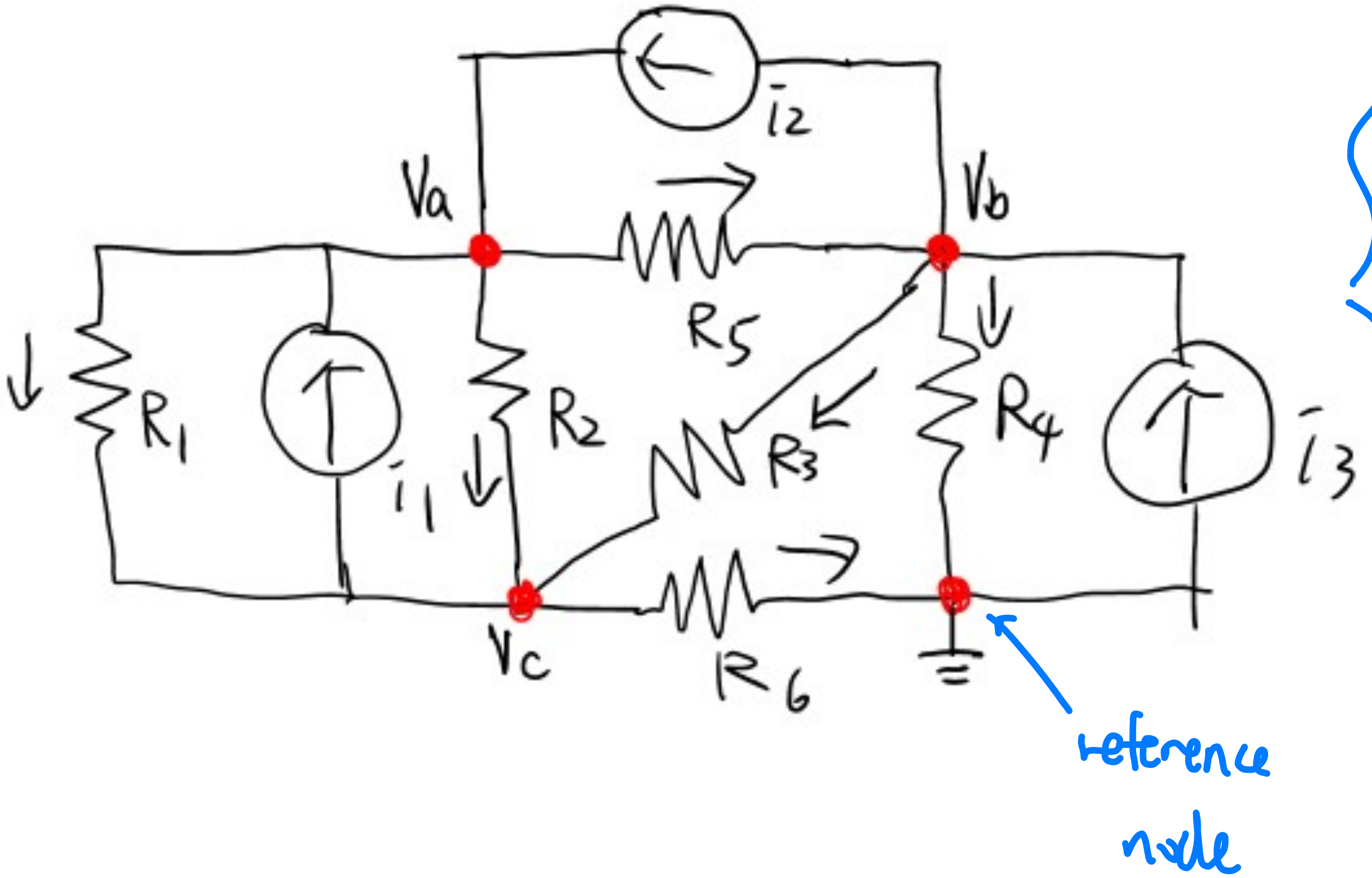
Methods of analysis of resistive circuits
Large scale circuits by 1 st order linear equations
Algorithm to analyze circuit using computers

