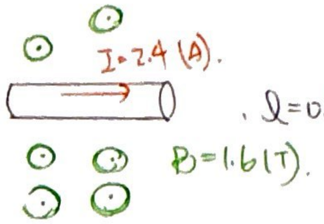
Extra w7-8

21.

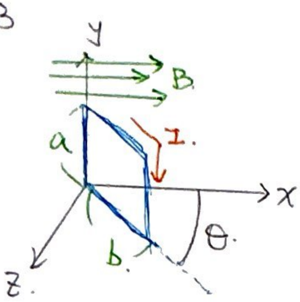


, $l=0.75\,(m)$

$$\begin{cases} \vec{F}_B=(q\vec{v}\times\vec{B})\cdot nAL \\ \vec{F}_B = I\vec{L}\times\vec{B} \end{cases}$$

$\vec{F}_B = 2.4\times0.75\times1.6 = 2.88\,(-\hat{j})$ #

33



$N=100$
$a=0.4\,(m)$.
$b=0.3\,(m)$.
$\theta=30^\circ$

(a). $B=0.8\,(T)$, $I=1.2\,(A)$

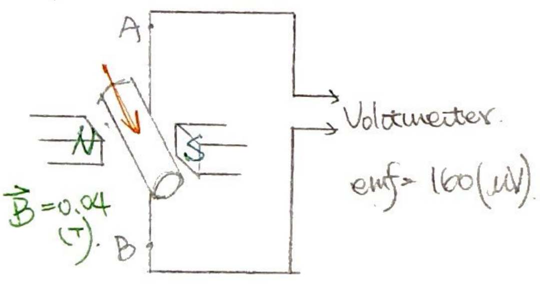
$$\begin{cases} \vec{\tau}=\vec{\mu}\times\vec{B} \\ \vec{\mu}=I\vec{A} \end{cases} \Rightarrow \vec{\tau}=(I\vec{A})\times\vec{B}\times N$$

$$=(1.2\cdot ab)(0.8)\underline{\sin(90-\theta)}^{*}\times100$$

$$=1.2\times0.3\times0.4\times0.8\times\sin60^\circ\times100$$

$$\cong 9.98\,(N\cdot m)$$ #

47.



(a). speed of blood. $\Delta V_H = v_d B d$

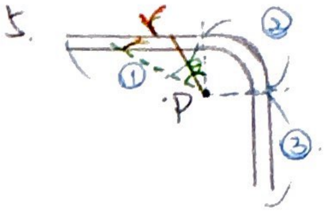
$$\begin{cases} \vec{F}_E=\vec{F}_B \\ qE = qv_dB \\ \Delta V_H = E_H\cdot d \end{cases}$$

$\Leftarrow$ $160\times10^{-6} = v_d\times0.04\times3\times10^{-3}$

$\Rightarrow$ $v_d \cong 1.33\,(m/s)$ #

Biot-Savart Law

5.



$$\begin{cases} \text{長直導線}: B = \dfrac{\mu_0 I}{2\pi a} \\ \text{Biot-Savart}: d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\vec{l} \times \hat{r}}{r^2} \end{cases}$$

for ①: $B = \frac{\mu_0 I}{4\pi a}(-\sin\theta)\Big|_{\theta_\infty}^{\theta=0}$

$= \frac{\mu_0 I}{4\pi a}(-\sin\theta)\Big|_{\frac{\pi}{2}}^{0}$

$dl = r\,d\theta$

$= \frac{\mu_0 I}{4\pi a}$, $a = r$

Similarly for ③

$B = \frac{\mu_0 I}{4\pi a}(-\sin\theta)\Big|_{0}^{\theta_\infty = -\frac{\pi}{2}}$

$= \frac{\mu_0 I}{4\pi a}$, $a = r$.

for ②: $d l = \cancel{2\pi r}\, d\theta$. *

$\hookrightarrow B = \frac{\mu_0 I}{4\pi} \cdot \frac{2\pi r \cdot \frac{1}{4}}{r^2}$

$= \frac{\mu_0 I}{8r}$

$B_{total} = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{8r} + \frac{\mu_0 I}{4\pi r}$

$= \frac{\mu_0 I}{2r}\left(\frac{1}{\pi} + \frac{1}{4}\right)$ (T) #

29. $r = 1.25$ (cm)

$l = 30$ (cm)

