system transfer function

△ Frequency response . circuit gain in frequency domain

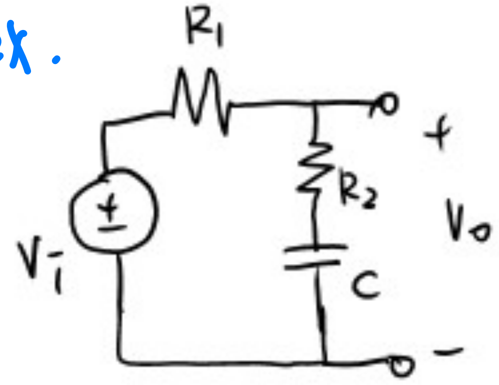
$$H(\omega) = \frac{V_o(\omega)}{V_I(\omega)} = \frac{|V_o| \angle V_o}{|V_I| \angle V_I} = \frac{|V_o| e^{j\phi_o}}{|V_I| e^{j\phi_I}} = \left|\frac{V_o}{V_I}\right| e^{j(\phi_o - \phi_I)}$$

(V_o : output voltage
V_I : input voltage)

增益
$$\text{gain} = |H(\omega)| = \frac{|V_o|}{|V_I|}$$

phase shift. $\angle H(\omega) = \angle V_o - \angle V_I$

ex.



$$H(\omega) = \frac{V_o}{V_I} = \frac{R_2 + Z_C}{R_1 + R_2 + Z_C} = \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{1 + j\omega C R_2}{1 + j\omega C(R_1 + R_2)}$$

$$\text{gain} = |H(\omega)| = \frac{\sqrt{1 + \omega^2 C^2 R_2^2}}{\sqrt{1 + \omega^2 C^2 (R_1 + R_2)^2}}$$

phase shift $= \angle H(\omega) = \tan^{-1}(\omega C R_2) - \tan^{-1}(\omega C(R_1 + R_2))$

$f \to 0$ (DC) gain $\doteq 1$, $f \uparrow$ (高頻) $\omega^2 C^2 R_2^{2} \gg 1 \to f \gg \frac{1}{2\pi C R_2}$. gain $\cong \frac{R_2}{R_1 + R_2} < 1$

$R_1 = 8k\Omega$, $R_2 = 16k\Omega$, $C = 0.23\mu F$, . $H(\omega) = \frac{1 + j\omega(3.68 \times 10^{-3})}{1 + j\omega(5.52 \times 10^{-3})}$ $f = 1000\,Hz$, $\omega = 6280\ rad/s$

gain $= 0.67$, phase $= -0.8°$

$$H(w) = \frac{1 + j\frac{w}{z}}{1 + j\frac{w}{p}}$$

$$1 + j = \sqrt{2} \angle 45^\circ$$

$$H(w) = \frac{(1+j\frac{w}{z_1})(1+j\frac{w}{z_2}) \cdots}{(1+j\frac{w}{p_1})(1+j\frac{w}{p_2}) \cdots}$$

z is called zero of the circuit

$w \ll z$ gain remains the same. , phase shift $\sim 0^\circ$

$w \simeq z$, gain increase $\sqrt{2}$ fold. , phase shift. increase 45°

$w \gg z$, gain increase as. $\frac{w}{z}$, phase shift increase 90°

$$H(w) = \frac{1 + jw(3.68 \times 10^{-3})}{1 + jw(5.52 \times 10^{-3})}$$

p is called. pole. of the circuit.

one zero at $f_z = \frac{1}{2\pi(3.68 \times 10^{-3})} = \frac{271.74}{2\pi}$

$\simeq 43.25\text{Hz}$

$w \ll p$, gain reamain the same. , phase shift $\sim 0^\circ$

$w \simeq p$. , gain decrease $\sqrt{2}$ fold , phase shift decrease 45°

$w \gg p$, gain decrease as $\frac{w}{p}$, phase shift decrease 90°

one pole at $f_p = \frac{1}{2\pi(5.52 \times 10^{-3})} = \frac{181.16}{2\pi}$

$\simeq 28.83\text{Hz}$

Bode plot: a chart of gain and phase shift versus frequency

describes the frequency response

gain is usually shown in decibels

frequency is usually shown in logarithm scale.

$$H(w) = \frac{1}{1+j\frac{w}{w_0}} = \frac{1}{\sqrt{1+(\frac{w}{w_0})^2}} \angle \left[-\tan^{-1}\left(\frac{w}{w_0}\right)\right]$$

