Lecture 09 – chapter 32

Inductance

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Before Lecture

Week 10

The General Form of Faraday's Law

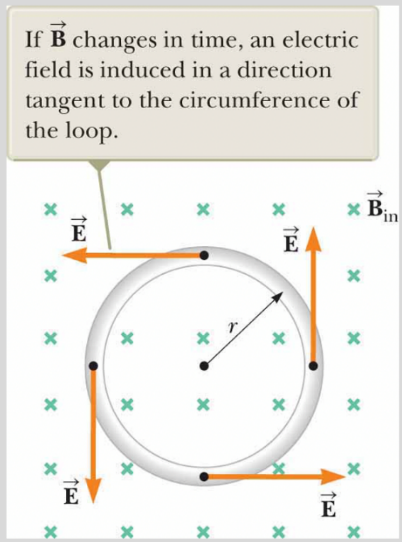
~~Why no negative sign?~~

$$\Delta V = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$E \cdot \oint ds = -\frac{d}{dt}(BA)$$

$$\Rightarrow E(2\pi r) = -\frac{dB}{dt}(\pi r^2)$$

$$\Rightarrow E = -\frac{r}{2}\frac{dB}{dt}$$



CJLI

Week 3

Electric Potential (電位)

- A charged particle moving in an electric field will experience a change in potential

A negative sign

$$\Delta V \equiv \frac{\Delta U}{q} = -\int_A^B \vec{E} \cdot d\vec{s} \qquad (25.2)\ (25.3)$$

  - The potential energy per unit charge, U/q_0
  - Has a value at every point in an electric field
  - Scalar quantity, Units: **V (voltage, volt, 伏特)**

It indicates that the V_B<V_A.

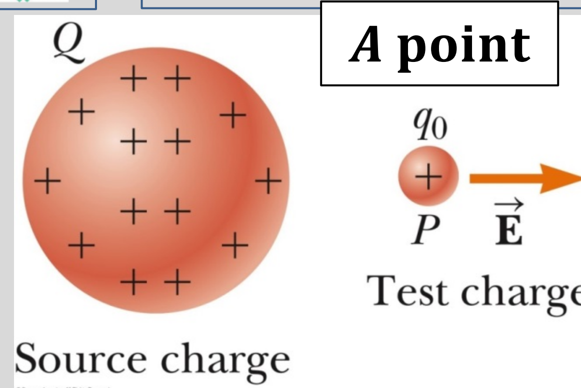
- Electron-Volts
  - Unit of energy commonly used in atomic and nuclear physics
  - 1 eV = 1.60×10^{-19} C·V = 1.60×10^{-19} J

Section 25.1

20

A point

B point



Electric field lines always point in the direction of decreasing electric potential

Before Lecture

Week 10

The General Form of Faraday's Law

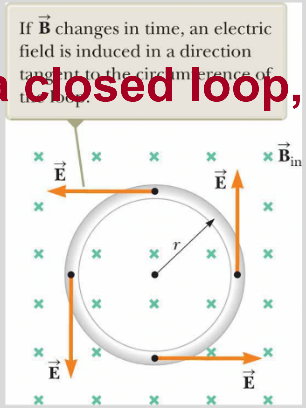
$$\Delta V = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

If A=B, a closed loop, $\Delta V \neq 0$

$$E \cdot \oint ds = -\frac{d}{dt}(BA)$$

$$\Rightarrow E(2\pi r) = -\frac{dB}{dt}(\pi r^2)$$

$$\Rightarrow E = -\frac{r}{2}\frac{dB}{dt}$$



CJLI

Week 3

Electric Potential (電位)

- A charged particle moving in an electric field will experience a change in potential

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- Electron-Volts
  - Unit of energy commonly used in atomic and nuclear physics
  - 1 eV = 1.60×10^{-19} C·V = 1.60×10^{-19} J

If A=B, $\Delta V = 0$

Section 25.1 20

Induced emf and Electric Fields

- Changing magnetic flux → An electric field in the conductor
  - Physical loop built up by conductor $\quad \varepsilon = -\frac{d\Phi_B}{dt}$
- Changing magnetic field → An electric field in empty space
  - For **any closed path,** the emf can be expressed as the **line integral of electrical field over the path** $\quad \varepsilon = \oint \vec{E} \cdot d\vec{s}$
- Faraday's law written in a general form:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

