internal resistance of the power source

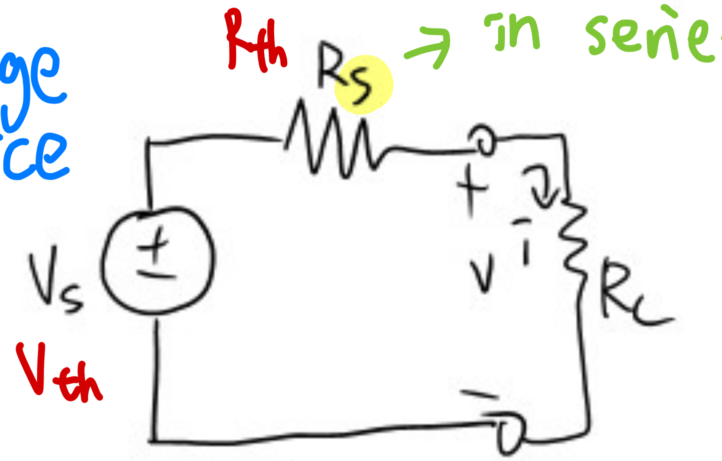
Circuit theorems: a way to reduce the complexity of the circuit

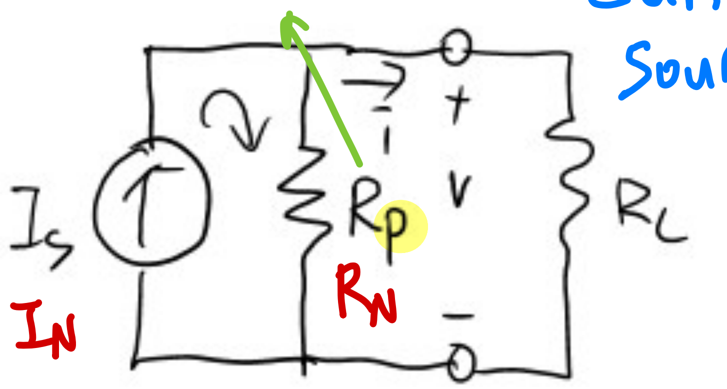
R_L: load resistance 負載电阻

⓪ source transformation — model for practical voltage or current source

R_{th} R_s → in series — in parallel

voltage source — current source





internal resistance lowers the real voltage and current than we expect.

Thevinin's Law — Norton's Law

① $R_L = \infty$ (open circuit)

$V = V_s$ $\qquad V = I_s \cdot R_p$ $\qquad V_s = I_s \cdot R_p \Rightarrow R_p = \dfrac{V_s}{I_s}$

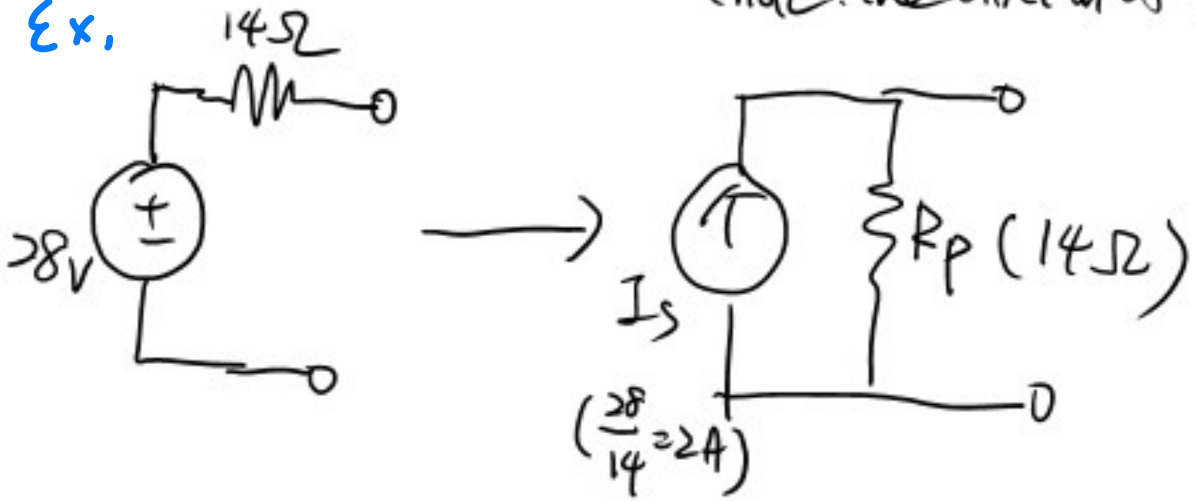
$$\boxed{R_p = R_s = \frac{V_s}{I_s}}$$

② $R_L = 0$ (short circuit)

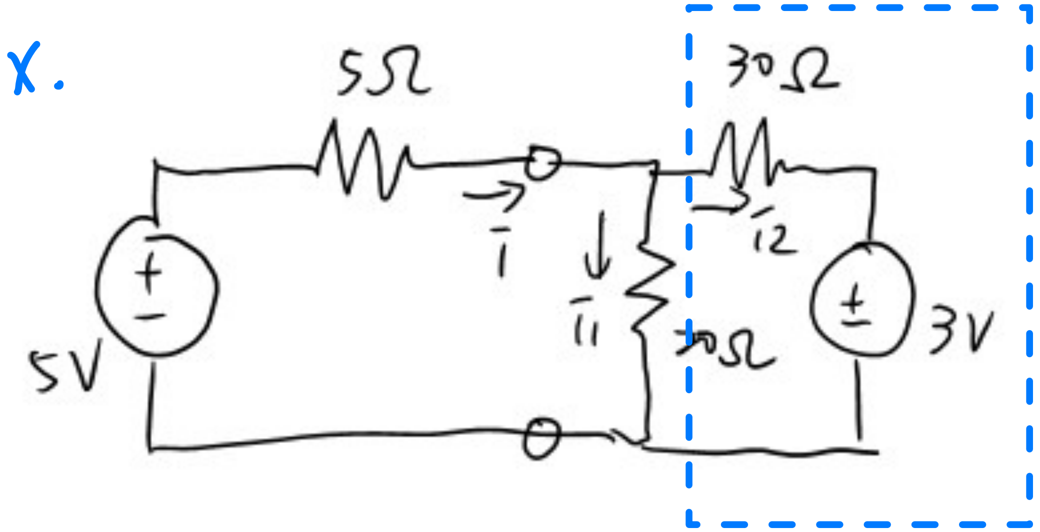
$I = \dfrac{V_s}{R_s}$ $\qquad I = I_s$ $\qquad \dfrac{V_s}{R_s} = I_s \Rightarrow R_s = \dfrac{V_s}{I_s}$

