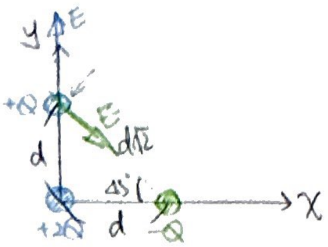
EF

11



$\hat{y}$: $\frac{k_e 2Q}{d^2} + \left(\frac{-k_e Q}{2d^2}\sin 45^\circ\right)$ $\qquad$ $\hat{x}$: $\frac{k_e Q}{2d^2}\cos 45^\circ$

$= \frac{k_e Q}{d^2}\left[2 - \frac{1}{2\sqrt{2}}\right]$ $\qquad$ $= \frac{k_e Q}{d^2}\cdot\frac{1}{2\sqrt{2}}$

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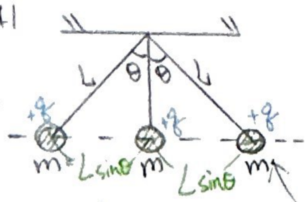
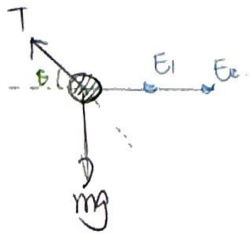


$m = 0.1$ kg

$\theta = 45^\circ$

$L = 0.3$ (m)



∵ 靜力平衡 $\cdot\, \Sigma \vec{F} = 0$

∴ $\begin{cases} mg = T\sin\theta \\ \vec{F}_{E_1} + \vec{F}_{E_2} = T\cos\theta \end{cases}$

$\Rightarrow$ ∵ $\sin 45^\circ = \cos 45^\circ$ ∴ $F_{E_1} + F_{E_2} = mg$

$$\Leftrightarrow \frac{k_e q\cdot q}{(L\sin\theta)^2} + \frac{k_e q\cdot q}{(2L\sin\theta)^2} = mg$$

$$\frac{9\times10^9\cdot q^2}{\left(\frac{0.3}{\sqrt{2}}\right)^2} + \frac{9\times10^9\cdot q^2}{\left(\frac{0.6}{\sqrt{2}}\right)^2} = 0.1\times10 \Rightarrow q \cong 2\times10^{-6}\,(C) = 2\,(\mu C)$$ #

Charge. Gauss's Law.

12 $\vec{E} = ay\hat{i} + bz\hat{j} + cx\hat{k}$

flux through surface in xy-plane.

Gauss's Law: $\phi_E = \int_S \vec{E}\cdot d\vec{A}$

1°. 找到 $d\vec{A}$

∵ xy-plane ∴ $d\vec{A} = dx\cdot dy\cdot\hat{k}$

2° $\vec{E}\cdot d\vec{A} = (ay\hat{i} + bz\hat{j} + cx\hat{k})\cdot(dx\,dy\,\hat{k})$

$= cx\,dx\,dy$

3° $\int_0^h\int_0^w cx\,dx\,dy$

$= c\cdot\int_0^h \left(\frac{x^2}{2}\right)\Big|_0^w dy$

$= c\cdot\left(\frac{x^2}{2}\right)\Big|_0^w\cdot y\Big|_0^h$

$= \frac{cw^2h}{2}$ #

33 $R=0.4\,(m)$

$Q=26\,(\mu C)$

(a) $d=0 \rightarrow E=0$

(b) $d=0.1 \rightarrow$ 球內

$$E=\frac{k_eQd}{R^3}=\frac{9\times10^9\cdot26\times10^{-6}\cdot0.1}{(0.4)^3}\cong \underline{3.65\times10^5\,(N/C)}\ \#$$

(c) $d=0.4 \rightarrow$ 球殼上.

① $E=\dfrac{k_eQd}{R^3}$

$R=0.4,\ d=0.4$ 代入.