Node voltage analysis
1. Unknowns are node voltage (n-1)
2. Express current as function of node voltage
3. Apply KCL to all nodes except reference (n-1)

① Resistors + independent current source.



node a : $-\dfrac{V_a-V_c}{R_1}+i_1-\dfrac{V_a-V_c}{R_2}+i_2-\dfrac{V_a-V_b}{R_5}=0$

node b : $-i_2+\dfrac{V_a-V_b}{R_5}-\dfrac{V_b-V_c}{R_3}-\dfrac{V_b}{R_4}+i_3=0$

node c : $\dfrac{V_a-V_c}{R_1}-i_1+\dfrac{V_a-V_c}{R_2}+\dfrac{V_b-V_c}{R_3}-\dfrac{V_c}{R_6}=0$

node a $\quad -\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}\right)V_a+\frac{1}{R_5}V_b+\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_c=-i_1-i_2$

b $\quad \frac{1}{R_5}V_a-\left(\frac{1}{R_5}+\frac{1}{R_3}+\frac{1}{R_4}\right)V_b+\frac{1}{R_3}V_c=i_2-i_3$

c $\quad \left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_a+\frac{1}{R_3}V_b-\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right)V_c=i_1$

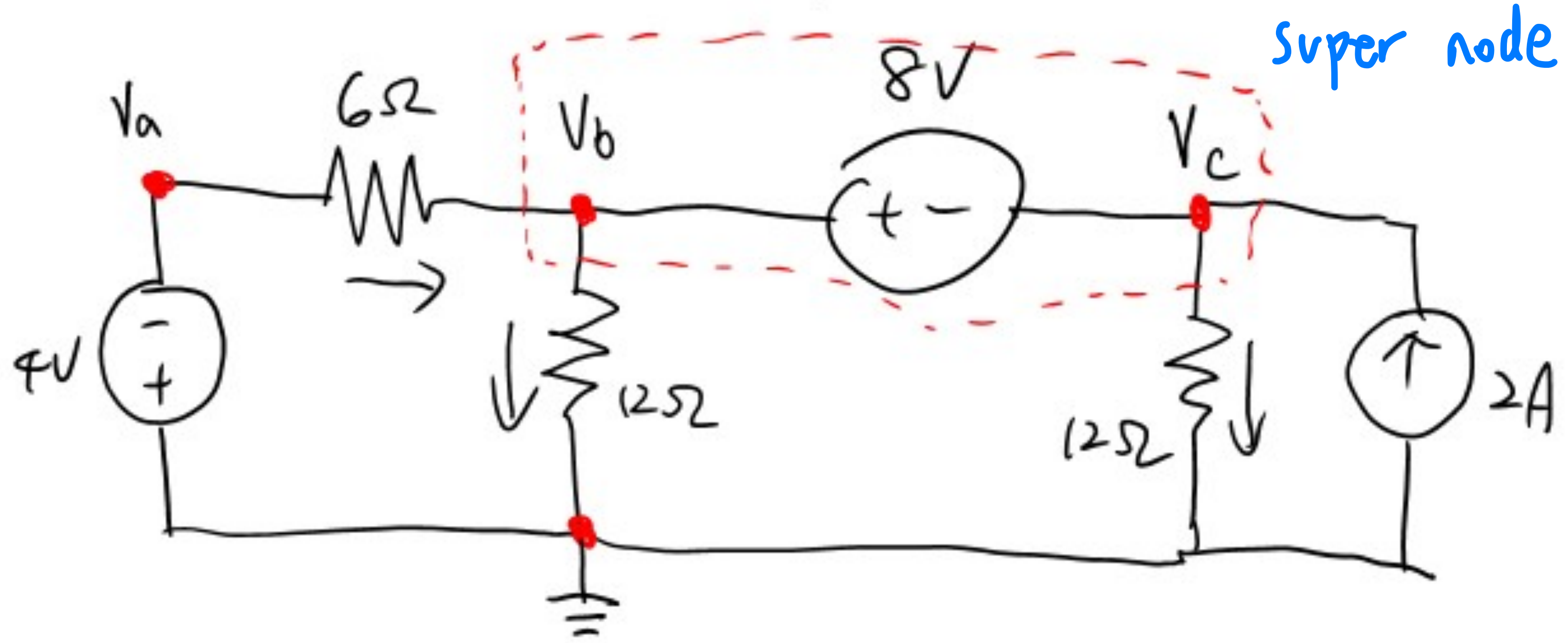
$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5} & -\frac{1}{R_5} & -\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \\ \frac{1}{R_5} & -\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right) & \frac{1}{R_3} \\ \frac{1}{R_1}+\frac{1}{R_2} & \frac{1}{R_3} & -\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right) \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} I_1+I_2 \\ I_2-I_3 \\ I_1 \end{bmatrix}$$

