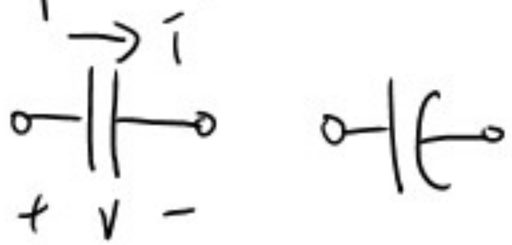
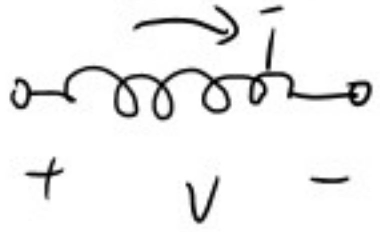
△ time response of AC signal

⓪ energy storage element

	capacitor	inductor
symbol		
i-v	$i = C \cdot \frac{dv}{dt}$	$v = L \cdot \frac{di}{dt}$
power	$P_c = C \cdot v \frac{dv}{dt}$	$P_L = L i \frac{di}{dt}$
energy	$W_c = \frac{1}{2} C v^2$	$W_L = \frac{1}{2} L i^2$
instaneous change (AC)	i	v

$C \to L$

$\Longrightarrow$

$V \to I$

	C	L
in steady state (DC)	open → no I	short → no V
series	$\frac{C_1 C_2}{C_1 + C_2}$	$L_1 + L_2$
parallel	$C_1 + C_2$	$\frac{L_1 L_2}{L_1 + L_2}$

In AC analysis, we need determine the initial condition when switch change at "t_0"

1. Analyze DC circuit that exist before t_0 ($t < t_0$)

2. Analyze capacitor voltage, inductor current change as AC source after t_0 ($t > t_0$)

Response of RC circuit

= highest derivative of voltage or current in the circuit's differential equation

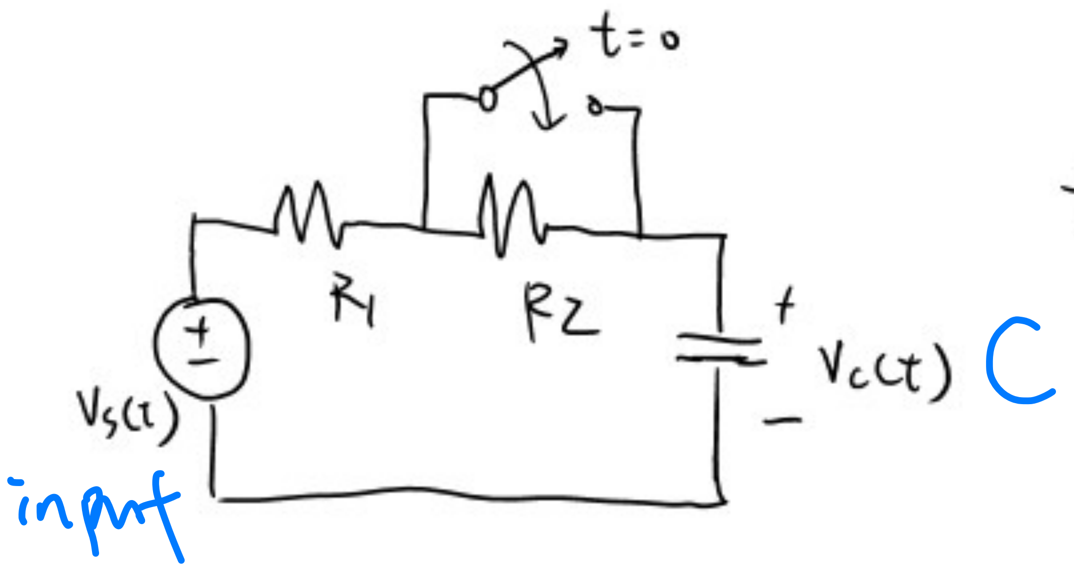
- Circuit that contains capacitors can be represented by differential equations. and the order of the circuit is usually equeal to the number of capacitors (and inductors)

- time roponse of the circuit, means the change of output for an input change varies with respect to time

