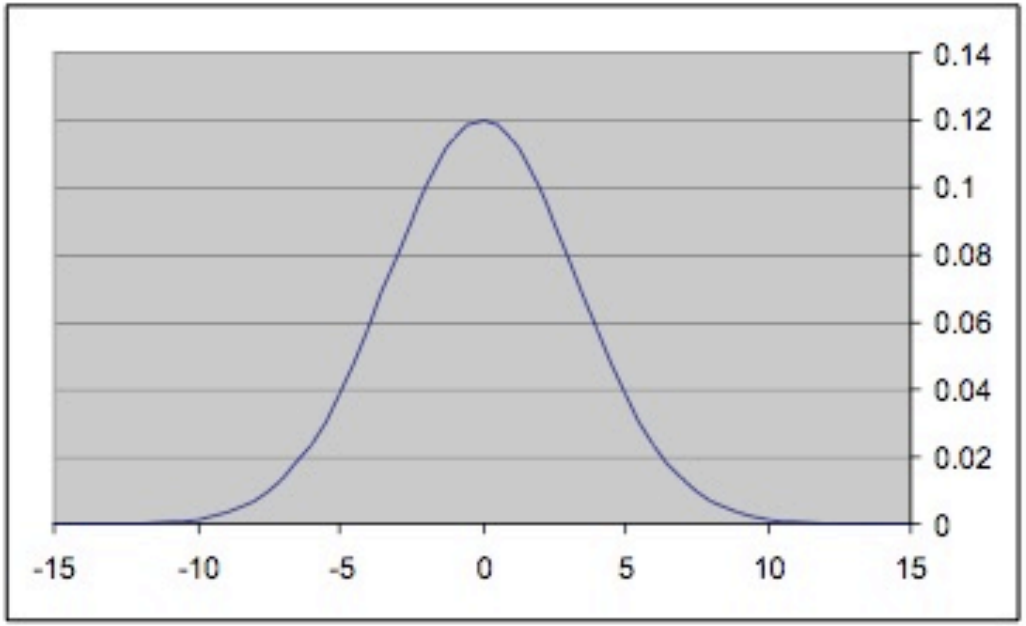


Fig. 23.

Professor Bronzin 1908, Option Pricing:



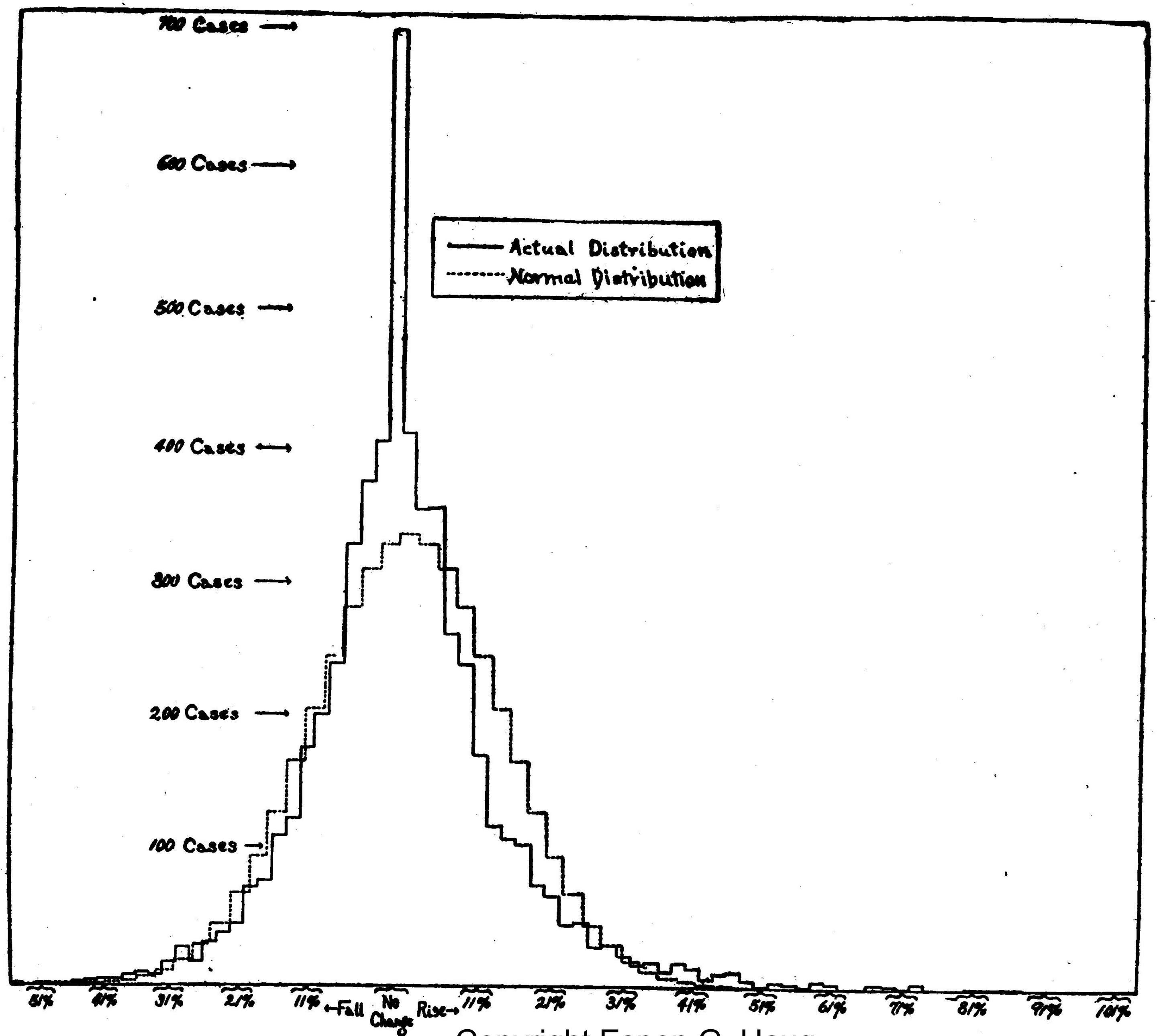


Fat-Tails in Price Data

Wesley Clair Mitchell
1874-1948

“The Making and Using of Index Numbers” published in 1915

CHART 2.—DISTRIBUTION OF 5,578 PRICE VARIATIONS (PERCENTAGES OF RISE OR FALL FROM PRICES OF PRECEDING YEAR).



Mills rejects the Gaussian hypothesis.

"A distribution may depart widely from the Gaussian type because the influence of one or two extreme price changes."

Mills, F. C. (1927): The Behaviour of Prices. New York: National Bureau of Economic Research, Albany: The Messenger Press.