② $$E=\frac{k_eQ}{d^2}=\frac{9\times10^9\cdot26\times10^{-6}}{(0.4)^2}\cong \underline{1.46\times10^6\,(N/C)}\ \#$$

(d) $d=0.6 \rightarrow$ 球外

$$E=\frac{k_eQ}{d^2}=\frac{9\times10^9\cdot26\times10^{-6}}{(0.6)^2}\cong \underline{6.5\times10^5\,(N/C)}\ \#$$

E. Potential

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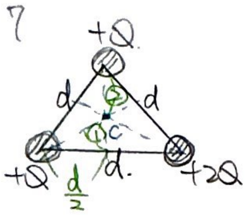

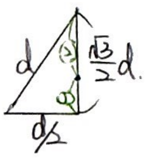

$$V=\frac{k_eQ}{r} \Rightarrow V_c=\frac{k_e\cdot Q}{\frac{\sqrt3}{2}d\cdot\frac23}+\frac{k_eQ}{\frac{\sqrt3}{2}d\cdot\frac23}+\frac{k_e\,2Q}{\frac{\sqrt3}{2}d\cdot\frac23}$$

$$=k_eQ\cdot\frac{1}{\frac{\sqrt3}{3}d}\cdot(1+1+2)\cong \underline{6.93\,\frac{k_eQ}{d}}\ \#$$

44. $\vec{E}=E_0\hat{k}$

$V(x,y,z)=V_0$ inside

$$V=V_0-E_0z+\frac{E_0a^3z}{(x^2+y^2+z^2)^{3/2}}$$

outside.

$E=-\nabla V\ \left(E=\frac{-dV}{dx}\right)$

$$\nabla V=\left(\frac{\partial V}{\partial x},\frac{\partial V}{\partial y},\frac{\partial V}{\partial z}\right)$$

(a) $\nabla V_{inside}=\nabla V_0=0 \Rightarrow E$ inside the sphere is 0 #

(b). $$V_{out}=V_0+E_0z+\frac{E_0a^3z}{(x^2+y^2+z^2)^{3/2}}$$

$$E_x=\frac{-\partial}{\partial x}\left[\frac{E_0a^3z}{(x^2+y^2+z^2)^{3/2}}\right]=\underline{\frac{E_0a^3z\ 3x}{(x^2+y^2+z^2)^{5/2}}}\ \#$$

$$E_y=-\frac{\partial}{\partial y}\left[\frac{E_0a^3z}{(x^2+y^2+z^2)^{3/2}}\right]=\underline{\frac{E_0a^3z\cdot3y}{(x^2+y^2+z^2)^{5/2}}}\ \#$$

$$E_z=-\frac{\partial}{\partial z}\left[-E_0z+\frac{E_0a^3z}{(x^2+y^2+z^2)^{3/2}}\right]=E_0-E_0a^3\left[\frac{(x^2+y^2+z^2)^{\frac32}\cdot1-z\cdot\left[\frac32\cdot2z\cdot(x^2+y^2+z^2)^{\frac12}\right]}{(x^2+y^2+z^2)^3}\right]$$

$$=E_0-E_0a^3\left[\frac{(x^2+y^2+z^2)-3z^2}{(x^2+y^2+z^2)^{5/2}}\right]$$ 上下同乘 $(x^2+y^2+z^2)^{1/2}$

$$=\underline{E_0-\frac{E_0a^3(x^2+y^2-2z^2)}{(x^2+y^2+z^2)^{5/2}}}\ \#$$

Capacitance and Dielectrics

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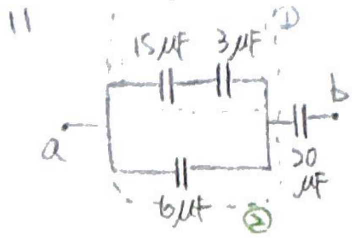