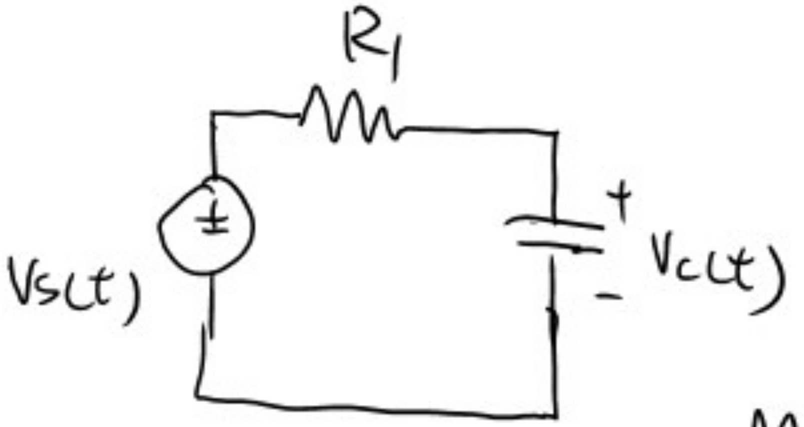
- Complete response = transient response (natural response) + steady-state response (forced response)

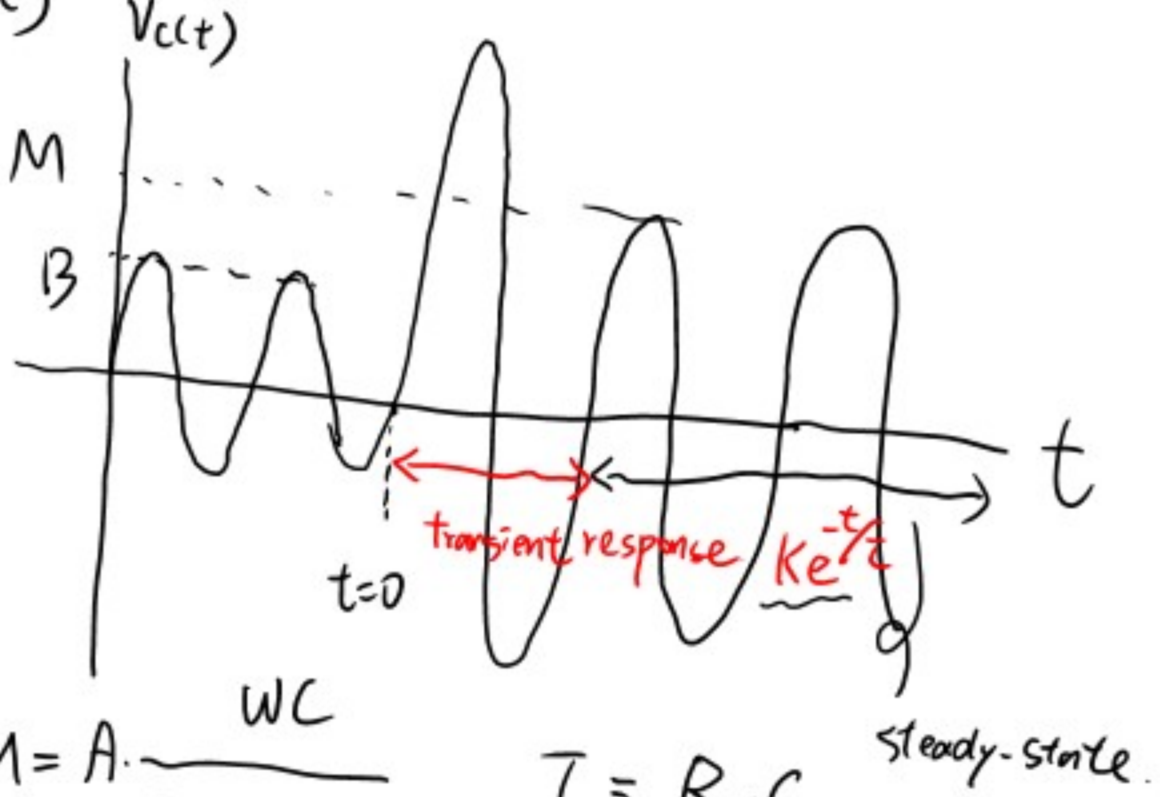
- note, in electrical engineering or signal process, sometimes "transient" is used for "complete rosponse."