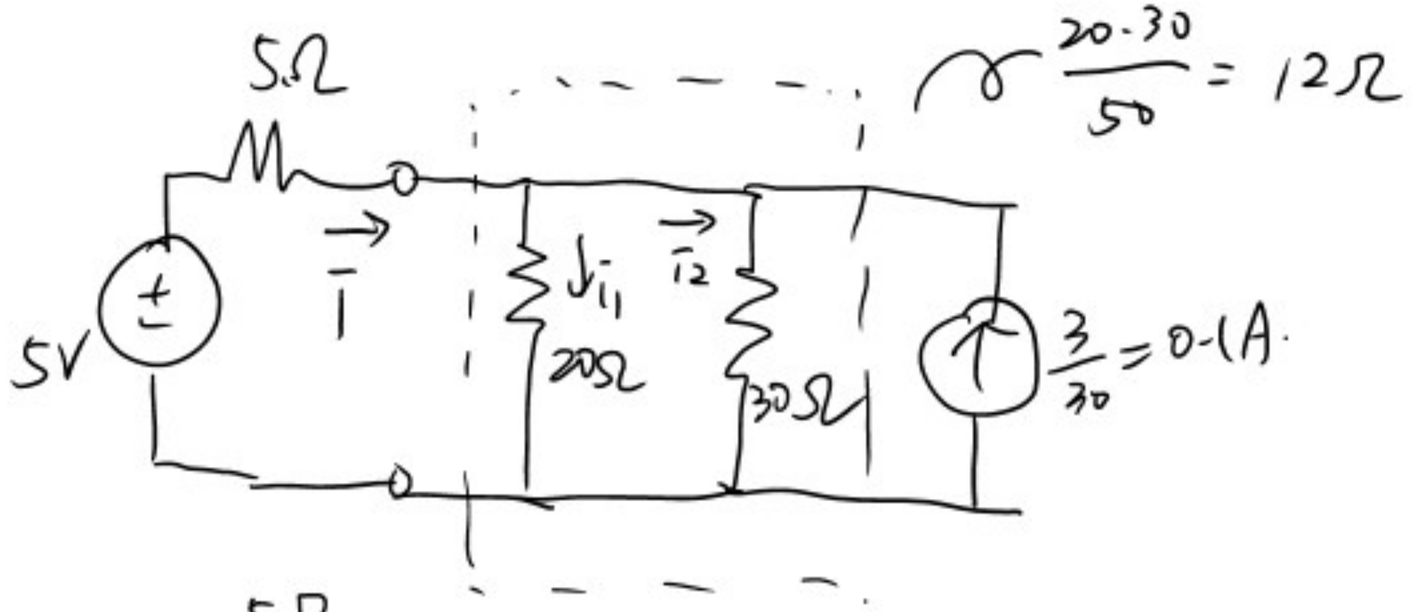
Ex,

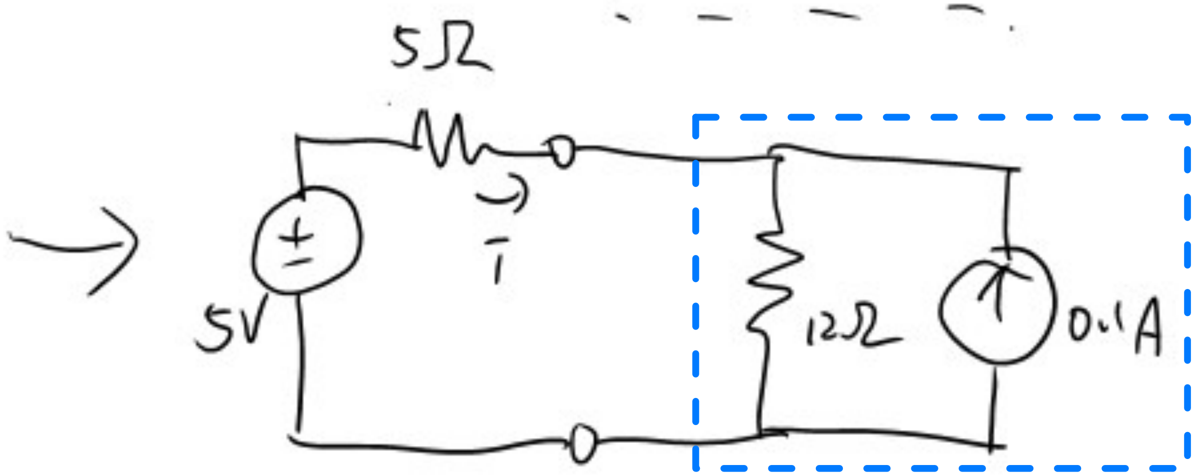
(note the direction of the current)

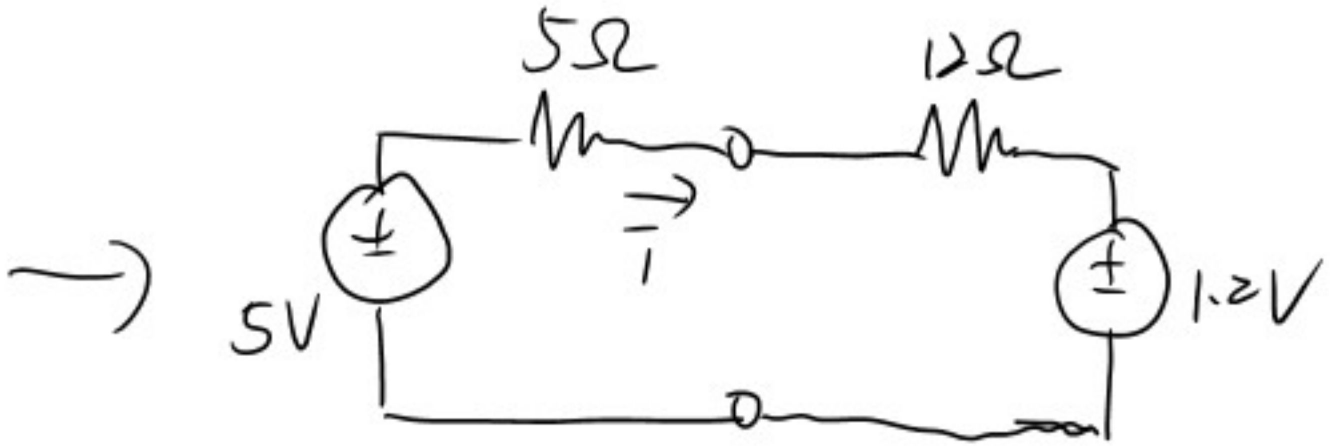


Ex.