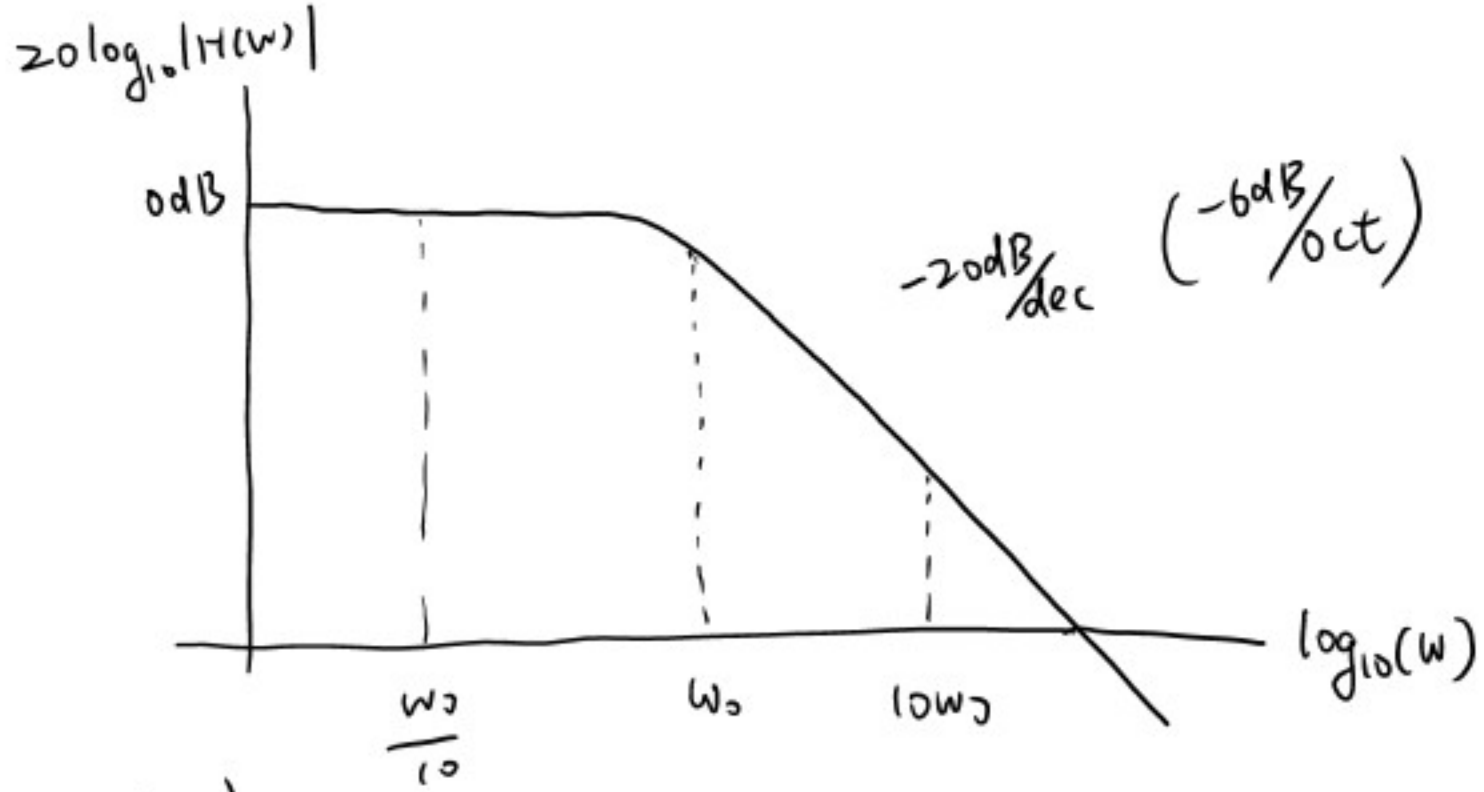
w_0 is the pole.

$$20\log_{10}|H(w)| = 20\log_{10}\left(\frac{1}{\sqrt{1+(\frac{w}{w_0})^2}}\right) = -20\log_{10}\left(\sqrt{1+(\frac{w}{w_0})^2}\right) = -10\log_{10}\left[1+\left(\frac{w}{w_0}\right)^2\right]$$

$$\begin{cases} w \ll w_0, & 20\log_{10}|H(w)| \cong 0\,dB \\ w \gg w_0, & 1+\left(\frac{w}{w_0}\right)^2 \cong \left(\frac{w}{w_0}\right)^2 \quad 20\log_{10}|H(w)| = -20\log_{10}\left(\frac{w}{w_0}\right) = 20\log_{10} w_0 - 20\log_{10} w \end{cases}$$

