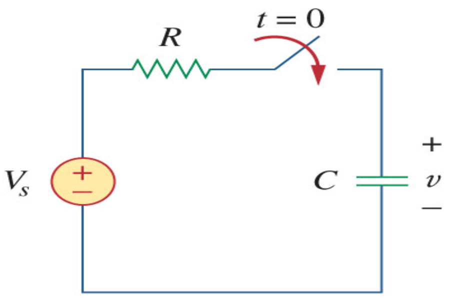
Lab 01: RC and RLC Circuit

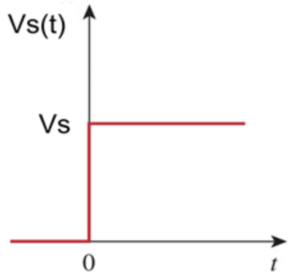
【Purpose】

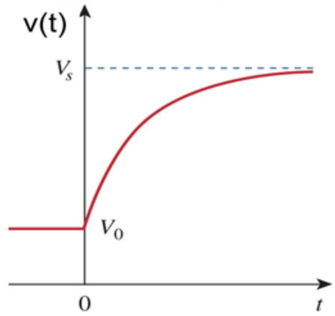
Understand the basic analysis of AC circuit and characteristic of passive circuit element. Observe charge and dis-charge of RC circuit and frequency response of RLC circuit.

【Theory】

RC circuit







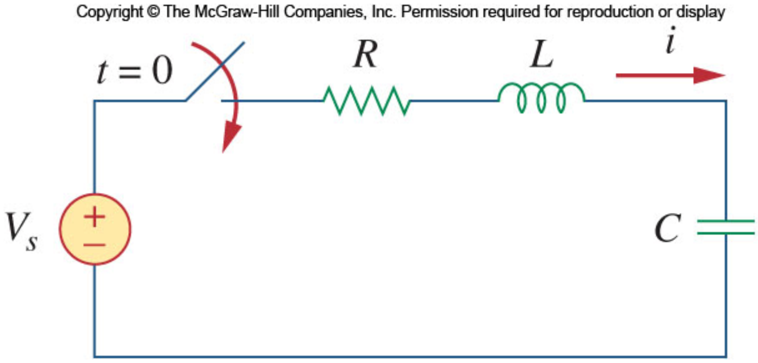
$$RC\frac{dv(t)}{dt} + v(t) = Vs(t) \;\Rightarrow\; \frac{dv(t)}{dt} + \frac{v(t)}{RC} = \frac{Vs(t)}{RC}$$

$$v(t) = \begin{cases} Vs(0) = 0 & t < 0 \\ Vs + (0 - Vs)e^{-t/\tau} & t > 0 \end{cases} \qquad \tau = RC$$

time constant

$= Vs(1 - e^{-t/\tau})$

RLC circuit



$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = Vs(t) \;\Rightarrow\; \frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{t} + \frac{1}{LC}i(t) = \frac{Vs(t)}{L}$$

$$Z = R + jLw + \frac{1}{jCw} = R + j\left(Lw - \frac{1}{Cw}\right) \qquad V_R = R\frac{Vs}{Z} = \frac{R}{R + j\left(Lw - \frac{1}{Cw}\right)}Vs$$

$$amplitude = \frac{R}{\sqrt{R^2 + \left(Lw - \frac{1}{Cw}\right)^2}}Vs \qquad \emptyset = -\tan^{-1}\left(\frac{Lw - \frac{1}{Cw}}{R}\right)$$

$w_0 = {}^{1}/_{\sqrt{LC}}$ $\quad$ $\Delta w = {}^{R}/_{L}$ → curve width at half-width point

↳ resonance angular frequency

【Instruments】

Oscilloscope(示波器)、Function generator(訊號產生器)、Resistor(10KΩ,100Ω)、Capacitor (0.1 uF)、Inductor (10mH)

RC Circuit

1° Definition of Capacitance

Capacitance is defined as the ability of a capacitor to store charge per unit voltage

$$C = \frac{Q}{V} \quad \left(\begin{array}{l} C: \text{capacitance (Farads)} \\ Q: \text{charge stored on capacitor (Coulombs)} \\ V: \text{voltage across the capacitor (volts)} \end{array} \right)$$

$$\Rightarrow Q = CV$$

2° Current as the Rate of Change of Charge

$$i \text{ (current)} = \frac{dQ}{dt} \xrightarrow[Q=CV]{} i = \frac{d(CV)}{dt} = C\frac{dV}{dt}$$