$N = 300$

$I = 12$ (A)

(a) $R = 5$ (cm)

$$\begin{cases} \oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 N I \\ B = \mu_0 \frac{N}{l} I \end{cases}$$

$$\left(\Phi_B = \int \vec{B} \cdot d\vec{A} \right)$$

$\Rightarrow B = \mu_0 \frac{300}{0.3} \cdot 12$

$= 48\pi \times 10^{-4}$ (T)

$(\mu_0 = 4\pi \times 10^{-7})$

$\Phi_B = (48\pi \times 10^{-4})(\pi r^2)$ *

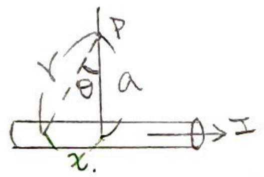
$= (48\pi \times 10^{-4})(\pi (1.25 \times 10^{-3})^2)$

$\simeq 7.4 \times 10^{-6}$ (Wb) #

(b). Area $= \pi(b^2 - a^2) = 4.8\pi \times 10^{-5}$

$\Phi_B = (48\pi \times 10^{-4})(4.8\pi \times 10^{-5})$

$\simeq 2.27 \times 10^{-6}$ (Wb) #



$dB = \frac{\mu_0 I}{4\pi} \frac{dx \sin(\frac{\pi}{2} - \theta)}{r^2}$

$= \frac{\mu_0 I}{4\pi} \frac{dx \cos\theta}{r^2}$

$$\begin{cases} \cos\theta = \frac{a}{r}, \; x = a\tan\theta \\ dx = a \cdot \sec^2\theta\, d\theta \\ \sec\theta = \frac{1}{\cos\theta} = \frac{r}{a} \end{cases}$$

$\Rightarrow dB = \frac{\mu_0 I}{4\pi} \cdot \frac{a\, d\theta}{\cos^2\theta} \cdot \cos\theta \cdot \left(\frac{\cos\theta}{a}\right)^2$

$= \frac{\mu_0 I}{4\pi} \cdot \frac{\cos\theta}{a} d\theta$

$B = \frac{\mu_0 I}{4\pi} \int_{\theta_\infty}^{\theta'} \frac{\cos\theta}{a} d\theta$

$= \frac{\mu_0 I}{4\pi a}(-\sin\theta)\Big|_{\theta_\infty}^{\theta'}$

$\theta' = \theta_\infty \Rightarrow \frac{\mu_0 I}{4\pi a}(-\sin\theta)\Big|_{\frac{\pi}{2}}^{-\frac{\pi}{2}}$

$= \frac{\mu_0 I}{4\pi a}\left(\sin\frac{\pi}{2} - \sin\frac{-\pi}{2}\right)$

$= \frac{\mu_0 I}{2\pi a}$

Faraday's Law

4. Solenoid. $n = 400\left(\text{turns}/\text{m}\right)$

$I = 30\left(1 - e^{-1.6t}\right)$ (A)

Coil: $R = 6\times10^{-2}$ (m)

$N = 250$

$\Rightarrow B_{solenoid} = \mu_0 n I$

$= 12\mu_0\times10^{3}\left(1 - e^{-1.6t}\right)$ (T)

152.72×10^{-4}

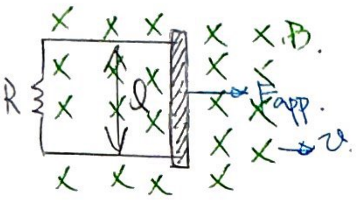
$\Rightarrow \varepsilon = -\dfrac{d\phi_B}{dt},\ \phi_B = \vec{B}\cdot\vec{A}$

$$\varepsilon = \frac{-d\left\{\left[12\mu_0\times10^{3}\left(1-e^{-1.6t}\right)\right]\left[\pi\left(6\times10^{-2}\right)^2\right]\cdot 250\right\}}{dt}$$

$$= \left[12\mu_0\times10^{3}\cdot 36\pi\times10^{-4}\ 250\right]\cdot \underbrace{\frac{d\left(1-e^{-1.6t}\right)}{dt}}_{=1.6\times e^{-1.6t}}$$

$$\simeq 2.72\times10^{-2}\, e^{-1.6t}\ \text{(V)}$$ #

13. Find the distance in terms of m, ℓ, R, B, v



令移動距離 x

$$\varepsilon = -\frac{d\phi_B}{dt} = \frac{-d(B\ell x)}{dt} = -B\ell v$$

$$I = \frac{|\varepsilon|}{R} = \frac{B\ell v}{R}\ \ \&\ \ F_B = I\vec{L}\times B = I\ell B \Rightarrow F_B = \frac{B^2\ell^2 v}{R}$$