w increase 10 fold. ~ $20\log_{10}|H(w)|$ decrease $-20dB$ $[-20dB/dec]$

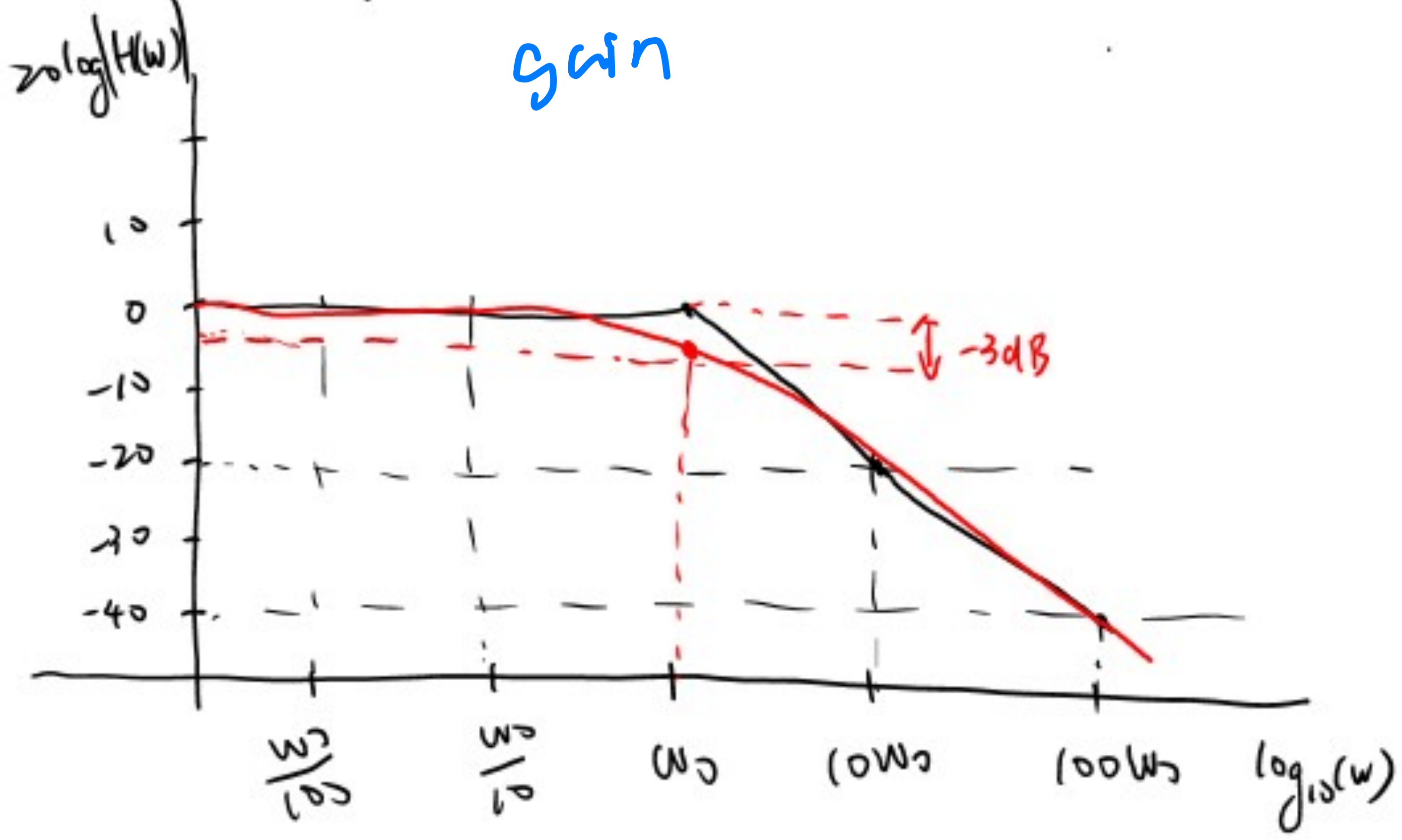
2 fold ~ $20\log_{10}|H(w)|$ decrease $-6dB$ $[-6dB/oct]$