3° Kirchhoff's Voltage Law

(the sum of the voltage drop across the resistor and capacitor must equal the supplied voltage)

$$V_R + V_C = V_S(t)$$

1. Ohms Law
   $V = R\,i(t)$
2. $i(t) = C\dfrac{dv(t)}{dt}$

$$RC\frac{dv(t)}{dt} + v(t) = V_S(t)$$

(1st-order linear differential equation)

4° Using standard method for solving 1st-order linear differential equations, the solution is

$$v(t) = V_s + (V_0 - V_s)\, e^{-t/\tau}$$

V_0 (initial capacitor voltage) = 0

$$v(t) = V_s\,(1 - e^{-t/\tau})$$

where $\tau = RC$ (time constant, determining how fast the capacitor charges)

This equation shows:

- **At** $t = 0$, $v(0) = 0$ (initially uncharged).
- **At** $t = \tau$, $v(\tau) = 0.63V_s$ (63% charged).
- **At** $t = 5\tau$, $v(5\tau) \approx 0.99V_s$ (almost fully charged).

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3. Meaning of $\tau = RC$ **(Time Constant)**

- The time constant τ determines how quickly the capacitor charges or discharges.
- **Larger** τ (large R or C) → **slower charging**.
- **Smaller** τ (small R or C) → **faster charging**.

Conclusion

The equation

$$RC\frac{dv(t)}{dt} + v(t) = V_s(t)$$

describes how the capacitor charges or discharges over time. The time constant $\tau = RC$ dictates the speed of this process. At high frequencies, the capacitor does not have enough time to fully charge, which is why its voltage response depends on frequency.

RLC Circuit

1. Kirchoff's Voltage Law

$$V_R + V_L + V_C = V_s(t)$$

$V_R = R\,i(t)$

$V_L = L\dfrac{di(t)}{dt}$

$V_C = \dfrac{1}{C}\int i(t)\,dt$

$$R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt = V_s(t)$$

2° Taking the derivative of the entire equation

$$R\frac{di(t)}{dt} + L\frac{d^2 i(t)}{dt^2} + \frac{1}{C}i(t) = \frac{dV_s(t)}{dt}$$

(2nd-order differential equation)

3° Impedance Representation (Frequency Domain)

Using phasor analysis in the frequency domain, the impedance of each component is:

Resistor $Z_R = R$

Inductor $Z_L = j\omega L$

Capacitor $Z_C = \dfrac{1}{j\omega C}$

The total impedance of the series RLC circuit is:

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L + \frac{1}{\omega C}\right)$$

(This shows the reactance of the inductor (ωL) & capacitor ($1/\omega C$) oppose each other)

4° V_R (Voltage across resistor) $= RI$

$I = \dfrac{V_s}{Z}$

$$V_R = \frac{R}{Z}V_s$$

Expanding Z

$$V_R = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}V_s$$

5°

amplitude $|V_R| = \frac{R}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}} |V_S|$

phase shift $\phi = -\tan^{-1}\left(\frac{wL - \frac{1}{wC}}{R}\right)$

At resonance frequency W_0, the inductive reactance X_L & capacitive reactance X_C cancel out:

$$W_0 L = \frac{1}{W_0 C} \Rightarrow W_0 = \frac{1}{\sqrt{LC}}$$

The corresponding resonance frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \ (Hz)$$

At resonance, the circuit behaves like a pure resistor, and the output voltage is maximized

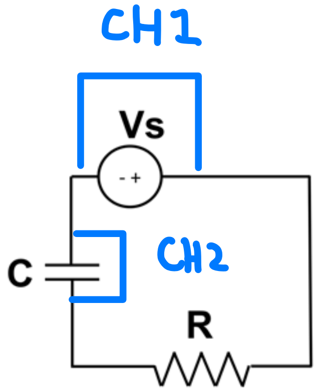
6° The bandwidth (Δw) of the circuit, determining how selective the resonance peak is, is:

$$\Delta W = \frac{R}{L}$$

A higher resistance R results in broader bandwidth (less selective resonance), and vice versa.

c: capacitor, R: resistor, V_s: signal generator

【Steps】

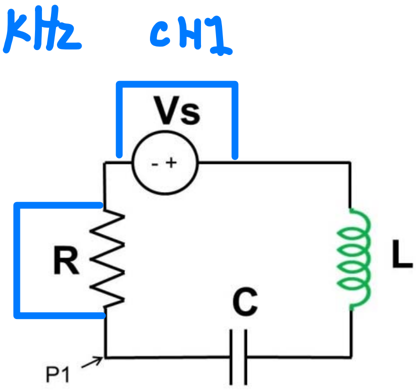


RC circuit

1. Connect the circuit. (R=10 KΩ，C= 0.1 uF)
2. Set function generator to square wave (Vs). Set the frequency to the inverse of time constant of RC circuit (τ =1 msec) and adjust signal level to get around 2 V peak-to-peak
3. Observe the output voltage across capacitor. Adjust the frequency to observe waveform of charge and ~~ids-charge~~ discharge pattern. Plot the waveform.
4. Change the frequency of square wav. Plot the waveform and measure the peak-to-peak voltage or peak voltage of the output volatge.

