1. 偶函數微分後為奇函數：

pf.1：利用Chain Rule.

Let $f(x)$ be an even function, we have $f(-x)=f(x)$

$\frac{d}{dx}f(-x)=f'(-x)\cdot\frac{d}{dx}(-x)=-f'(-x)$

∴ We prove that if $f(x)$ is an even function, then the derivative of $f(x)$ is an odd function.

pf.2：利用微分之定義

Let $f(x)$ be an even function, we have $f(-x)=f(x)$

$f'(-x)=\lim_{h\to 0}\frac{f(-x-h)-f(-x)}{-h}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{-h}$

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=-\lim_{h\to 0}\frac{f(x+h)-f(x)}{-h}=-f'(x)$

∴ We prove that if $f(x)$ is an even function, then the derivative of $f(x)$ is an odd function.

※奇函數微分完為偶函數，可自己嘗試看看