$$\bar{I} = \frac{5 - 1.2}{17} = 0.224A$$

superposition: output of the linear circuit can be expressed as a linear combination of its input

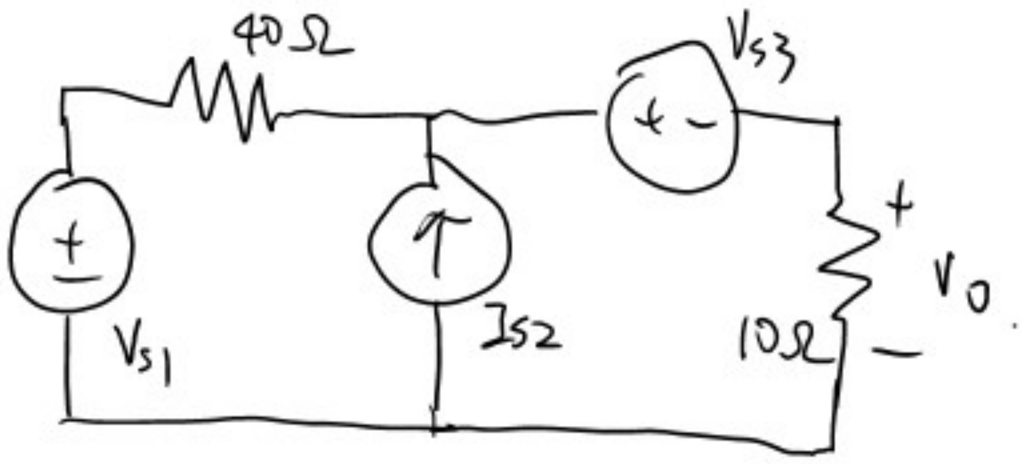
Linear circuit: (satisfies Ohm's law : $V = IR \Rightarrow V \propto I$)

1. consist of only resistor and source (both depedent and independent)
2. inputs are independent sources
3. output can be v or i of an element → circuit element

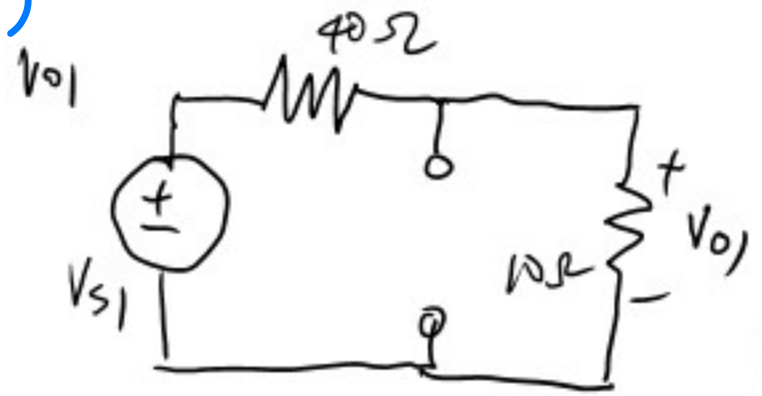
$$V_o \ (I_o) = \underbrace{a_1 V_{s1}}_{V_{o1}} + \underbrace{a_2 V_{s2}}_{V_{o2}} + \underbrace{a_3 I_{s3}}_{V_{o3}} \cdots\cdots a_n V_{sn} \ (\text{or } I_{sn})$$

linear combination

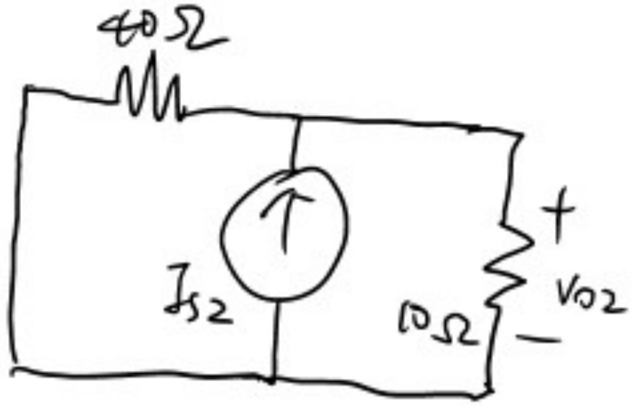
$V_{o1} = a_1 V_{s1}$, when $V_{s2}, I_{s3} \cdots\cdots V_{sn} \ (\text{or } I_{sn}) = 0$



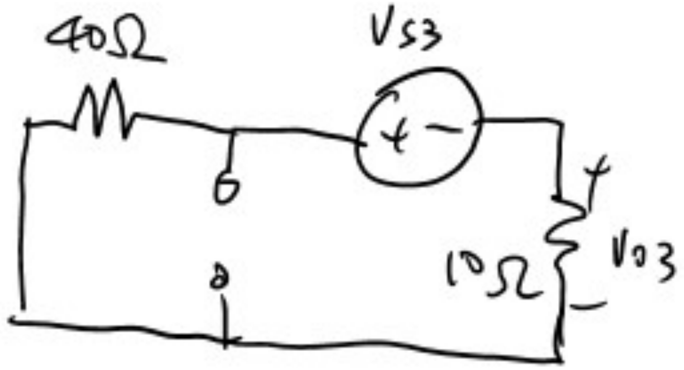
$$V_o = \frac{1}{5} V_{s1} + 8 I_2 - \frac{1}{5} V_{s3}$$



$$V_{o1} = V_{s1} \cdot \frac{10}{50} = \frac{1}{5} V_{s1}$$



$$V_{o2} = I_{s2} \cdot \frac{40 \cdot 10}{40 + 10} = 8 I_{s2}$$



$$V_{o3} = -V_{s3} \cdot \frac{10}{40 + 10} = \frac{-1}{5} V_{s3}$$

* Analyzing the effect of each energy source one at a time, setting others to zero, and summing the contributions.

1. Turning off a Voltage Source → Short Circuit

An **ideal voltage source** always maintains a fixed voltage difference, regardless of the current.

- **Turning it off** means setting its voltage to 0V.
- A 0V voltage source acts like a **short circuit** — there's no voltage drop, so it's just like a wire.