Osborne 1958

"Brownian Motion in Stock Market"

This nearly normal distribution in the changes of logarithm of prices suggests that it may be a consequence of many independent random variables contributing to the changes in values (as defined by the Weber-Fechner law). The normal distribution arises in many stochastic proc-

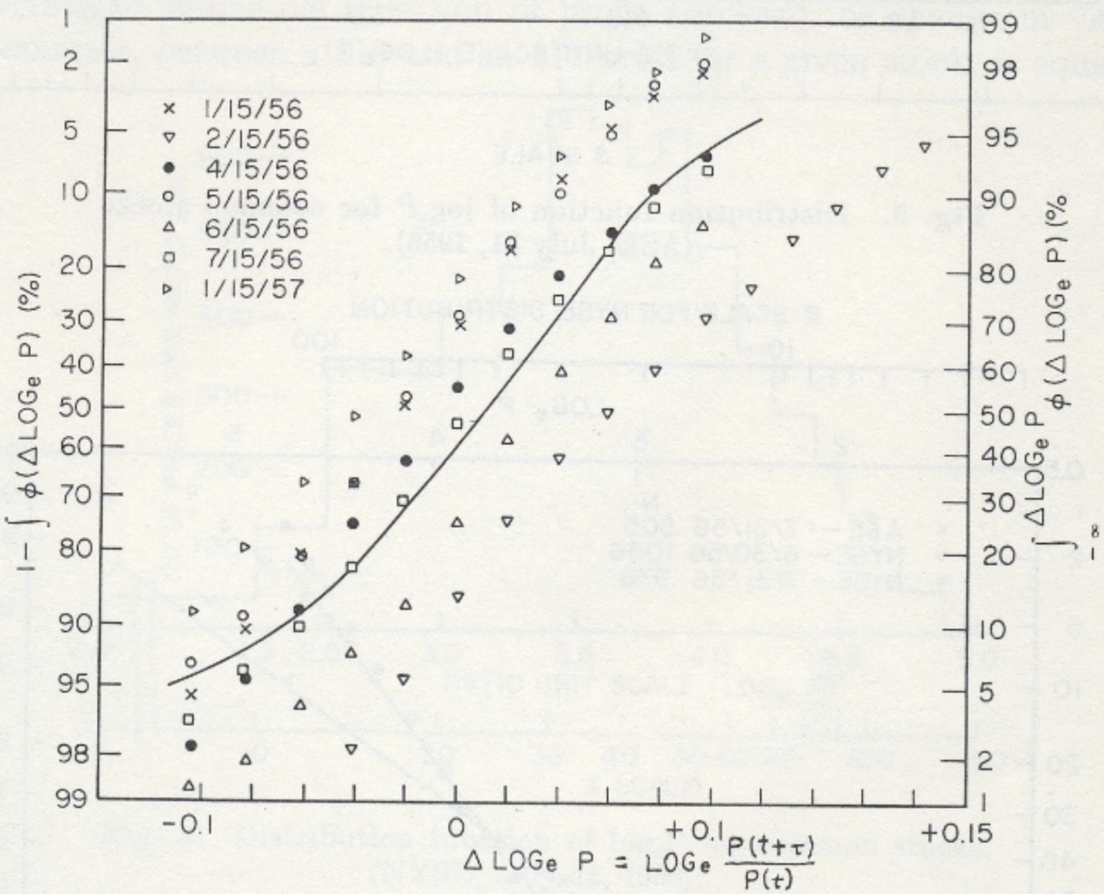


Fig. 7. Cumulated distributions of $\Delta\log_e P = \log_e[P(t+\tau)/P(t)]$ for $\tau = 1$ month (NYSE common stocks). These, and also Fig. 8, may be regarded as distributions of $S(\tau)$ for fixed $M^*(\tau)$. The solid line is the distribution of $Z(\tau) \approx M(\tau)$, transcribed from Fig. 12 for comparison.

esses involving large numbers of independent variables, and certainly the market place should fulfill this condition, at least.

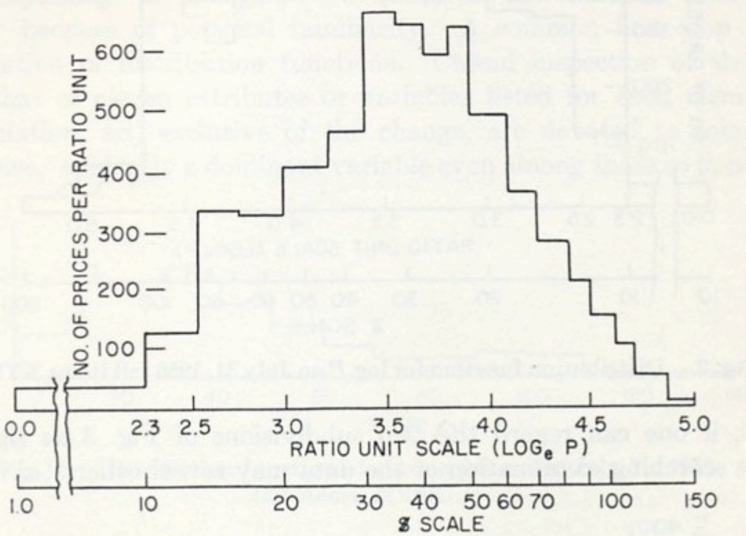


Fig. 4. Distribution function of $\log_e P$ for common stocks (NYSE, July 31, 1956).

Osborne (1959) detects fat-tails in price data, but basically ignores them and seems to be a strong believer in normal distributed returns.

