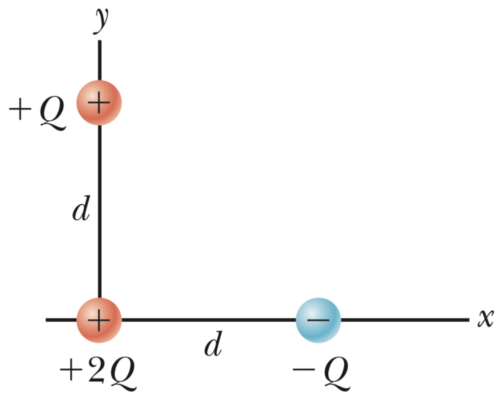
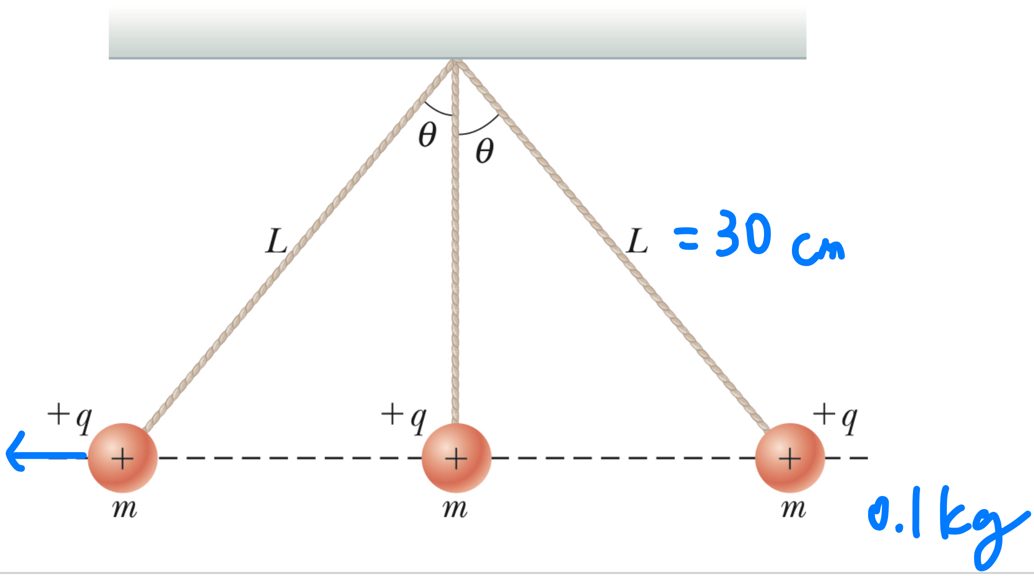
Electric Field

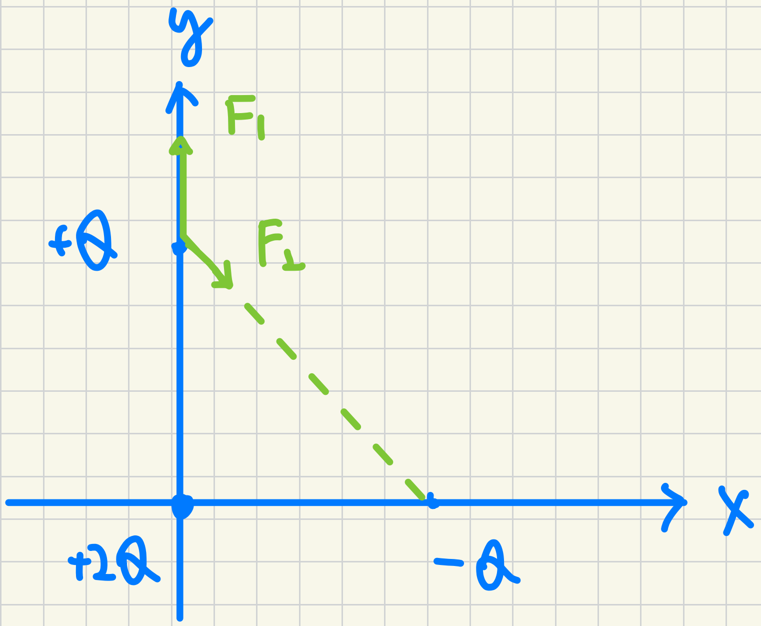
11. **S** A point charge $+2Q$ is at the origin and a point charge $-Q$ is located along the x axis at $x = d$ as in Figure P22.11. Find a symbolic expression for the net force on a third point charge $+Q$ located along the y axis at $y = d$.