$$AV = b \Rightarrow V = A^{-1}b$$

$I_1 = 1A$, $I_2 = 2A$, $I_3 = 3A$. $R_1 = R_5 = 5\Omega$, $R_2 = R_6 = 2\Omega$ $R_3 = 10\Omega$, $R_4 = 4\Omega$

$$A = \begin{bmatrix} 0.9 & -0.2 & -0.7 \\ 0.2 & -0.55 & 0.1 \\ 0.7 & -0.1 & -1.3 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \quad V = \begin{bmatrix} 7.16 \\ 5.05 \\ 3.47 \end{bmatrix}$$

② Resistor + independent "voltage" source



node a : $V_a = -4V$

node b : current in the ind. voltage source can not be expressed by node voltage

node b, c combine as "super node" (view as the same node)

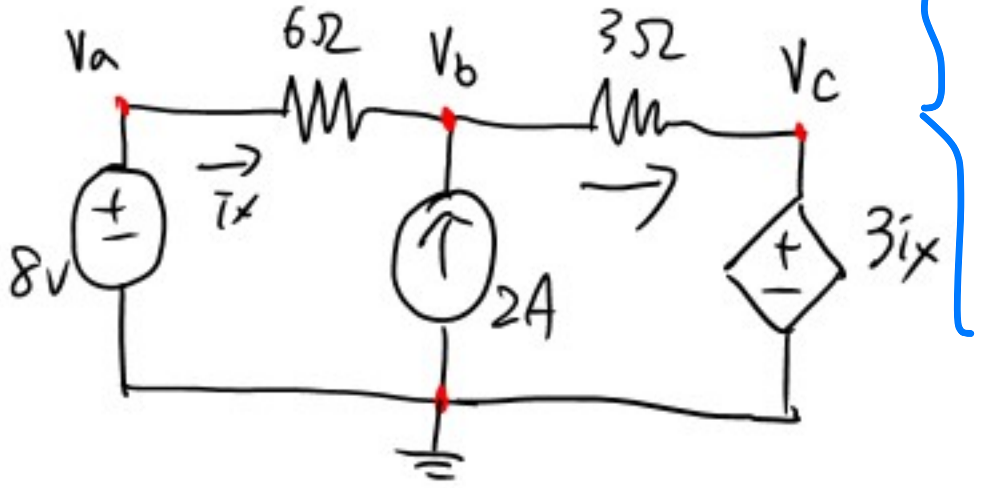
$$\frac{V_a - V_b}{6} - \frac{V_b}{12} - \frac{V_c}{12} + 2 = 0 \Rightarrow V_b = 6V,\ V_c = -2V$$