(a) ① $\frac{1}{15}+\frac{1}{3}=\frac{6}{15}=\frac{2}{5}$

$C_{eq①}=\frac{5}{2}$

② $C_{eq②}=C_{eq①}+6$

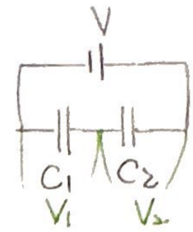
$=\frac{5}{2}+6=\frac{17}{2}$

$\Rightarrow \frac{1}{\frac{17}{2}}+\frac{1}{20}=\frac{57}{340}$

$C_{eq}=\frac{340}{57}\cong 5.96\,(\mu F)$ #

(b) $C=\frac{Q}{|\Delta V|}$

推導 考慮:



$C_{eq}=\frac{C_1\cdot C_2}{C_1+C_2}$

∵ 電荷守恆 and $Q=C\cdot|\Delta V|$

∴ $C_{eq}\cdot V=C_1\cdot V_1=C_2\cdot V_2$

$\Rightarrow \frac{C_1\cdot C_2}{C_1+C_2}\cdot V=C_1\cdot V_1$

$\frac{V_1}{V}=\frac{C_1\cdot C_2}{C_1+C_2}\cdot\frac{1}{C_1}=\frac{C_2}{C_1+C_2}$

繼續(b).

for 20(μF): $V=\frac{\frac{17}{2}}{\frac{17}{2}+20}\cdot 15\cong 4.47\,(V)$

$\hookrightarrow Q=20\times 4.47\cong 89.4\,(\mu C)$

for 6(μF) $V=\frac{20}{\frac{17}{2}+20}\cdot 15\cong 10.53\,(V)\rightarrow Q=6\times 10.53\cong 63.2\,(\mu C)$.

for 3(μF): $V=\frac{15}{15+3}\cdot 10.53\cong 8.78\,(V)\rightarrow Q=3\times 8.78\cong 26.3\,(\mu C)$

for 15(μF): $V=\frac{3}{15+3}\cdot 10.53\cong 1.76\,(V)\rightarrow Q=15\times 1.76\cong 26.3\,(\mu C)$.

也說明了電荷守恆.

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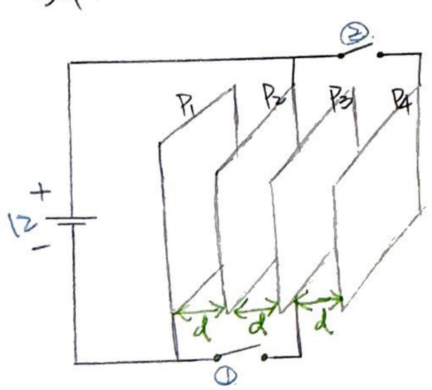
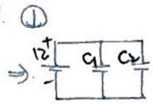


$d=1.19\times 10^{-3}\,(m)$

$A=7.5\times 10^{-4}\,(m^2)$

$V=12\,(V)$

(a) ①



P_1, P_2 間形成 C_1

P_2, P_3 間形成 C_2

and $C_1 // C_2$.

$C=\frac{\varepsilon_0 A}{d}$

$C_1=C_2=\frac{8.85\times 10^{-12}\cdot 7.5\times 10^{-4}}{1.19\times 10^{-3}}$

$\cong 5.58\times 10^{-12}\,(F)$

$C_1 // C_2$ ∴ $C_{eq}=C_1+C_2=11.2\,(pF)$ #

(b) $C=\frac{Q}{|\Delta V|}$

$Q=C\cdot|\Delta V|$

$=11.2\times 12=134.4\,(pC)$ #

(c). 與(a)同理

P_1, P_2 間 $\rightarrow C_1$

P_2, P_3 間 $\rightarrow C_2$

P_3, P_4 間 $\rightarrow C_3$

and $C_1 // C_2 // C_3$

thus. $C_{eq}=C_1+C_2+C_3$

$=5.58\times 3$

$=16.74\,(pF)$ #

(d)