41. Three identical point charges, each of mass $m = 0.100$ kg, hang from three strings as shown in Figure P22.41. If the lengths of the left and right strings are each $L = 30.0$ cm and the angle θ is 45.0°, determine the value of q.



11.



1° $\vec{F_1} = \dfrac{k(Q)(2Q)}{d^2}\hat{j} = \dfrac{2kQ^2}{d^2}\hat{j}$

$\vec{F_2} = \dfrac{k(Q)(Q)}{(\sqrt{2}d)^2} \cdot \dfrac{\langle 1,-1\rangle}{\sqrt{2}}$

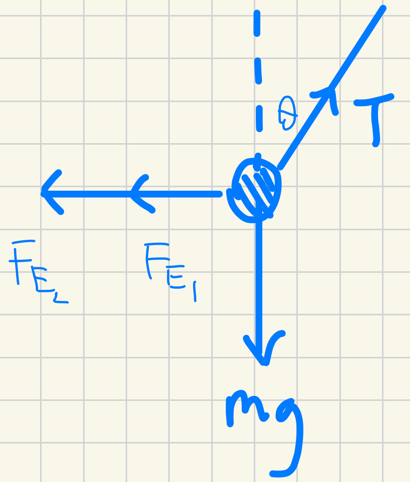
$= \dfrac{kQ^2}{2\sqrt{2}d^2}\hat{i} - \dfrac{kQ^2}{2\sqrt{2}d^2}\hat{j}$

2° $\overrightarrow{F_{total}} = \vec{F_1} + \vec{F_2}$

$= \dfrac{kQ^2}{2\sqrt{2}\,d^2}\hat{i} + \left(2 - \dfrac{1}{2\sqrt{2}}\right)\dfrac{kQ^2}{d^2}\hat{j}$

#

41.



mechanical equilibrium

$$\begin{cases} mg = T\cos 45^\circ \\ F_{E_1} + F_{E_2} = T\sin 45^\circ \end{cases}$$

$k \approx 9\times 10^9 \left(N\cdot m^2/C^2\right)$, $g \approx 9.8\ (m/s^2) \approx 10\ (m/s^2)$

$$F_{E_1} + F_{E_2} = mg$$

$$\Rightarrow \frac{kq^2}{(L\sin 45^\circ)^2} + \frac{kq^2}{(2L\sin 45^\circ)^2} = mg$$

$$\Rightarrow \frac{kq^2}{\left(\frac{0.3}{\sqrt{2}}\right)^2} + \frac{kq^2}{\left(\frac{0.6}{\sqrt{2}}\right)^2} = (0.1)(10)$$

$$\Rightarrow \frac{2kq^2}{0.09} + \frac{2kq^2}{0.36} = 1$$

$$\Rightarrow \frac{10kq^2}{0.36} = 1$$

$$\Rightarrow q = \sqrt{\frac{0.36}{10\times 9\times 10^9}} = 2\times 10^{-6}\ (C)$$

$$= 2\ (\mu C)\ \#$$

Charge, Gauss's Law

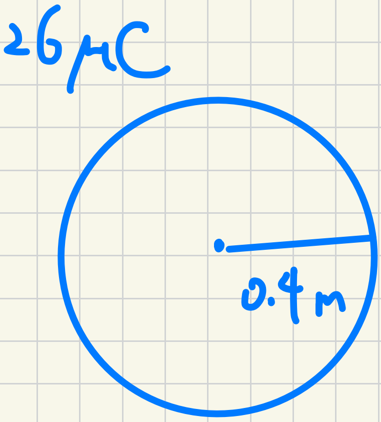
33. **V** A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.

12. **S** A nonuniform electric field is given by the expression

$$\vec{\mathbf{E}} = ay\,\hat{\mathbf{i}} + bz\,\hat{\mathbf{j}} + cx\,\hat{\mathbf{k}}$$

where a, b, and c are constants. Determine the electric flux through a rectangular surface in the xy plane, extending from $x = 0$ to $x = w$ and from $y = 0$ to $y = h$.