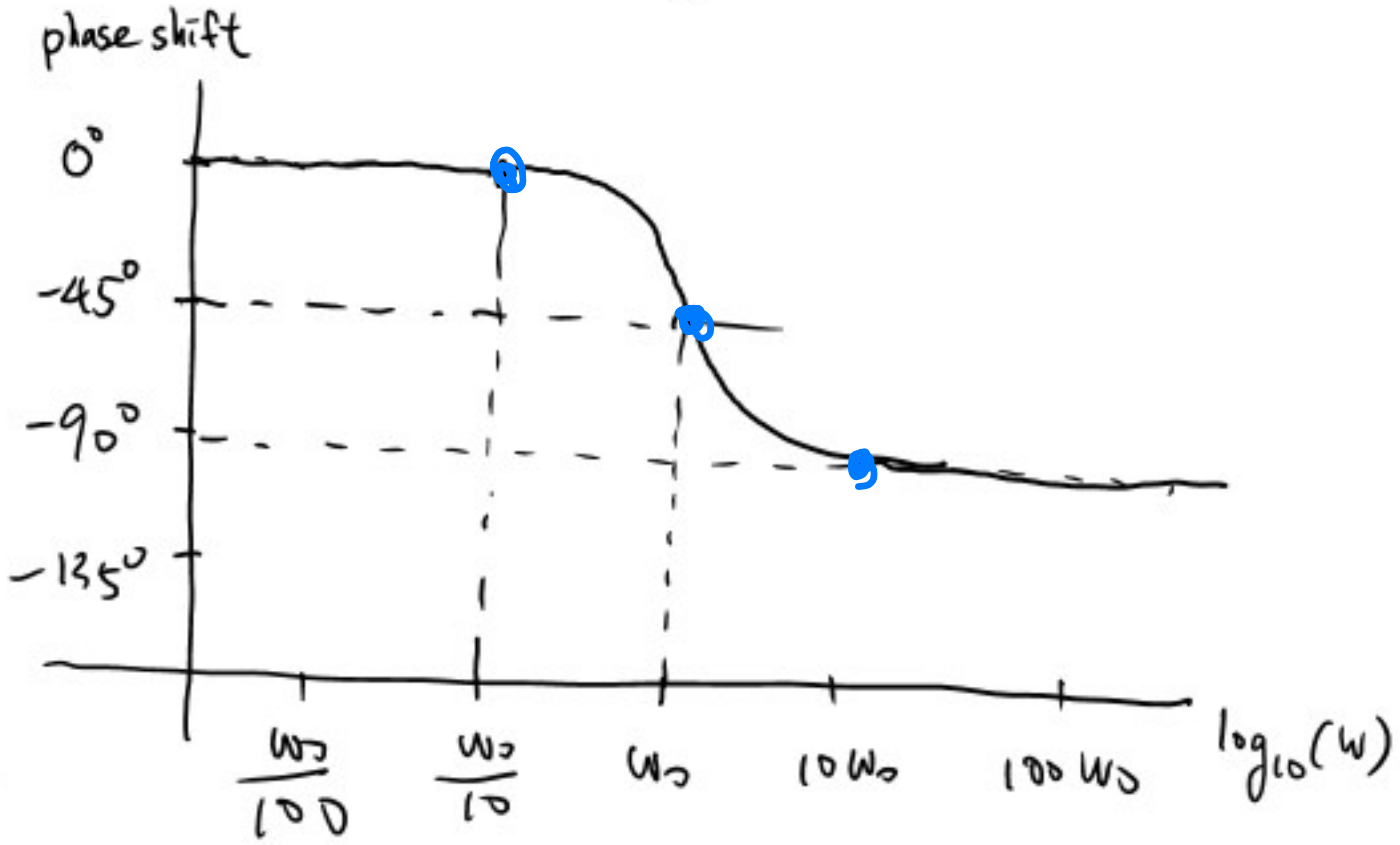


a straight line.

$\omega \cong \omega_0$ corner frequency (cut-off freq)
3dB freq

$$20\log_{10}|H(\omega)| = -10\log_{10}\left[1+\left(\frac{\omega}{\omega_0}\right)^2\right] = -3dB$$





phase shift

ex 1.

$$H(w) = K \cdot \frac{jw}{1 + j\frac{w}{p}}$$

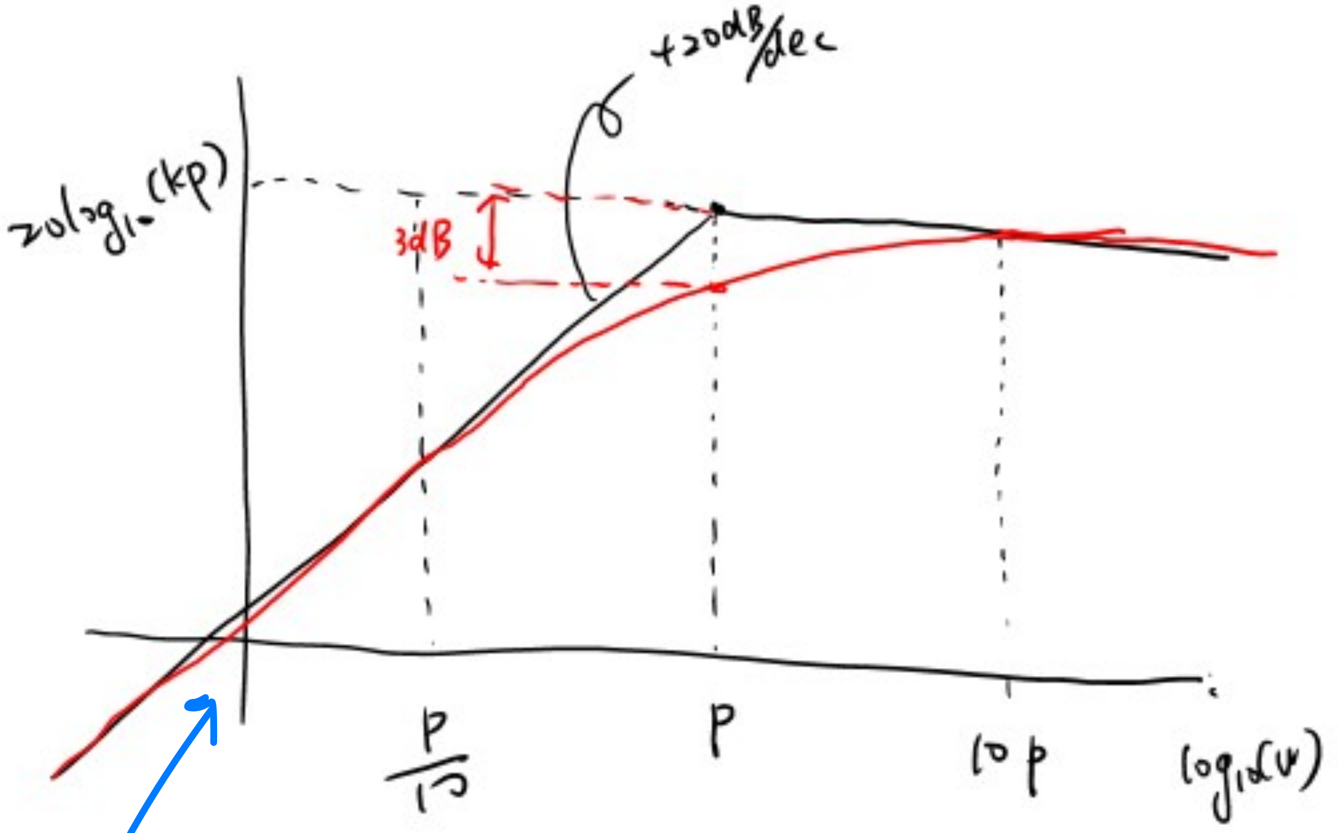
$$20\log_{10}|H(w)| = 20\log_{10}K + 20\log_{10}(w) - 20\log_{10}\sqrt{1+(\frac{w}{p})^2},$$