Mandelbrot 1962

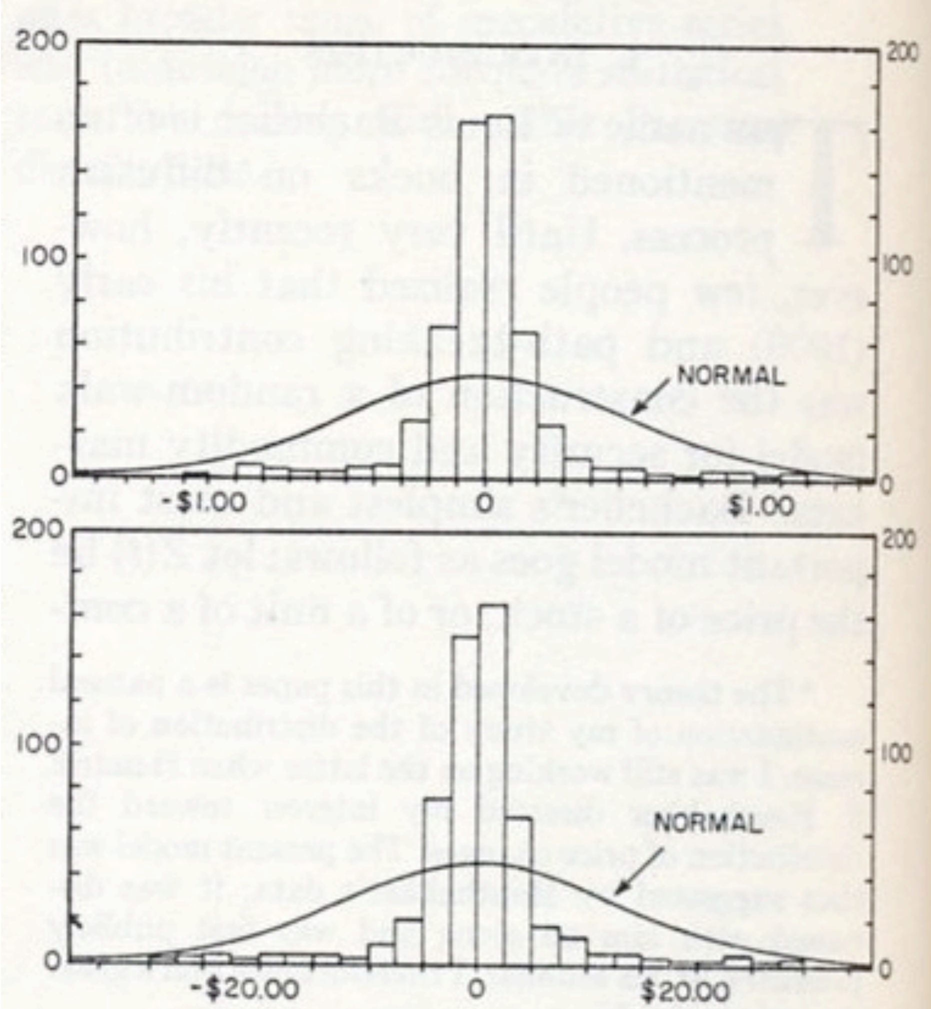


FIG. 1.—Two histograms illustrating departure from normality of the fifth and tenth difference of monthly wool prices, 1890–1937. In each case, the continuous bell-shaped curve represents the Gaussian "interpolate" based upon the sample variance. Source: Gerhard Tintner, *The Variate-Difference Method* (Bloomington, Ind., 1940).

"The Variation of Certain Speculative Prices"

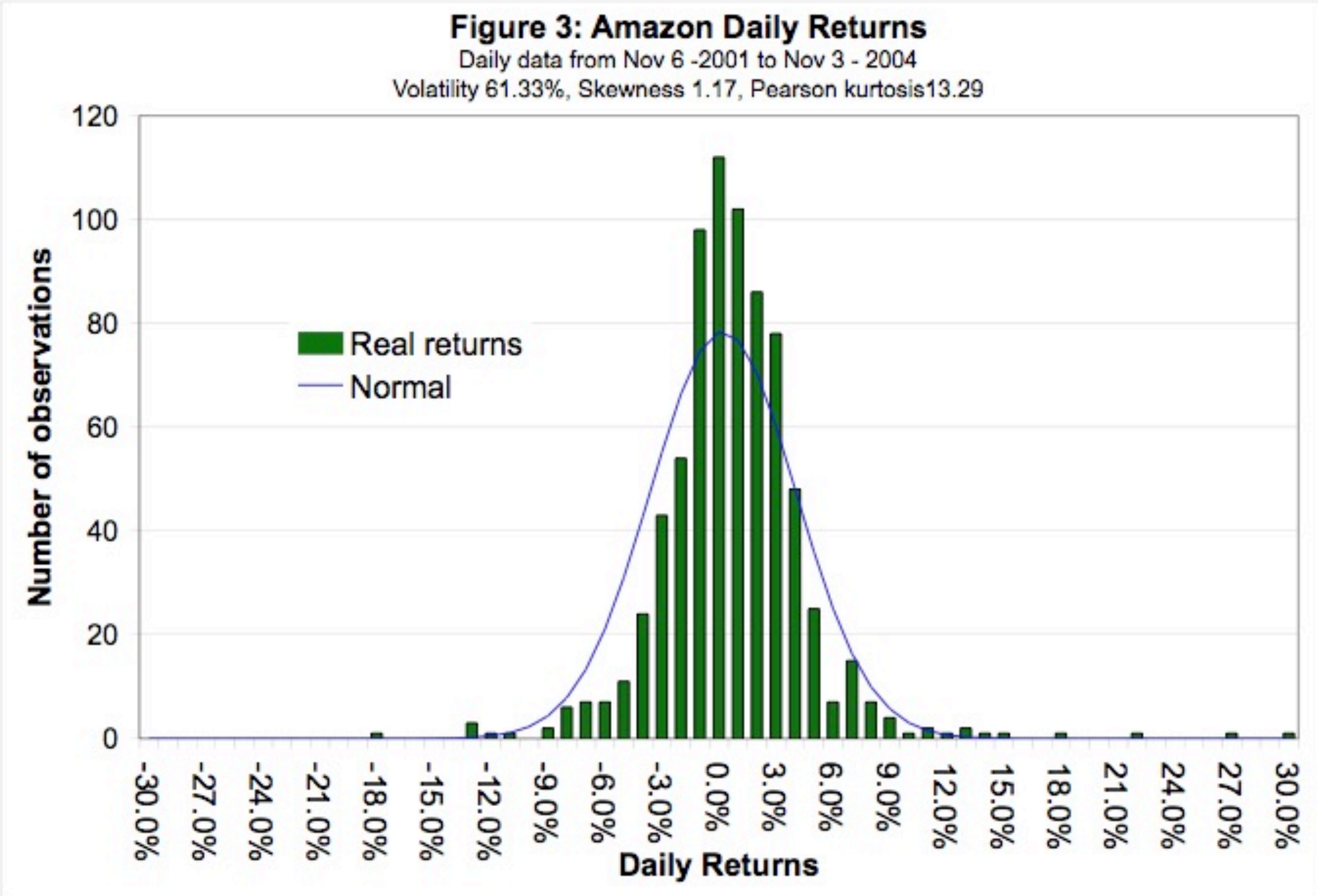


Figure 3: Amazon Daily Returns
Daily data from Nov 6 -2001 to Nov 3 - 2004
Volatility 61.33%, Skewness 1.17, Pearson kurtosis13.29
Number of observations
0
20
40
60
80
100
120
Real returns
Normal
-30.0%
-27.0%
-24.0%
-21.0%
-18.0%
-15.0%
-12.0%
-9.0%
-6.0%
-3.0%
0.0%
3.0%
6.0%
9.0%
12.0%
15.0%
18.0%
21.0%
24.0%
27.0%
30.0%
Daily Returns

If Gaussian

We can measure all risk by variance/standard deviation σ.

Easy to make models.

Consistent models

Cost: we are loosing out on important information if non-Gaussian

Some of the BIG Ideas in Finance

CAPM: Based on Gaussian!

Sharpe Ratio: Based on Gaussian!

Black-Scholes-Merton: Based on Gaussian!

