CQF Exam One
Portfolio and Risk Techniques

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Question 1

- To solve for the weight in minimum variance portfolio with a target return m, we formulate

$$\underset{w}{min}\frac{1}{2}w'\Sigma w$$

subject to

$$\begin{aligned} w'\mu &= m \\ w'1 &= 1 \end{aligned} \tag{1}$$

The Lagrangian function is

$$L(w, \lambda, \gamma) = \frac{1}{2}w'\Sigma w + \lambda(m - w'\mu) + \gamma(1 - w'1) \tag{2}$$

Its partial derivatives are

$$\frac{\partial L}{\partial w}(w, \lambda, \gamma) = \Sigma w - \lambda\mu - \gamma 1 = 0 \tag{3}$$

- From (3), the optimal weight solution has

$$w^* = \Sigma^{-1}(\lambda\mu + \gamma 1) \tag{4}$$

Bring this into (1), we have

$$\begin{cases} \lambda = \frac{Am-B}{AC-B^2} \\ \gamma = \frac{C-Bm}{AC-B^2} \end{cases} \tag{5}$$

subject to

$$\begin{cases} A = 1'\Sigma^{-1}1 \\ B = 1'\Sigma^{-1}\mu \\ C = \mu'\Sigma^{-1}\mu \end{cases} \tag{6}$$

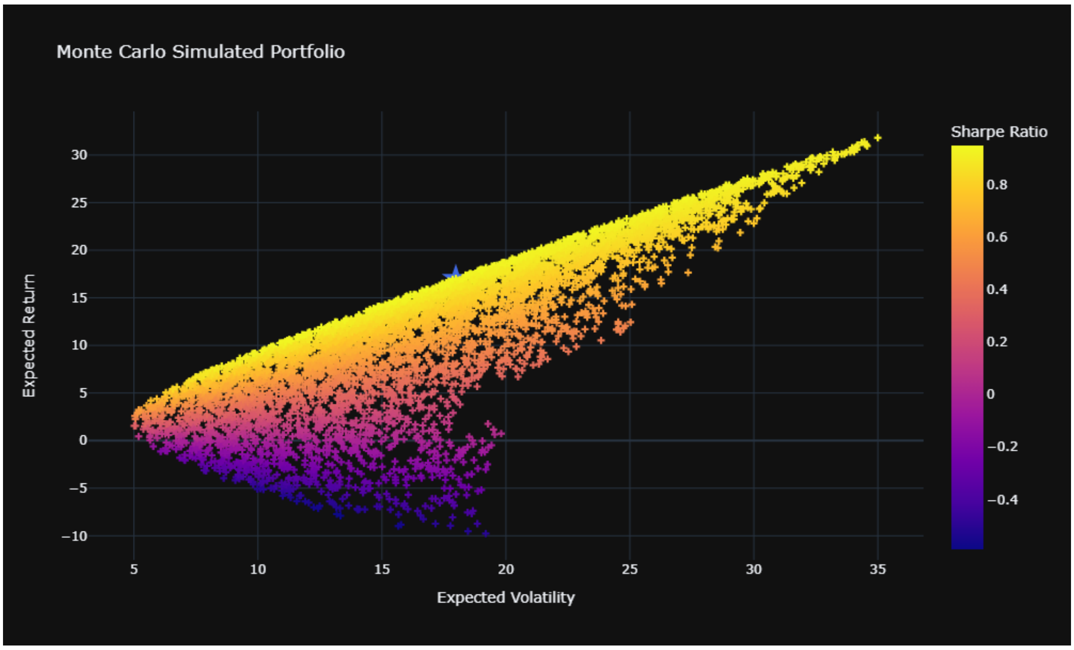
Now we obtain w^*:

$$w^* = \frac{1}{AC - B2}\Sigma^{-1}[(A\mu - B1)m + (C1 - B\mu)] \tag{7}$$

Finally, calculate the optimal weight for the minimum variance portfolio and the standard deviation (please see the attachment for relevant codes). The summary tables is

The allocation w^* and portfolio risk σ_π					
	σ_π	w^*			
x1	0.05840091	0.78511066	0.05386419	0.13355472	0.02747042
x1.25	0.0607102	0.81818944	-0.00940302	0.17896585	0.01224773
x1.5	0.06109091	0.87617647	-0.14612952	0.32570145	-0.0557484