  - **The induced electric field** by magnetic field, not by the charges
  - The induced field **cannot** be an **electrostatic field** and is **non-conservative**

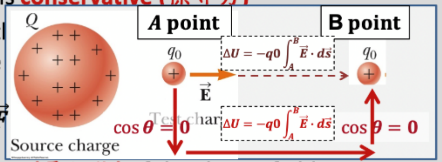
The induced electric field is different in nature from the electric field from the stationary charge

Section 31.4 53

Electric Potential Energy

- Electrostatic force is **conservative (保守力)**
- The work done with field on the charge

$\vec{F}$



- **The potential energy (電位能)** of the charge-field system

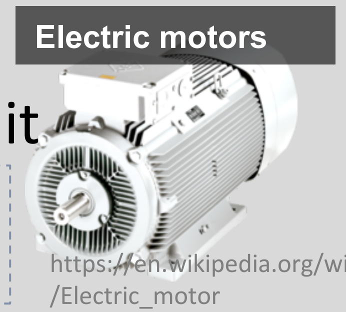
$$\Delta U = -q\int_A^B \vec{E} \cdot d\vec{s}$$

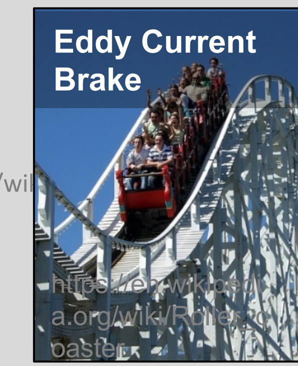
Because the force is conservative, the line integral does not depend on the path taken by the charge

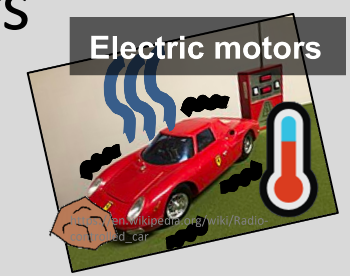
CJLI Introduction 18

Previous Lecture

- ○ Some History $\varepsilon = -\dfrac{d\Phi_B}{dt}$
- Faraday's Law of Induction
- Motional emf
  - – A moving conductor in the $\vec{B}$ field
  - – If the above is part of a closed circuit
- Lenz's law $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$
- The General Form of Faraday's Law
- ○ Generators & Motors
- ○ Eddy Currents

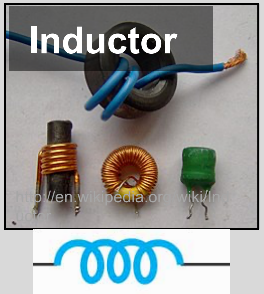


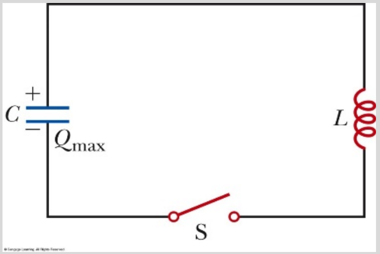


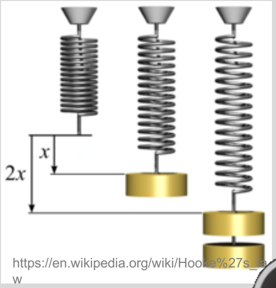


This Lecture

- Inductance
  - Self-Inductance
- *RL* Circuit
  - Energy in a Magnetic Field
- Energy Storage Summary
  - Charged capacitor, Inductor, Resistor
- Mutual Inductance
- *LC* Circuits
- *RLC* Circuit







Inductance (電感)

- A time-varying current in a circuit or a coil produces a changing magnetic flux
  - Self-inductance : an induced emf on self opposing initial emf 自感
  - Mutual induction : an induced emf on another coil 互感

- The electrical circuit element called an *inductor*
  - Energy is stored in the magnetic field
  - Electrical field in capacitor