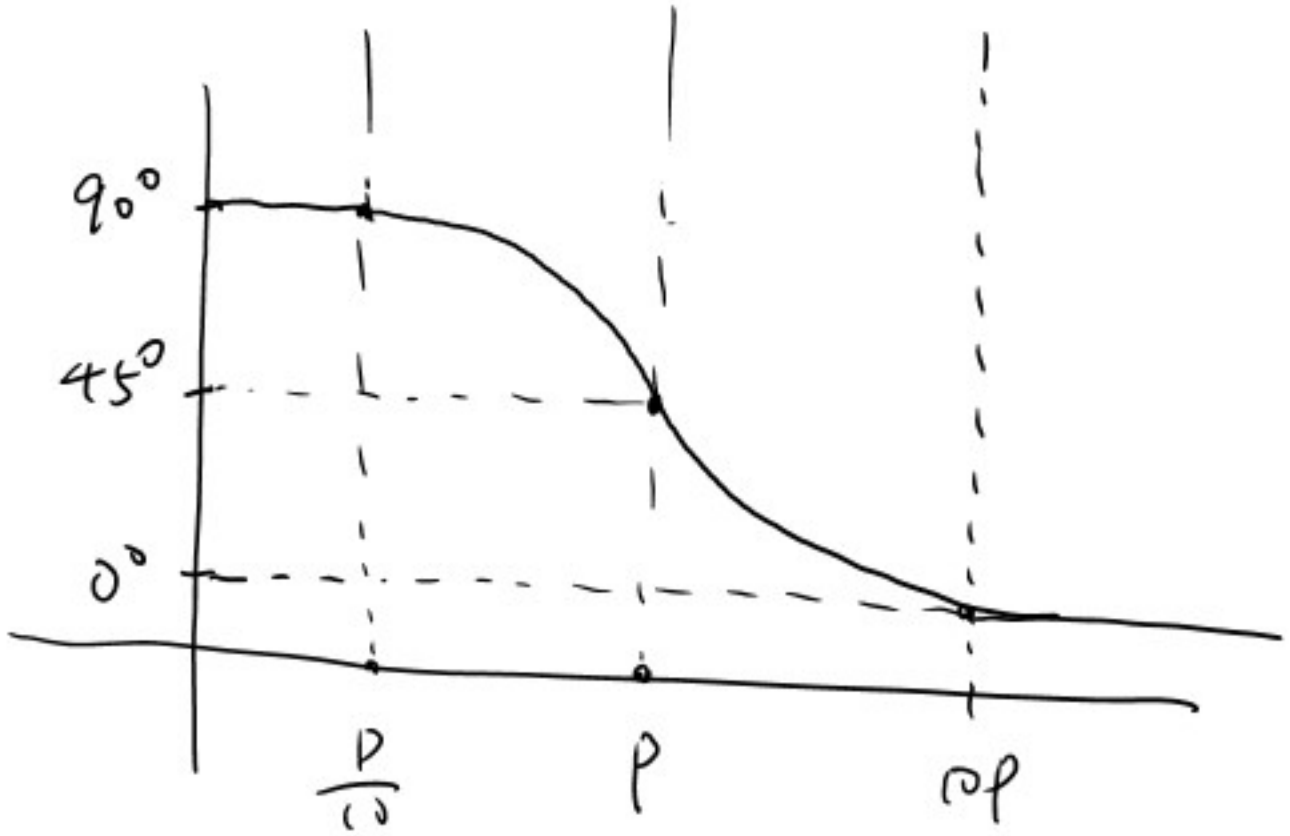
one pole, p_1, one zero at 0.