力平衡: $F_{app} = F_B \Rightarrow ma = \dfrac{-B^2\ell^2 v}{R},\ a = \dfrac{dv}{dt}$

$$\Rightarrow \frac{1}{v}dv = \frac{-B^2\ell^2}{mR}dt$$

$$\Rightarrow \int_{v}^{v(t)} \frac{1}{v}dv = \int_0^t \frac{-B^2\ell^2}{mR}dt$$ #

$$\ln\left(\frac{v(t)}{v}\right) = \frac{-B^2\ell^2}{mR}t$$

$$\Rightarrow v(t) = v\,e^{\left(\frac{-B^2\ell^2}{mR}\right)t}$$ ... v(t)

$$x = \int_0^\infty v(t)dt = \int_0^\infty v\cdot e^{\left(\frac{-B^2\ell^2}{mR}\right)t}dt$$

$$= v\cdot\frac{-mR}{B^2\ell^2}e^{\left(\frac{-B^2\ell^2}{mR}\right)t}\Big|_{t=0}^{t=\infty} = \frac{mRv}{B^2\ell^2}$$ #

25. rotating loop in AC generator

$l = 0.1\ (m) \Rightarrow Area = 10^{-2}\ (m^2)$

(f) $\nu = 60\ (Hz)$

$B = 0.8\ (T)$

(a) $\phi_B = \vec{B}\vec{A}\cos\theta,\ \theta = \omega t,\ \omega = 2\pi\nu$

$\phi_B(t) = 0.8 \cdot 10^{-2} \cdot \cos(2\pi 60\, t)$

$= 8\times10^{-3} \cos(120\pi t)\ (Wb)$ #

(b) $\varepsilon = -\dfrac{d\phi_B}{dt} = -\dfrac{d}{dt}\left[8\times10^{-3}\cos(120\pi t)\right]$

$= 9.6\pi\times10^{-1}\sin(120\pi t)$ #

(c) $I(t) = \dfrac{\varepsilon}{R} = \dfrac{9.6\pi\times10^{-1}\sin(120\pi t)}{1}$ #

(d) $P_{average} = \dfrac{1}{2}\dfrac{\varepsilon_{max}^2}{R}$

$= \dfrac{1}{2}(9.6\pi\times10^{-1})^2$

$\cong 4.6\ (W)$ #

or

$P = \dfrac{\varepsilon^2}{R} \cong 9.1\sin^2(120\pi t)$ # 課本解答

(e) $\tau_{average} = \dfrac{P_{avg}}{\omega} = \dfrac{4.6}{120\pi} \cong 1.2\times10^{-2}\ (N\cdot m)$

or

$\tau = \dfrac{P}{\omega} \cong 2.4\times10^{-2}\sin^2(120\pi t)$

linear	$P = F \cdot v$
rotational	$P = \tau \cdot \omega$

39. $a = 0.5\ (m)$, $A = \frac{1}{2}a^2\theta$

$B = 0.5\ (T)$

$\omega = 2\ (rad/s)$

(a) $\varepsilon = -\dfrac{d\phi_B}{dt} = -\dfrac{d\left(0.5\ \frac{1}{2}(0.5)^2\theta\right)}{dt}$

$= -\frac{1}{2}(0.5)^3 \cdot \dfrac{d\theta}{dt}$, $\because \dfrac{d\theta}{dt} = \omega = 2$

