$\boxed{0}$ Intro & Electric Fields

- $q = \pm Ne$ (quantization of electric charges, elementary charge $e = 1.6 \times 10^{-19}$ C) permittivity of free space $= 8.854 \times 10^{-12}$ (F/m)

- $|\vec{F}| = \dfrac{k|q_1||q_2|}{r^2}$ (Coulomb's Law, $k\varepsilon_0 = \dfrac{1}{4\pi}$)

  unit = N/C = V/m

- $\vec{E} = \dfrac{\vec{F_E}}{q_0} = k_e \dfrac{q}{r^2}\hat{r}$ $\underline{\underline{\text{continuous charge}}}$ distribution $k_e \displaystyle\int \frac{dq}{r^2}\hat{r}$

$\boxed{1}$ Gauss's Law

- charge densities :

rho ρ (volume) $= Q/V$ $\xrightarrow[\text{distributed charge}]{\text{non-uniformly}}$ $dq = \rho\, dV$

sigma σ (surface) $= Q/A$ $dq = \sigma\, dA$

λ (linear) $= Q/l$ $dq = \lambda\, dl$

- Electric Flux

  * $\Phi_E = EA_\perp = \vec{E} \cdot \vec{A}$

  * general case:

  $$\Phi_E \approx \sum \vec{E}_i \cdot \Delta \vec{A}_i = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

  * through any close surface

  $$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \int E_n \, dA$$

- Gauss's Law : $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \dfrac{q_{in}}{\varepsilon_0}$

$\boxed{2}$ Electric Potential

- work done within a charge-field system by the electric field on the charge $W = \vec{F} \cdot d\vec{s} = q \vec{E} \cdot d\vec{s}$

- electric potential energy : $\Delta U = -W = -q \int_A^B \vec{E} \cdot d\vec{s}$

  ↳ electrostatic force F_E is conservative

$V = \dfrac{kq}{r}$

- electric potential difference $\Delta V = \dfrac{\Delta U}{q} = -\int_A^B \vec{E} \cdot d\vec{s}$

  (potential energy per unit charge)

  $\Rightarrow \quad W = -\Delta U = -q \Delta V$ ☆

$U = \dfrac{kq_1 q_2}{r}$

$V_\infty = 0$

- electron-volt (energy unit)

  $1\,eV = 1.6 \times 10^{-19}\, C \cdot V = 1.6 \times 10^{-19}\, J$

- $V \rightarrow E$

  $$E_x = -\frac{dV}{dx},\quad E_y = -\frac{dV}{dy},\quad E_z = -\frac{dV}{dz} \;\Big/\; E_r = -\frac{dV}{dr}$$

- continuous charge distribution

  sum → integral

  * if charge distribution is known:

    $$dV = k_e \frac{dq}{r}$$

    $$V = k_e \int \frac{dq}{r}$$

  * if electric field is known:

    $$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

- conductor in electrostatic equilibrium:

V is the same anywhere inside the conductor (including the surface)

1. $E_{inside} = 0$ $\Rightarrow$ $\phi_E = 0$ $\Rightarrow$ $q_{inside} = 0$
2. Charge resides on its surface
3. The electric field at a point just outside a charged conductor is perpendicular to the surface and $|\vec{E}| \approx \sigma / \varepsilon_0$
4. For irregularly-shaped conductor, σ is greatest where radius is of the smallest curvature

$\boxed{3}$ Capacitance & Dielectrics

- $C = \dfrac{Q}{\Delta V}$ (2 conductors: charges of equal magnitude but opposite directions)

$$= \frac{\varepsilon_0 A}{d}$$ (parellel plate capacitor)

- equivalent capacitance

① in series : $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

② in parellel : $C_{eq} = C_1 + C_2$

- Energy stored in a capacitor :

$$U_E = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{CV^2}{2}$$

* for a parellel-plate capacitor : $\frac{CV^2}{2}$, $C = \frac{\varepsilon_0 A}{d}$, $V = Ed$

$$U_E = \frac{\varepsilon_0 A d E^2}{2}$$, energy density $u_e = \frac{\varepsilon_0 E^2}{2} = \frac{U_E}{Ad}$