Self-Inductance 自感

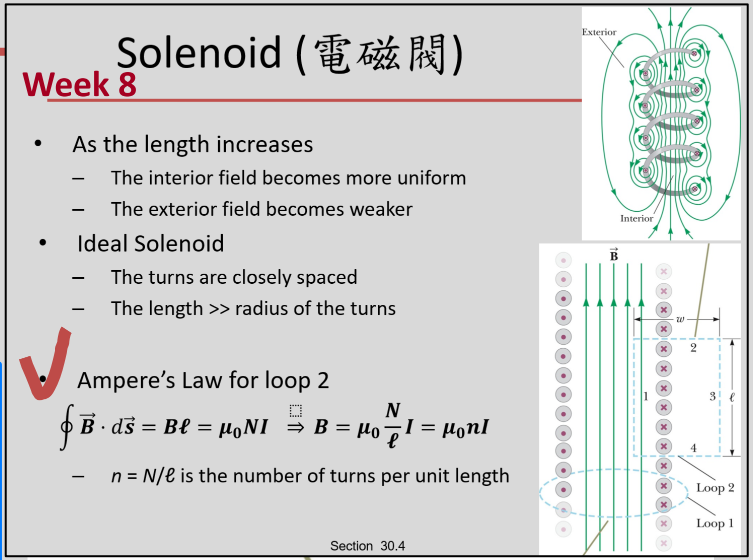
- When the switch is closed
  - Before reaching maximum, the increasing flux creates an induced emf opposite the direction of the emf of the battery by Lenz's law
- L is called the **inductance (電感)** of the coil

$$\boxed{L=\frac{\Phi_B}{i}\Rightarrow \varepsilon_L=-\frac{d\Phi_B}{dt}=-L\frac{di}{dt} \qquad \textbf{(32.1)}}$$

  - SI unit of inductance is the **henry (H)**
- Example of solenoid (電磁閥，螺管線圈)
  - N turns and length ℓ
  - A common circuit element act like capacitor

$$\Phi_B=NBA=N\mu_0 niA=N\mu_0\frac{N}{l}iA=\frac{\mu_0 N^2 iA}{l}$$

$$L=\frac{\Phi_B}{i}=\mu_0\frac{N^2}{l}A=\mu_0 n^2 V$$


Solenoid (電磁閥)

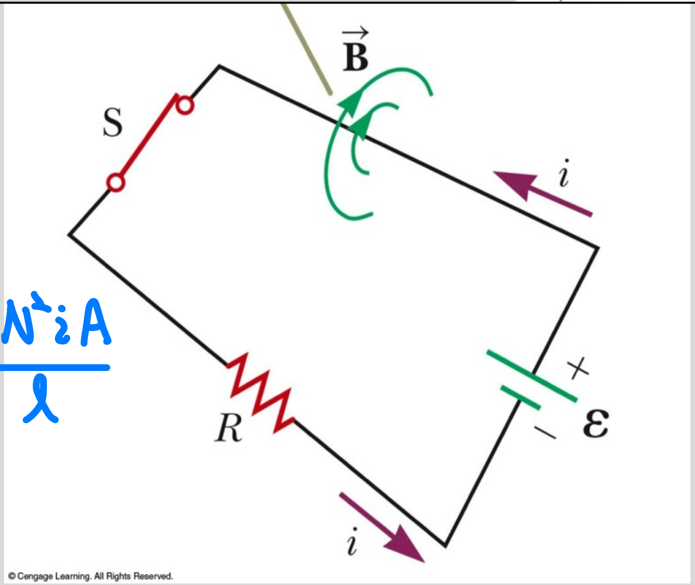
Week 8

- As the length increases
  - The interior field becomes more uniform
  - The exterior field becomes weaker
- Ideal Solenoid
  - The turns are closely spaced
  - The length >> radius of the turns
- Ampere's Law for loop 2

$$\oint \vec{B}\cdot d\vec{s}=B\ell=\mu_0 NI \Rightarrow B=\mu_0\frac{N}{\ell}I=\mu_0 nI$$