$$\begin{cases} w \ll p & 20\log_{10}K + 20\log_{10}w, \quad \text{gain increase with slope } +20dB/dec\ (+6dB/oct) \\ w \gg p & 20\log_{10}K + 20\log_{10}w - 20\log_{10}\frac{w}{p} = 20\log_{10}K + \underline{20\log_{10}w} - \underline{20\log w} + 20\log_{10}p = \underline{20\log_{10}(Kp)}, \text{ a constant} \\ w \cong p & 20\log_{10}K + 20\log_{10}w - 20\log_{10}\sqrt{2} = 20\log_{10}(Kp) - 3dB \end{cases}$$

phase shift $\angle H(w) = 90° \begin{cases} w \ll p & 90° \\ w \gg p & 0° \\ w \cong p & 45° \end{cases}$

$$H(\omega) = K \cdot \frac{j\omega}{1 + j\frac{\omega}{p}}$$

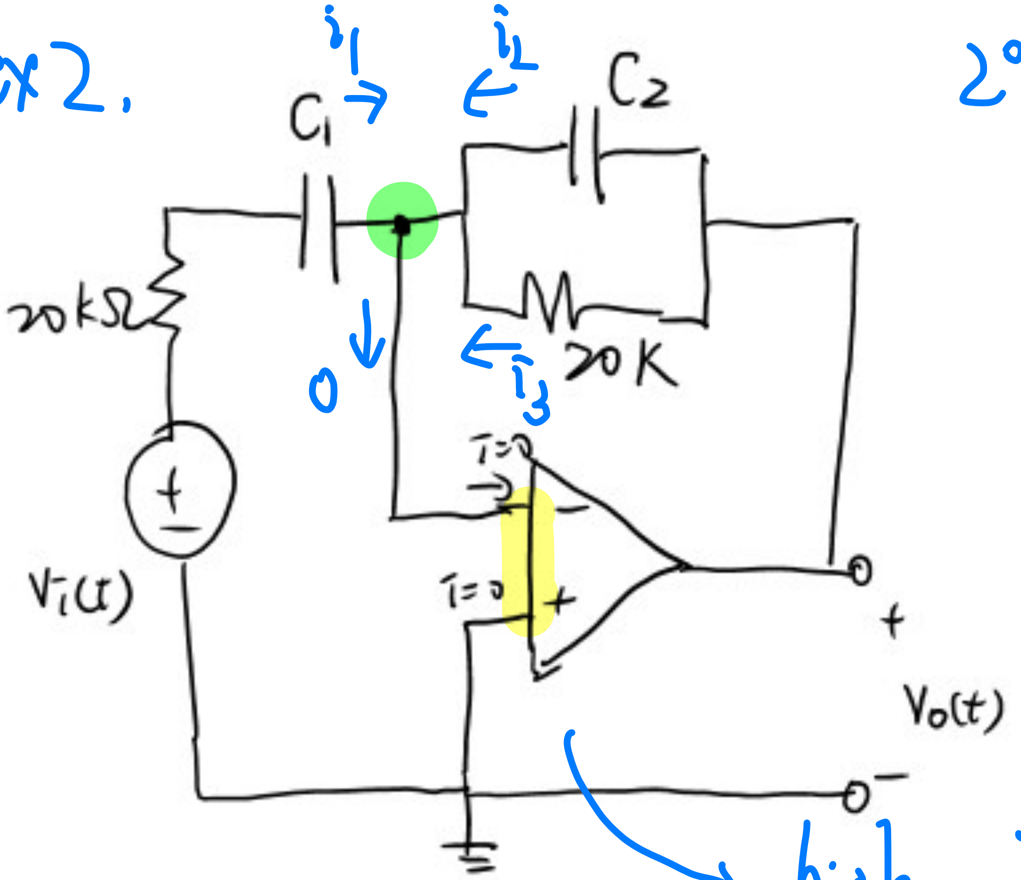




- a capacitor gives a pole or a zero
- Two poles may lie together with -40dB/dec
- zero, positive +20dB/dec and +90°
- pole negative -20dB/dec and -90°