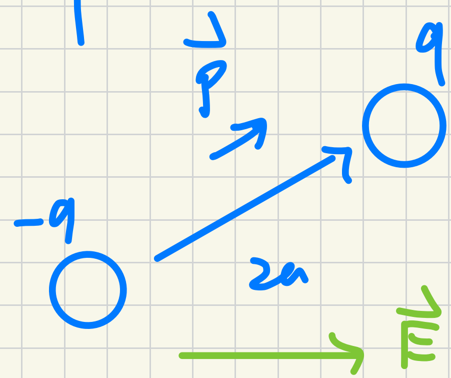
- capacitors with dielectrics:

$C = K C_0$ (K: dielectric constant)

parrellel-plate capacitor: $\dfrac{K \varepsilon_0 A}{d}$

- Electric Dipole in Electric Field

  * electric dipole moment $\vec{p}$ $\begin{cases} \text{magnitude: } 2aq \\ \text{direction: } -q \rightarrow +q \end{cases}$
  * net force $F = 0$
  * net torque $\vec{\tau} = \vec{p} \times \vec{E}$
  * potential energy $U = -\vec{p} \cdot \vec{E}$



- $E = E_0 - E_{ind}$, $\vec{E} = \dfrac{\vec{E}_0}{k}$

$\vec{E}_{ind} = \dfrac{k-1}{k} \vec{E}_0$, $\sigma_{ind} = \dfrac{k-1}{k} \sigma$ (Dielectrics)

$\boxed{4}$ Current & Resistance

- $I_{ave} = \frac{\Delta Q}{\Delta t} = \frac{(nA\Delta x)q}{\Delta t} = nqA\,v_d$ drift velocity

- $J = \frac{I}{A} = nq\,v_d \;\underset{\text{materials}}{\overset{\text{ohmic}}{=\!=}}\; \sigma E = \frac{E}{\rho}$

  current density, conductivity, resistivity

- $\Delta V = E\ell = \frac{\ell J}{\sigma} = \left(\frac{\ell}{\sigma A}\right) I = RI$

  $R = \frac{V}{I} = \rho \frac{d}{A}$

- Drude Model for Conduction

* $\vec{V}_f = \vec{V}_i + \underbrace{\frac{q\vec{E}}{m}}_{}\, t$ ($\vec{a}$)

* $\vec{V}_{f,avg} = \vec{V}_d = \frac{q\vec{E}}{m_e}\tau$

τ: average time interval b/w successive collisions

* $I_{avg} = nq\left(\frac{qE}{m_e}\tau\right)A = \frac{nq^2E}{m_e}\tau A$ (v_d)

* $J = nqv_d = \frac{nq^2E}{m_e}\tau$

* resistivity $\rho = \frac{m_e}{nq^2\tau}$

Drude model fails to predict the temperature dependence of resistivity

$J = \frac{E}{\rho}$

Note : $\tau = l_{avg} / V_{avg}$

temperature coefficient of resistivity (α)

- resistivity & temperature : $\rho = \rho_0 \left[1 + \alpha (T - T_0)\right]$

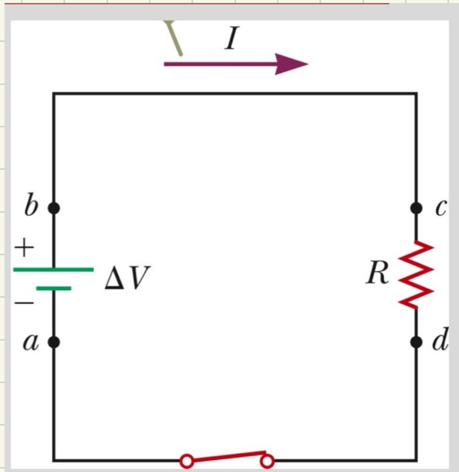
- rate of energy delivered to a resistor