  - $n = N/\ell$ is the number of turns per unit length

Section 30.4



Section 32.1

Self-Inductance

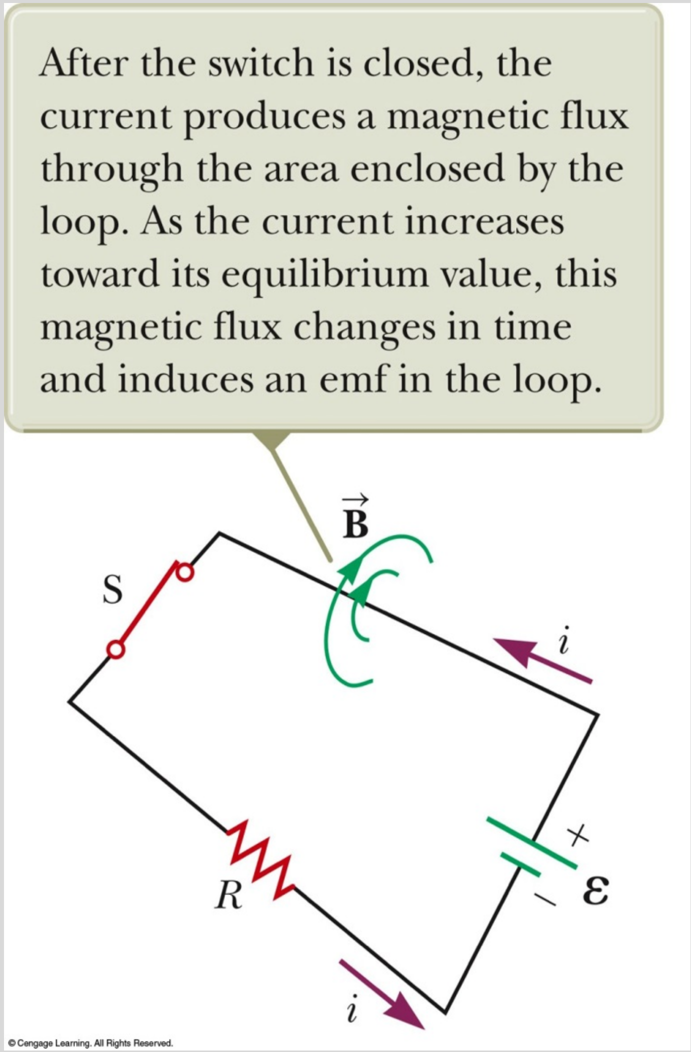
- When the switch is closed
  - Before reaching maximum, the increasing flux creates an induced emf opposite the direction of the emf of the battery by Lenz's law
- *L* is called the **inductance (電感)** of the coil

$$L = \frac{\Phi_B}{i} \Rightarrow \varepsilon_L = -\frac{d\Phi_B}{dt} = -L\frac{di}{dt} \qquad \textbf{(32.1)}$$

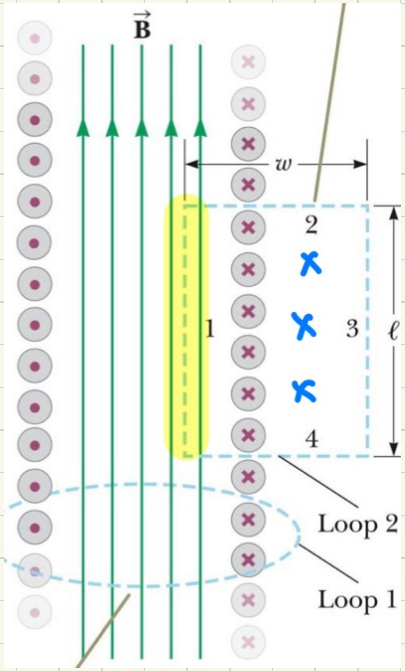
  - SI unit of inductance is the **henry (H)**
- Example of solenoid (電磁閥，螺管線圈)
  - *N* turns and length ℓ
  - A common circuit element act like capacitor

$$\Phi_B = NBA = N\mu_0 niA = N\mu_0\frac{N}{l}iA$$

$$L = \frac{\Phi_B}{i} = \mu_0\frac{N^2}{l}A = \mu_0 n^2 V$$



Self Inductance of a Solenoid



1° B in a solenoid (Ampere's Law)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

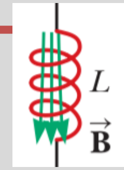
$$\Rightarrow B \cdot l = \mu_0 (NI)$$

$$\Rightarrow B = \mu_0 \frac{N}{l} I = \mu_0 n I$$

2° $\phi_B = N(BA) = N(\mu_0 n i)A$

3° $L = \frac{\phi_B}{i} = N\mu_0 n = \mu_0 \frac{N^2}{l^2}(Al) = \mu_0 n^2 V$ #

RL Circuit

- Inductor (電感)  

