1.3 Karnaugh Maps

1 Simplifying Logic Functions of Different Numbers of Variables

- provides a convenient way to simplify functions of 3 ~ 4 variables
- 5-variable function : 2 × 4-variable K-maps can be used to simplify the function.
- > 5 variables : map-entered variable (MEV)

2 Procedure to Obtain a min. SoP from a K-Map

1. Choose a minterm (a 1) that hasn't yet been covered.
2. Find all 1's and X's adjacent to that minterm.

   ⇒ Prime Implicant (PI)

3. If a single term covers the minterm and all the adjacent 1's and X's, then that term is an essential prime implicant (EPI) ⇒ select this term

4. Repeat 1~3 until all EPIs have been chosen

5. Find a minimum set of PIs that cover the remaining 1's on the map.

3 K-Map + Map-Entered Variables

- For functions of ≥ 5 variables. when to use?
- map-entered variable (MEV): A variable P_i is placed in square m_j of a map of a function F.

⇒ $F = 1$ when $P_i = 1$ and the variables are chosen so that $m_j = 1$

Example : $G(A, B, C, D, E, F)$

$= m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9$

$+ m_{11} + m_{15} + d_1 + d_{10} + d_{13}$

				IMEVs		output
A	B	C	D	E	F	G
0	0	0	0	X	X	1
0	0	0	1	X	X	X
0	0	1	0	X	X	1
0	0	1	1	X	X	1
0	1	0	1	1	X	1
0	1	1	1	1	X	1
1	0	0	1	X	1	1
1	0	1	0	X	X	X
1	0	1	1	X	X	1
1	1	0	1	X	X	X
1	1	1	1	X	X	1

G = 0 when (A, B, C, D) =

(1,1,1,1) >

(0,1,1,0) >

(1,0,0,0) >

(1,1,0,0) >

(1,1,1,0) >

partial truth table (input combinations not specified result in an output of 0.)

- General method of simplifying a K-map w/ MEVs P_is:
  - Given a map w/ variables $P_1, P_2, \ldots$ entered into some of the squares, the (minimum) sum-of-products form of F:

$$F = MS_0 + P_1 MS_1 + P_2 MS_2$$

MS_0: the minimum sum obtained by setting $P_i = 0$ $(i = 1, 2, \ldots)$ setting all MEVs to 0

MS_i: the minimum sum obtained by setting $P_i = 1$, $P_j = 0$, and replacing all 1's on the map w/ don't care's $(j = 1, 2, \ldots, i \neq j)$

Note: F is always correct, and F is minimum SoP if the MEVs are independent

CD \ AB	00	01	11	10
00	1			
01	X	E	X	F
11	1	E	1	1
10	1			X

$\Downarrow$

$P_1 = E = 1$,
other $= 0$

① MS_0 ($E = F = 0$)

CD \ AB	00	01	11	10
00	1			
01	X		X	
11	1		1	1
10	1			X

$MS_0 = A'B' + ACD$

② MS_1 ($E = 1$, $F = 0$, 1's $\rightarrow$ X's)

CD \ AB	00	01	11	10
00	X			
01	X	1	X	
11	X	1	X	X
10	X			X

$MS_1 = A'D$

③ MS_2 ($E = 0,\ F = 1$; $1's \to X's$)

CD \ AB	00	01	11	10
00	X			
01	X		X	1
11	X		X	X
10	X			X

$MS_2 = AD$

$\Downarrow$

$$G = MS_0 + P_1 MS_1 + P_2 MS_2$$

$$= A'B' + ACD$$

$$+ EA'D$$

$$+ FAD$$

✡ Exercise of 1-3

For the following function, find the min. SoP using 4-variable maps with MEVs.

(m_i represents the minterm of A, B, C, D)

$$Z(A, B, C, D, E, F, G)$$
$$= \sum m(2, 5, 6, 9)$$
$$+ \sum d(1, 3, 4, 13, 14)$$
$$+ E(m_{11} + m_{12})$$
$$+ F(m_{10})$$
$$+ G(m_0)$$