- Generate 5,000 random allocations sets and satisfy $w'1 = 1$



Question 2

- To solve for risk minimization with N risky assets and a risk- free asset, we formulate

$$\min_w \frac{1}{2} w' \Sigma w$$

subject to

$$r + w'(\mu - r1) = m \tag{8}$$

The Lagrangian function is

$$L(w, \lambda) = \frac{1}{2} w' \Sigma w + \lambda(m - r - w'(\mu - r1)) \tag{9}$$

Its partial derivatives are

$$\frac{\partial L}{\partial w}(w, \lambda) = \Sigma w - \lambda(\mu - r1) = 0 \tag{10}$$

- From (10), the optimal weight solution has

$$w^* = \Sigma^{-1}(\mu - r1) \tag{11}$$

Substituting the value of w^* into the constraint (8), we solve for λ:

$$\lambda = \frac{m - r}{(\mu - r1)'\Sigma^{-1}(\mu - r1)} \tag{12}$$

Finally, we get w^*:

$$w^* = \frac{(m - r)\Sigma^{-1}(\mu - r1)}{(\mu - r1)'\Sigma^{-1}(\mu - r1)} \tag{13}$$

Because the tangency portfolio is fully invested in risky assets, then its asset allocation must satisfy the budget equation:

$$1'w^* = 1 \tag{14}$$

We get:

$$w = \frac{\lambda\Sigma^{-1}(\mu - r1)}{B - Ar} \tag{15}$$

and

$$\begin{aligned} m &= w'\mu \\ \sigma &= \sqrt{w'\Sigma w} \end{aligned} \tag{16}$$

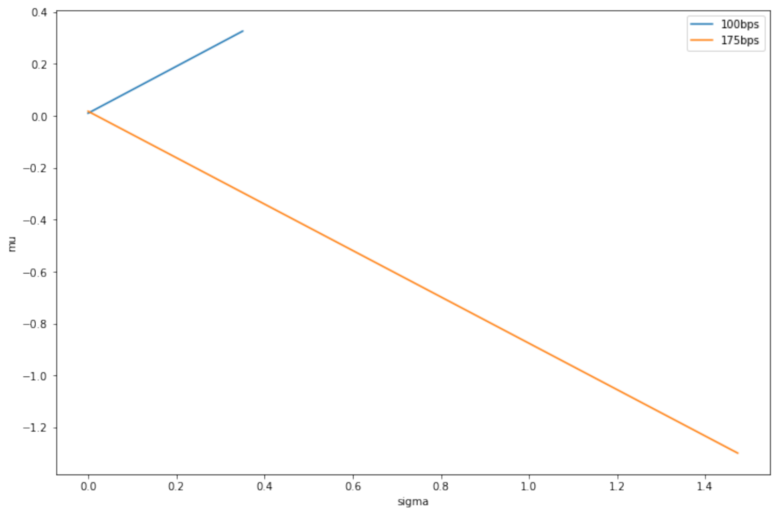
subject to

$$\begin{cases} A = 1'\Sigma^{-1}1 \\ B = 1'\Sigma^{-1}\mu \end{cases} \tag{17}$$

The summary tables is (please see the attachment for relevant codes)

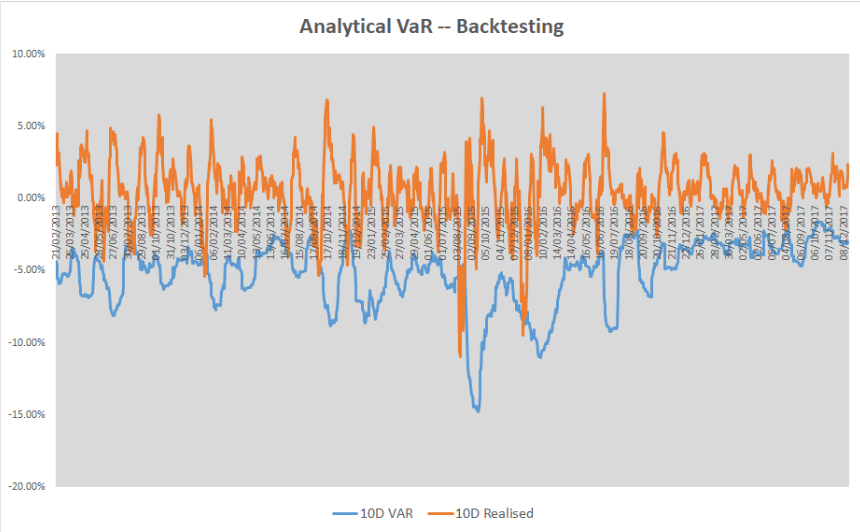
bps	mean	sigma	weights
50	0.18607	0.196511	[0.0168352 -0.22936698 0.81434026 0.39819152]
100	0.32613	0.350665	[-0.74593711 -0.51056937 1.49024934 0.76625714]
150	1.77653	1.97239	[-8.64485405 -3.42257114 8.48965087 4.57777433]
175	-1.2988	1.47351	[8.10350247 2.75185052 -6.3514309 -3.50392209]

