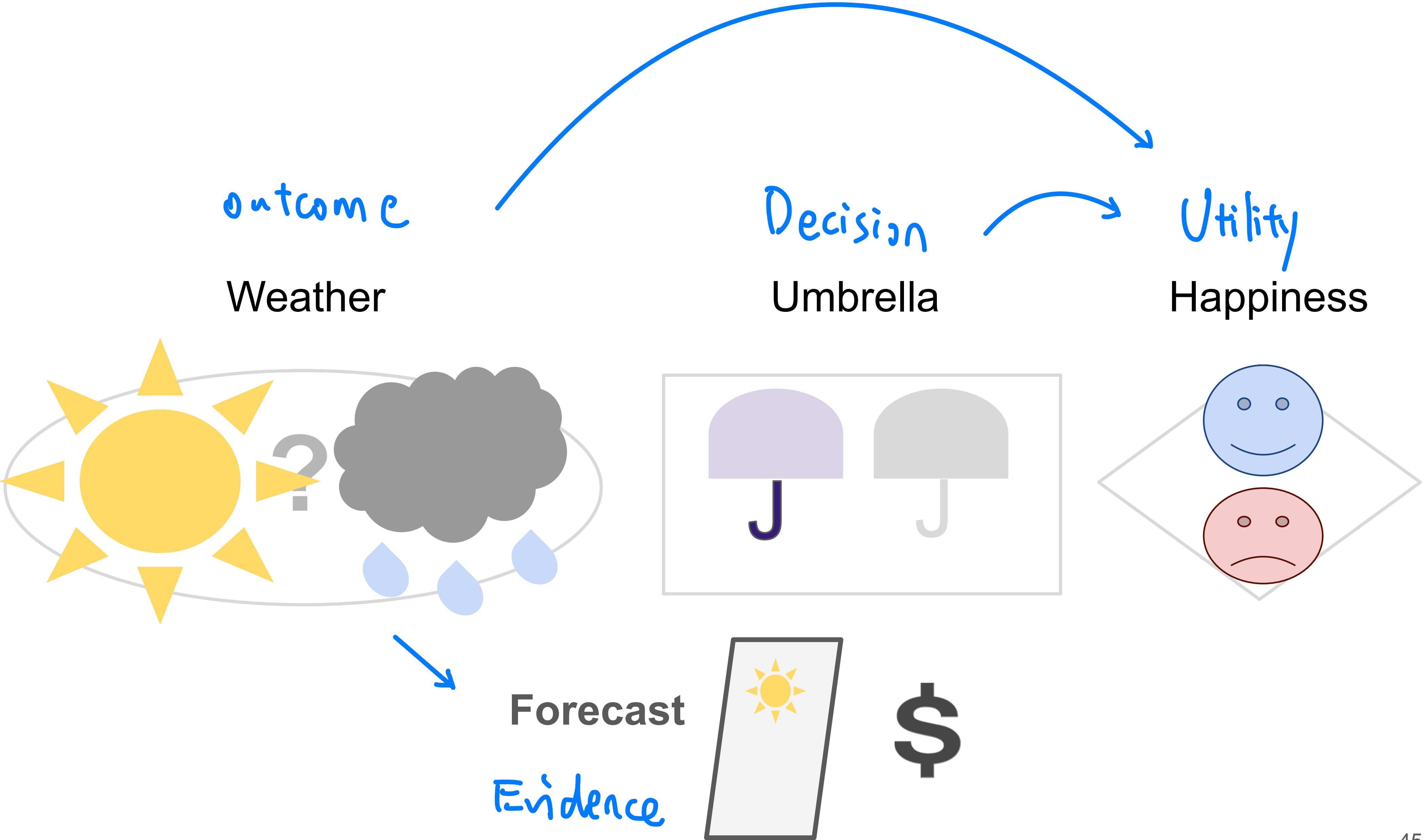
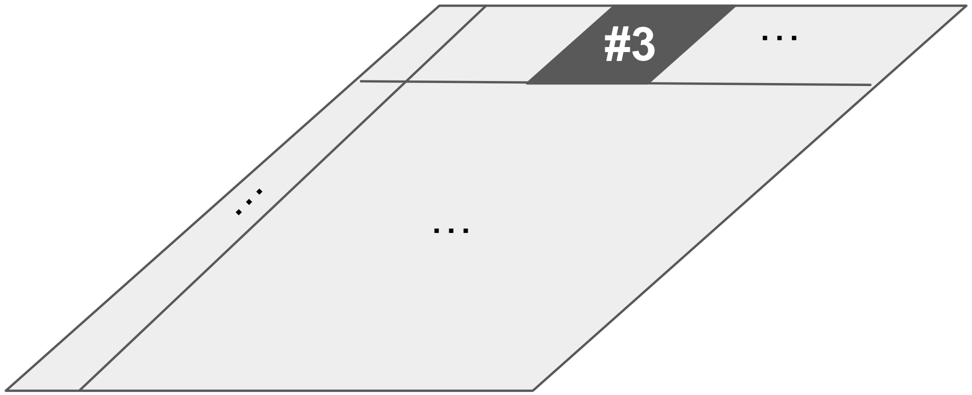