33.



Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

★ sphere

· surface area $A = 4\pi r^2$

· volume $V = \frac{4}{3}\pi r^3$

(a) 0 (V/m)

(b) Inside the sphere

$$E \cdot 4\pi r^2 = \frac{\frac{r^3}{R^3} \cdot Q}{\varepsilon_0}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$\Rightarrow E \cdot \cancel{4\pi} \cancel{r^2} = \frac{r^3 Q \cancel{4\pi} k}{R^3}$$

$$\Rightarrow \frac{1}{\varepsilon_0} = 4\pi k$$

$$\Rightarrow E = \frac{rkQ}{R^3} = \frac{0.1 \times 9\times10^9 \times (26\times10^{-6})}{(0.4)^3}$$

$$= 3.66 \times 10^5 \text{ (N/C)}$$

(c) On the sphere

$$E(\cancel{4\pi} r^2) = \cancel{4\pi} kQ$$

$$\Rightarrow E = \frac{kQ}{r^2} = \frac{9\times10^9 \times (26\times10^{-6})}{(0.4)^2}$$

$$= 1.46 \times 10^{6} \ (N/C)$$

(d) outside the sphere

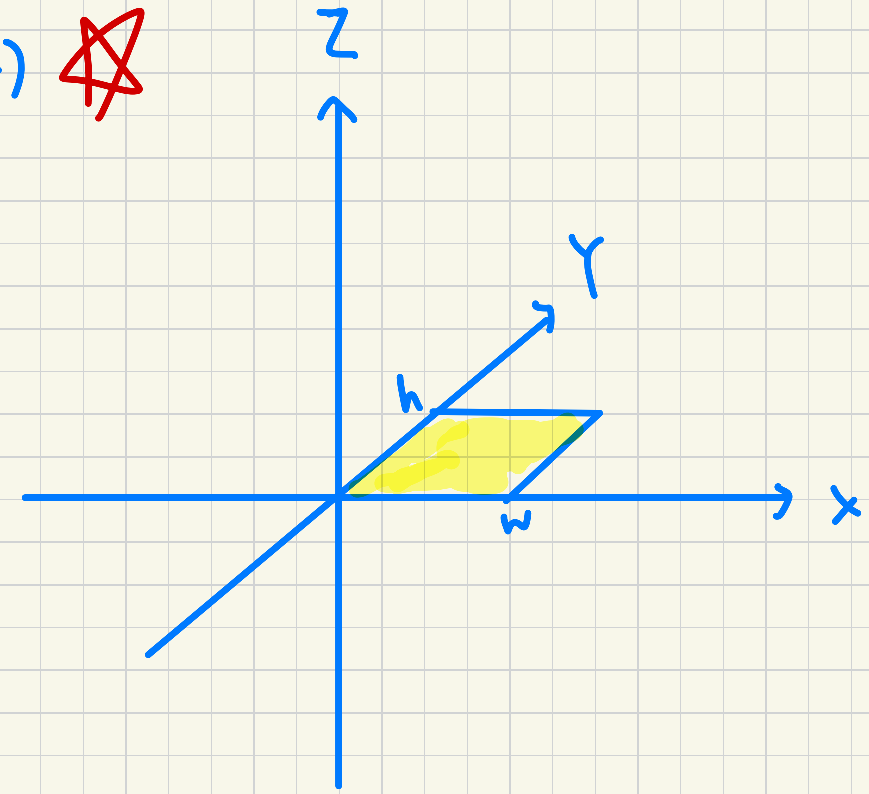
$$E(\cancel{4\pi} d^2) = \cancel{4\pi} kQ$$