(can be used generally)

$$\frac{dU_E}{dt} = \frac{d}{dt}(Q\,\Delta V) = \frac{dQ}{dt}\,\Delta V = I\,\Delta V$$

$$P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

(can be used on a resistor)



Note: transmit electricity by power companies usually have High voltages and low currents to minimize power losses

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$\boxed{5}$ DC Circuits

- Kirchhoff's Rules

1. junction rule : $\sum_{\text{junction}} I = 0$

2. loop rule : $\sum_{\text{loop}} \Delta V = 0$

- Combination of Resistors

1. in series: $R_{eq} = R_1 + R_2$

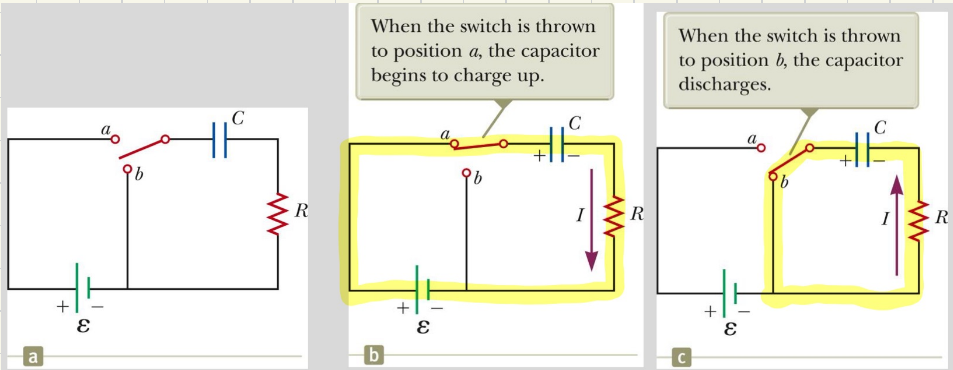
2. in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

- RC Circuits

(1) set up equations:

$*$ charging: $\varepsilon = V_c(t) + IR = V_c(t) + C\frac{dV_c(t)}{dt}R$

$*$ discharging: $0 = V_c(t) - IR = V_c(t) - C\frac{dV_c(t)}{dt}R$



(4) result

$RC = \tau$ (time constant)

* charging :

$$q(t) = C\varepsilon\left(1 - e^{-t/RC}\right) = Q_{max}\left(1 - e^{-t/RC}\right)$$

$$V_c(t) = \varepsilon\left(1 - e^{-t/RC}\right)$$

$$i(t) = \frac{\varepsilon}{R} e^{-t/RC}$$

* discharging :

$$q(t) = C\varepsilon\, e^{-t/RC} = C\varepsilon\, e^{-t/RC}$$

$$V_c(t) = \varepsilon\, e^{-t/RC}$$

$$i(t) = -\frac{Q_i}{RC} e^{-t/RC}$$

6 Magnetic Fields

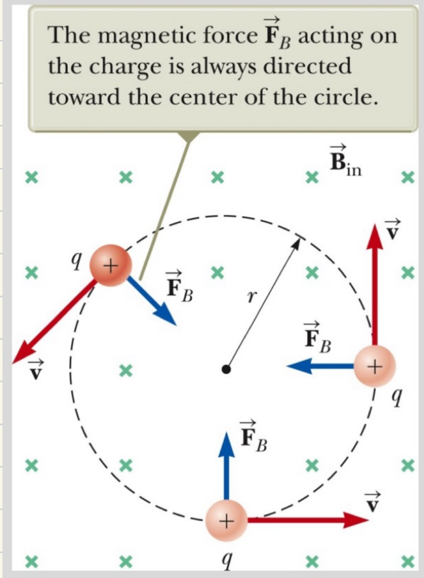
- magnetic flux $\phi_B = \vec{B} \cdot \vec{A}$