outcome
Decision
Utility
Weather
Umbrella
Happiness
?
Forecast
Evidence
$

Value of Information

Example

- n blocks
- Each block costs \$C/n
- Exactly one block containing oil - profit: \$C



Survey of block number 3:

Case 1:
Oil in #3 with probability 1/n
Case 2:
No oil in #3 with probability (n-1)/n

Q: How much should the oil company be willing to pay for the information?

Oil in #3, buy #3:
Profit: C - C/n = (n-1)C/n
Oil not in #3, buy one of others:
Profit: C/(n-1) - C/n = C/(n(n-1))
Expected profit:

$$\frac{1}{n} \times \frac{(n-1)C}{n} + \frac{n-1}{n} \times \frac{C}{n(n-1)} = C/n.$$

(The information is worth C/n)

We don't know
what the evidence will be ahead of time.

Value of Information

the expected utility of taking action a

- The value of the current best action α is

new evidence e_j

best

$$EU(\alpha) = \max_a \underline{\sum_{s'} P(\textsc{Result}(a) = s')\, U(s')}$$

Expected Utility

- The value of the new best action (given the new evidence $E_j = e_j$)

$$EU(\alpha_{e_j} | e_j) = \max_a \sum_{s'} P(\textsc{Result}(a) = s' \mid e_j)\, U(s')$$

Value of Information

- Idea
  - Compute value of acquiring evidence by using the decision network
- **Value of perfect information (VPI)**

w/ evidence

w/o evidence

$$VPI(E_j) = \left(\sum_{e_j} P(E_j = e_j) \; EU(\alpha_{e_j} | E_j = e_j) \right) - EU(\alpha)$$

summing out each evidendence

where
best action α,
random variable E_j,
evidence $E_j = e_j$, and new best action α_{ej}

Maximum Expected Utility:

MEU (Evidence set)

Decision Network

Example: VPI

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

No evidence

MEU(∅) = $\max_a \mathrm{EU}(a)$= 70

If forecast is bad

MEU(F=bad) = $\max_a \mathrm{EU}(a|\mathrm{bad})$= 53

If forecast is good

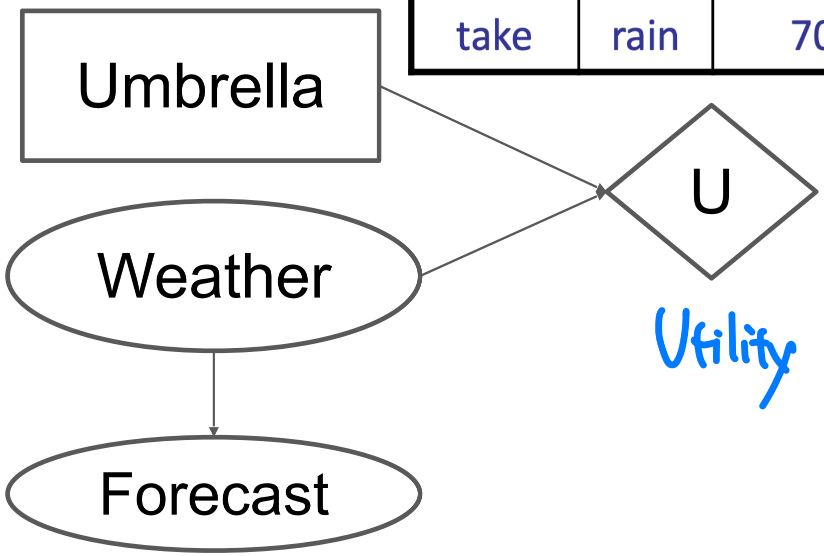
MEU(F=good) = $\max_a \mathrm{EU}(a|\mathrm{good})$= 95