PSPice



$t>0$





$V_s(t) = A\cos(1000\cdot t + \theta)$ $\qquad \omega = 1000\ \text{rad}/s$

$$\begin{cases} V_c(t) = B\cos(1000\cdot t + \phi_1) & (t<0) \\ V_c(t) = \underbrace{K\cdot e^{-t/\tau}}_{\text{transient response}} + \underbrace{M\cdot\cos(1000\cdot t + \phi_2)}_{\text{steady-state response}} & (t>0) \end{cases}$$

$$B = A\cdot\frac{\omega C}{\sqrt{(R_1+R_2)^2+\omega^2C^2}}$$

$$M = A\cdot\frac{\omega C}{\sqrt{R_1^2+\omega^2C^2}}, \quad \tau = R_1\cdot C$$

Stability

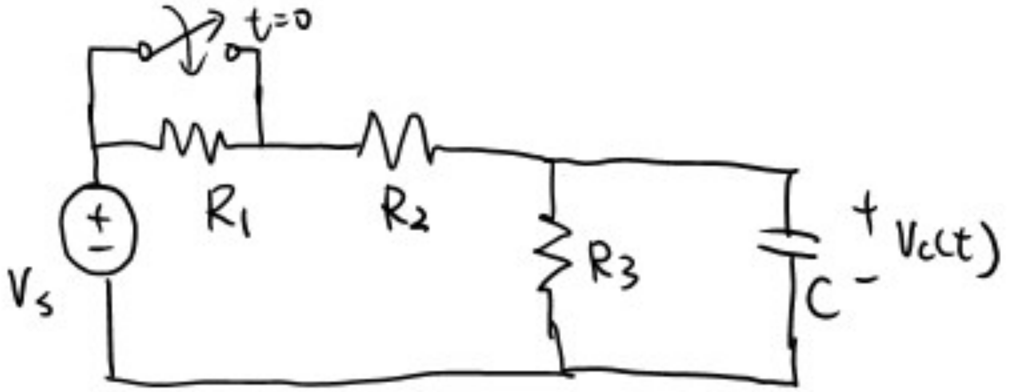
transient decays to 0

$$\begin{cases} \tau > 0, \quad x_n(t) = K e^{-t/\tau} \to 0, \quad \text{as } t \to \infty \implies \text{stable.} \\ \\ \tau < 0, \quad x_n(t) = K e^{-t/\tau} \to \infty. \quad \text{as } t \to \infty \implies \text{unstable} \end{cases}$$

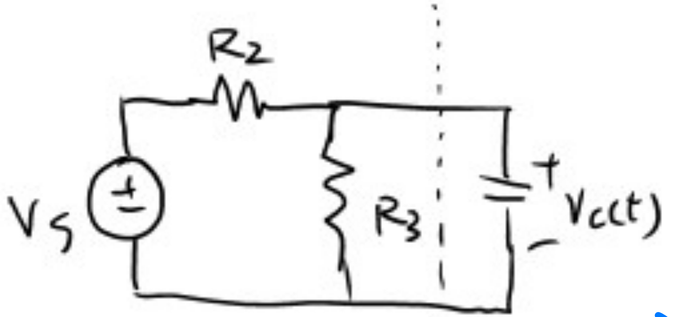
transient grows unbounded

- Steady-state response depends on the input, which contains the information of the input.
- In unstable circuit, information is lost because transient response is too large.

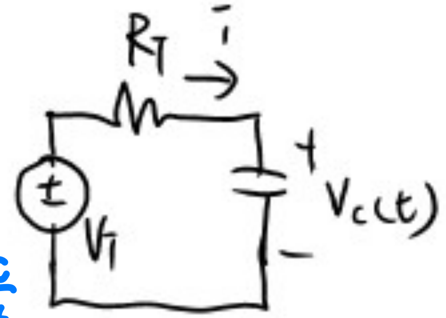
⑤ 1st order RC circuit to a constant input