Before Black-Scholes-Merton

Option valuation by discounting expected value

$$dS = \mu S_t dt + \sigma S_t dz$$

Bachelier/Sprenkle/Boness/Thorp (1964/1969)

formula: Exact Boness formula

$$c = SN(d_1) - Xe^{-rT}N(d_2)$$

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad d_2 = d_1 - \sigma\sqrt{T}$$

Before Black-Scholes-Merton

Option valuation by discounting expected value

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Bachelier/Sprenkle/Boness/Thorp (1964/1969) formula: Exact Boness formula

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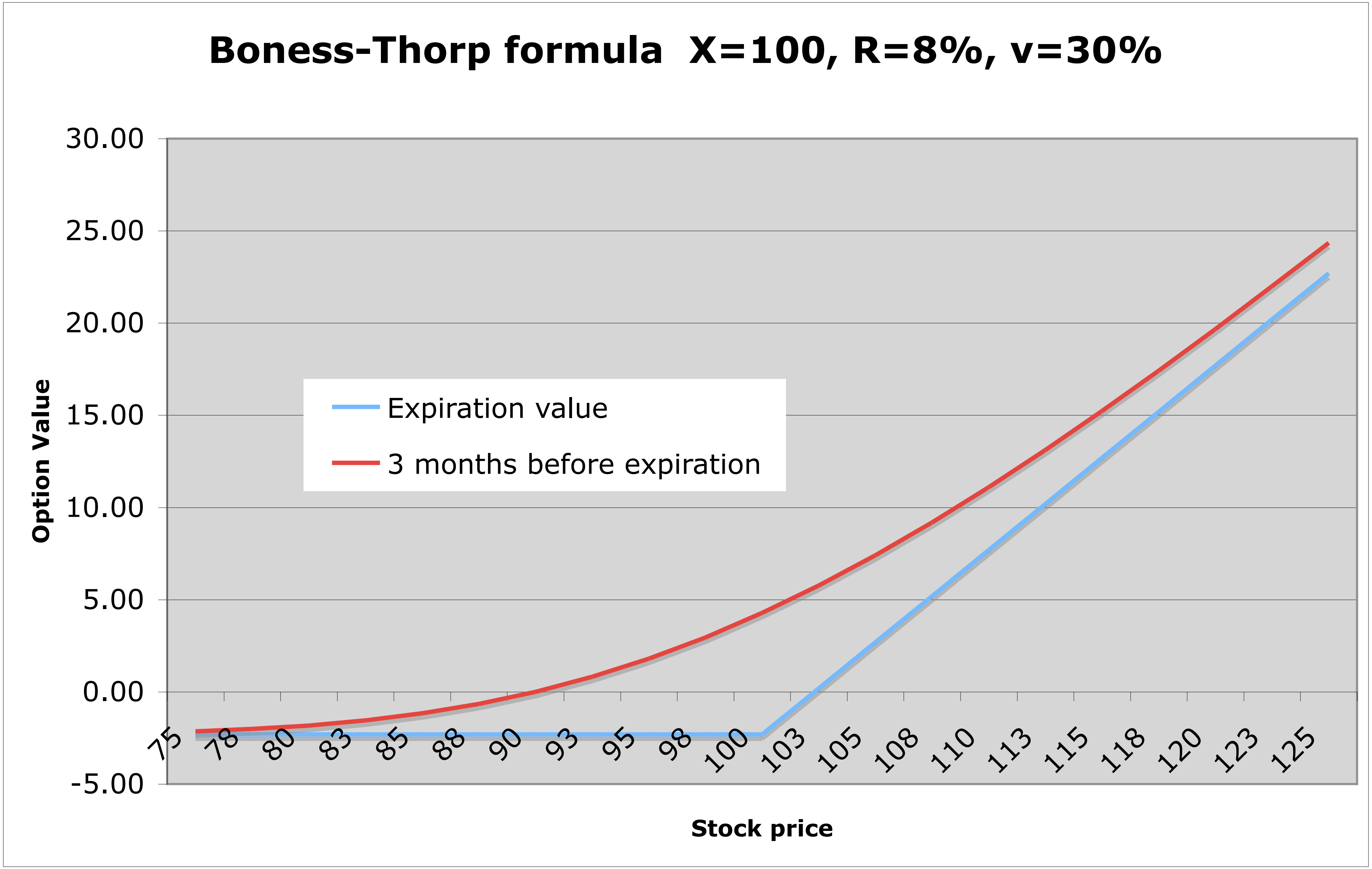
$$d_1 = \frac{\ln(S/X) + (\mu + \sigma^2/2)T}{\sigma\sqrt{T}} \quad d_2 = d_1 - \sigma\sqrt{T}$$



Boness-Thorp formula X=100, R=8%, v=30%
30.00
25.00
20.00
15.00
10.00
5.00
0.00
-5.00
Option Value
Expiration value
3 months before expiration
75
78
80
83
85
88
90
93
95
98
100
103
105
108
110
113
115
118
120
123
125
Stock price

MODERN HISTORY DELTA HEDGING

Ed Thorp (2002) 1966: "We understood static hedging, dynamic hedging, and delta hedging,in particular market-neutral delta hedging."

Thorp and Kassouf (1967) "Beat the Market"

Thorp (1969) "Optimal Gambling Systems for Favorable Games"

Rubinstein (2006) "Thorp came close since he early on understood the idea of dynamically delta hedging an option with a position its underlying asset…."

Thorp 1967

“If, when the common changed price, the warrant moved along this line, then a 1 point increase in the common would result in a 1/3 point increase in the warrant. If we are short 3 warrants to one common long, then the gain on the common is completely offset by the loss on the warrant.”

Thorp 1969

“We have assumed so far that a hedge position is held unchanged until expiration, then closed out. This static or ‘desert island’ strategy is not optimal. In practice intermediate decisions in the spirit of dynamic programming lead to considerably superior dynamic strategies. The methods, technical details, and probabilistic summary are more complex so we defer the details for possibly subsequent publication.”

Arnold Bernhard & Co., 1970

EVALUATION OF WARRANT

	[16]	% Overvalued (+) Undervalued (−) Footnote (See Next Page) [17]	[18]	Wt Leverage Projections (for these changes in the price of the underlying security) +50% [19]	+25% [20]	−25% [21]	−50% [22]	Hedge Rank [23]	Hedge Ratio [24]	Effective Per Share Exercise Price [25]	Per Share Cost of Option [26]	Total $ Premium [27]	% Premiu… Tangible Value [28]	[29]	[3…	
1	Y.WS		+40%	+45%	+23%	−40%	−70%	F	90	3.75	5.13	2.00	3.13	29%	Y	
2			+21	+65	+35	−40	−70	D	40	37.52	4.00	4.00	.00	28	ADP	
3		1	−1	+75	+35	−30	−60	C	50	12.00	2.25	2.25	.00	35	AMO	
4	T.WS	2	+25	+140	+60	−55	−85	C+	40	52.00	8.50	8.50	.00	19	T	
5			+55	+45	+25	−45	−75	F	55	28.00	6.50	6.50	.00	43	APL	
6			−16	+125	+70	−30	−65	A	85	47.50	42.00	−5.63	47.63	−6	ARA	NYS
7	ARCW	3	−10	+115	+50	−35	−65	B	35	110.00	11.25	11.25	.00	18	ARC	NYS
8	AZ>W	4	−4	+60	+30	−25	−55	C−	65	6.25	1.38	1.38	.00	55	AZ	NYS
9	ATOW	5	+30	+60	+30	−40	−70	D	35	30.00	2.13	2.13	.00	27	ATO	NYS
10	AV.W		+7	+80	+40	−35	−65	C−	35	33.56	2.88	2.88	.00	24	AV	NYS
	NKW	6	+75	+45	+25	−45	−75	F	30	55.00	1.88	1.88	.00	24	BNK	NYS
		7	+17	+70	+35	−40	−70	D	60	12.50	5.50	5.25	.25	41	BIW	NYS
		8	+995	−100	−100	−100	−100	F	25	11.98	.50	.50	.00	8	BKY	NYS
			−16	+100	+45	−30	−60	B	135	11.43	2.50	7.50	.00	29	BNF	NYS
		9	−18	+105	+50	−30	−60	B	45	7.26	1.38	1.38	.00	28	BWN	NYS
	Y	10	+18	+70	+35	−40	−70	D	45	14.00	2.38	2.38	.00	32	BGT	NYS
	CNY	11	+50	+60	+30	−45	−75	F	30	23.73	1.19	1.25	.00	21	CCN	NY
	CTCW		−13	+105	+50	−35	−60	B	50	22.38	6.25	6.25	.00	31	CTC	N
19	DLNW	12	−4	+90	+45	−35	−65	C+	60	12.94	5.38	4.19	1.19	30	DLN	
20		13	−7	+90	+45	−35	−65	C+	35	45.00	4.50	4.50	.00	23	DRY	