$$\Rightarrow E = \frac{kQ}{d^2}$$

$$= \frac{(9\times10^9)(2.6\times10^{-6})}{(0.6)^2} = 6.5\times10^{5} \ (N/C)$$

#

(e) ☆



$$1° \ \phi_E = \vec{E} \cdot \vec{A}$$

$$= (ay\hat{i} + bz\hat{j} + cx\hat{k}) \cdot (dx\,dy\,\hat{k})$$

$$= cx\,dx\,dy$$

2° $\Phi_{E\,total} = \int_0^h \int_0^w cx\,dx\,dy$

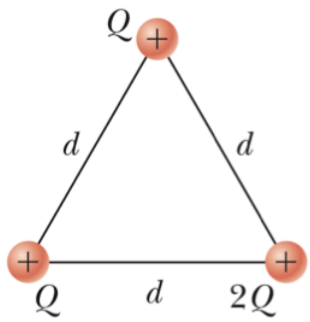
$$= \int_0^h c\left[\frac{x^2}{2}\right]_0^w dy$$

$$= \frac{cw^2}{2}\int_0^h dy$$

$$= \frac{hcw^2}{2} \quad \#$$

Electric Potential

7. **S** Three positive charges are located at the corners of an equilateral triangle as in Figure P24.7. Find an expression for the electric potential at the center of the triangle.

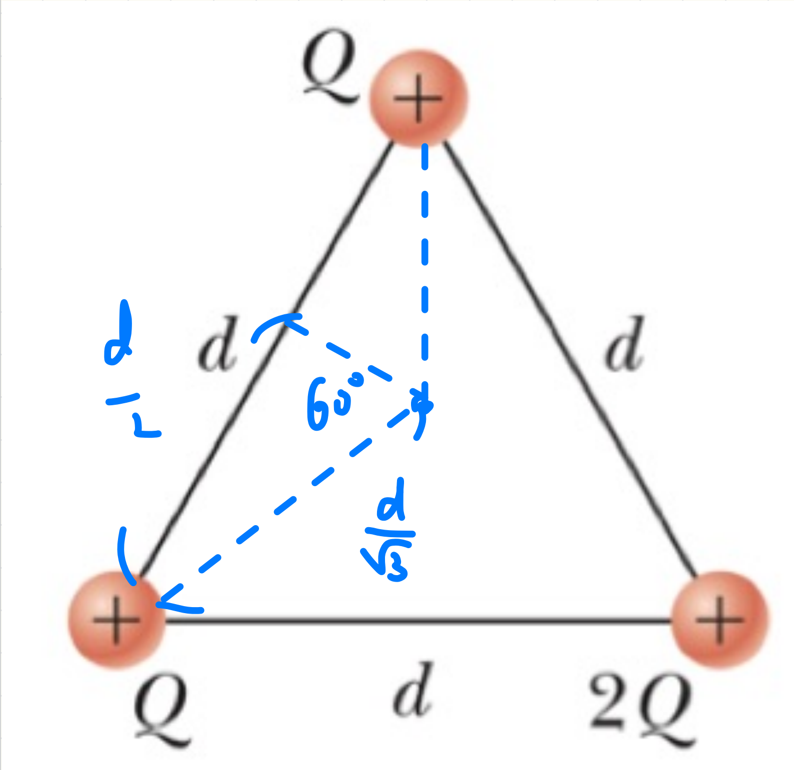


44. **S** When an uncharged conducting sphere of radius a is placed at the origin of an xyz coordinate system that lies in an initially uniform electric field $\vec{\mathbf{E}} = E_0\hat{\mathbf{k}}$, the resulting electric potential is $V(x, y, z) = V_0$ for points inside the sphere and

$$V(x, y, z) = V_0 - E_0 z + \frac{E_0 a^3 z}{(x^2 + y^2 + z^2)^{3/2}}$$

for points outside the sphere, where V_0 is the (constant) electric potential on the conductor. Use this equation to determine the x, y, and z components of the resulting electric field (a) inside the sphere and (b) outside the sphere.