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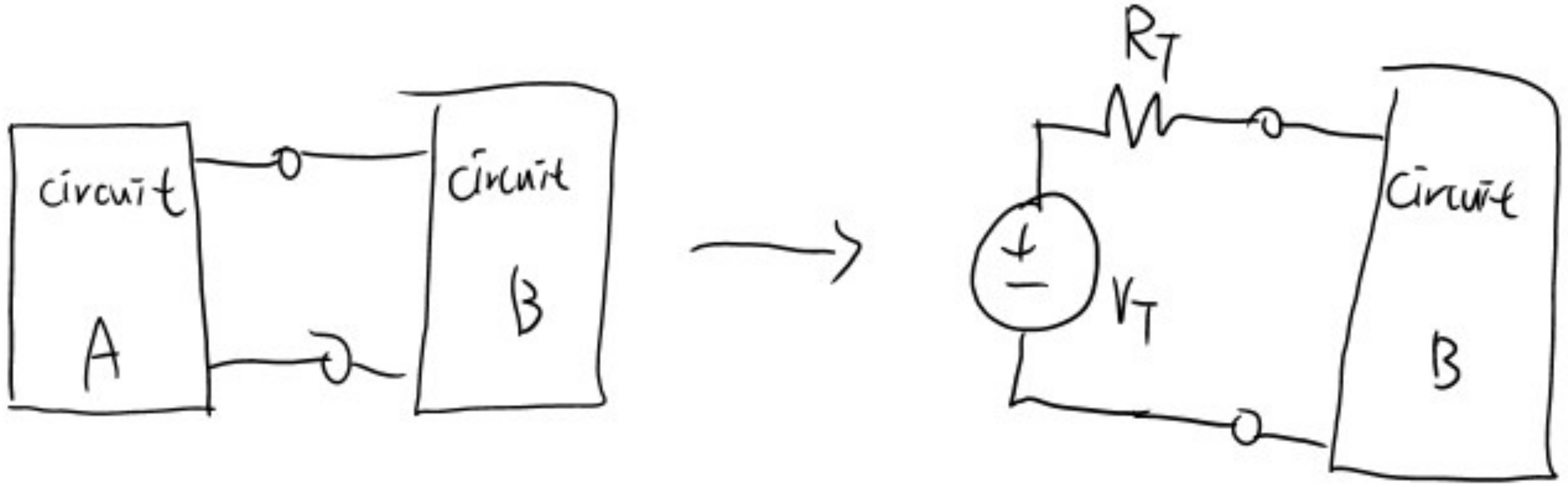
2. Turning off a Current Source → Open Circuit

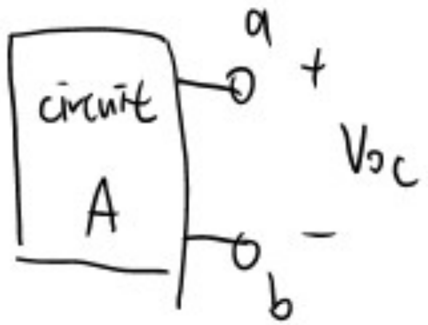
An **ideal current source** always forces a specific current, regardless of the voltage across it.

- **Turning it off** means setting the current to 0A.
- A 0A current source is equivalent to **no current flow**, which is an **open circuit** — as if the wire is cut.

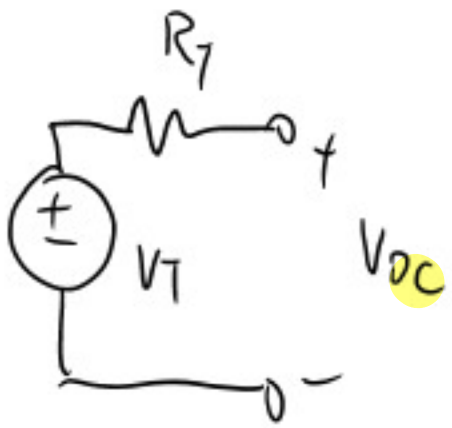
This convention ensures the rest of the circuit behaves naturally when isolating individual sources for superposition analysis.