Black-Scholes reference to Thorp and Kassouf

One of the concepts that we use in developing our model is expressed by Thorp and Kassouf (1967). They obtain an empirical valuation formula for warrants by fitting a curve to actual warrant prices. Then they use this formula to calculate the ratio of shares of stock to options needed to create a hedged position by going long in one security and short in the other. What they fail to pursue is the fact that in equilibrium, the expected return on such a hedged position must be equal to the return on a riskless asset. What we show below is that this equilibrium condition can be used to derive a theoretical valuation formula.

Black-Scholes 1973

"If the hedge is maintained continuously, then the approximations mentioned above become exact, and the return on the hedged position is completely independent of the change in the value of the stock. In fact, the return on the hedged position becomes certain. This was pointed out to us by Robert Merton."

HISTORY

Black-Scholes (1973) and Merton (1973): "First" to publish the brilliant idea of removing all risk by holding the right combination of options and stocks, where the number of stocks are continuously rebalanced, known as dynamic: delta hedging, dynamic replication, dynamic spanning…

$$dS = \mu S_t dt + \sigma S_t dz$$

Ito's Lemma

$$dc = \left[\frac{\partial c}{\partial t} + \frac{\partial c}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 c}{\partial S^2}\sigma^2 S^2\right] dt + \frac{\partial c}{\partial S}\sigma S dz$$

Idea: risk free portfolio $$-c+\frac{\partial c}{\partial S}S$$

Change in value : $$dV=-dc+\frac{\partial c}{\partial S}dS$$

$$dV=-\frac{\partial c}{\partial t}dt-\frac{1}{2}\frac{\partial^2 c}{\partial S^2}\sigma^2 S^2 dt$$

$$dV=rV\,dt$$

$$\left[\frac{\partial c}{\partial t}+\frac{1}{2}\frac{\partial^2 c}{\partial S^2}\sigma^2 S^2+r\frac{\partial c}{\partial S}S\right]dt=rc.$$

DYNAMIC DELTA HEDGING

-Works perfectly under the theoretical assumptions of continuous time continuous price. Brilliant mathematical idea!!

- Finance is not pure mathematics!

-Models are only models: we can not claim a model is bad simply because there are breaks on it’s assumptions!

- ROBUST?? : to what we really can do and how real markets behave?

Merton 1998

“A broader, and still open, research issue is the robustness of the pricing formula in the absence of a dynamic portfolio strategy that exactly replicates the payoffs to the option security.”

DELTA HEDGING IN PRACTICE

-Discrete time and price steps.

- Transaction costs, bid-offer spreads, etc.

- Empirically we have fat tailed distributions.

- Jumps in prices as well as stochastic volatility, and even jumps in volatility.

-Boyle and Emanuel (1980)

-Gilster (1990) Systematic risk

-Mello Neuhaus (1998)

-Derman and Kamal (1999)

-Wilmott (2000)

-Derman-Taleb (2005)

Substantial Transaction Costs

Basele, Shows, Thorpe (1983)

Gilster and Lee (1984)

Lealand (1985)

Hoggard, Whalley, and Wilmott (1994)

Kabanov and Safarian (1997)

Grandits and Schachinger (2001)

Not path dependent, only S at maturity counts

Need reasonable speed (100 000 simulations)

Monte Carlo simulation put option no hedge and static delta hedge, 100 000 simulations.

($S = 100$, $T = 30/365$, $r = b = 0$)

Vol	Strike	Delta	Value	Stdev %	Max error	Stdev %	Max error
				No Hedging		Static Delta Hedge	
10%	100.0000	-49.4%	1.14369	144.2%	856.2%	75.7%	457.9%
10%	98.1252	-25.0%	0.43383	231.9%	2204.3%	166.3%	1520.6%
10%	96.4322	-10.0%	0.13736	395.9%	5740.3%	339.5%	4897.5%
30%	100.0000	-48.3%	3.43014	139.1%	855.5%	75.6%	548.0%
30%	94.7136	-25.0%	1.34026	221.0%	1817.0%	159.9%	1239.6%
30%	89.8953	-10.0%	0.42222	374.2%	5390.1%	321.4%	4600.8%
60%	100.0000	-46.6%	6.85394	132.8%	680.4%	75.5%	619.1%
60%	90.3727	-25.0%	2.80468	202.2%	1380.2%	148.6%	981.0%
60%	81.4117	-10.0%	0.87655	335.6%	3620.1%	291.3%	3036.7%

VERY RISKY BUSINESS!!

Risk can be good or Bad!

Delta Hedging Replication:

Now path dependent

Need good random number generator

Need a lot of speed C++ (100 000 simulations)

3 to 30 million random number per number in table

Table 1: Monte Carlo simulation of dynamic delta replication put option
($S = 100$, $T = 30/365$, $r = b = 0$)

Vol	Strike	Delta	Value	Stdev %	Max	Stdev %	Max	Stdev %	Max
				$n = 30$		$n = 60$		$n = 300$	
10%	100.0000	-49.4%	1.14369	15.6%	100.7%	11.2%	70.2%	5.1%	35.0%
10%	98.1252	-25.0%	0.43383	34.9%	304.6%	25.3%	246.7%	11.4%	88.9%
10%	96.4322	-10.0%	0.13736	76.9%	854.8%	55.1%	663.0%	24.8%	275.6%
30%	100.0000	-48.3%	3.43014	15.6%	123.1%	11.2%	80.4%	5.1%	37.9%
30%	94.7136	-25.0%	1.34026	34.2%	304.2%	24.4%	216.4%	11.0%	77.6%
30%	89.8953	-10.0%	0.42222	73.4%	763.6%	52.0%	518.6%	23.7%	269.4%
60%	100.0000	-46.6%	6.85394	15.7%	127.6%	11.1%	93.8%	5.1%	35.4%
60%	90.3727	-25.0%	2.80468	31.9%	436.7%	22.7%	352.2%	10.3%	83.7%
60%	81.4117	-10.0%	0.87655	67.0%	661.5%	48.0%	534.9%	21.9%	262.9%