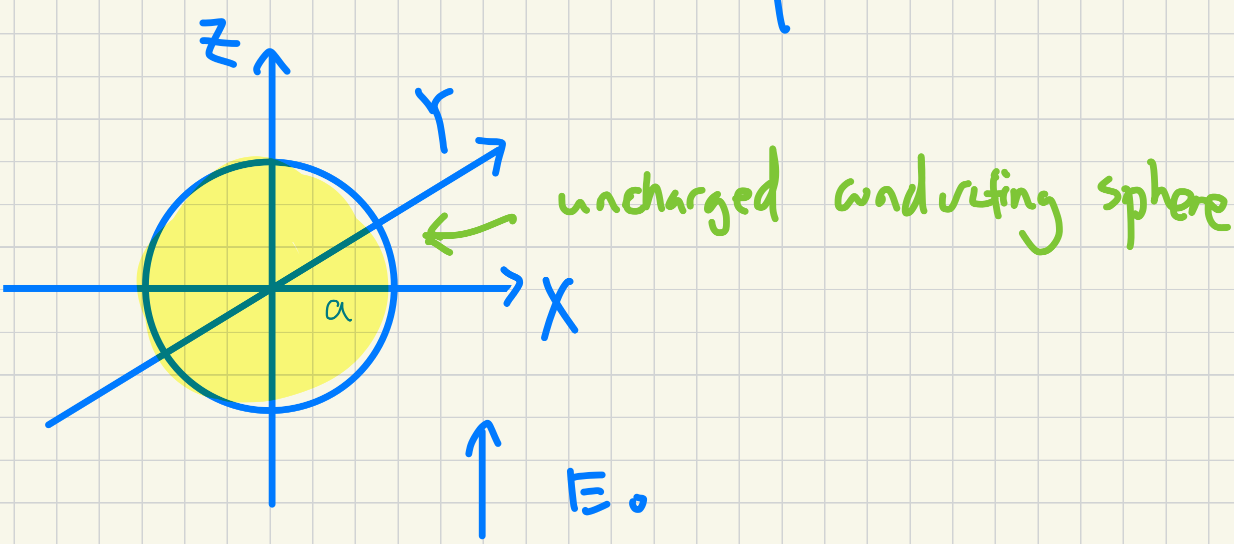
7.



$$V = k \sum \frac{q}{r}$$

$$= k\left(\frac{(1+1+2)Q}{d/\sqrt{3}}\right) = \frac{4\sqrt{3}kQ}{d}$$

44. ☆ electrostatic equilibrium



$$V = \begin{cases} \text{inside the sphere} : V_0 \\ \text{outside the sphere} : V_0 - E_0 z + \dfrac{E_0 a^3 z}{(x^2+y^2+z^2)^{3/2}} \end{cases}$$

$$E = -\nabla V \quad \text{gradient}$$

$$= -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right)$$

(a) $E = -\nabla V = 0 = E_x = E_y = E_z$ #

(b)

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial\left(V_0 - E_0 z + \dfrac{E_0 a^3 z}{(x^2+y^2+z^2)^{3/2}} \right)}{\partial x}$$

$$= -\frac{E_0 a^3 z \cdot 3x}{(x^2+y^2+z^2)^{5/2}} \quad \#$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial\left(V_0 - E_0 z + \dfrac{E_0 a^3 z}{(x^2+y^2+z^2)^{3/2}} \right)}{\partial y}$$

$$= -\frac{E_0 a^3 z \cdot 3y}{(x^2+y^2+z^2)^{5/2}} \quad \#$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial\left(V_0 - E_0 z + \frac{E_0 a^3 z}{(x^2+y^2+z^2)^{3/2}}\right)}{\partial z}$$

$$= E_0 - E_0 a \frac{\partial\left(\frac{z}{(x^2+y^2+z^2)^{3/2}}\right)}{\partial z}$$

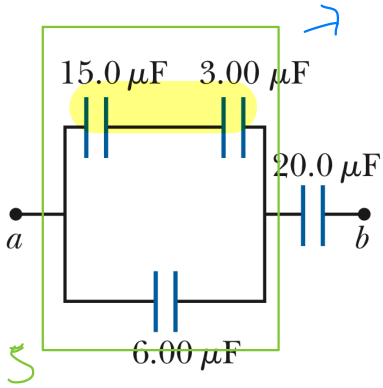
$$\boxed{f(x) = \frac{u(x)}{v(x)} \Rightarrow f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}}$$

$$= E_0 - E_0 a \frac{1 \cdot (x^2+y^2+z^2)^{3/2} - z \cdot \frac{3}{2} \cdot (x^2+y^2+z)^{\frac{1}{2}} \cdot 2z}{(x^2+y^2+z^2)^3}$$

$$= E_0 - E_0 a \frac{x^2+y^2-2z^2}{(x^2+y^2+z^2)^{5/2}}$$ #

Capacitance and Dielectric

11. **T** Four capacitors are connected as shown in Figure P25.11. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor, taking $\Delta V_{ab} = 15.0$ V.