$f = 1/2\pi\sqrt{LC} \approx 5.04$ kHz

RLC Circuit



1. Connect the circuit. (R=100Ω，C=0.1 uF，L=10 mH)
2. Set function generator to sinusoidal wave (Vs)，Set the frequency to the resonance frequency of RLC circuit and adjust signal level to get around 2 V peak-to-peak

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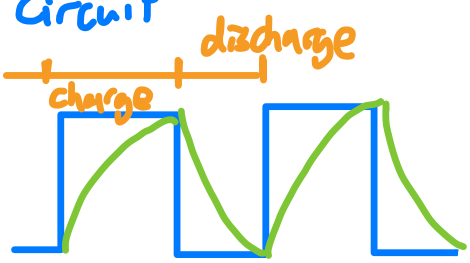
3. Adjust the frequency in the range between 500Hz to 10Khz and observe the wave form of output voltage across resistor. Measure the peak voltage (or peak-to-peak voltage) and phase for different frequencies.
4. Make a plot with the peak voltage (y-axis) versus frequency (x-axis).

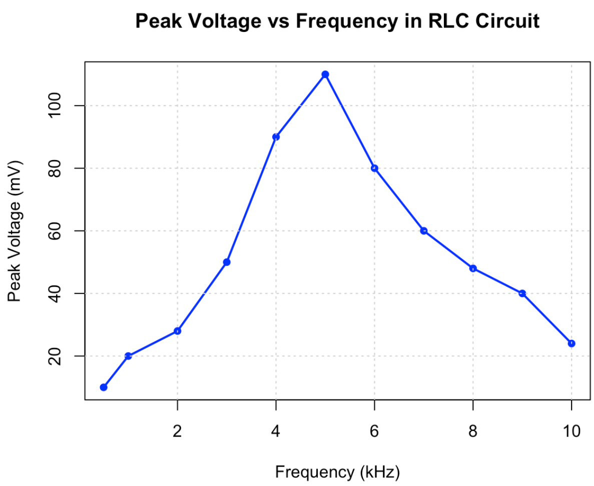
L: Inductor

【Questions】

1. In the RC circuit, what is the influence of the frequency of square wave on the peak voltage of capacitor?
2. In the RLC circuit, how to estimate the frequency when the voltage decrease to $1/\sqrt{2}$ of it max value (As $\omega_{\pm}$ in figure 2) ?
3. The RLC circuit is considered as a band pass filter. The resonance frequency is central frequency and bandwidth is defined as the difference of frequency between $1/\sqrt{2}$ points (As Δw in figure 2). How to estimate the bandwidth?

RC circuit



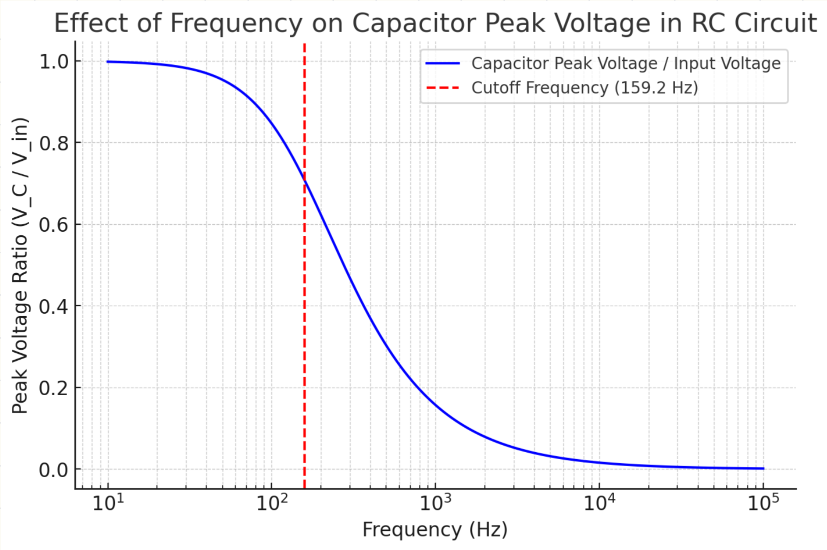


RLC circuit 3.