Thevenin's theorem



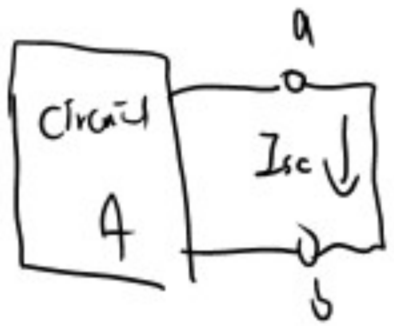


open circuit →

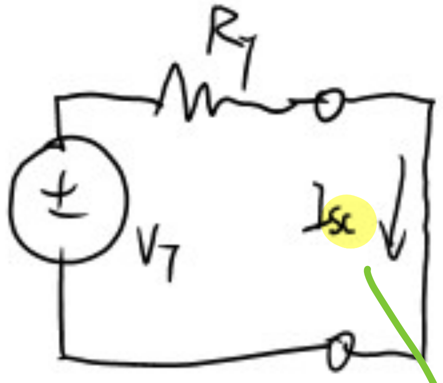


→ open circuit

① $\Rightarrow V_T = V_{oc}$



short circuit →



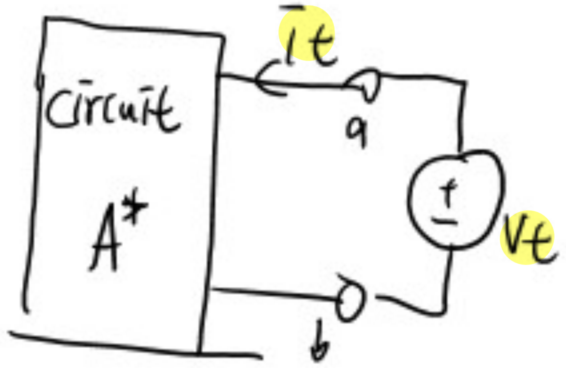
short circuit

② $\Rightarrow V_T = I_{sc} \cdot R_T$

$$R_T = \frac{V_{oc}}{I_{sc}}$$

↓ unsuitable when I_{sc} is difficult to work out

②*



→ test current

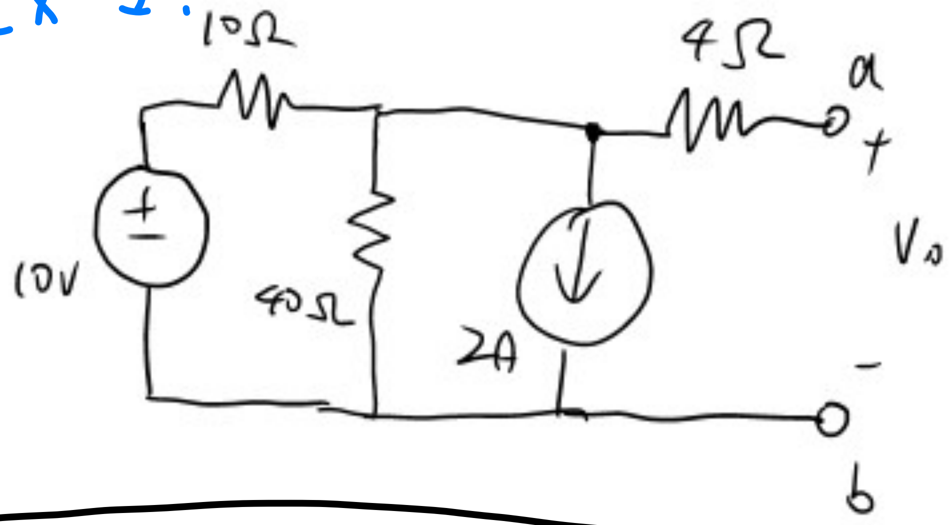
→ test voltage

A^*: all independent sources to zero

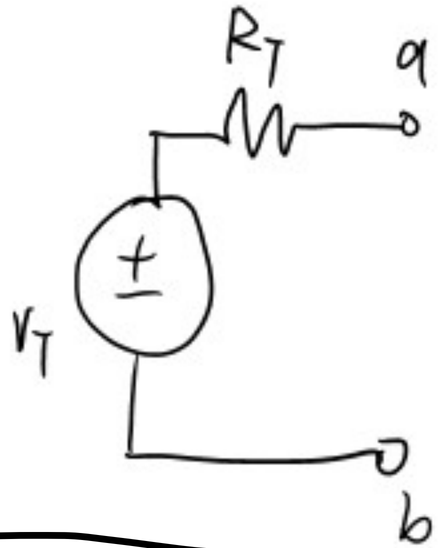
$$R_T = \frac{V_t}{i_t}$$

↓ unsuitable when there are dependent sources in the circuit

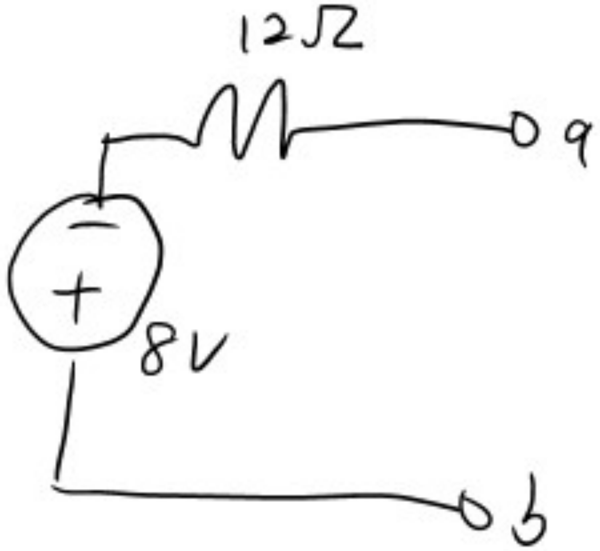
Ex 1:



$\longrightarrow$

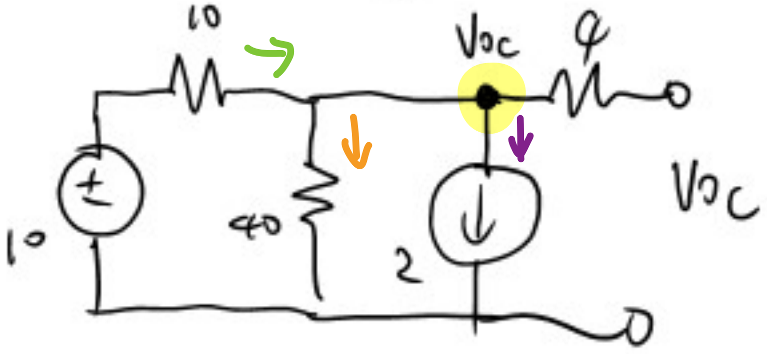


$\longrightarrow$



equivalent circuit

open circuit voltage V_{oc}

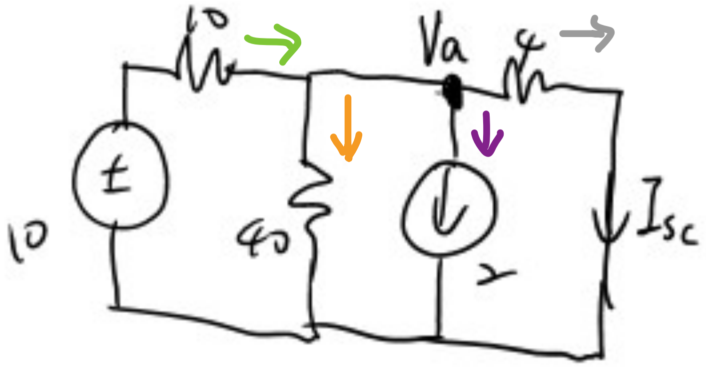


①

KCL: $\dfrac{10 - V_{oc}}{10} = \dfrac{V_{oc}}{40} + 2$

$V_{oc} = -8V$

short circuit current I_{sc}



②

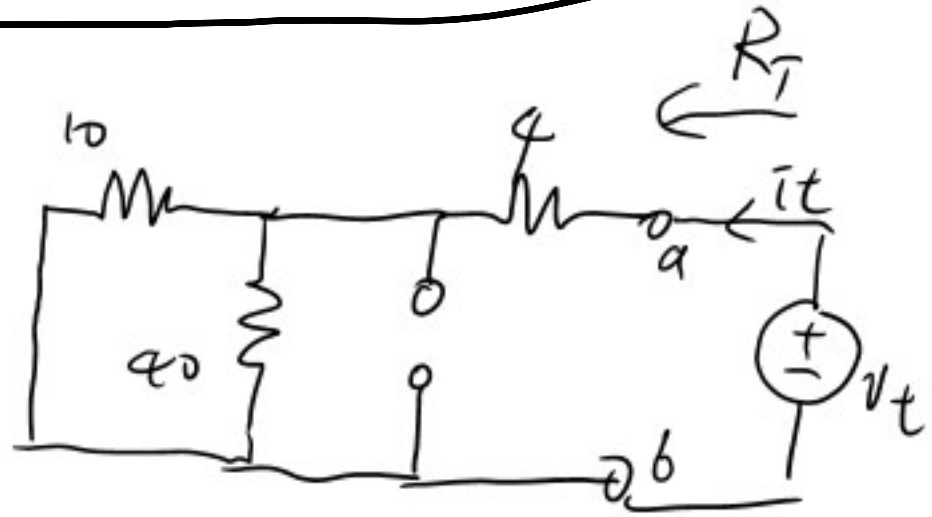
KCL: $\dfrac{10 - V_a}{10} = \dfrac{V_a}{40} + 2 + \dfrac{V_a}{4}$