$$V_b - V_c = 8$$

$V = IR$

$\Rightarrow I = \frac{V}{R}$

③ Resistor + dependent source



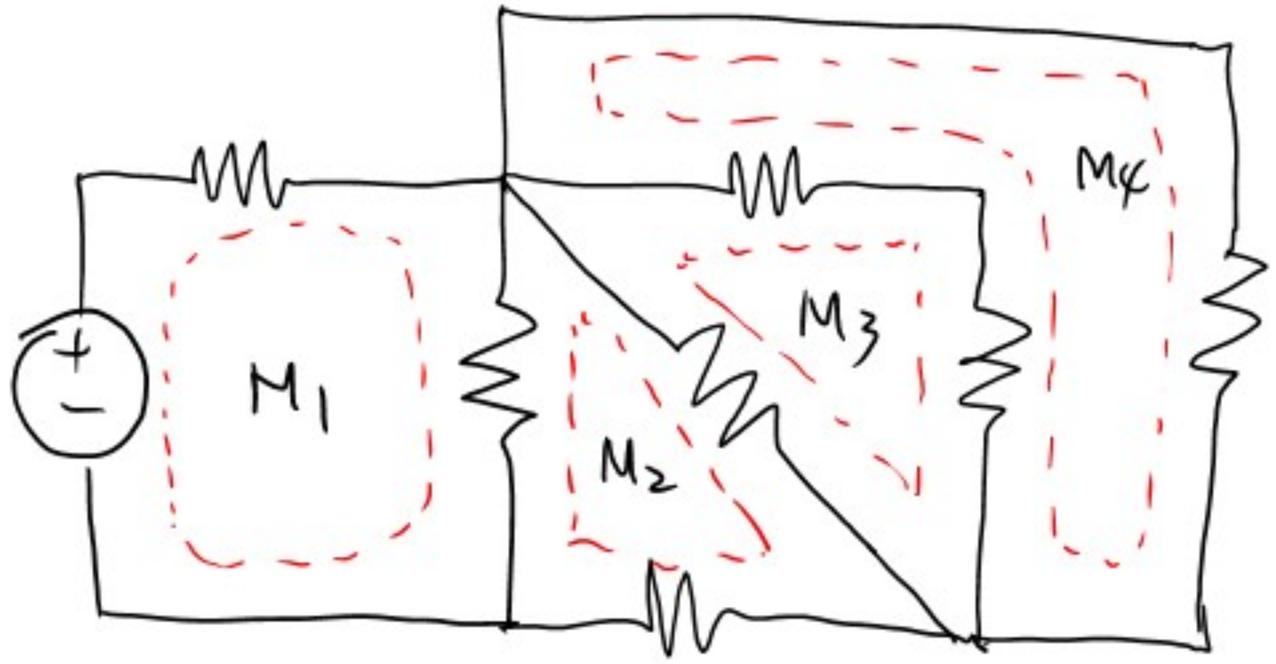
$$\begin{cases} \text{node } a, \quad V_a = 8V \\ \text{node } b, \quad \dfrac{V_a - V_b}{6} + 2 - \dfrac{V_b - V_c}{3} = 0 \\ \text{node } c, \quad V_c = 3i_x = 3 \cdot \dfrac{V_a - V_b}{6} \end{cases}$$

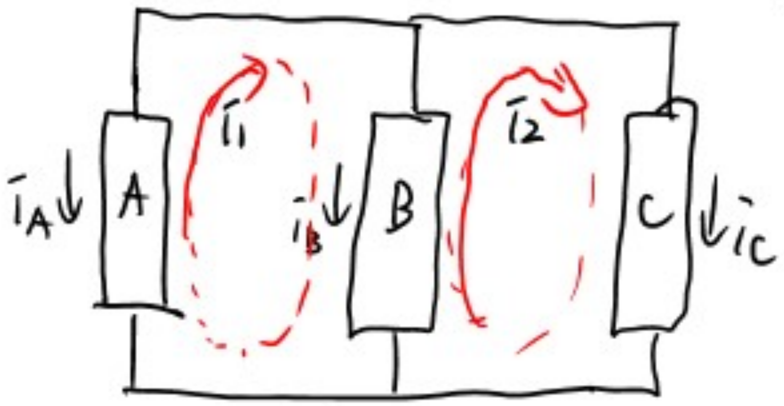
$$\begin{cases} V_a + 0V_b + 0V_c = 8 \\ \frac{1}{6}V_a - \frac{1}{2}V_b + \frac{1}{3}V_c = -2 \\ \frac{1}{2}V_a - \frac{1}{2}V_b - V_c = 0 \end{cases}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{6} & -\frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{2} & -1 \end{bmatrix} \quad b = \begin{bmatrix} 8 \\ -2 \\ 0 \end{bmatrix} \quad V = \begin{bmatrix} 8 \\ ? \\ \frac{1}{2} \end{bmatrix}$$