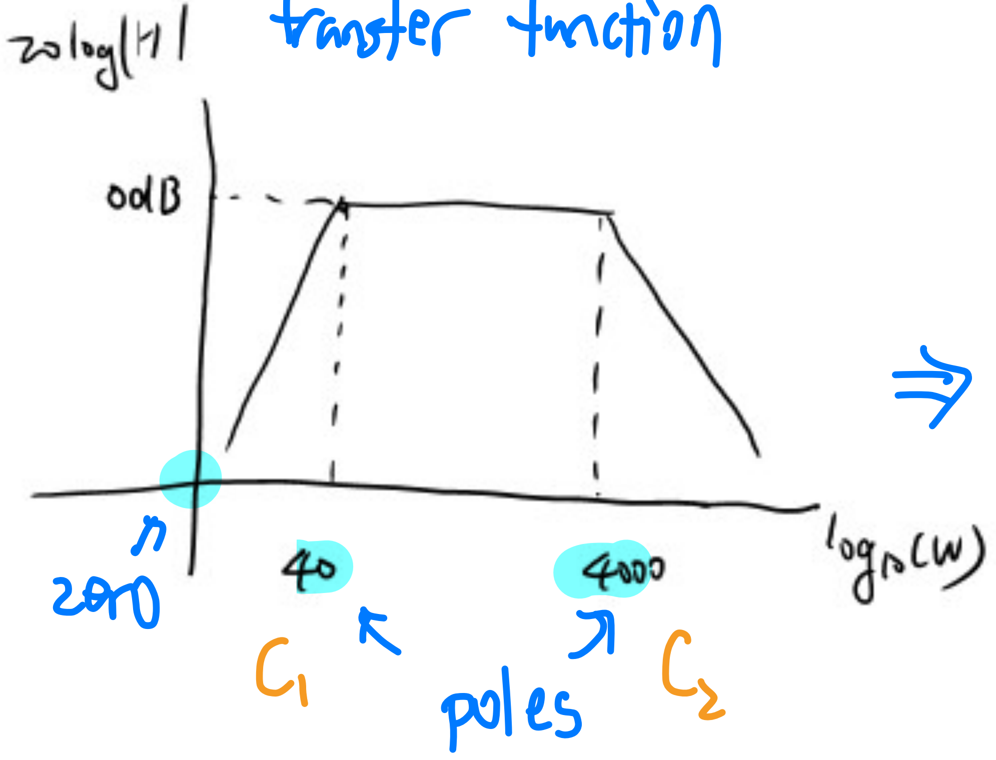
does not necessarily pass the origin

ex2.



high Z_{input} for op-amps

2° transfer function



$$\Rightarrow H(w) = A\frac{jw}{(1+j\frac{w}{40})(1+j\frac{w}{4000})}$$

$40 \le w \le 4000$ midband $\quad |H(w)| = A\frac{|jw|}{|(1+j\frac{w}{40})|} \cong 40A$

$$20\log_{10}(40A) = 0 \rightarrow 40A = 1 \rightarrow A = 0.025$$

1° $V^+ = V^- = 0$

$i^+ = i^- = 0$

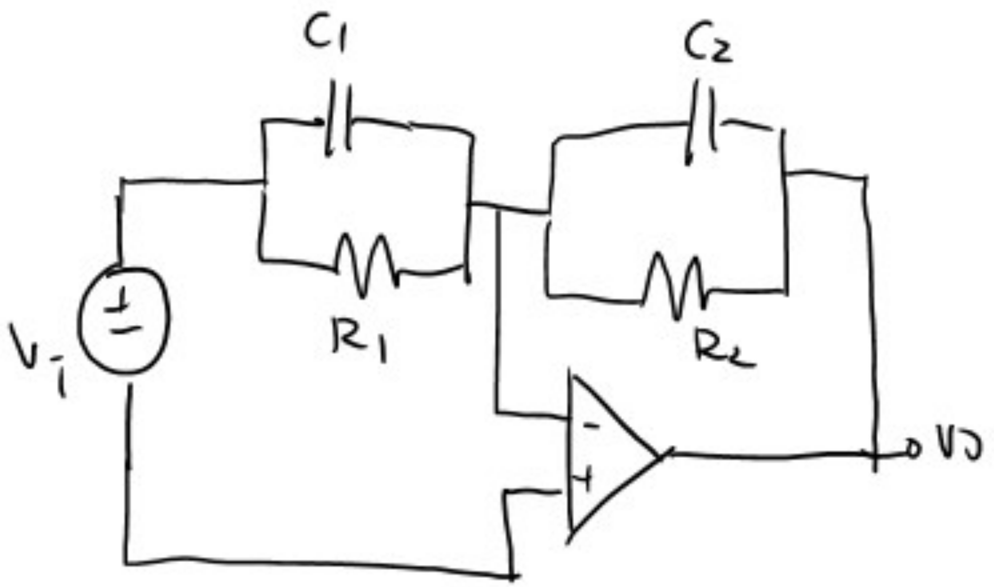
$$\underbrace{\frac{V_1 - 0}{20k + \frac{1}{jwC_1}}}_{i_1} + \underbrace{\frac{V_0}{\frac{1}{jwC_2}}}_{i_2} + \underbrace{\frac{V_0}{20k}}_{i_3} = 0$$

