Homework Week 4

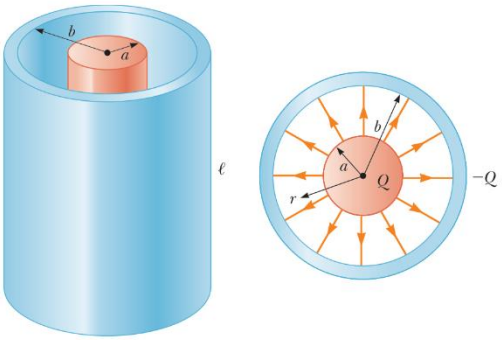
113-2 General Physics II

Due before 4:10 PM on March 17, 2025

Name 徐宏宇 資科四 110703056

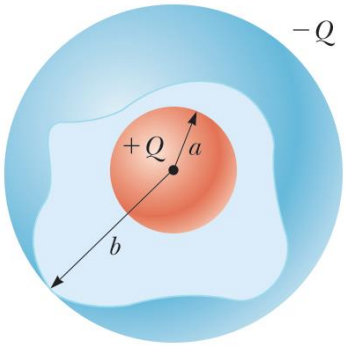
1. [20 points] **Example 25.1 The Cylindrical Capacitor**

   A solid cylindrical conductor of radius a is coaxial with *a* cylindrical shell of negligible thickness and radius $b > a$. Find the capacitance of this cylindrical capacitor if its length is $l \gg b$.



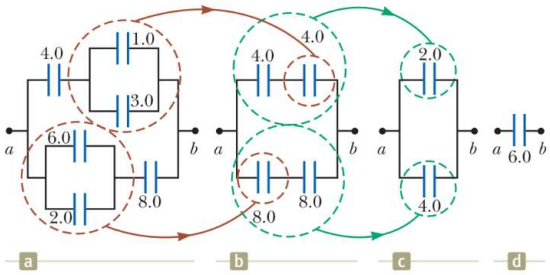
2. [20 points] **Example 25.2 The Spherical Capacitor**

   A spherical capacitor consists of a spherical conducting shell of radius *b* concentric with a smaller conducting sphere of radius *a*. Find the capacitance of this device.



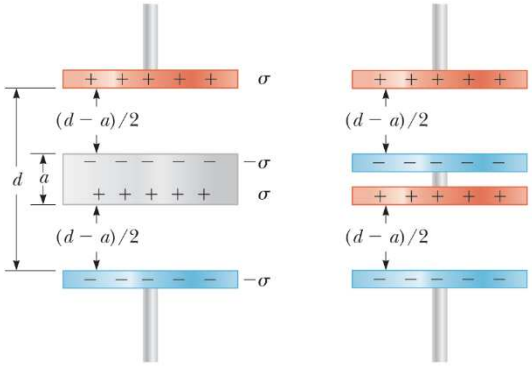
3. [20 points] **Example 25.3** Equivalent Capacitance

   Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in the figure below. All capacitances are in microfarads.



4. [10 points] **Example 25.7 Effect of a Metallic Slab**

   A parallel-plate capacitor has a plate separation *d* and plate area A. An uncharged metallic slab of thickness *a* is inserted midway between the plates. (A) Find the capacitance of the device. (B) Show that the capacitance of the original capacitor is unaffected by the insertion of the metallic slab if the slab is infinitesimally thin.



5. According to our course schedule, what topics will be covered in the next lecture? _____ [5 points] and _____ [5 points].

6. [20 points] 嘗試問一個生活中跟物理有關的問題。比如:

   庫侖定律為什麼不是

$$\frac{kQq}{r^{2.000000000000000000000000000000000000000000000000000000001}}?$$

   萬有引力為什麼不是

$$\frac{GMm}{r^{2.000000000000000000000000000000000000000000000000123456789}}?$$

有問就給分，盡量好奇地去問

勇敢地提出笨的問題

有一天就會問到對的問題

[0 points] **Extra1**

An air-filled parallel-plate capacitor has plates of area 2.30 cm^2 separated by 1.50 mm. (a) Find the value of its capacitance. The capacitor is connected to a 12.0-V battery. (b) What is the charge on the capacitor? (c) What is the magnitude of the uniform electric field between the plates? The dielectric constant of the air ~ 1.