$t=0,\ V_c = V_s \cdot \frac{R_3}{R_1+R_2+R_3} = V_c(0)$



Thevinin's Thm $\longrightarrow$



$V_T = V_s \cdot \frac{R_3}{R_2+R_3}$

$R_T = R_2 /\!/ R_3 = \frac{R_2 R_3}{R_2+R_3}$

$Q = CV_c \Rightarrow i = C\frac{dV_c}{dt}$

time constant

$$V_T = i \cdot R_T + V_c(t) = R_T \cdot C\frac{dV_c}{dt} + V_c \Longrightarrow \frac{dV_c}{dt} + \frac{V_c}{R_T \cdot C} = \frac{V_T}{R_T \cdot C} \quad \left(R_T C = \tau \quad \frac{V_T}{R_T \cdot C} = K \right) \text{ Let}$$

$$\frac{dV_c}{dt} + \frac{1}{\tau}\cdot V_c = K \rightarrow \left[\frac{dV_c}{dt} = \frac{K\tau - V_c}{\tau} \rightarrow \int \frac{1}{K\tau - V_c}\, dV_c = \int \frac{1}{\tau}\, dt \rightarrow \ln(V_c - \tau) = -\frac{t}{\tau} + T_0 \right]$$

$$\underline{V_c(t) = e^{-\frac{t}{\tau}} e^{T_0} + K\tau = A \cdot e^{-t/\tau} + K\tau}$$

solve differential equation

$$V_c(t) = A e^{-t/\tau} + k\tau.$$

$$\begin{cases} t=0, & V_c(0) = A + k\tau \rightarrow A = V_c(0) - k\tau \\ t=\infty, (t>3\tau) & V_c(\infty) = k\tau \end{cases}$$

$$V_c(t) = (V_c(0) - k\tau)e^{-t/\tau} + k\tau = \underbrace{V_c(\infty)}_{\text{steady-state}} + \underbrace{(V_c(0) - V_c(\infty))e^{-t/\tau}}_{\text{transient}} \quad \tau = R_T \cdot C.$$

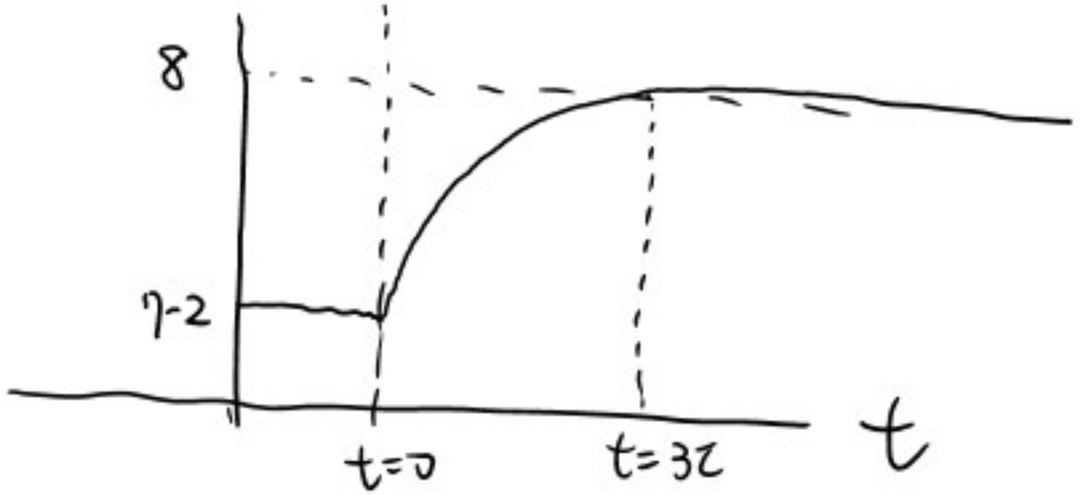
Ex $V_s = 12V$, $R_1 = 10k\Omega$, $R_2 = 30k\Omega$, $R_3 = 60k\Omega$, $C = 2\mu F$