- magnetic force $\vec{F}_B = q\,\vec{v} \times \vec{B}$

- $F_B = q v B = \frac{r v^2}{m} = F_c$

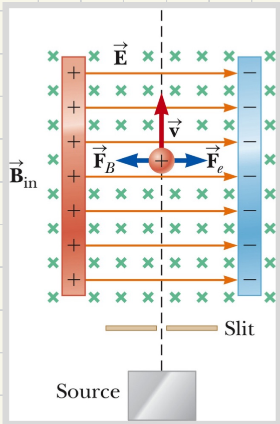
  $\Rightarrow r = \frac{mv}{qB}$ radius of curvature

- angular velocity $\omega = \frac{v}{r} = \frac{qB}{m}$



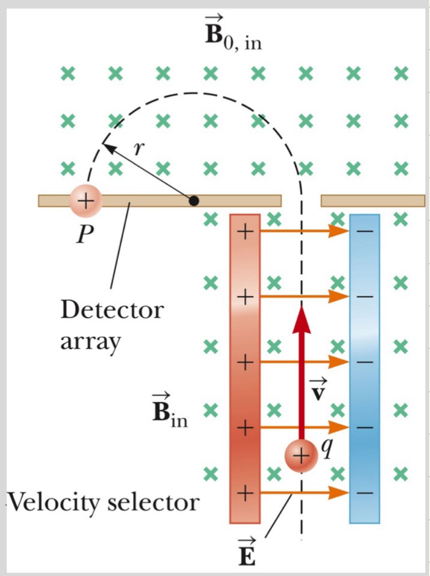
- Lorentz Force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

- velocity selector: $qE = qvB \Rightarrow v = \frac{E}{B}$



- mass spectrometer :

$$r = \frac{mv}{qB_0} = \frac{mE}{qBB_0}$$



- Total Force of a Current-carrying Wire

$$F_B = (q\vec{v} \times \vec{B})\, nAL$$

$$= I\,\vec{L} \times \vec{B}$$

$$d\vec{F}_B = I\, d\vec{s} \times \vec{B}$$

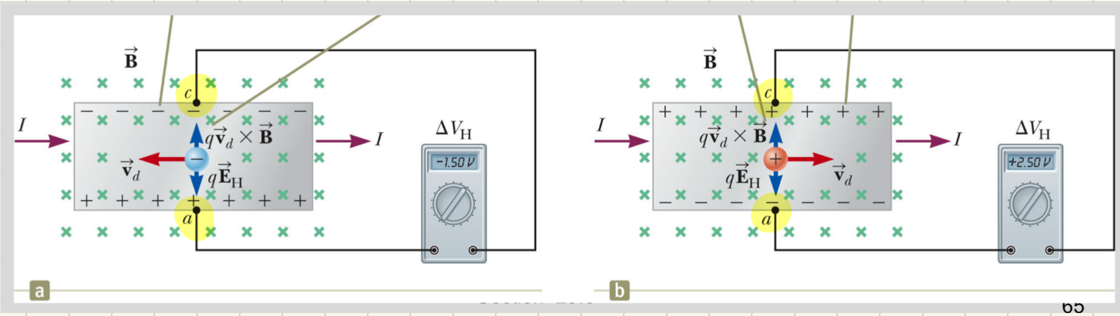
$$\vec{F}_B = I \int_A^B d\vec{s} \times \vec{B}$$

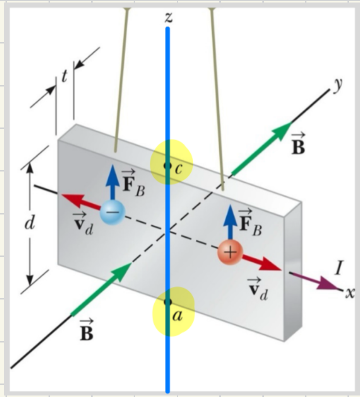
- Magnetic (Dipole) Moment: $\vec{\mu} = I\vec{A}$
- Torque on a Magnetic Dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$
- Magnetic Potential Energy: $U_B = -\vec{\mu} \cdot \vec{B}$