KCL

3° $$H(w) = \frac{V_0}{V_i} = \frac{-jwC_1(20k)}{[1+jwC_1(20k)][1+jwC_2(20k)]} = -C_1(20k)\frac{jw}{[1+jwC_1(20k)][1+jwC_2(20k)]}$$

$$C_1(20k) = A = 0.025 \rightarrow C_1 = \frac{0.025}{20\times10^3} = 1.25\times10^{-6}F = 1.25\,uF \quad \#$$

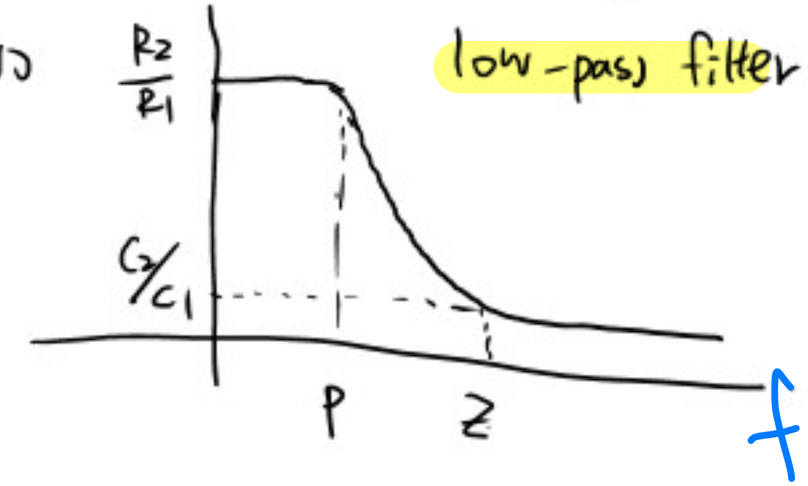
$C_1(20k) = \frac{1}{40}$ valid $\qquad C_2(20k) = \frac{1}{4000} \rightarrow C_2 = \frac{1}{20\times10^3\times4\times10^3} = 1.25\times10^{-8}F = 12.5\,nF \quad \#$

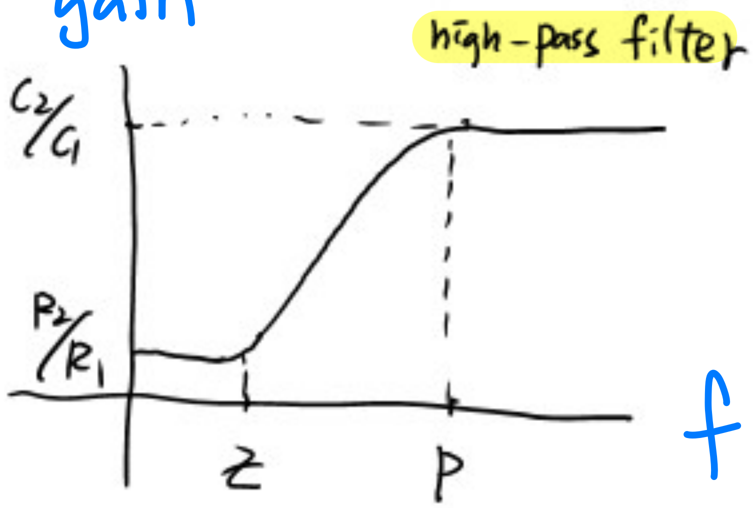


$$H(w) = \frac{R_2}{R_1}\,\frac{1 + j\frac{w}{z}}{1 + j\frac{w}{p}} \qquad z = \frac{1}{C_1 R_1}, \quad p = \frac{1}{C_2 R_2}$$

gain

gain





pole < zero

zero < pole

2nd order filter

$$H(\omega) = \frac{K\omega_0^2}{\underbrace{(j\omega)^2 + j2\zeta\omega_0\omega + \omega_0^2}_{\text{double root}}} = \frac{K\cdot\omega_0^2}{\omega_0^2 - \omega^2 + j\cdot 2\zeta\omega_0\omega}$$

(ζ : zeta)

$K=1$

$$\begin{cases} \omega \ll \omega_0 & |H(\omega)| \cong 1 \quad 20\log_{10}|H| = 0\,dB \\ \omega \gg \omega_0 & |H(\omega)| = \left(\frac{\omega_0}{\omega}\right)^2 \quad 20\log_{10}|H| = 40\log_{10}\omega_0 - 40\log_{10}\omega \quad [-40dB/dec,\ -12dB/oct] \\ \omega \cong \omega_0 & |H(\omega)| \cong \frac{1}{2\zeta} \quad 20\log_{10}|H| = -20\log_{10}(2\zeta) = -6 - 20\log_{10}(\zeta)\,dB \end{cases}$$