$V_a = \dfrac{-8}{3}$

$I_{sc} = \dfrac{V_a}{4} = \dfrac{-2}{3}A$

$R_T = \dfrac{-8}{-2/3} = 12\Omega$ #

③*

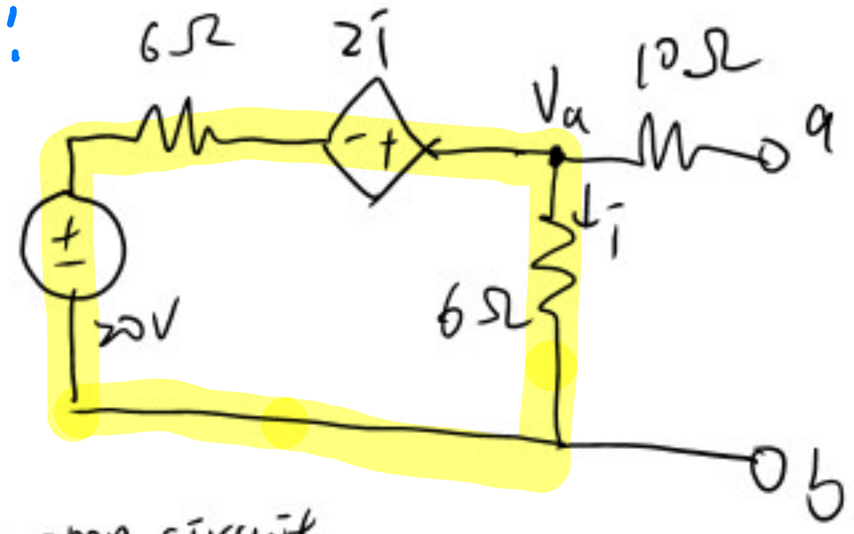


$R_T = \dfrac{V_t}{i_t} = 8 + 4 = 12\Omega$ #

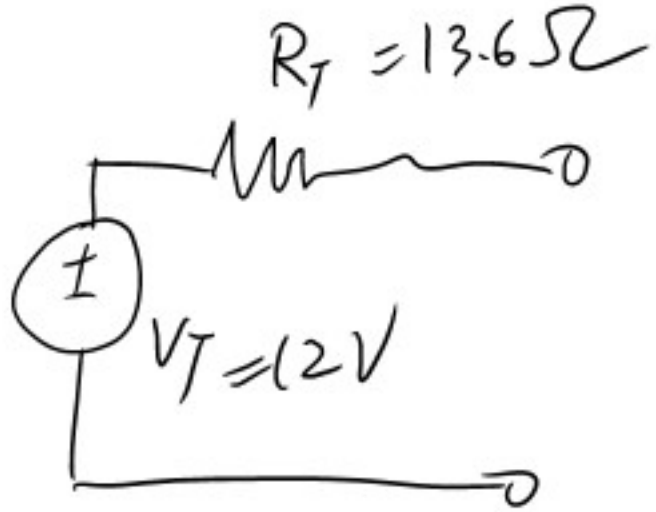
$\downarrow$

$\dfrac{1}{\frac{1}{10} + \frac{1}{40}}$

Ex 2:



$\rightarrow$



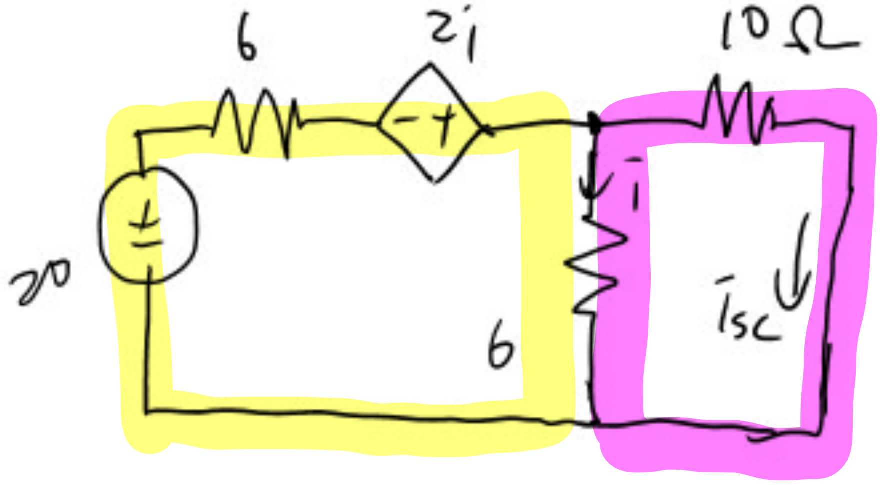
Result

① open circuit

KVL: $20 - 6i + 2i - 6i = 0 \Rightarrow i = 2$ $V_a = V_{oc} = 2 \cdot 6 = 12V = V_T$

→ the equivalent resistance is difficult to work out when $V_s = 0$

② short circuit

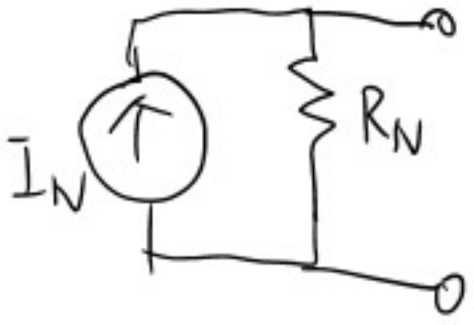


$$\begin{cases} 20 - (i + i_{sc})6 + 2i - 6i = 0 \Rightarrow 20 = 16 i_{sc} - \frac{10}{3} i_{sc} + 10 i_{sc} \\ 6i = 10 i_{sc} \end{cases}$$

$$i_{sc} = \frac{60}{68} = \frac{15}{17} A$$

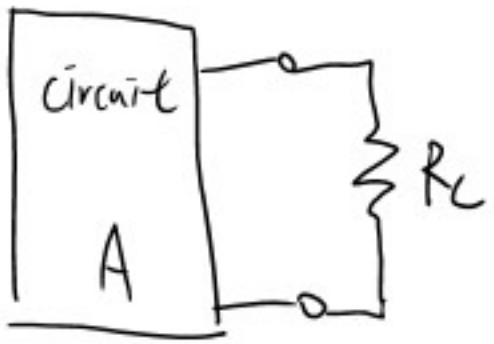
$$R_T = \frac{12}{(15/17)} = \frac{68}{5} = 13.6\Omega \ \#$$