- Hall Effect

$$F_E = qE = qvB = F_B$$

$$\rightarrow \Delta V_H = E_H d = (v_d B)\, d = \frac{IB}{nqt}$$





$\boxed{7}$ Source of Magnetic Fields

- Biot - Savart Law

permenbility of free space

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

- Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

magnetic flux

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

unit $= T \cdot m^2$

$= \text{Weber}$

- Gauss's Law in Magnetism: $\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$

- $I = \frac{e}{T} = \frac{ev}{2\pi r}$

magnetic moment $\mu = \left(\frac{e}{2m_e} \right) L$

quantization of angular momentum

$\Rightarrow$?

spin $S = \frac{\sqrt{3}}{2} \hbar$ ($\hbar$: Planck's constant)

$\mu_{spin} = \frac{e\hbar}{2m_e} = \mu_B$ (Bohr magneton)

8 Faraday's Law

- changing E $\underset{\text{Faraday's Law}}{\overset{\text{Biot-Savart Law}}{\rightleftarrows}}$ changing B

moving charge / current

emf

* leads to Maxwell's Equation

- Faraday's Law of Induction → used in a closed loop setting

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Lenz's law

: General Form $E = -\frac{r}{2}\frac{dB}{dt}$

$$\varepsilon = \oint E \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

- magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$

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9 Inductance

	i.V	P	W	AC	DC	series	parellel
C	$i = C\frac{dV}{dt}$	$CV\frac{dV}{dt}$	$\frac{CV^2}{2}$	i	open circuit	$\frac{C_1C_2}{C_1+C_2}$	C_1+C_2
L	$V = L\frac{di}{dt}$	$Li\frac{di}{dt}$	$\frac{Li^2}{2}$	V	short circuit	L_1+L_2	$\frac{L_1L_2}{L_1+L_2}$

- $L = \frac{\Phi_B}{i} \Rightarrow \varepsilon_L = \frac{-d\Phi_B}{dt} = -L\frac{di}{dt}$

unit: Henries (H)

- RL Circuit :

(1) charging : $i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$

(2) discharging : $i = \frac{\varepsilon}{R} e^{-\frac{t}{\tau}}$

time constant $\tau = \frac{L}{R}$

Comparison :

$U_c = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{CV^2}{2}$

- Energy in a Magnetic Field, $U_B = \frac{1}{2} L i^2$

- Mutual Inductance $M = \frac{N_2 \phi_{12}}{i_1} = \frac{N_1 \phi_{21}}{i_2}$

- $\varepsilon_1 = -M \frac{di_2}{dt}$, $\varepsilon_2 = -M \frac{di_1}{dt}$

- Time Functions of a LC Circuit

  - $q = Q_{max} \cos(\omega t + \phi)$ charge on the capacitor

  - $i = \frac{dq}{dt} = - \underbrace{\omega Q_{max}}_{I_{max}} \sin(\omega t + \phi)$

  - $\underbrace{U}_{\text{total energy}} = U_E + U_B = \frac{Q_{max}^2}{2C} \cos^2 \omega t + \frac{L I_{max}^2}{2} \sin^2 \omega t$

  - $\frac{q}{C} = -L \frac{di}{dt} = -L \frac{d^2 q}{dt^2}$ (KVL)

$\Rightarrow \frac{d^2 q}{dt^2} = -\frac{1}{LC} q = -\omega^2 q \Rightarrow \underbrace{\omega}_{\text{angular frequency}} = \frac{1}{\sqrt{LC}}$