Derman-Kamal analytic approximation 1998

$$\sigma_{\text{P\&L}} \approx \sqrt{\frac{\pi}{4}} S e^{(b-r)T} n(d_1)\sqrt{T}\frac{\sigma}{\sqrt{N}} = \sqrt{\frac{\pi}{4}}\text{Vega}\frac{\sigma}{\sqrt{N}}$$

Table 2: Derman-Kamal theoretical dynamic delta hedging replication error put option

($S = 100$, $T = 30/365$, $r = 0$)

Vol	Strike	Delta	Value	Stdev % $n = 30$	Stdev % $n = 60$	Stdev % $n = 300$
10%	100.0000	-49.4%	1.14369	16.2%	11.4%	5.1%
10%	98.1252	-25.0%	0.43383	34.0%	24.0%	10.7%
10%	96.4322	-10.0%	0.13736	59.3%	41.9%	18.7%
30%	100.0000	-48.3%	3.43014	16.2%	11.4%	5.1%
30%	94.7136	-25.0%	1.34026	33.0%	23.3%	10.4%
30%	89.8953	-10.0%	0.42222	57.8%	40.9%	18.3%
60%	100.0000	-46.6%	6.85394	16.1%	11.4%	5.1%
60%	90.3727	-25.0%	2.80468	31.5%	22.3%	10.0%
60%	81.4117	-10.0%	0.87655	55.7%	39.4%	17.6%

A) VOLATILITY IS NOT CONSTANT IN DESCRETE TIME!!!

"sampling error" ? (yes and no)

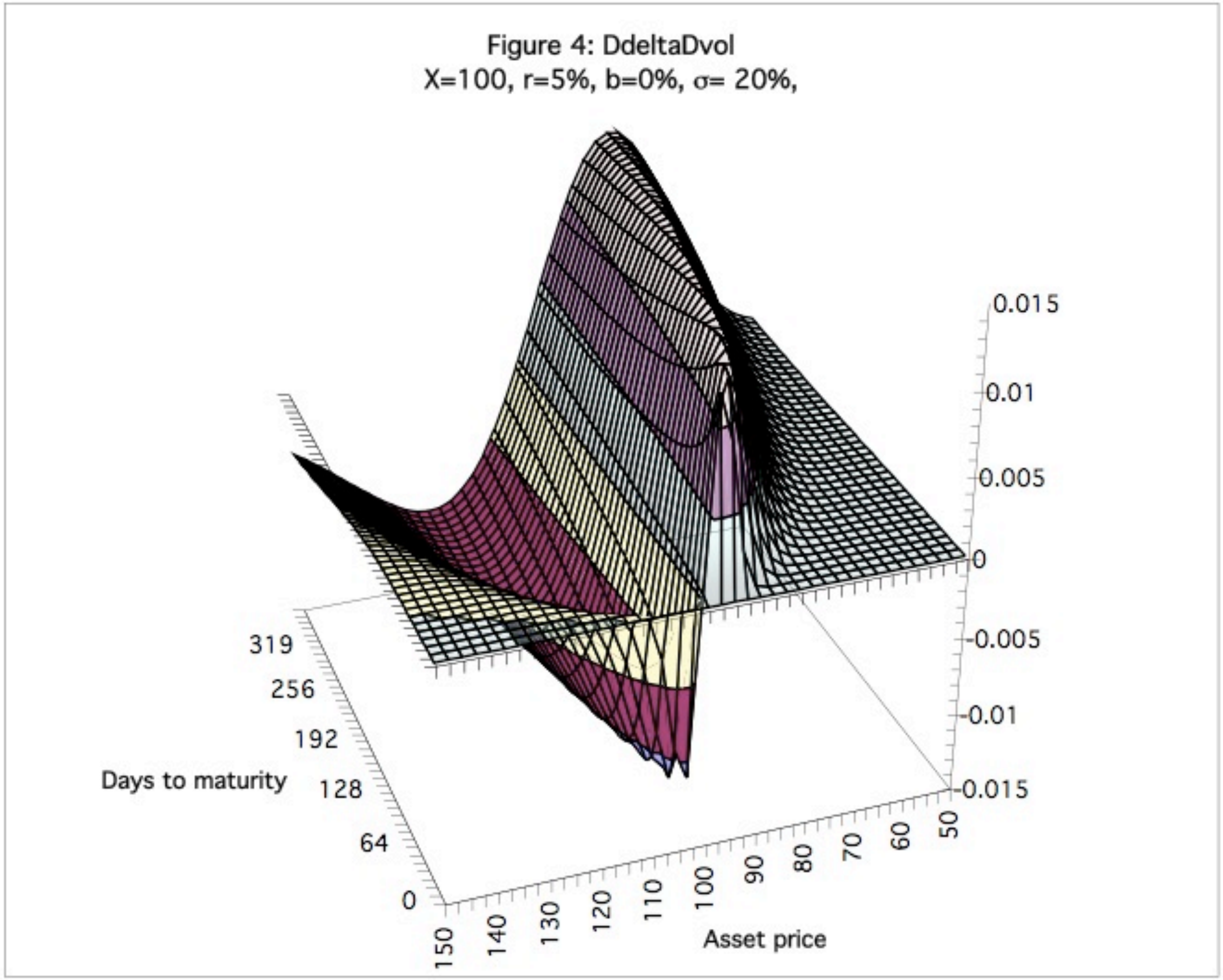
Not really: Real effect: we can not hedge continuous in time!!

$$P\left[s\sqrt{\frac{(n-1)}{\chi^2_{(n-1;\alpha/2)}}} \leq \sigma \leq s\sqrt{\frac{(n-1)}{\chi^2_{(n-1;1-\alpha/2)}}}\right] = 1-\alpha$$

Example: N=50, 95% confidence interval, 30% volatility: 25.06% to 37.38%

N=300 : 95% confidence: 27.78% to 32.61%

B) Delta for OTM options very sensitive to volatility. High DDeltaDVol



SO FAR ONLY LOOKED AT GBM

What about stochastic volatility, jumps etc.?

We already got weak-form stochastic volatility or small price jumps from discrete time hedging.

Can only make things worse when we add jumps or stochastic volatility using BSM delta.