- Plot the true Eficient Frontier by $(0, rf_{100bps})$ $(\sigma_{100bps}, \mu_{100bps})$, $(0, rf_{175bps})$ $(\sigma_{175bps}, \mu_{175bps})$

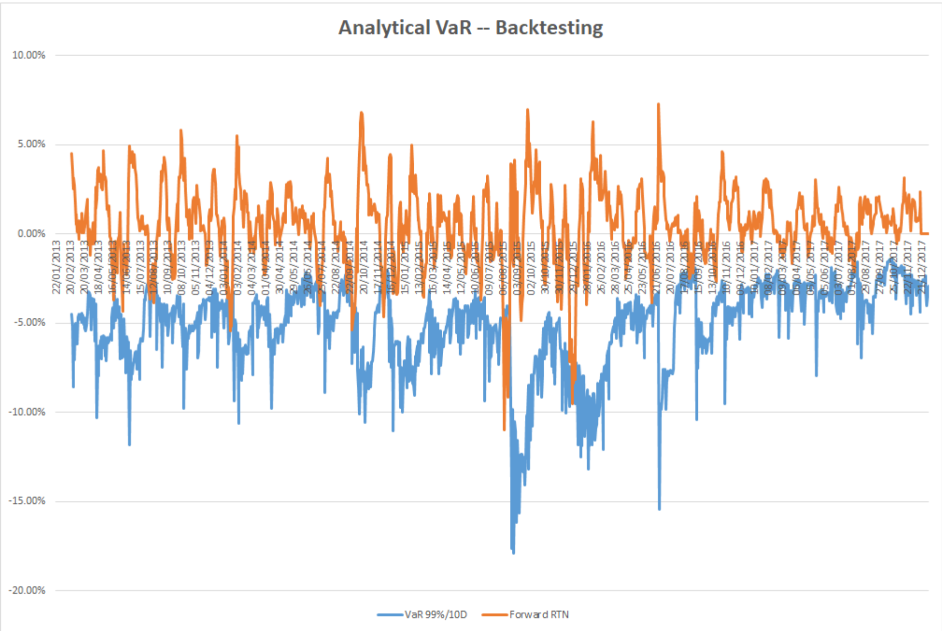


Question 3

- a) the percentage of VaR breaches is 2.05%
  b) the number of consecutive breaches is 14
  c) provide a plot which clearly identifies breaches



- a) the percentage of VaR breaches is 2.47%
  b) the number of consecutive breaches is 15
  c) provide a plot which clearly identifies breaches



*The calculation process and results are shown in the attached excel file.

Question 4

- The summary tables is

LVaR(million $)	VaR(million $)	VaR proportion	Liquidity	Liquidity proportion
1.2636	0.9565	0.7569	0.3071	0.2431

- The summary tables is

bid-ask spread	LVaR(million $)	VaR(million $)	VaR proportion	Liquidity	Liquidity proportion
15 bps	2.8212	2.7912	0.9894	0.0300	0.0106
125 bps	3.0412	2.7912	0.9178	0.2500	0.0822

If the bid-ask spread increases to 125 bps, the VaR remains unchanged, and the Liquidity increases greatly from 0.0300 to 0.2500, which increases the LVaR from 2.8212 to 3.0412.

*The calculation process and results are shown in the attached juypter notebook file.