$\therefore \varepsilon = -\frac{1}{2}(0.5)^3 \cdot 2 = -0.125\ (V)$ #

(b) $\lambda_R = 5\ (\Omega/m)$, $t = 0.25\ (s)$

total length of the loop

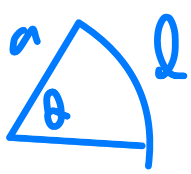
$= 2a + a\theta\ (t = 0.25)$

$= 2\times0.5 + 0.5\times\omega\times0.25$

$= 1 + 0.25 = 1.25$

$R = 5\times1.25 = 6.25$

$I = \dfrac{-0.125}{6.25} = -0.02\ (A)$ #



- $l = A\theta$
- $A = \frac{1}{2}a^2\theta$

Inductance

3. $\mathcal{E} = 24\ (mV)$

$N = 500$

$\frac{di}{dt} = 10\ (A)$

$\mathcal{E} = -L\frac{di}{dt} = -N\frac{d\phi_B}{dt}$

1° $L = \frac{\mathcal{E}}{\frac{di}{dt}} = \frac{24\times10^{-3}}{10}$

$= 24\times10^{-4}$

2° $L = \frac{\phi_B}{I}N$, $I=4$

$24\times10^{-4} = \frac{\phi_B}{4}\,500$

$\Rightarrow \phi_B = 1.92\times10^{-5}\ (Wb)$ #

5. $\mathcal{E}_L = \mathcal{E}_0 e^{-kt}$

$\mathcal{E}_L = -L\frac{di}{dt},\ i = \frac{dq}{dt}$

$\mathcal{E}_0 e^{-kt} = -L\frac{di}{dt}$

$\int -\frac{1}{L}\mathcal{E}_0 e^{-kt}\,dt = \int di$, $t=0 \to \mathcal{E}=\mathcal{E}_0$; $t=\infty \to \mathcal{E}=0$

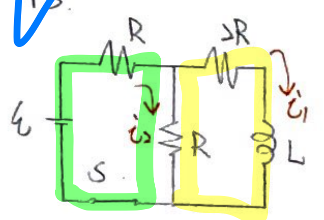
$i = \frac{1}{Lk}\mathcal{E}_0 e^{-kt}$

$\int \frac{1}{Lk}\mathcal{E}_0 e^{-kt}\,dt = \int dq$

$q = \frac{-1}{Lk^2}\mathcal{E}_0 e^{-kt}$

total charge $= \frac{\mathcal{E}_0}{Lk^2}$ #

15.



$R = 4\ (\Omega)$

$L = 1\ (H)$

$\mathcal{E} = 10$

(a) {charging L: $i(t) = \frac{\mathcal{E}}{R}\left(1-e^{-\frac{R}{L}t}\right)$}

$$\begin{cases} i_1 2R + L\frac{di_1}{dt} - i_2 R = 0 \\ \mathcal{E} - (i_1+i_2)R - i_2 R = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 10 - i_1 R - 2i_2 R = 0 \\ 10 - 4i_1 - 8i_2 = 0 \\ 8i_1 + \frac{di_1}{dt} - 4i_2 = 0 \\ i_2 = \frac{5}{4} - \frac{1}{2}i_1 \end{cases}$$

$\Rightarrow 8i_1 + \frac{di_1}{dt} - 5 + 2i_1 = 0$

$\frac{di_1}{dt} + 10i_1 - 5 = 0$

令 $u = 10i_1 - 5$

$du = 10\,di_1$

$di_1 = \frac{1}{10}du$

$\Rightarrow \frac{1}{10}du = -u\,dt$

$\frac{1}{u}du = -10\,dt$

$\int_{u_0}^{u}\frac{1}{u}du = \int_0^t -10\,dt$

$\ln\left(\frac{u}{u_0}\right) = -10t$

$u_0 = 10i_1(t=0) - 5 = -5$

$\frac{10i_1 - 5}{-5} = e^{-10t}$

$2i_1 - 1 = -e^{-10t}$

$i_1 = \frac{1}{2}\left(1 - e^{-10t}\right)$ #

(b) 問 $i_1 + i_2$.

$i_1 + i_2 = i_1 + \frac{5}{4} - \frac{1}{2}i_1$

$= \frac{5}{4} + \frac{1}{2}i_1$

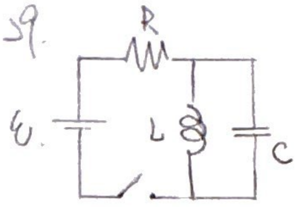
$= \frac{5}{4} + \frac{1}{4}\left(1 - e^{-10t}\right)$ #

or

$= \frac{3}{2} - \frac{1}{4}e^{-10t}$ #

RLC Circuit.

39.



$\mathcal{E} = 50\,(V)$

$R = 250\,(\Omega)$

$C = 0.5\,(\mu F)$

$V_{C\max} = 150\,(V)$

1° close for a long time

↳ C 完全放電 $\Rightarrow V_C = 0$

L 完全充電 $\Rightarrow V_L$.