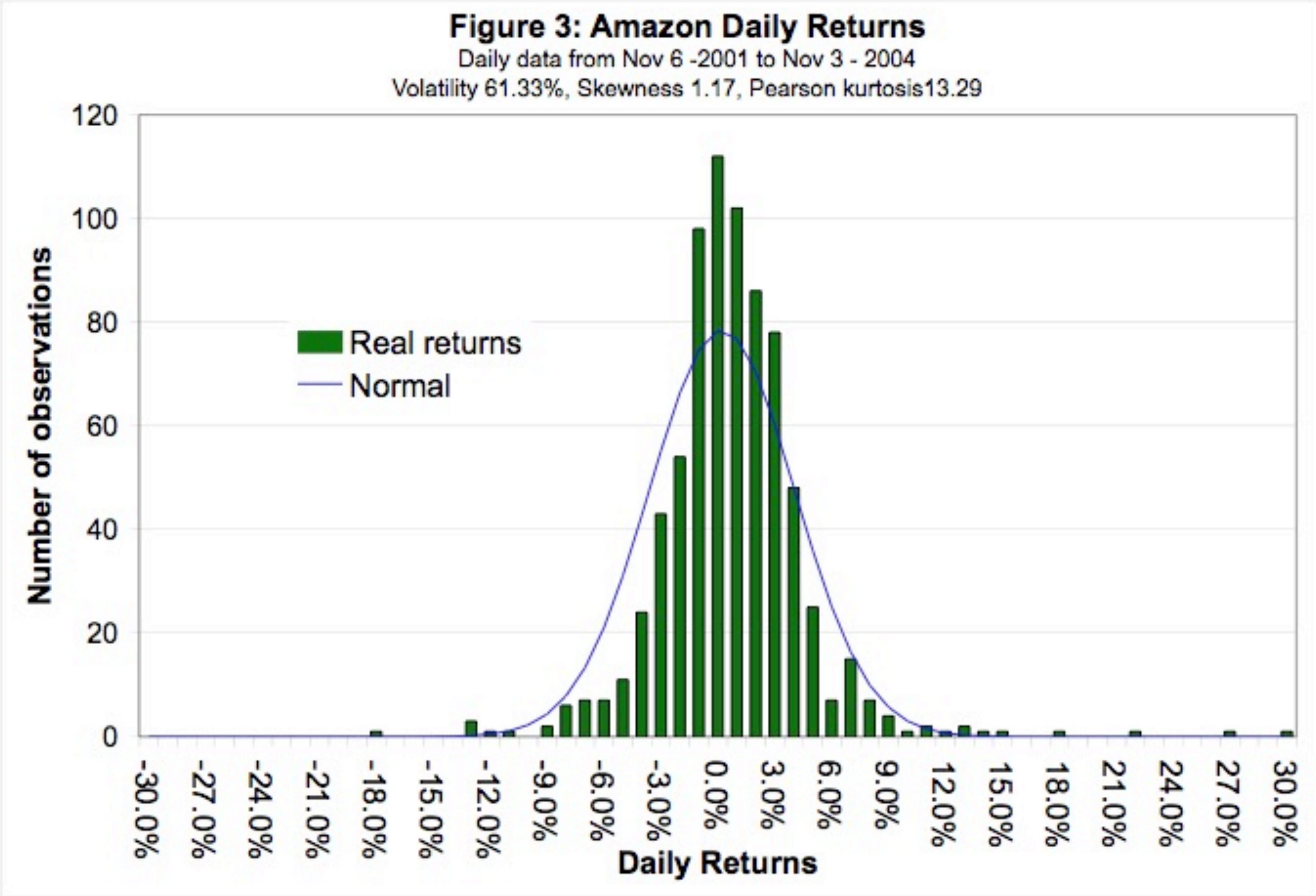


Figure 3: Amazon Daily Returns
Daily data from Nov 6 -2001 to Nov 3 - 2004
Volatility 61.33%, Skewness 1.17, Pearson kurtosis13.29
Number of observations
120
100
80
60
40
20
0
Real returns
Normal
-30.0%
-27.0%
-24.0%
-21.0%
-18.0%
-15.0%
-12.0%
-9.0%
-6.0%
-3.0%
0.0%
3.0%
6.0%
9.0%
12.0%
15.0%
18.0%
21.0%
24.0%
27.0%
30.0%
Daily Returns

What about jumps

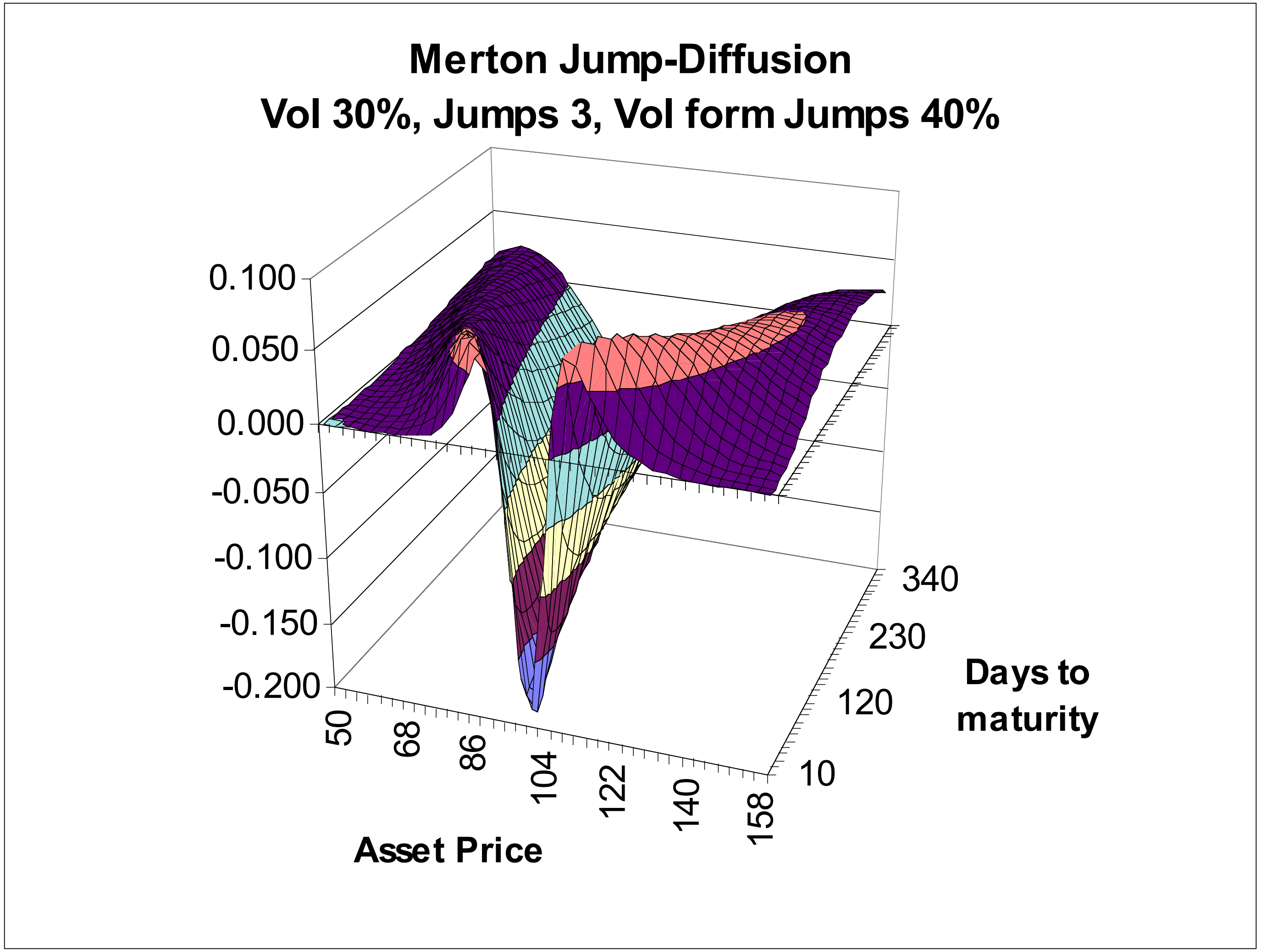
Jumps at random time during option lifetime

