$$e^x = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$$

pf.: ① Let $f(x) = \ln x$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\ln(x+h) - \ln x}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\ln\frac{x+h}{x}$$

$$= \lim_{h\to 0}\ln\left(\frac{x+h}{x}\right)^{\frac{1}{h}}$$

$\because f'(x) = \frac{d}{dx}[\ln x] = \frac{1}{x}$

$$\Rightarrow f'(1) = \lim_{h\to 0}\ln(1+h)^{\frac{1}{h}} = \ln\left[\lim_{h\to 0}(1+h)^{\frac{1}{h}}\right], \text{ and } \boxed{f'(1) = \frac{1}{1} = 1} \Rightarrow 1 = \ln\left[\lim_{h\to 0}(1+h)^{\frac{1}{h}}\right]$$

② $$e = e^1 = e^{\ln\left[\lim_{h\to 0}(1+h)^{\frac{1}{h}}\right]} = \lim_{h\to 0}(1+h)^{\frac{1}{h}}$$

※ 此过程較为繁瑣，故过程看过可以直接記下 $e = \lim_{h\to 0}(1+h)^{\frac{1}{h}}$

(除非你要念数学系)
妳