↳ open $\Rightarrow i_L(t=0) = \frac{\mathcal{E}}{R}$

2° 能量儲存: $\begin{cases} U_C = \frac{1}{2}CV^2, \\ U_L = \frac{1}{2}LI^2. \end{cases}$

$i_L(t=0) = \frac{50}{250} = \frac{1}{5}\,(A)$

Energy stored in L

↓

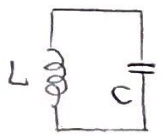
stored in C

$\frac{1}{2}LI^2 = \frac{1}{2}CV^2$

$L \cdot \left(\frac{1}{5}\right)^2 = 0.5\times10^{-6} \cdot 150^2$

$\underline{L \doteqdot 0.281\,(H)}$ //.

37



$\begin{cases} U_C = \frac{1}{2}CV^2 \\ U_L = \frac{1}{2}LI^2 \end{cases}$

initial: $C\to Q$. $\Rightarrow$ total Energy $= \cancel{\frac{1}{2}C\left(\frac{Q}{C}\right)^2}$

$= \frac{1}{2}\frac{Q^2}{C}$

when $C \to \frac{Q}{2}$, $U_C = \frac{1}{2}\frac{1}{C}\left(\frac{Q}{2}\right)^2$ $\frac{Q^2}{2C}$

$= \frac{Q^2}{8C}$

the rest is in L.

$\Rightarrow \frac{Q^2}{2C} - \frac{Q^2}{8C} = \frac{1}{2}LI^2$

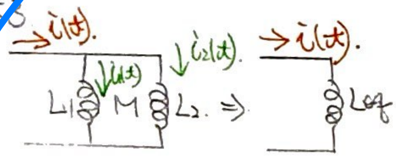
$\Rightarrow I = \sqrt{\left(\frac{3Q^2}{8C}\right)\left(\frac{2}{L}\right)} = \frac{Q}{2}\sqrt{\frac{3}{CL}}$

又 $L = \frac{\phi_B}{I} \Rightarrow \underbrace{\phi_{B\,total}}_{N\cdot\phi_{B\,each}} = L\cdot I$.

$\phi_{Bt} = L\cdot\frac{Q}{2}\sqrt{\frac{3}{CL}}$

$\phi_{B\,each} = \frac{L}{N}\left(\frac{Q}{2}\sqrt{\frac{3}{CL}}\right) = \frac{Q}{2N}\sqrt{\frac{3L}{C}}$ ※

40.



$\begin{cases} i(t) = i_1(t) + i_2(t). \\ V(t) = V_1(t) = V_2(t). \end{cases}$

and $\begin{cases} V_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ V_2(t) = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \end{cases}$

↳ thus. $L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$

$\Rightarrow (L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$.

$\Rightarrow \frac{di_1}{dt} = \frac{L_2 - M}{L_1 - M}\frac{di_2}{dt}$

又 $i(t) = i_1(t) + i_2(t) \Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

$\Rightarrow$ 得. $\frac{di}{dt} = \left(\frac{L_2 - M}{L_1 - M} + 1\right)\frac{di_2}{dt}$

↳ $\frac{di_2}{dt} = \left(\frac{L_1 - M}{L_1 + L_2 - 2M}\right)\frac{di}{dt}$.

Goal. ∵ $V = L_{eq}\cdot\frac{di}{dt}$.

$\Rightarrow V_2(t) = L_2\frac{di_2}{dt} + M\left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt}$ 代入.

$\Rightarrow V_2(t) = V(t) = \underbrace{\left(L_2 + \frac{M(L_2 - M)}{L_1 - M}\right)\left(\frac{L_1 - M}{L_1 + L_2 - 2M}\right)}_{\text{即 } L_{eq}}\frac{di}{dt}$

$L_{eq} = \frac{L_1L_2 - \cancel{ML_2} + \cancel{ML_2} - M^2}{\cancel{L_1 - M}} \cdot \frac{\cancel{L_1 - M}}{L_1 + L_2 - 2M}$

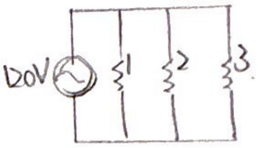
$= \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$ #

AC Circuit

$\Rightarrow I = \frac{P}{V}$

Extra week 12-13.

