Neural Networks – Part II PHYS 250 (Autumn 2018) – Lecture 16

David Miller

Department of Physics and the Enrico Fermi Institute University of Chicago

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Outline

- Reminders
 - Reminders from Lecture 15
- 2 Historical perspective
 - Brief History of Machine Learning Generally
 - Brief History of Neural Networks
- Structure of Neural Networks
 - Single layer perceptron
 - Training a single layer perceptror
 - Training a Multi-Layer Perceptron (MLP)

Reminders from last time

We embarked on a whirlwind introduction to neural networks.

Neural networks and machine learning

- Context and perspective
 - We discussed the general issue of training computers to discover, identify, and analyze patterns of interest in datasets
 - Categorized tasks that make use of this idea: classification, regression, generation, clustering, anomaly detection
- Neural networks as a tool
 - Introduced both the **modeling** perspective as well as the **biological** perspective on what a neural network achieves
 - Described the **structure and function** of a neuron
 - Began discussing the mathematical properties of a neural network

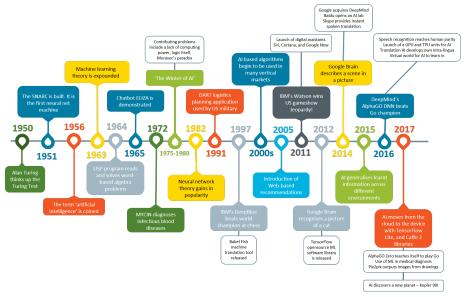
Today we will build our own networks! But first, I just wanted to follow-up on some points and questions from last time.

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Brief history of machine learning

Taken from Harry Ide on InnovationLaboratory.com (18 May 2018):



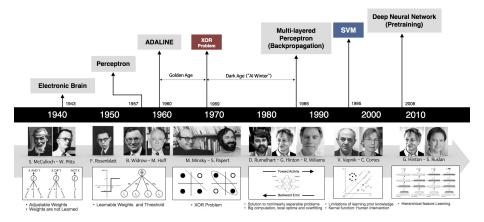
Brief history of machine learning

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Brief history of neural networks

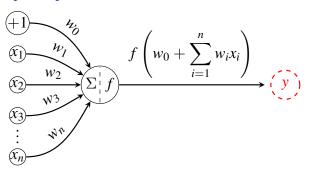
Taken from this talk on SlideShare:



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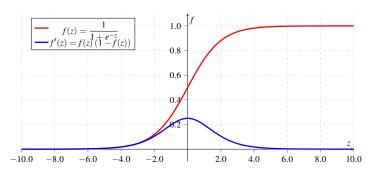
Single layer perceptron



- $\vec{x} = (x_1, x_2, \dots, x_n)$ is an input feature vector of length n i.e. the attributes of the data, e.g. voltages
- $\vec{w} = (w_1, w_2, \dots, w_n)$ is the weight vector with w_0 reserved as a bias becomes a matrix for multiple layers
- Σ indicates summation (or matrix mult.): $z = \sum w_i x_i$ ($x_0 = 1$)
- \bullet f is the activation function, or non-linearity: f(z)
- y = f(z) is the output

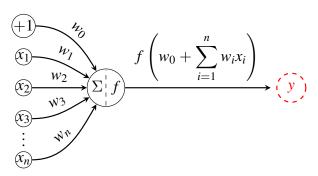
Sigmoid as activation function

As we discussed, a typical function for a **single layer perceptron** is the **sigmoid**.



Here, we plot both the function itself, as well as its derivative, since that will be important when evaluating the **backpropagation** of weights in order to update the neural network.

Training a single layer perceptron

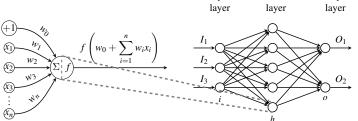


Given j objects \vec{x}_j in dataset, each with **known values of** f, d_j

- Calculate the output: $y_i = f(\vec{w} \cdot \vec{x}_i)$
- Determine the error: $\epsilon_j = d_j y_j$
- Update the weights: $w_i^{\text{new}} = w_i + r(\epsilon_j \cdot \vec{x}_j)_i$

Choosing the learning rate r is where the derivative is used. It's not important for the single-layer perceptron, but is **essential** for a network.

Multi-layer perceptron (MLP)



Input

Hidden

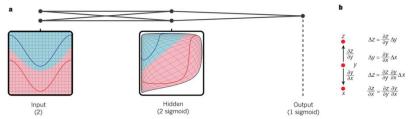
Output

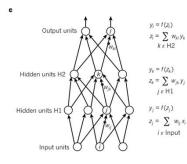
Given j objects \vec{l}_j in dataset, each with features $\vec{l}=(I_1,I_2,\cdots,I_n)$ and known outputs \vec{d}_j at each output node o, $\vec{d}=(d_1,d_2,\cdots,d_o)$

- Calculate the h outputs of hidden layer: $v_h = f(\sum_i w_{ih}I_i)$
- Calculate the o outputs of output layer: $y_o = f(\sum_h w_{ho}v_h)$
- Determine the error at output each node o: $\epsilon_o = d_o y_o$
- Determine the total error for data object j: $\mathcal{E}_j = \frac{1}{2} \sum_o \epsilon_o^2$
- Determine change in weights for output neuron y_o : $\Delta w_{oh} = -\eta \frac{\partial \mathcal{E}}{\partial z_o} v_h = \eta \epsilon_o f'(z_o)$

LeCun, Bengio, Hinton, "Deep learning"

Nature volume 521, pages 436-444 (28 May 2015)





 $\frac{\partial E}{\partial y_k} = \sum_{z \text{ out}} \frac{w_{kl}}{\partial z_l} \frac{\partial E}{\partial z_l}$ $\frac{\partial E}{\partial z_k} = \frac{\partial E}{\partial y_k} \frac{\partial y_k}{\partial z_k}$