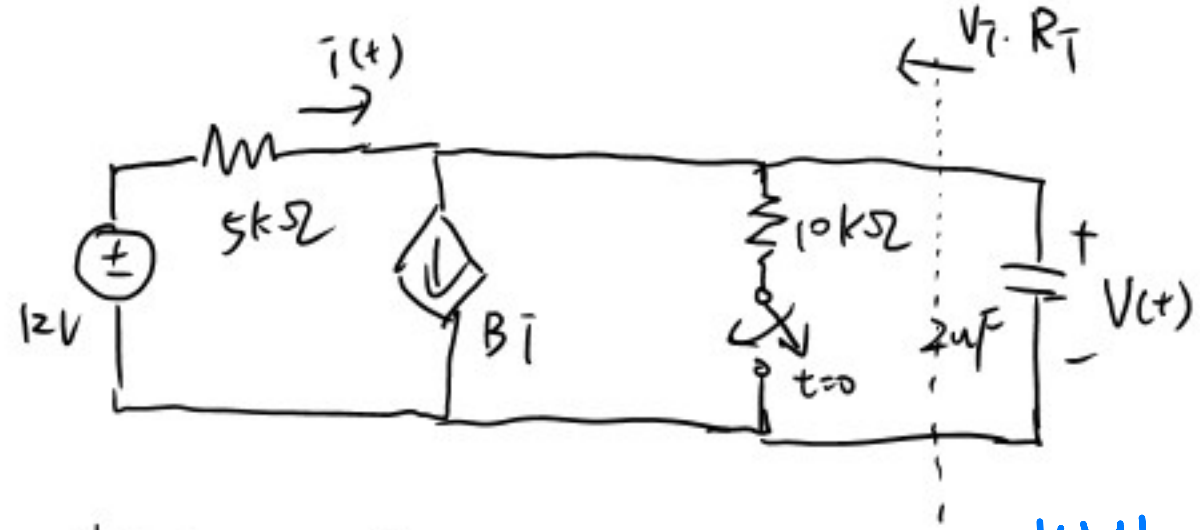
$$V(0) = 12 \cdot \frac{60}{10+30+60} = 7.2V, \quad \tau = R_T \cdot C = \frac{R_2 \cdot R_3}{R_2 + R_3} \cdot C = \frac{30(60)}{30+60} \times 10^3 \times 2\times10^{-6} = 40\times10^{-3} = 40\,ms$$

$$V(\infty) = k\cdot\tau = V_T = \frac{60}{30+60} \cdot 12 = 8V$$

$$V_c(t) = 8 + (7.2 - 8)e^{-t/40ms}$$

$$= 8 - 0.8\, e^{-t/0.04} \quad (V)$$

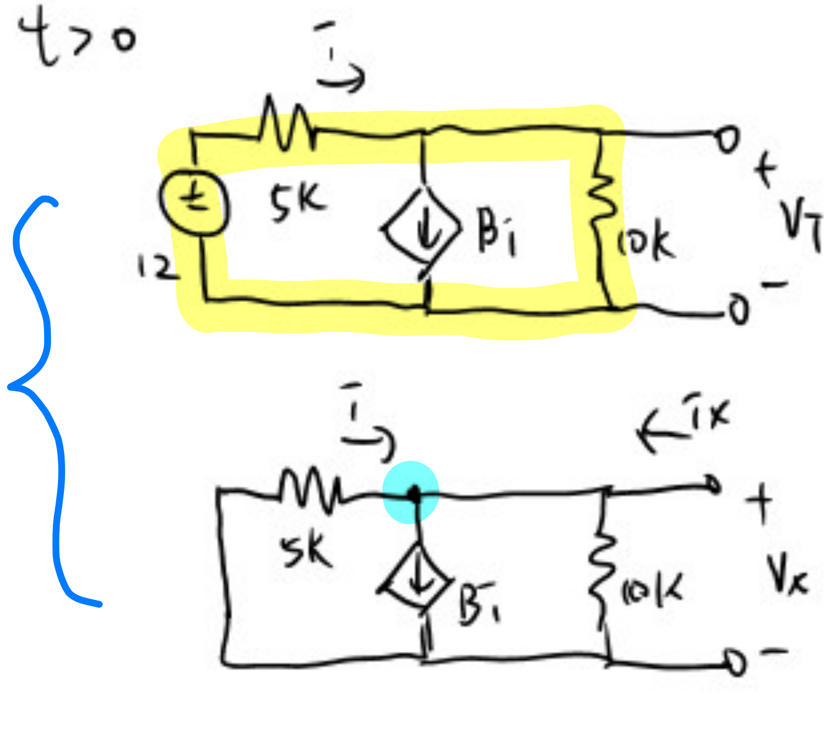




Design B so that the circuit is stable with $\tau = 20ms$

$V(0) = 12V$

t>0



KVL

$$12 = 5i + 10(i - Bi) \rightarrow i = \frac{12}{15-10B}, \quad V_T = 10(i - Bi) = \frac{120(1-B)}{15-10B} = \frac{24(1-B)}{3-2B}$$