4.



1. 2. $\Rightarrow$ 150 (W)

3 $\rightarrow$ 100 (W)

(rms power)*

(a) $\underline{P = IV \text{ (r.m.s for each term)}}$

1: $I = \frac{150}{120} = 1.25$ (A)

2: $I = \frac{150}{120} = 1.25$ (A)

3: $I = \frac{100}{120} \simeq 0.833$ (A) #

(b) $\underline{P = \frac{V^2}{R}} \Rightarrow R = \frac{V^2}{P}$

$R_1 = \frac{120^2}{150} = 96\ (\Omega)$

$R_2 = \frac{120^2}{150} = 96\ (\Omega)$

$R_3 = \frac{120^2}{100} = 144\ (\Omega)$ #

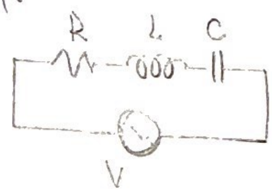
(c) $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$= \frac{1}{96} + \frac{1}{96} + \frac{1}{144}$

$= \frac{1}{36}$

$\hookrightarrow R_{total} = 36\ (\Omega)$ #.

19.



$R = 150\ (\Omega)$

$L = 460 \times 10^{-3}$ (H)

$C = 21 \times 10^{-6}$ (F)

$V = 120$ (V), 60 (Hz).

(a) $\begin{cases} \text{inductive reactance}: X_L = \omega L \\ \text{capacitive reactance}: X_C = \frac{1}{\omega C} \\ \text{phase angle} = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \end{cases}$

$\Rightarrow \omega = 2\pi f = 2\pi \cdot 60 = 120\pi.$

$\tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{120\pi \times 460 \times 10^{-3} - \frac{1}{120\pi \times 21 \times 10^{-6}}}{150}\right)$

$\simeq \tan^{-1}\left(\frac{173.4 - 126.4}{150}\right) \simeq \underline{\tan^{-1}(0.313)} \Rightarrow \underline{\phi \simeq 17.4^\circ}$ or #

(b) current lags $\rightarrow$ thus voltage reaches its Max earlier.

35. $P = 20$ (kW). $\Rightarrow P_{loss} = 200$ (W).

a pair of wire $\Rightarrow$ total distance $= 2 \times 18$ (km) $= 36$ (km).

$\Delta V_{r.m.s} = 1.5 \times 10^3$, 令直徑 $= d$

$\Rightarrow P = IV \cdot 20 \times 10^3 = I \cdot 1.5 \times 10^3 \Rightarrow I \simeq 13.33$ (A)

$P_{loss} = I^2 R \cdot 200 = (13.33)^2 \cdot R \Rightarrow R \simeq 1.125\ (\Omega)$

$\Rightarrow$ *$\underline{R = 1.125 \text{ for both wire, thus } \frac{1.125}{2} \text{ for each wire.}}$

Review. $\left[R = \rho\frac{l}{A},\ \rho = \rho_{copper},\ l = 18 \text{ (km)},\ A = \pi \cdot \left(\frac{d}{2}\right)^2 \text{ for this case} \right]$.

$\Rightarrow R = \rho\frac{l}{A}$, $A = \frac{\pi d^2}{4}$

$\hookrightarrow \underline{d = \sqrt{\frac{4\rho l}{\pi R}}}$ #

$\rho_{copper} = 1.68 \times 10^{-8}\ (\Omega \cdot m) \Rightarrow$ 查表.

$\hookrightarrow$ 都代入數字會得.

$\underline{d \simeq 2.6 \times 10^{-2}\ (m)}$. #

Transformer

31 $V = 5000\ (V)\ (r.m.s)$

$f = 60\ (Hz)$

$C_s = 20\times10^{-12}\ (F)$

$R_b = 50\times10^{3}\ (\Omega)$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\, 60\cdot 20\times10^{-12}} \approx 132.6\times10^{\cancel{4}\ 8}\ (\Omega)$$

$\begin{cases} V_R = IR_b \\ V_C = IX_c \end{cases}$ but * For C, Voltage lags current by 90°.

For R, Voltage and current are in phase.