Mesh current analysis
1. Unknowns is mesh current (I)
2. Express voltage of each branch as function of mesh current
3. Apply KVL to all meshes (I)

mesh = independent loop



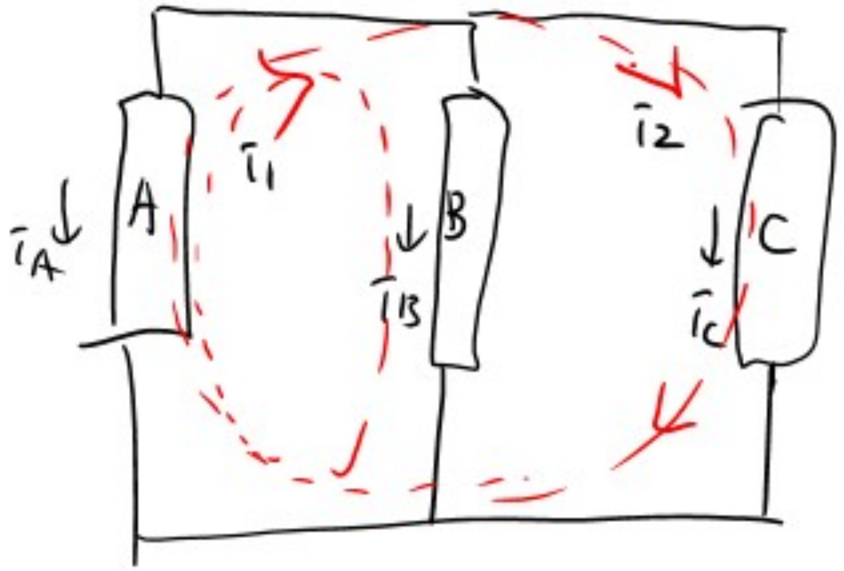


mesh current, i_1, i_2

$i_A = -i_1$

$i_B = i_1 - i_2$

$i_C = i_2$



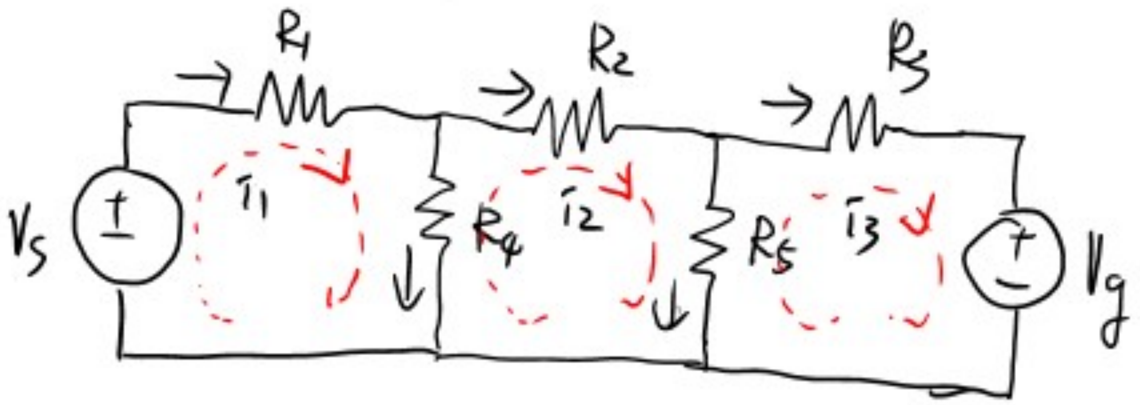
mesh current: i_1 i_2

$i_A = -i_1 - i_2$

$i_B = i_1$

$i_C = i_2$

resistor + voltage source ... ②



$$\begin{cases} \text{mesh 1}: & +V_s - i_1 R_1 - (i_1 - i_2) R_4 = 0 \\ \text{mesh 2}: & (i_1 - i_2) R_4 - i_2 R_2 - (i_2 - i_3) R_5 = 0 \\ \text{mesh 3}: & (i_2 - i_3) R_5 - i_3 R_3 - V_g = 0 \end{cases}$$