$Q=C\cdot|\Delta V|$

$=16.74\times 12$

$=200.88\,(pC)$ #

Current and Resistence

7

$A = 2\times10^{-4}\ (m^2)$

$q(t) = 4t^3 + 5t + 6$

(a) $I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

$I(t) = q'(t) = 12t^2 + 5$

$I(t=1) = 12\cdot1^2 + 5 = \underline{17\ (A)}$ #

(b). $J = \frac{I}{A}$

$J(t=1) = \frac{I(t=1)}{A}$

$= \frac{17}{2\times10^{-4}} = \underline{8.5\times10^{4}\ (A/m^2)}$ #

34

$P_A = 25\ (W),\ V = 120\ (V)$

$P_B = 100\ (W),\ V = 120\ (V)$

Cost: $0.11\ (\$/kw\cdot h)$

days: 30

(a) $\left(\begin{array}{l} V = IR,\ P = IV \\ \Rightarrow P = I^2R = \frac{V^2}{R} \end{array} \right)$

$R = \frac{V^2}{P} \Rightarrow R_A = \frac{120^2}{25} = 576\ (\Omega)$

$R_B = \frac{120^2}{100} = 144\ (\Omega)$ #

(b) $I_A = \frac{V}{R_A} = \frac{120}{576} = \frac{5}{24}$

$I = \frac{\Delta Q}{\Delta t}$

$\Delta t = \frac{\Delta Q}{I} = \frac{1}{\frac{5}{24}} = 4.8\ (s)$ #.

(c) No

(d) $P = \frac{\Delta E}{\Delta t}$

$\Delta t = \frac{\Delta E}{P} = \frac{1}{25} = 0.04\ (s)$ #

(e) enter: electric potential difference.

exit: thermal energy and radiation.

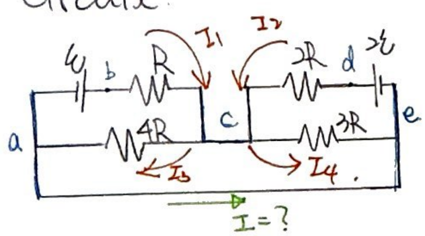
(f) $\Delta E = P\times\Delta t = 0.025\ (kw)\times(30\times24)\ (hr)$

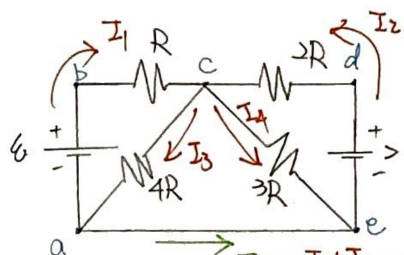
$= 18\ (kw\cdot hr)$

$\text{Cost} = 0.11\times18 = 19.8\ (\$)$ #.

DC Circuit

19.





$I = -I_1 + I_3 = I_2 - I_4$ 求=?

$\varepsilon = 250\ (V)$

$R = 1\ (k\Omega)$

Loop 1: $a\to b\to c\to a$

$\varepsilon - I_1R - I_3\cdot4R = 0$

$250 - I_1 - 4I_3 = 0$

Loop 2: $e\to d\to c\to e$

$2\varepsilon - I_2\cdot2R - I_4\cdot3R = 0$

$500 - 2I_2 - 3I_4 = 0$

Loop 3: $c\to a\to e\to c$

$I_3\cdot4R - I_4\cdot3R = 0$

$4I_3 - 3I_4 = 0$

Node: c

$I_1 + I_2 = I_3 + I_4$.

$\left\{ \begin{array}{l} 250 - I_1 - 4I_3 = 0 \\ 500 - 2I_2 - 3I_4 = 0 \\ 4I_3 - 3I_4 = 0 \\ I_1 + I_2 = I_3 + I_4 \end{array} \right.$

$I_3 = \frac{3}{4}I_4$

$I_1 + I_2 = \frac{7}{4}I_4$

$I_1 + 3I_4 = 250$

$I_2 + \frac{3}{2}I_4 = 250$

$\frac{25}{4}I_4 = 500$

$\left\{ \begin{array}{l} I_4 = 80 \\ I_3 = 60 \\ I_2 = 130 \\ I_1 = 10 \end{array} \right.$

$I = -I_1 + I_3$

$= -10 + 60$

$= 50\ (mA)$.