↳ i.e. 等效 impedance $= \sqrt{R_b^2 + X_c^2}$ ※

thus $V_R = \left(\frac{R_b}{\sqrt{R_b^2+X_c^2}}\right)\cdot V$ *

$$= \frac{50\times10^{3}}{\sqrt{(50\times10^{3})^2 + (132.6\times10^{6})^2}}\cdot 5000 \approx 1.89\ (V)$$ #

↳ 從此ex.可以知，經過R_b的電流極小 ⇒ 幾乎無感，不會電到

EW.

15. $d = 1\times10^{-3}\ (m)$

$E_0 = 0.7\times10^{6}\ (V/m)$

$B = \frac{E}{C}$, $C = 3\times10^{8}$

$\mu_0 = 4\pi\times10^{-7}$

$\varepsilon_0 = 8.85\times10^{-12}$

(a) $B_0 = \frac{0.7\times10^{6}}{3\times10^{8}} \approx 2.33\times10^{-3}\ (T)$ #

(b). $\text{Intensity}\ (I) = S_{avg} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 C} = \frac{CB_{max}^2}{2\mu_0}$ ※

$$\Rightarrow I = \frac{1}{2}\cdot\frac{(0.7\times10^{6})^2}{4\pi\times10^{-7}\cdot 3\times10^{8}} \approx 6.51\times10^{8}\ (W/m^2)$$ #

(c) $A = \frac{\pi d^2}{4} = \frac{\pi\,(10^{-3})^2}{4} \approx 7.85\times10^{-7}\ (m^2)$

$P = IA = 6.51\times10^{8}\cdot 7.85\times10^{-7} \approx 511\ (W)$ #

Momentum, Radiation Pressure.

25

$I = 6\ (W/m^2)$

$A = 40\times10^{-4}\ (m^2)$

perfectly reflection *

(a) $P_{momentum} = \frac{E}{C}$, $E = 2\cdot IA$

mirror

$$P = \frac{2\cdot 6\cdot 40\times10^{-4}}{3\times10^{8}} \approx 1.6\times10^{-10}\ (N\cdot s)$$ #

(b) ∵ momentum per second = force

∴ $F = 1.6\times10^{-10}\ (N)$

(c) $F = \frac{dP}{dt}$

43

mirror diameter $D = 1 (m)$

absorbing plate $r = 2\times10^{-2} (m)$

water : $1 (L) = 1 (kg)$.

$20^\circ C \to 100^\circ C$

$I_{sun} = 1000 \left(W/m^2\right)$

(a)

$$A_{mirror} = \frac{\pi D^2}{4} = \frac{\pi}{4} 1^2 = \frac{\pi}{4}$$

$$\Rightarrow P_{collected} = I_{sun} A_{mirror}$$

$$= 1000 \cdot \frac{\pi}{4} = 250\pi$$

$$A_{plate} = \pi r^2 = \pi \left(2\times10^{-2}\right)^2 = 4\times10^{-4}\pi$$

$$\Rightarrow I_{plate} = \frac{P_{collected}}{A_{plate}}$$

$$= \frac{250\pi}{4\times10^{-4}\pi} = \underline{62.5\times10^4 \left(W/m^2\right)}\ \#$$

(b).

$$I = \frac{E_{max}^2}{2\mu_0 C} \Rightarrow E_{max} = \sqrt{2\mu_0 C I} = \sqrt{2\left(4\pi\times10^{-7}\right)\left(3\times10^8\right)\left(6.25\times10^4\right)} \cong \underline{2.17\times10^4 \left(V/m\right)}\ \#$$

(c).

$$I = \frac{C B_{max}^2}{2\mu_0} \Rightarrow B_{max} = \sqrt{\frac{2\mu_0 I}{C}} = \sqrt{\frac{2\left(4\pi\times10^{-7}\right)\cdot\left(62.5\times10^4\right)}{3\times10^8}} \cong \underline{7.23\times10^{-5} (T)}\ \#$$

or

$\left(\text{by (b). and } B = \frac{E}{C}\right)$.

(d). 補 : specific heat of water : 4186 J/kg·c

Heat required : $Q = m\, s\, \Delta T = 1\,(4186)\cdot(100-20) = 334880\ (J)$.

$$P_{absorbed} = P_{collected} \times 40\% = 0.4\times250\pi = 100\pi\ (W)$$

$$\text{time} : \Delta t = \frac{Q}{P_{absorbed}} = \frac{334880}{100\pi} \cong \underline{1066\ (s) \cong 17.8\ (mins)}\ \#$$