不用交

[0 points] **Extra2**

Find the equivalent capacitance between points a and b in the combination of capacitors shown in Figure P25.13.

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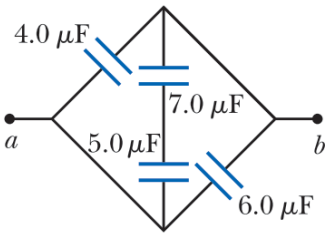


Figure P25.13

1. 1° Find E_r (Gauss's Law)

$$\oint E_r \cdot dA = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E_r (2\pi r l) = \frac{\lambda l}{\varepsilon_0}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$\Rightarrow E_r (2\pi r) = 4\pi k \lambda l \Rightarrow \frac{1}{\varepsilon_0} = 4\pi k$$

$$\Rightarrow E_r = \frac{2k\lambda}{r}$$

2° Find V from E

$$\Delta V = -\int_a^b E_r \, dr$$

$$= -\int_a^b \frac{2k\lambda}{r} \, dr$$

$$= -2k\lambda \int_a^b \frac{dr}{r}$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$= -2k\lambda \ln\left(\frac{b}{a}\right)$$

3° Find Capacitance

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{2k\lambda \ln\left(\frac{b}{a}\right)}$$

$\lambda = \frac{Q}{l}$

$$= \frac{Q}{2k\left(\frac{Q}{l}\right)\ln\left(\frac{b}{a}\right)}$$

$$= \frac{l}{2k\ln\left(\frac{b}{a}\right)} \quad \#\checkmark$$

2. 1° Find E_r (Gauss's Law)

$$\oint E_r \cdot dA = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E_r(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$k = \frac{1}{4\pi\varepsilon_0}$

$$\Rightarrow E_r(4\pi r^2) = 4\pi k Q$$

$\Rightarrow \varepsilon_0 = 4\pi k$

$$\Rightarrow E_r = \frac{kQ}{r^2}$$

2° Find ΔV from E

$$\Delta V = -\int_a^b E_r \, dr$$

$$= -\int_a^b \frac{kQ}{r^2} \, dr$$

$$= -kQ \int_a^b r^{-2} \, dr$$

$$= kQ \left[\frac{1}{r}\right]_a^b$$

$$= kQ \, \frac{a-b}{ab}$$

3° $$C = \frac{Q}{|\Delta V|} = \frac{Q}{kQ \frac{a-b}{ab}}$$

$$= \frac{ab}{k(b-a)}$$ #✓

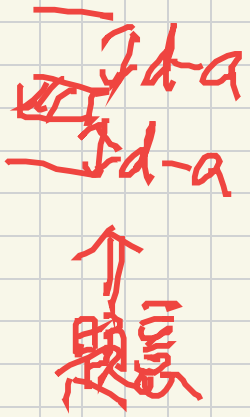
3. $$\frac{1}{\frac{1}{4} + \frac{1}{1+3}} + \frac{1}{\frac{1}{6+2} + \frac{1}{8}}$$

$$= 2 + 4 = 6 \ (\mu F)$$ ✓#

4. (A) $\frac{1}{C} = \frac{1}{\frac{k\varepsilon_0 A}{d-a}} + \frac{1}{\frac{\varepsilon_0 A}{d-a}}$

$= \frac{a + k(d-a)}{k\varepsilon_0 A}$

$= \frac{a + kd - ka}{k\varepsilon_0 A}$



題意

$\Rightarrow C = \frac{k\varepsilon_0 A}{a + kd - ka}$ ✗

(B) $C = \frac{k\varepsilon_0 A}{kd + a(1-k)}$ ✗

$a \ll k$

$\approx \frac{k\varepsilon_0 A}{kd}$

$= C_0$ (original capacitor)

(Q.E.D.)

