1. $v_1 \cdot v_2 = 1\cdot 1 + 2\cdot 1 + 3(-1) = 0$

$v_2 \cdot v_3 = 1\cdot 5 + 1(-4) + (-1)1 = 0$

$v_1 \cdot v_3 = 1\cdot 5 + 2(-4) + 3\cdot 1 = 0$

$\Rightarrow$ 三向量兩兩正交

2. $s \cdot v_1 = a_1 |v_1|^2 = 14 a_1 = 14 \Rightarrow a_1 = 1$

$s \cdot v_2 = a_2 |v_2|^2 = 3 a_2 = 3 \Rightarrow a_2 = 1$

$s \cdot v_3 = a_3 |v_3|^2 = 42 a_3 = 84 \Rightarrow a_3 = 2$

$\Rightarrow a_1 = 1,\ a_2 = 1,\ a_3 = 2$

3. $s = -2(1,2,3) + 3(1,1,-1) + 4(5,-4,1)$

$= (21, -17, 5)$

4. $b_3 = (1, \omega^3, \omega^6, \omega^9)$

$\omega = e^{\frac{i\cdot 2\pi}{4}} = e^{i\cdot\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$\Rightarrow b_3 = (1, \bar{i}^3, \bar{i}^2, \bar{i})$

$= (1, -\bar{i}, -1, \bar{i})$

5.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \begin{pmatrix} -2 \\ -3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4-2i \\ 4 \\ -4+2i \end{pmatrix}$$

6.

$$a_j = \frac{s \cdot b_j}{n}$$

$$a_0 = (1, i, -5, -i) \cdot (1, 1, 1, 1) / 4$$

$$= -1$$

$$a_1 = (1, i, -5, -i) \cdot (1, i, -1, -i) / 4$$

$$= [1 \cdot 1 + i(-i) + (-5)(-1) + (-i)(i)] / 4$$

$$= 2$$

$$a_2 = (1, i, -5, -i) \cdot (1, -1, 1, -1) / 4$$

$$= -1$$

$$a_3 = (1, i, -5, -i) \cdot (1, -i, -1, i) / 4$$

$$= [1 \cdot 1 + i \cdot i + (-5)(-1) + (-i)(-i)] / 4$$

$$= 1$$

$$\Rightarrow (a_0, a_1, a_2, a_3) = (-1, 2, -1, 1)$$

7.

(a) While Fourier was working on heat equations, Fourier wished to express any function, even a step function, as a sum of sine/cosine waves, provided the frequencies of those waves satisfy boundary conditions.

(b) discrete: sampled data

(c) Both analyze frequency content of signals

(d) $\int f(t)\, e^{-i\omega t}\, dt$