* VPI(F) = MEU(F) - MEU(∅)

= (Σ_f P(F=f) MEU(F=f)) - MEU(∅)

= (0.59·95 + 0.41·53) - 70 = 7.8

Decision



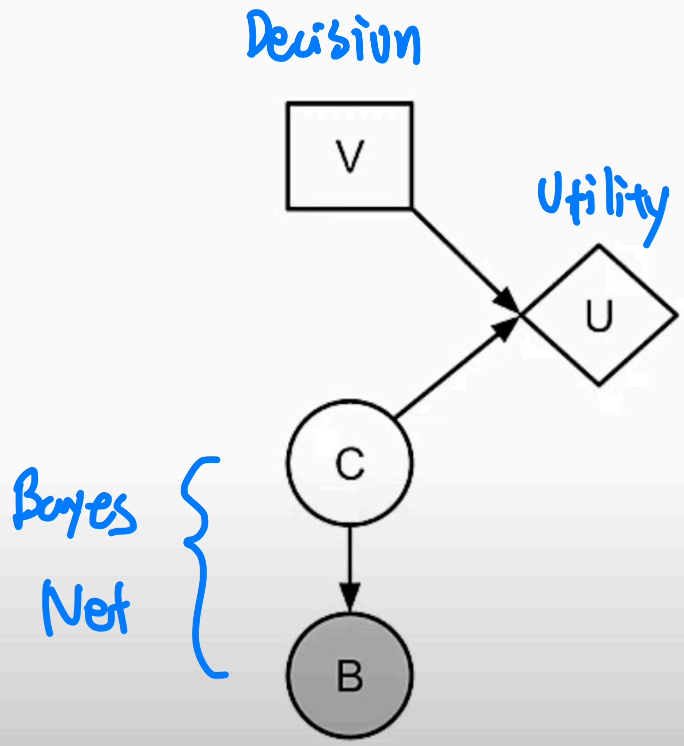
Bayes Net

Utility

F	P(F)
good	0.59
bad	0.41

Decision Networks

- Expected Utility
  - $EU(+v) = \sum_c P(c)U(+v, c)$
  - $EU(+v|+b) = \sum_c P(c|+b)U(+v, c)$
- Maximum Expected Utility
  - $MEU(\varnothing) = \max_v EU(v)$
  - $MEU(+b) = \max_v EU(v|+b)$
  - $MEU(B) = \sum_b P(b)MEU(b)$
  - Generally:
  - $MEU(e_1 \dots e_n) = \max_v EU(v|e_1 \dots e_n)$
  - $MEU(e_1 \dots e_n, E) = \sum_e P(e|e_1 \dots e_n)MEU(e, e_1 \dots e_n)$
- Value of Perfect Information
  - $VPI(E'|e) = MEU(e, E') - MEU(e)$



VPI Properties

Note. Evidence: E_i, E_j

- Non-negative

$$\forall\, j \qquad VPI(E_j) \geq 0$$

- Not additive (in general)

$$VPI(E_j, E_k) \neq VPI(E_j) + VPI(E_k)$$

- Order-independent

$$VPI(E_j, E_k) = VPI(E_k, E_j)$$

Claim: $\forall\, j \qquad VPI(E_j) \geq 0$

$$VPI(E_j) = \left(\sum_{e_j} P(E_j = e_j) \quad EU(\alpha_{e_j} | E_j = e_j) \right) - EU(\alpha)$$

Because

$$EU(\alpha) = \Sigma_{e_j} P(E_j = e_j) EU(\alpha \,|\, E_j = e_j)$$

and

$$EU(\alpha_{e_j} \,|\, E_j = e_j) \geq EU(\alpha \,|\, E_j = e_j)$$

$\Rightarrow \quad VPI(E_j) \geq 0$