Norton's equivallent circuit

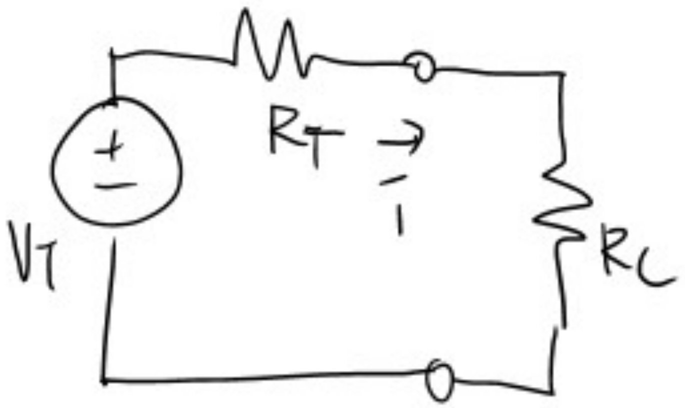


$R_N = R_T$

$I_N = \frac{V_T}{R_T}$



R_L : load resistance



maximum power transfer

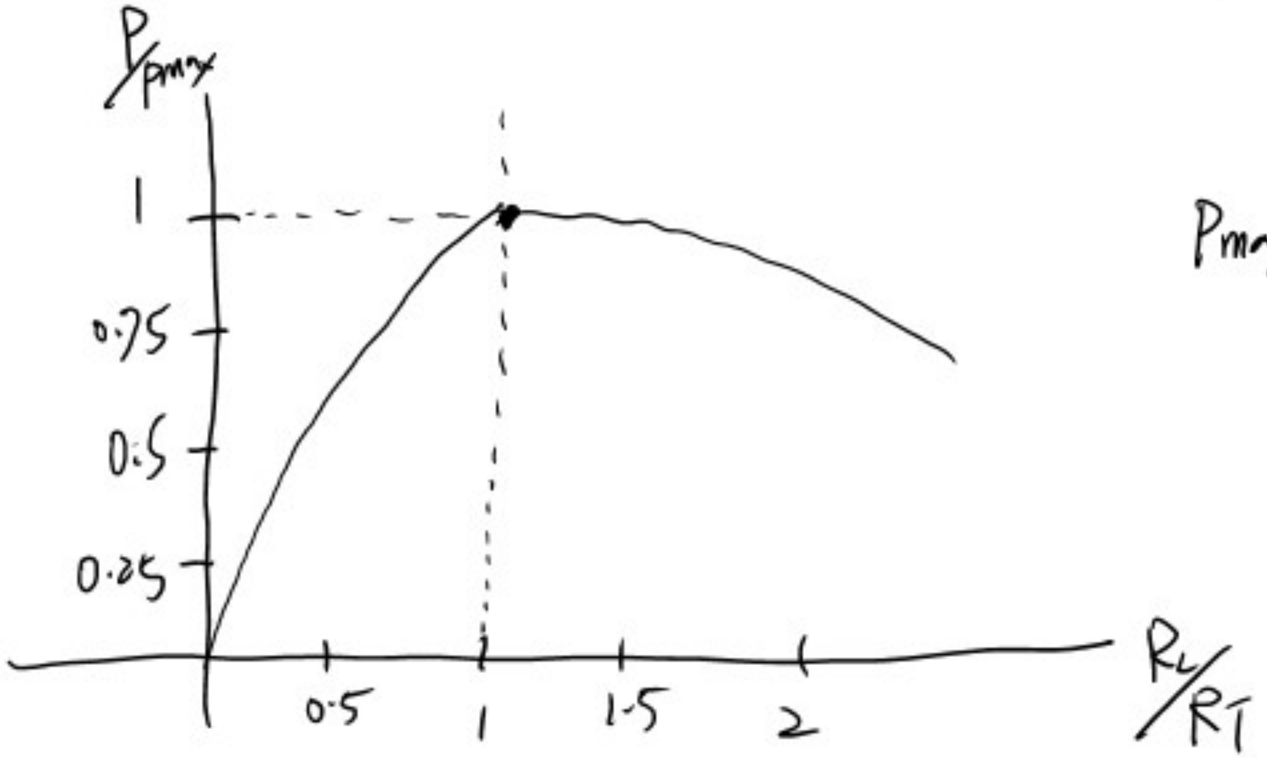
in the case of signal transmission, the problem is to attain maximum signal strength at the load

$$P = i^2 \cdot R_L = \left(\frac{\cancel{R_L}}{R_T + R_L} V_T\right)^2 \cdot R_L$$

let $\frac{dP}{dR_L} = 0$ to find max.

$$\frac{dP}{dR_L} = V_T^2 \left[\frac{1}{(R_T + R_L)^2} - 2\frac{R_L}{(R_T+R_L)^3} \right] \rightarrow R_T - \overset{R_L}{\cancel{R_T}} = 0$$

$$\rightarrow R_T = R_L$$



$$P_{max} = \frac{V_s^2}{4R_T}$$

impedance matching

1. Express the power P in terms of R_L:

- Current in the circuit:

$$i = \frac{V_T}{R_T + R_L}$$

- Power delivered to the load:

$$P = i^2 R_L = \left(\frac{V_T}{R_T + R_L}\right)^2 R_L$$

Expand:

$$P = \frac{V_T^2 R_L}{(R_T + R_L)^2}$$

---

2. Differentiate P with respect to R_L to find maximum:

Using the quotient rule:

$$\frac{dP}{dR_L} = \frac{V_T^2 (R_T + R_L)^2 - V_T^2 R_L \cdot 2(R_T + R_L)}{(R_T + R_L)^4}$$

