1. What is the limiting behaviour of
$$\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^{nt},$$
where r represents the annual risk-free rate, n is the number of periods per year and t is the number of years.

2. (a) Prove using the definition of the derivative that
$$\frac{d}{dx}\left(\frac{1}{x}\right)=-\frac{1}{x^2}.$$
(b) Consider the function
$$y=\frac{f(x)}{g(x)}.$$
Using the product rule and chain rule, derive the quotient rule given by
$$\frac{dy}{dx}=\frac{f'(x)\,g(x)-f(x)\,g'(x)}{g^2(x)}.$$
Hence use the quotient rule to differentiate the function
$$y=\frac{2^x}{\sin x}.$$
(c) Find the equation of the tangent to the curve $x=2y^3+\ln y$ at the point $(2,1)$, giving your solution in the form $px+qy+r=0$.

(d) Show that the Taylor series expansion of $f(x)=\sinh x$, can be written in the form
$$\sum_{n=0}^{\infty}a_n x^{2n}$$
where a_n should be given. Hence use this result to obtain the series expansion for $g(x)=\cosh x$

3. (a) Obtain a general solution of the following differential equation
$$\frac{dy}{dx}=e^{-y}(2x-4).$$
(b) Find the particular solution of the following initial value problem
$$\frac{dy}{dx}-y=e^{-x},\ y(0)=1$$
and show that this can be written as
$$2y=3e^x-e^{-x}.$$
(c) Solve the differential equation
$$xy''-y'=3x^2$$
giving the general solution in the form $y=g(x)$.

4. **This question is on applications of complex numbers, where $i=\sqrt{-1}$**

(a) Solve the differential equation
$$y''-5y'+6y=e^{-3x}$$
(b) Writing $\sin x$ as the imaginary part of (5), calculate
$$\int e^x\sin x\,dx$$

Question 5

The expression $\sinh(ix)$ is equal to

A. $-i\sin(ix)$ B. $\sin(x)$ C. $\sin(x/i)$ D. $\sin(ix)$ E. $i\sin(ix)$ F. none of these

Question 6

If $f(x) = \log\left(\frac{1}{2}\cosh\left(x^3\right)\right)$, then $f'(x)$ is equal to

A. $2\tanh\left(x^3\right)$ B. $2/\tanh\left(3x^2\right)$ C. $6x^2\tanh\left(x^3\right)$ D. $6x^2\tanh\left(x^3\right)$ E. $3x^2\tanh\left(x^3\right)$
F. none of these

Question 7

For $z = \exp(2 - i\pi/4)$, z^5 equals

A. $\exp\left(10 - i\frac{\pi}{4}\right)$ B. $\exp\left(5 + i\frac{5\pi}{4}\right)$ C. $e^{10} + i\frac{5\pi}{4}$ D. $e^{10}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$ E. $-e^{10}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$
F. none of these

Question 8

For $z = \exp(2 - i3\pi/4)$, z^5 equals

A. $\exp\left(10 - i\frac{\pi}{4}\right)$ B. $\exp\left(5 + i\frac{5\pi}{4}\right)$ C. $e^{10} + i\frac{5\pi}{4}$ D. $e^{10}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$ E. $-e^{10}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$
F. none of these

Question 9

The integral $\displaystyle\int_0^1 e^{\sqrt{x}}dx$ is

A. 0 B. 1 C. 2 D. e E. does not exist F. none of these

Hint: Consider a substitution $y^2 = x$ and use integration by parts.

Question 10

For $|x| < \dfrac{1}{2}$, the first two terms of the Taylor expansions for $f(x) = \ln\left(1 - 4x^2\right)$ about $x = 0$ are

A. $-4x^2 - 8x^4$ B. $2x + \frac{8}{3}x^3$ C. $-4x^2 - 16x^4$ D. $-4x^2 + 96x^4$ E. $2x^2 - 48x^4$
F. none of these

Question 11

Let $I = \displaystyle\int_3^4 \frac{3x-5}{(x-2)^2}dx$. Then I equals

A. $\ln 8 + \frac{1}{2}$ B. $\ln 8 - \frac{1}{2}$ C. $\ln 8 + 2$ D. $\ln 8 - 2$ E. $\ln 8 + \frac{3}{2}$
F. none of these

Question 12

Consider the function $f(x, y) = xe^{xy}$, where $x = t^2$ and $y = t^{-1}$. What is the value of $\dfrac{df}{dt}$ at $t = 1$?

A. 0 B. e C. $2e^2$ D. $3e$ E. 2 F. none of these

13. Solve the equation $x^2 + 4x + 20 = 0$, giving your answers in the form $c + di$ where $c, d \in \mathbb{Z}$.

14. Simplify the complex numbers by writing in the form $x + iy$

    i. $\dfrac{1}{i-2}$

    ii. $\dfrac{1-5i}{3+4i}$

15. If the complex number $z = \dfrac{2+3i}{1-i}$, then calculate $z/\overline{z}$.

16. What does the argument $\arg z$ of the complex number $z = 4\exp(-i\pi/6)$ equal?

17. Simplify $\cos(ix)$

18. Given $z = \exp(2 - i\pi/4)$, express z^5 in the form $x + iy$

19. What does the ratio

$$\frac{e^{i\sqrt{x}} - 1}{e^{i\sqrt{x}} + 1}$$

simplify to?

20. Recall the argument of z is given as the principal value, i.e. $\arg z \in [-\pi, \pi]$. What is $\arg z$ for $z = -2ie^{-i\pi}$?