- Damped RLC Circuit
  - $q = Q_{max}\, e^{-\frac{Rt}{2L}} \cos \omega_d t$

    $\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

  - oscillation: damped | critically damped | overdamped

    critical resistance $R_c = \sqrt{\frac{4L}{C}}$

$\boxed{10}$ AC Circuit

- AC Circuit = time-varying signal represented as sum of sinusoids
  - periodic function : $f(t) = \sum_{n=0}^{\infty} A_n \sin(\omega t + \phi)$
  - any function : $f(t) = \int_0^{\infty} A(\omega) \sin(\omega t + \phi)$
- sinusoids $v(t) = V_{max} \sin(kx + \omega t + \phi)$

angular frequency
$\omega = 2\pi f = 2\pi / T$

- Phasor Diagram
  - sinusoids

time domain $V_R(t) = V_{max} \sin(\omega t + \phi)$

$\downarrow$ simplify

phase space

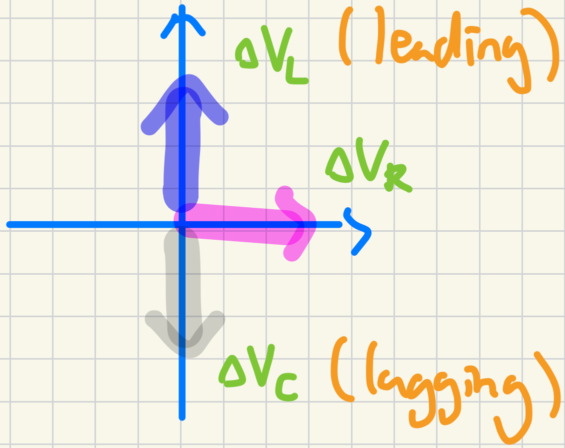
phasors

frequency domain

$$V_R = V_{max} \angle \phi = V_{max} e^{j\phi}$$
$$= V_{max} \cos\phi + j V_{max} \sin\phi$$

- AC Power Source : $\Delta V = \Delta V_{max} \sin \omega t$ → here, voltage is the reference
  - resistor : $i = \dfrac{\Delta V_{max}}{R} \sin \omega t$
  - inductor : $i = \dfrac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \dfrac{\pi}{2}\right)$
    ↳ ωL → X_L (inductive reactance)
  - capacitor : $i = \dfrac{\Delta V_{max}}{\frac{1}{\omega C}} \sin\left(\omega t + \dfrac{\pi}{2}\right)$
    ↳ $\frac{1}{\omega C}$ → X_C (capacitive reactance)

- RLC Circuit
  - $V_L = i\, \omega L \sin\left(\omega t + \dfrac{\pi}{2}\right)$
  - $V_C = i\, \dfrac{1}{\omega C} \sin\left(\omega t - \dfrac{\pi}{2}\right)$



- $V = IZ$

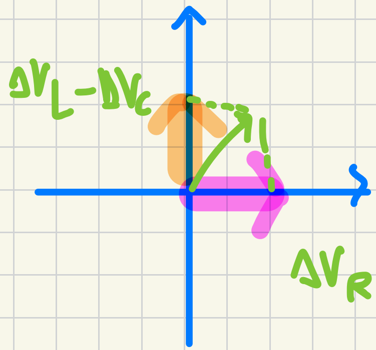
$X_L = X_C \Rightarrow$ pure resistive

- Impedance $\quad Z = R + j(X_L - X_C) \quad (\Omega)$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= A \angle \phi$$

$$A = \sqrt{R^2 + (X_L - X_C)^2} \qquad \phi = \frac{X_L - X_C}{R}$$



- $I = \dfrac{V}{Z} = \dfrac{V_{max} \angle 0}{A \angle \phi} = \dfrac{V_{max}}{A} \angle(-\phi)$

$$i(t) = \frac{V_{max}}{A(\omega)} \sin(\omega t - \varphi(\omega))$$

- reactance, $X_L \uparrow = \omega \uparrow L$, $\quad X_C \downarrow = \dfrac{1}{\omega \uparrow C}$ $\quad (\omega \propto f)$