$\rightarrow \frac{1}{\frac{1}{3}+\frac{1}{15}} = 2.5$

$2.5+6=8.5$

34. Four parallel metal plates P_1, P_2, P_3, and P_4, each of area 7.50 cm^2, are separated successively by a distance d = 1.19 mm as shown in Figure P25.34. Plate P_1 is connected to the negative terminal of a battery, and P_2 is connected to the positive terminal. The battery maintains a potential difference of 12.0 V. (a) If P_3 is connected to the negative terminal, what is the capacitance of the three-plate system $P_1P_2P_3$? (b) What is the charge on P_2? (c) If P_4 is now connected to the positive terminal, what is the capacitance of the four-plate system $P_1P_2P_3P_4$? (d) What is the charge on P_4?

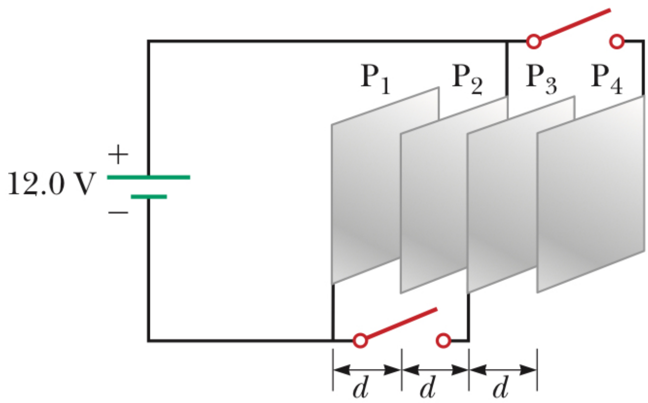


Figure P25.34

11. (a)

$$C_{eq} = \cfrac{1}{\cfrac{1}{\cfrac{1}{\frac{1}{15}+\frac{1}{3}} + 6} + \cfrac{1}{20}} \approx 6$$

(b)

$$C = \frac{Q}{|\Delta V|}$$, conservation of charge

$$\Rightarrow Q = C|\Delta V|$$

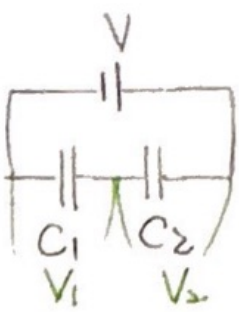
$$\Rightarrow |\Delta V| = \frac{Q}{C}$$

$$Q_{total} = C \cdot |\Delta V|$$

$$= 6 \cdot 15 = 90\ (C)$$

(b) $C = \dfrac{Q}{|\Delta V|}$

推導 考慮:



$$C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

∵ 電荷守恆 and $Q = C \cdot |\Delta V|$

∴ $C_{eq} \cdot V = C_1 \cdot V_1 = C_2 \cdot V_2$

$$\Rightarrow \frac{C_1 \cdot C_2}{C_1 + C_2} \cdot V = C_1 \cdot V_1$$

$$\frac{V_1}{V} = \frac{C_1 \cdot C_2}{C_1 + C_2} \cdot \frac{1}{C_1} = \frac{C_2}{C_1 + C_2}$$

$$Q_{20\mu F} = 20 \cdot \left(\frac{8.5}{8.5+20} \times 15 \right) \approx 89.4\ (\mu C)$$

$$Q_{6\mu F} = 6 \cdot \left(\frac{20}{8.5+20} \times 15 \right) \approx 63.2\ (\mu C)$$

$$Q_{15\mu F} = 15 \cdot \left(\frac{20}{8.5+20} \times \frac{3}{15+3} \times 15 \right) \approx 26.3\ (\mu C)$$

$$Q_{3\mu F} = 3 \cdot \left(\frac{20}{8.5+20} \times \frac{15}{15+3} \times 15 \right) \approx 26.3\ (\mu C)$$