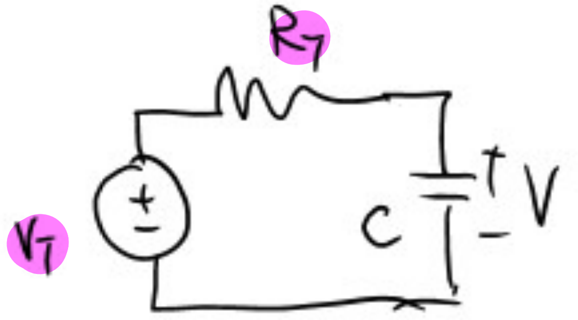
KCL

$$i + i_x = \frac{V_x}{10} + Bi \rightarrow i_x = \frac{V_x}{10} + (B-1)i = \frac{V_x}{10} + (B-1)\frac{-V_x}{5}$$

$$= V_x \cdot \frac{3-2B}{10}$$

$$V_x = -5i \rightarrow i = -\frac{V_x}{5},$$

$$R_T = \frac{V_x}{i_x} = \frac{10}{3-2B} \; (k\Omega)$$



$$\tau = R_T \cdot C > 0 \rightarrow \frac{10}{3-2B} \cdot C > 0 \rightarrow 3-2B > 0 \rightarrow B < 1.5 \quad \text{(stable)}$$

$$\tau = 20ms = \frac{10}{3-2B} \cdot C = \frac{10 \times 10^3}{3-2B} \cdot 2\times10^{-6} \rightarrow B = 1$$

$$V_T = \frac{24(1-B)}{3-2B} = 0 = V(\infty) \qquad \tau = 20\,ms$$

(steady state $V(\infty)$)

the circuit is stable when $\tau = 20\,ms$

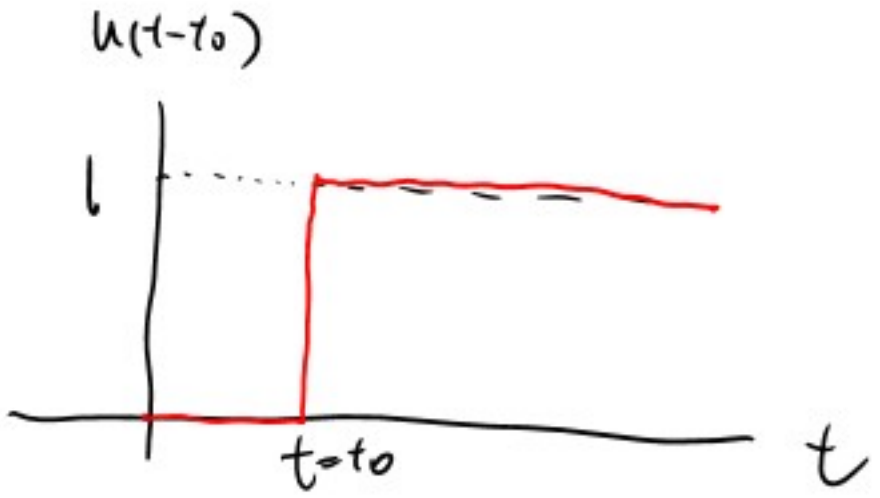
$$V(t) = \underset{V_c(\infty)}{0} + (\overset{V_c(0)}{12} - \underset{V_c(\infty)}{0}) \cdot e^{-t/\tau} = 12e^{-t/0.02}\ (V)$$