0.5	1	2	3	4	5	6	7	8	9	10	(kHz)
10	20	28	50	90	110	80	60	48	40	24	(mV)

[Questions]

1.



The plot shows how the **peak voltage across the capacitor (V_C)** changes as the frequency of the input square wave increases.

- **At low frequencies ($f \ll f_c$):**
  - The capacitor fully charges and discharges, so V_C **is nearly equal to** V_{in} (ratio ≈ 1).
- **At the cutoff frequency ($f_c = 159.2$ Hz, red dashed line):**
  - The capacitor voltage drops to $1/\sqrt{2}$ **(~0.707) of the input voltage**.
  - This marks the transition from full charging to incomplete charging.
- **At high frequencies ($f \gg f_c$):**
  - The capacitor has very little time to charge before the input switches.
  - The peak voltage across the capacitor **decreases significantly** (approaching zero).
  - The capacitor behaves more like a **short circuit**.

This explains why increasing the square wave frequency **reduces the peak voltage across the capacitor** in an RC circuit.

2.

In the RLC circuit, the **resonance frequency** f_0 is where the output voltage across the resistor is maximum. The frequencies at which the output voltage drops to $1/\sqrt{2}$ (or about **0.707 times**) of its maximum value are called the **half-power frequencies** f_+ and f_-.

To estimate these frequencies experimentally:

1. **Measure Peak Voltage at Resonance:**
   - Set the function generator to the resonance frequency f_0.
   - Measure the **peak voltage** V_{max} across the resistor.
2. **Find the -3 dB Points:**
   - Slowly **decrease** the frequency from f_0 until the voltage drops to $V_{max}/\sqrt{2}$. Record this frequency as f_-.
   - Slowly **increase** the frequency from f_0 until the voltage drops to $V_{max}/\sqrt{2}$. Record this frequency as f_+.

These frequencies are also called the **-3 dB frequencies** because a drop to $1/\sqrt{2}$ corresponds to a **3 dB decrease** in power.

3.

The **bandwidth** (Δf) of the RLC circuit, which acts as a **band-pass filter**, is defined as:

$$\Delta f = f_+ - f_-$$

where:

- f_+ is the **upper cutoff frequency** (higher $1/\sqrt{2}$ point),
- f_- is the **lower cutoff frequency** (lower $1/\sqrt{2}$ point).

Experimental Steps to Estimate Bandwidth:

1. **Find f_- and f_+ using the method in Question 2.**
2. **Calculate Bandwidth**: Use the equation:
$$\Delta f = f_+ - f_-$$
3. **Check Consistency with Theoretical Value:**
   - The theoretical bandwidth is given by:
$$\Delta f = \frac{R}{2\pi L}$$
   - Using $R = 100\Omega$, $L = 10$ mH:
$$\Delta f = \frac{100}{2\pi \times 10 \times 10^{-3}}$$
$$\approx 1.59 \text{ kHz}$$

Thus, comparing the experimental and theoretical bandwidths helps verify the circuit's behavior.

【Supplement】

Figure 1

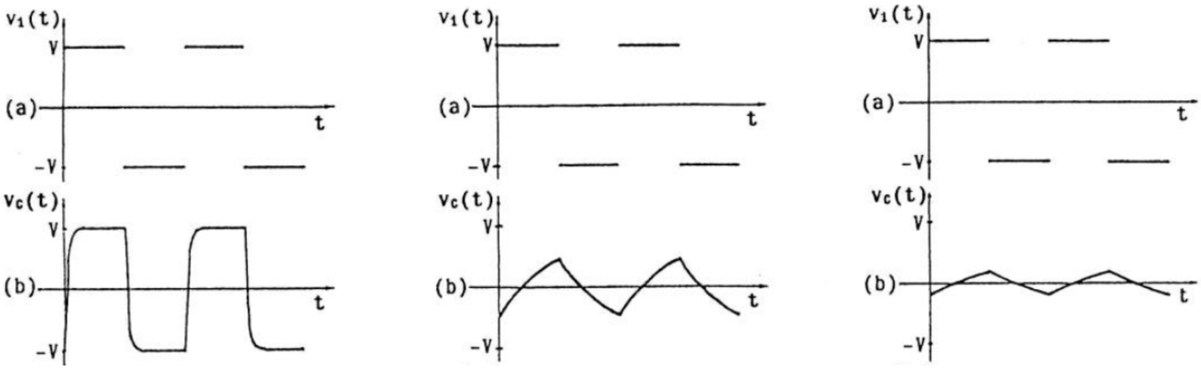


Figure 2