5. current & resistance ✓

6. How does a capacitor function in a filter?

★ $C \equiv \frac{Q}{\Delta V} = \frac{Q}{Ed}$,

1. 同軸電纜

$+Q \to -Q$
內 外

$\Delta V = V_b - V_a = -\int_a^b \vec{E}\, d\vec{s}$

$\Rightarrow -\int_a^b \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\, dr$ (無限長直導線E)

$= \frac{-\lambda}{2\pi\varepsilon_0} \ln r \Big|_a^b = \frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{b}{a}\right)$, $\lambda = \frac{Q}{\ell}$

$C \equiv \frac{Q}{|\Delta V|} = \frac{Q}{\frac{Q/\ell}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 \ell}{\ln\left(\frac{b}{a}\right)}$ ※ $\left(= \frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)}\right)$

2. $V_b - V_a = -\int_a^b \frac{keQ}{r^2}\, dr = keQ \frac{1}{r}\Big|_a^b = keQ\left(\frac{1}{b} - \frac{1}{a}\right)$

$C \equiv \frac{Q}{\Delta V} = \frac{Q}{\left|keQ\left(\frac{a-b}{ab}\right)\right|} = \frac{ab}{|ke(a-b)|} = \frac{ab}{ke(b-a)}$ ※

3. C：並聯 ⇒ 相加，串聯 ⇒ 倒相。

5 Current and Resistence

4. 平行板電容

(a). $\Delta V = -\int \frac{\sigma}{\varepsilon_0} d\vec{s} \Rightarrow \Delta V = \frac{Q/A}{\varepsilon_0} d$

$C \equiv \frac{Q}{\Delta V} = \frac{A\varepsilon_0}{d}$ 題：平行板間放入金屬板 ⇒ i.e. 兩個串聯電容。

$\Rightarrow \frac{1}{\frac{\varepsilon_0 A}{(d-a)/2}} + \frac{1}{\frac{\varepsilon_0 A}{(d-a)/2}} = \frac{1}{\frac{\varepsilon_0 A}{(d-a)}} \Rightarrow C_{total} = \frac{\varepsilon_0 A}{(d-a)}$ ※ (b) $a \to 0$.

$C_{total} = \frac{\varepsilon_0 A}{d}$ ※

Extra.

1 平行板C，$A = 2.3\,(cm^2)$，$d = 1.5\,(mm)$

↳ $A = 2.3\times10^{-4}\,(m^2)$，$d = 1.5\times10^{-3}\,(m)$ ※

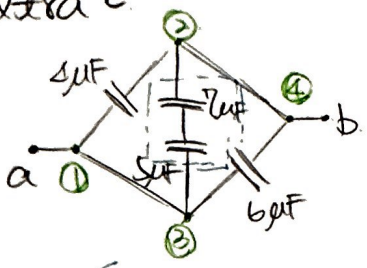
(a) $C = \frac{A\varepsilon_0}{d} = \frac{(2.3\times10^{-4})\cdot(8.85\times10^{-12})}{1.5\times10^{-3}} \cong 13.57\times10^{-13}\,(F) = 1.357\times10^{-12}\,(F)$

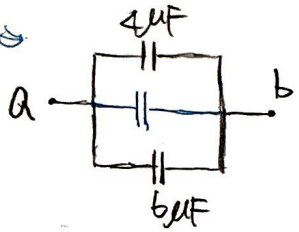
$= 1.357\,(pF)$ #

(b) $C \equiv \frac{Q}{\Delta V} \Rightarrow 13.57\times10^{-13} = \frac{Q}{12} \Rightarrow Q \cong 1.6284\times10^{-11}\,(C)$ ※

(c) $\Delta V = Ed \Rightarrow 12 = E\,1.5\times10^{-3} \Rightarrow E = 8\times10^{3}\,(V/m)$ #

Extra 2.





① and ③ 同電位.

② and ④ 同電位.

* 電容串聯 → 倒和 $\frac{1}{C_{合}}$

並聯 → 相加 Σ.

串: $\frac{1}{5}+\frac{1}{7}=\frac{12}{35} \Rightarrow C_{串}=\frac{35}{12}$.

$C_{等效}=4+\frac{35}{12}+6$.

$=12\frac{11}{12}\cong 12.9\,(\mu F)$.