$$\begin{cases} (R_1 + R_4) i_1 - R_4 \cdot i_2 + 0\, i_3 = V_s \\ -R_4 \cdot i_1 + (R_2 + R_4 + R_5) i_2 - R_5 \cdot i_3 = 0 \\ 0\, i_1 - R_5\, i_2 + (R_3 + R_5)\, i_3 = -V_g \end{cases}$$

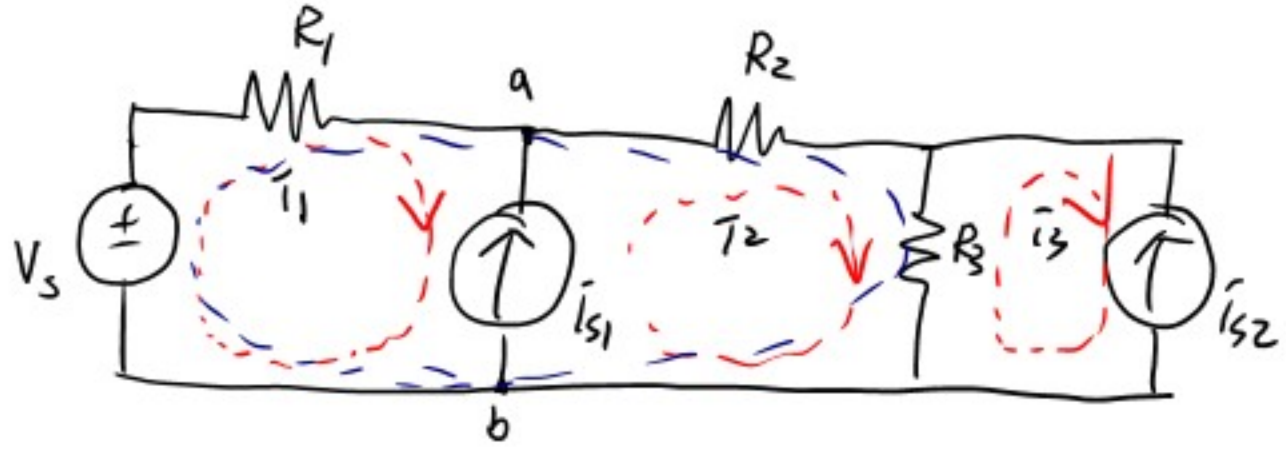
$$A = \begin{bmatrix} R_1 + R_4 & -R_4 & 0 \\ -R_4 & R_2 + R_4 + R_5 & -R_5 \\ 0 & -R_5 & R_3 + R_5 \end{bmatrix} \quad b = \begin{bmatrix} V_s \\ 0 \\ -V_g \end{bmatrix} \quad i = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$i_{R_1} = i_1$ $\quad i_{R_4} = i_1 - i_2$