$$V = IR \Rightarrow I = \frac{V}{R}$$

$$i(t) = \frac{12 - V(t)}{5} = 2.4 - 2.4e^{-t/0.02}\ (mA)$$

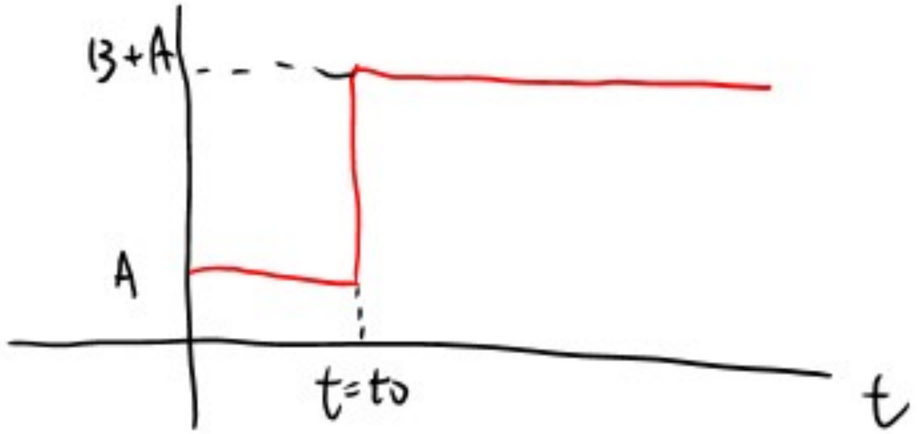
⑥ unit step response

$$u(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$



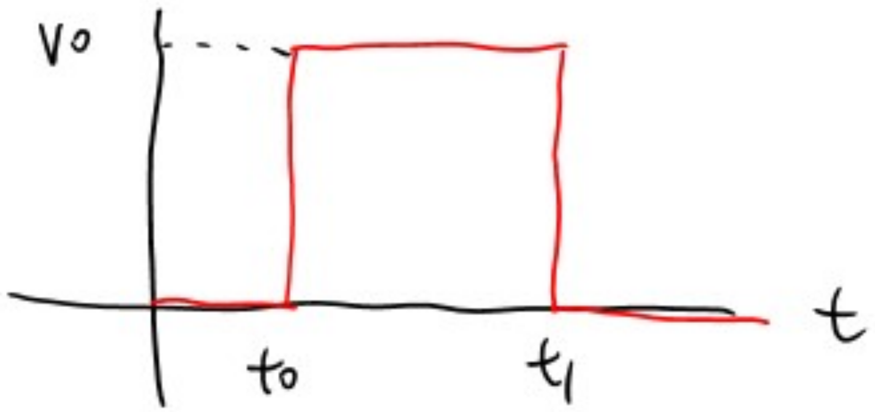
$$V_s(t) = A + B\,u(t-t_0)$$

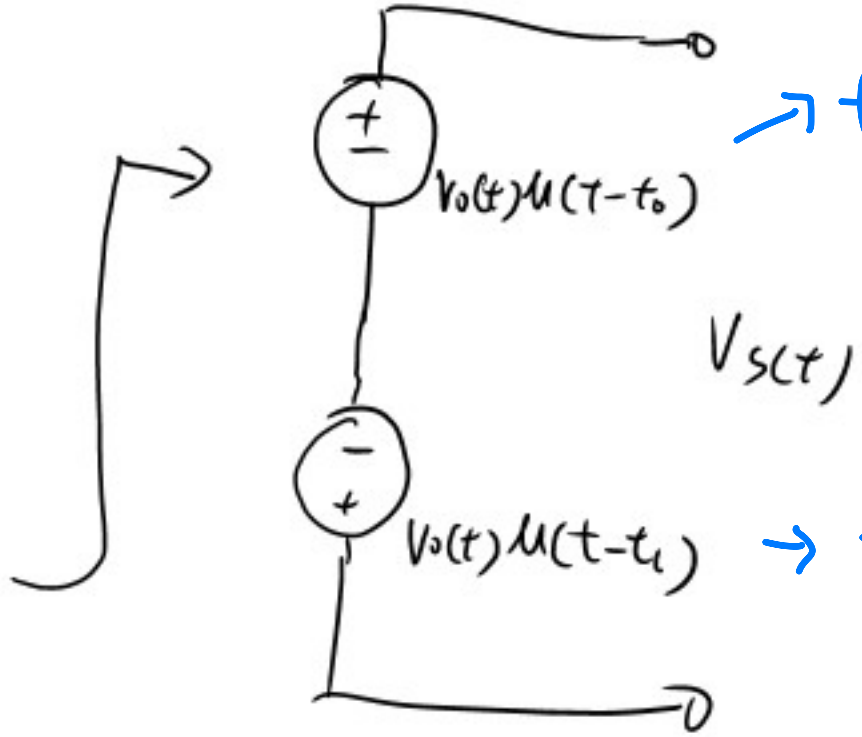
For switch $t_0 = 0$



pulse source

$$V_s(t) = V_0(t-t_0) - V_0(t-t_1)$$





→ turns on V_0 at t_0

→ turns on $-V_0$ at t_1