100 000 simulations per run

Table 2: Monte Carlo simulation of dynamic delta replication put option with jumps

($S = 100$, $T = 30/365$, $r = b = 0$, $\sigma = 0.30$)

Jump Size	Strike	Delta	Value	Stdev %	Max	Stdev %	Max	Stdev %	Max
				$n = 30$		$n = 60$		$n = 300$	
3%	100.0000	-48.3%	3.43014	17.7%	141.6%	13.7%	104.2%	9.1%	73.7%
3%	94.7136	-25.0%	1.34026	41.6%	349.7%	32.0%	252.0%	20.9%	191.0%
3%	89.8953	-10.0%	0.42222	97.4%	1014.9%	74.7%	731.8%	47.4%	619.0%
5%	100.0000	-48.3%	3.43014	25.6%	173.1%	23.1%	160.0%	20.8%	127.1%
5%	94.7136	-25.0%	1.34026	62.3%	394.6%	55.1%	403.1%	48.9%	312.8%
5%	89.8953	-10.0%	0.42222	149.1%	1504.2%	131.3%	1040.0%	114.1%	950.4%
10%	100.0000	-48.3%	3.43014	73.7%	287.9%	73.6%	275.2%	73.6%	268.8%
10%	94.7136	-25.0%	1.34026	189.1%	773.7%	188.5%	658.1%	188.1%	680.9%
10%	89.8953	-10.0%	0.42222	478.8%	2314.6%	473.3%	2026.8%	468.9%	2089.0%



Call

Asset price (S)	80.00
Strike price (X)	100.00
Time to maturity (T)	0.25
Risk-free rate (r)	5.00%
Volatility (σ)	30.00%
Jumps per year (λ)	3.00
Percent of total volatility (γ)	40.00%
Value	0.5255

Bates Jump-Diffusion

- Jumps are allowed to be asymmetric, i.e. with non-zero mean.
- Since we often have to do with options on stock index futures (e.g. S&P index options), it is hardly plausible to maintain Merton's simplifying assumption that jump risk is idiosyncratic and thus fully diversifiable.

The Bates (1991) jump-diffusion model is consistent with an asymmetric volatility smile (generated form a BSM type model). This is often what we observe in practice.

The “many world interpretation of Jumps”

Diversification, do the professional diversify?
Much less than many would like to think!!

ARCH, GARCH, STOCHASTIC VOL, JUMP MODELS OF ANY USE??

Yes: helps us understand what stochastic process causes fat tails, helps us in risk management..help us value out-of-the-money options. Help us improve delta-hedge.

Do not improved dynamic delta hedging enough to save risk-neutrality.

They are all good fudge models!! (rooted in Gaussian??)

Static/Semi-Static Hedging

Higgins 1902/ Nelson 1904 Put-Call Parity Fully Understood!!

Mello Neuhaus (1998) Hedging options with options

Carr and Wu (2002)

“Put Call Reversal” by Peter Carr and Jesper Andreasen April 25, 2002

Derman-Taleb 2005: only for European options “Complete Static Hedging Argument”

Exotic options: Carr 1994, Derman 1995, Haug 1998 and many others, also variance swaps and volatility swaps.

MODELS ARE ONLY MODELS In practice: semi-static hedging **not** same strike on call and put **different** maturities

BUT THIS IS A ROBUST MODEL!!!!!

Most brilliant ideas often diffuse start:
Black-Scholes-Merton not first model for pricing derivatives based on arbitrage and risk-neutrality:

Forward price must be risk-neutral* , due to covered arbitrage

$$F = Se^{bT}$$

John Maynard Keynes., A Tract on Monetary Reform, 1923 (2000) (Prometheus Books: Amherst).

Blau, G. "*Some Aspects of the Theory of Futures Trading.*" The Review of Economic Studies XII (1944-45), 1-30.

With no delta hedging: Derman-Taleb argument:

$$c = e^{-RT}(E(S-X)_+) = e^{-RT}[S^{\mu T}N(d_1) - XN(d_2)]$$

Put-call parity long call + short put = forward, so options must also be priced with discount rate = r and drift in stock =r

$$c = e^{-rT}[S^{rT}N(d_1) - XN(d_2)] = SN(d_1) - Xe^{-rT}N(d_2)$$

Derman-Taleb Criticized October 2006

Not consistent with specific well known equilibrium model! ?

Not consistent with different discount rate for call and put ?

Extended Argument: If hedging options with options and synthetic delta-replication is far from fully available then demand and supply of options will have to affect option values!

$$c = SN(d_1) - Xe^{-rT}N(d_2)$$

$$d_1 = \frac{\ln(S/X) + (r + \sigma_i^2/2)T}{\sigma_i\sqrt{T}}$$

$$d_2 = d_1 - \sigma_i\sqrt{(T)}$$

But can we just do this, is this not a crank model? Normally yes, but not in this case!!! All we say vol must not break with PCP

Black-Scholes/Merton: Dynamic Hedging model

Brilliant mathematical idea

NOT ROBUST IN PRACTICE!

Static/semi-static hedging:

VERY ROBUST IN PRACTICE!

Robust for discrete time and price moves

Robust for stochastic volatility

Robust for jumps

What traders actually try to use!

“In the end, a theory is accepted not because it is confirmed by conventional empirical tests, but because researchers persuade one another that the theory is correct and relevant.”

Fischer Black

Summary

- Dynamic delta hedging brilliant mathematical idea!

- Dynamic hedging reduces risk considerably compared to no hedging or static delta hedging!

- Dynamic delta hedging do not do what it promises!
Fischer Black was himself skeptical to dynamic hedging as solution to risk-neutrality argument (?)

- Dynamic delta hedging in practice is not in any way good enough to use risk-neutral valuation!

-Dynamic delta hedging works very poorly for OTM options.

What to do

- Hedging options with options, supply demand driven.
-Truncating your tails.
-Delta hedging to remove risk, but not to relay on risk-neutral valuation.
- Only stay long options? No, can take in option premium.
- Remove risk in robust way and/or construct portfolio in such a way that can live with risk.

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