$i_{R_2} = i_2$ $\quad i_{R_5} = i_2 - i_3$

$i_{R_3} = i_3$

resistor + independent current source



mesh1 : $+V_s - i_1 R_1 - V_{ab} = 0$ ——— ①

$-i_1 + i_2 = -i_{s1}$

mesh2 : $V_{ab} - i_2 R_2 - (i_2 - i_3) R_3 = 0$ ·· ②

mesh3 : $i_3 = -i_{s2}$

super mesh combine two meshes containing current source

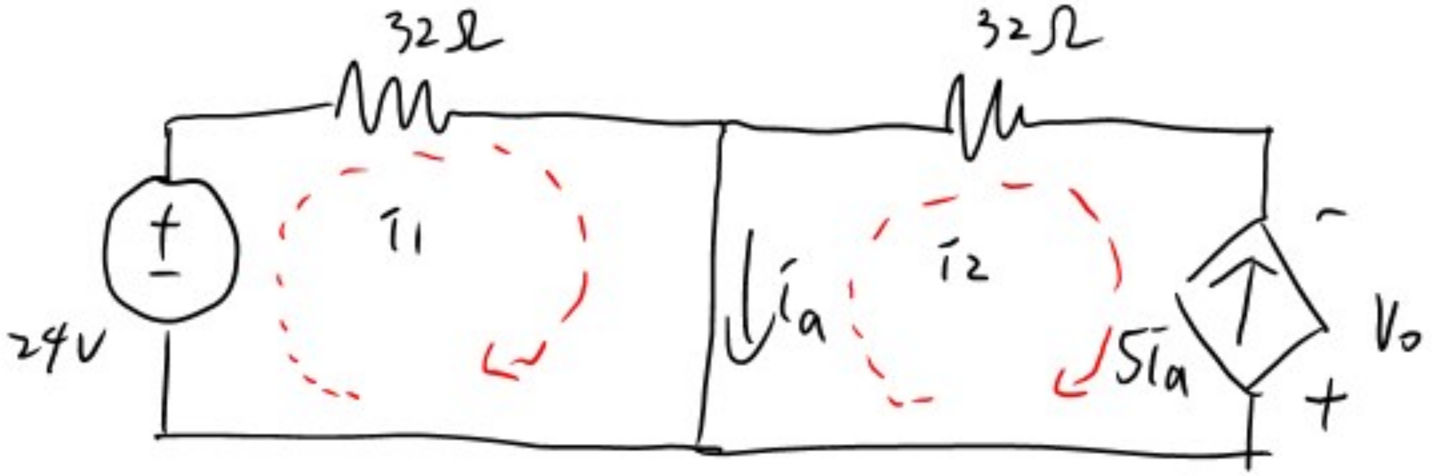
mesh 1.2 : $V_s - i_1 R_1 - i_2 R_2 - (i_2 - i_3) R_3 = 0$

$-i_1 + i_2 = i_{s1}$

mesh 3 : $i_3 = -i_{s2}$

Case 1: current source on branch of only one mesh
Case 2: current source is a common brahcn to two meshes
sol: assume voltage across current source as an unknown
sol: create supermesh containing two meshes

resistor with dependent source



$V_o = ?$

i_1, i_2, V_o

$$\begin{cases} 24 - 32 i_1 = 0 \\ -32 i_2 + V_o = 0 \\ 5 i_a = -i_2 = 5(i_1 - i_2) \end{cases}$$

$$i_1 = \frac{24}{32} = \frac{3}{4} A$$

$$5 i_1 - 4 i_2 = 0 \Rightarrow i_2 = \frac{5}{4} i_1 = \frac{5}{4} \cdot \frac{3}{4} = \frac{15}{16} A$$

$$V_o = 32 i_2 = 32 \cdot \frac{15}{16} = 30 V$$

$V = IR.$

mesh or node?

- mesh is easier for voltage source
- node is easier for current source

) avoid super nodes & meshes

↓ voltage & current sources

- use the one with fewer equations (meshes and nodes)
- consider what information is required, if current is of interest, mesh is easier