- Power in an AC Circuit
  - $I_{rms} = \frac{I_{max}}{\sqrt{2}}$, $V_{rms} = \frac{V_{max}}{\sqrt{2}}$

    power factor of the circuit

  - $P_{avg} = \frac{1}{2} I_{max} V_{max} \cos\phi = I_{rms} V_{rms} \cos\phi$

- For an RLC Circuit:
  - resonance frequency when $X_L = X_C$

    $\omega L = \frac{1}{\omega C} \Rightarrow \omega_{res} = \frac{1}{\sqrt{LC}}$

  - $I_{rms} = \frac{\Delta V_{rms}}{Z} = \frac{\Delta V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{rms}}{R} \Rightarrow Z\ \text{min},\ I_{max}$

    $\Rightarrow P\ \text{max}.$

- Quality Factor (Q Factor) $Q = \dfrac{\omega_0}{\Delta\omega} = \dfrac{\omega_0 L}{R}$

$$\Delta\omega = \frac{R}{L} \quad \text{(width of the curve at half-power pts.)}$$

- Transformer

- $$\frac{\Delta V_2}{\Delta V_1} \underset{\Delta V = -N\frac{d\phi_B}{dt}}{=} \frac{N_2}{N_1} \underset{P = IV}{\overset{\text{assume no power loss}}{=}} \frac{I_1}{I_2}$$

- $$R_{eq} = \frac{\Delta V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 \frac{\Delta V_2}{I_2}$$

$\boxed{\text{III}}$ EM Waves

Maxwell's Equations

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} \quad \text{(Gauss's Law)}$$

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \quad \text{(Gauss's Law in Magnetism)}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's Law)}$$

→ displacement current I_d

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{(Ampere-Maxwell Law)}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \quad \text{(Lorentz Force Law)}$$

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \\ \frac{\partial^2 B}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \end{aligned}\right\rangle \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \Rightarrow v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

The simplest solution to partial differential equations:

$$\begin{cases} E = E_{max} \cos(kx - \omega t) \\ B = B_{max} \cos(kx - \omega t) \end{cases}$$

- angular wave number $k = 2\pi/\lambda$
- angular frequency $\omega = 2\pi f$

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

- $$\frac{E_{max}}{B_{max}} = \frac{E}{B} = \frac{\omega}{k} = c$$

- Poynting Vector $\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu_0} \quad \left(\dfrac{W}{m^2} \right)$ → in the direction of propagation

  ↳ time-varying quantity

- Intensity is the time average of S over one or more cycles

$$I = S_{avg} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{c B_{max}^2}{2} \quad \left(\frac{W}{m^2} \right)$$

- Energy density

$$u_E = \frac{\varepsilon_0 E^2}{2}, \quad u_B = \frac{B^2}{2\mu_0}, \quad u = u_E + u_B = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$u_{avg} = \frac{\varepsilon_0 E^2}{2} = \frac{B^2}{2\mu_0}, \quad I = S_{avg} = c\,u_{avg}$$

- EM waves transport **momentum** $p = \dfrac{T_{ER}}{c}$ → total energy to a surface in Δt

  * ER: electromagnetic radiation

→ radiation / light pressure

- Pressure is exerted on a surface as momentum is absorbed

$$P_{pres} = \frac{F}{A} = (1+f)\frac{S}{c}$$

S → poynting vector

f → % (incident light reflected from the surface)

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12 The Nature of Light and the Laws of Geometric Optics

$$\boxed{\text{Refraction} \quad \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}}$$

$v = \cancel{f}\lambda$

Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Index of refraction (折射率)

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}$$

$n_{vacuum} = 1$

$\approx n_{air}$

$\boxed{13}$ Other Formulas

- long, straight conducting wire

  - $B = \frac{\mu_0 I}{2\pi r}$

- solenoid
  - $B = \mu_0 n I$
  - $L = \mu_0 n^2 V$
  - Energy $= \dfrac{B^2}{2\mu_0} V$, Energy Density $= \dfrac{B^2}{2\mu_0}$

- $\mu_0 = 4\pi \times 10^{-7}$ $(T \cdot m / A)$
- $F_B = i l B \sin\theta$
- E unit : V/m