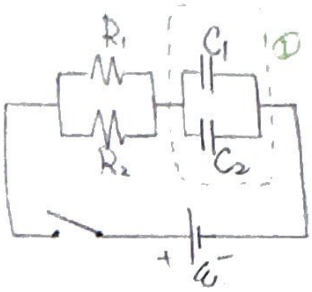
or

$I = I_2 - I_4$

$= 130 - 80$

$= 50\ (mA)$.

#1



$C_1 = 2(\mu F)$

$C_2 = 3(\mu F)$

$R_1 = 2(k\Omega)$

$R_2 = 3(k\Omega)$

$\varepsilon = 120(V)$

$C_{eq} = 2+3 = 5$

$\frac{1}{R_{eq}} = \frac{1}{2}+\frac{1}{3} = \frac{5}{6}$

$R_{eq} = \frac{6}{5}$

* $q(t) = C\varepsilon\left(1-e^{\frac{-t}{RC}}\right)$

$\Rightarrow q_{①} = 5\cdot 120\left(1-e^{\frac{-t}{5\cdot\frac{6}{5}}}\right)$

$= 600\left(1-e^{\frac{-t}{6}}\right)$

* $V_C = \frac{Q}{C}$

$V_{①} = \frac{1}{5}600\left(1-e^{\frac{-t}{6}}\right)$

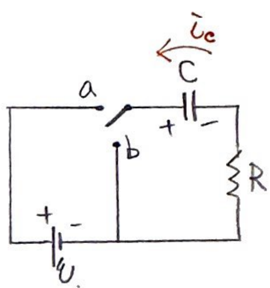
$= 120\left(1-e^{\frac{-t}{6}}\right)$

$\Rightarrow q_{C_1} = V_{①}\cdot C_1$

$= 240\left(1-e^{\frac{-t}{6}}\right)$ #

$q_{C_2} = V_{①}\cdot C_2$

$= 360\left(1-e^{\frac{-t}{6}}\right)$ #



* $\int \frac{1}{a-x}dx$, 令 $u=a-x$

$du=-dx$

$dx=-du$

$\Rightarrow \int \frac{-1}{u}du = -\ln(u)$

$\int \frac{1}{x-a}dx$, 令 $u=x-a$

$du=dx$

$\Rightarrow \int \frac{1}{u}du = \ln(u)$

ⓐ: charging

$\varepsilon - IR - V_c = 0$

$C \equiv \frac{Q}{V}$, $V=IR$

↳ $I_c = \frac{Q}{RC}$

$V_c = \frac{Q}{C}$ *

$\varepsilon - I(t)R - \frac{q(t)}{C} = 0$

↳ $I(t) = \frac{\varepsilon}{R} - \frac{q(t)}{RC}$

* actually $I(t) = \frac{dq(t)}{dt}$

$\frac{dq(t)}{dt} = \frac{C\varepsilon - q(t)}{RC}$

$\frac{1}{C\varepsilon - q(t)}dq(t) = \frac{1}{RC}dt$

$\int_0^q \frac{1}{q(t)-C\varepsilon}dq(t) = \int_0^t \frac{-1}{RC}dt$

$q=0$ when $t=0$

$q=q$ $t=t$

$\Rightarrow \ln\left(\frac{q(t)-C\varepsilon}{-C\varepsilon}\right) = \frac{-t}{RC}$

$q(t) - C\varepsilon = -C\varepsilon\cdot e^{\left(\frac{-t}{RC}\right)}$

$\Rightarrow q(t) = \boxed{C\varepsilon}\left(1-e^{\left(\frac{-t}{RC}\right)}\right)$

i.e. Q_{max}.

↳ $V_c(t) = \varepsilon\left(1-e^{\left(\frac{-t}{RC}\right)}\right)$

$I(t) = \frac{\varepsilon}{R}e^{\left(\frac{-t}{RC}\right)}$