Simplify numerator:

$$= V_T^2 (R_T + R_L)(R_T - R_L)$$

3. Solve for maximum:

Set:

$$\frac{dP}{dR_L} = 0$$

Thus:

$$(R_T + R_L)(R_T - R_L) = 0$$

→ Physically meaningful solution:

$$R_L = R_T$$

---

4. Maximum power:

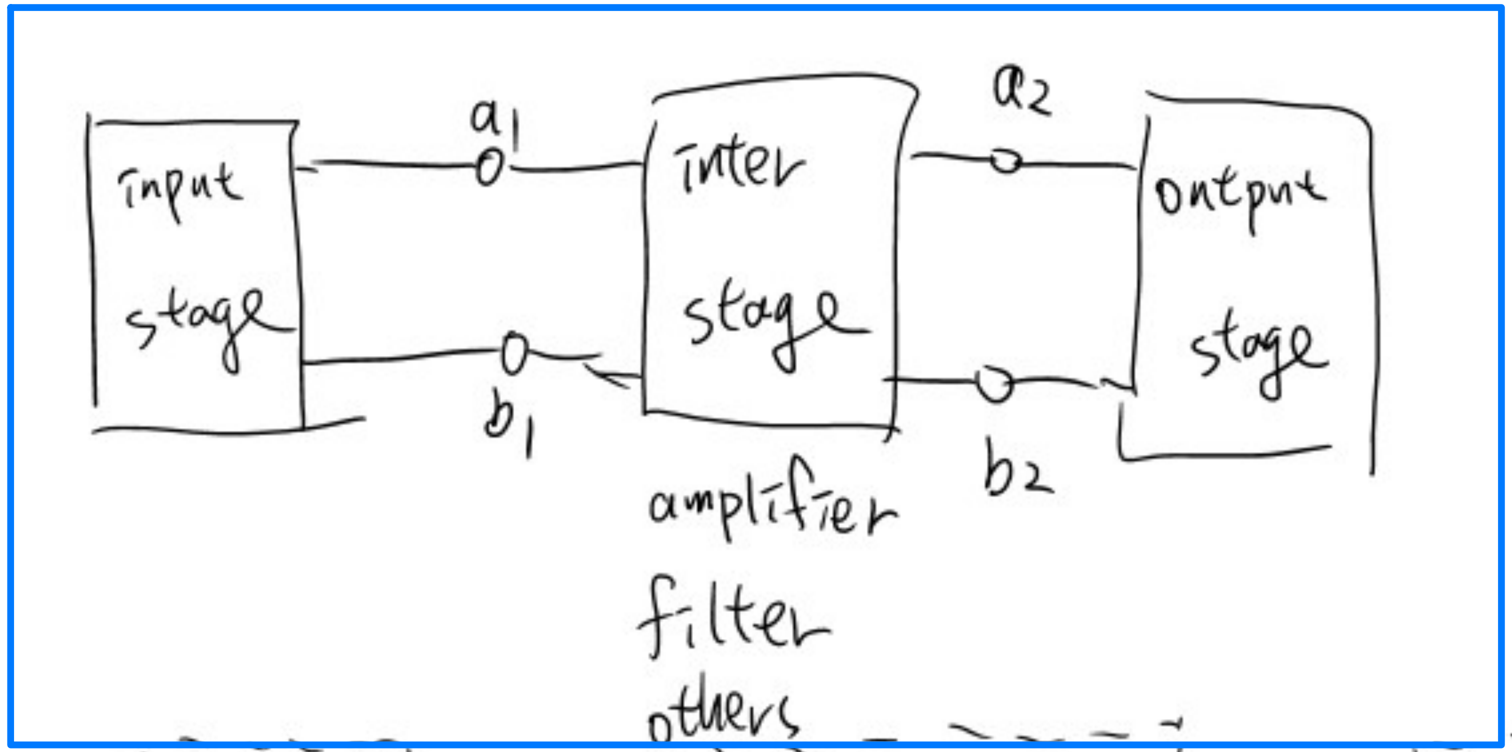
Substituting $R_L = R_T$ into P:

$$P_{\text{max}} = \frac{V_T^2 R_T}{(2R_T)^2} = \frac{V_T^2}{4R_T}$$

✨ Final Conclusion:

- **Condition:** $R_L = R_T$
- **Maximum power:** $P_{\text{max}} = \frac{V_T^2}{4R_T}$

system model and input/output resistance

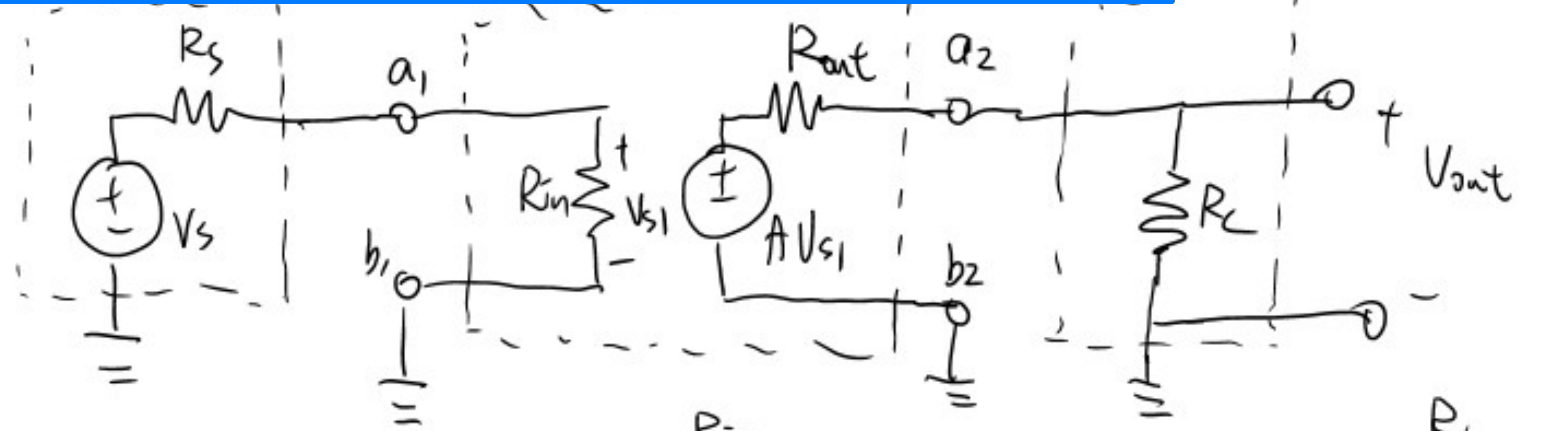


R_{in} as large as possible

$(V_{s1} \rightsquigarrow V_s)$

R_{out} as small as possible

$(V_{out} \rightsquigarrow A V_{s1}$



$$V_{s1} = V_s \frac{R_{in}}{R_s + R_{in}}$$

$$V_{out} = A V_{s1} \frac{R_L}{R_{out} + R_L}$$