  - Usually self-inductance of the rest of the circuit is negligible compared to the inductor
  - $v_L(t) = L\dfrac{di(t)}{dt}$ Because the inductance of a coil produces a back emf, an inductor in a circuit opposes any change in current and prevents the current from changing instantaneously.
- When switch S_1 close and S_2 set to a (at t = 0)

$$\boldsymbol{\varepsilon - i(t)R - L\frac{di(t)}{dt} = 0}$$

$$\frac{\mathcal{E}}{R} - i - \frac{L}{R}\frac{di}{dt} = 0$$

Let $x = (\mathcal{E}/R) - i$ and $dx = -di$

Rearrange this $x = -\dfrac{L}{R}\dfrac{dx}{dt}$ $\quad -\dfrac{R}{L}dt = \dfrac{dx}{x}$

$$\Rightarrow x + \frac{L}{R}\frac{dx}{dt} = 0$$

$$\int_{x_0}^{x}\frac{dx}{x} = -\frac{R}{L}\int_0^t dt \rightarrow \ln\frac{x}{x_0} = -\frac{R}{L}t$$

$$x = x_0 e^{-Rt/L}$$

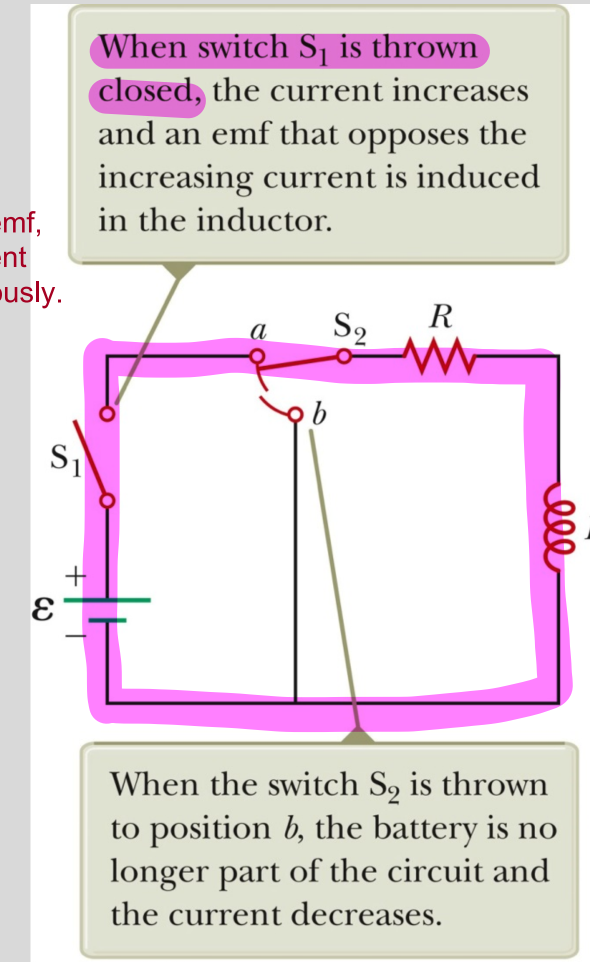
$$\Rightarrow \frac{\mathcal{E}}{R} - i = \frac{\mathcal{E}}{R}e^{-Rt/L}$$

$$\rightarrow i = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

$x = (\mathcal{E}/R) - i$

$i = 0$ at $t = 0$

$x_0 = \mathcal{E}/R$



Section 32.2

☆ We cannot use KVL here! As there is an inductor in the loop.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's Law})$$

$$\Rightarrow -\oint \vec{E} \cdot d\vec{l} = \frac{d\Phi_B}{dt}$$

1. $-\oint \vec{E} \cdot d\vec{l} = \Delta V$

2. $L = \frac{\Phi_B}{i} \Rightarrow \Phi_B = iL$

$$\Rightarrow \varepsilon - iR = L\frac{di}{dt}$$

$$\Rightarrow \frac{\varepsilon}{R} - i = \frac{L}{R}\frac{di}{dt}$$

Let $\frac{\varepsilon}{R} - i = x$

$\Rightarrow dx = -di$

$$\Rightarrow x = -\frac{L}{R}\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{R}{L}x \quad \text{(exponential decay equation)}$$

$$\Rightarrow x = x_0 e^{-Rt/L}$$

1. $x_0 = \frac{\varepsilon}{R} - i(0) = \frac{\varepsilon}{R}$

2. $x = \frac{\varepsilon}{R} - i$

$$\Rightarrow \frac{\varepsilon}{R} - i = \frac{\varepsilon}{R} e^{-Rt/L}$$

$$\Rightarrow i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$$

τ (time constant) $= \frac{L}{R}$

$$\Rightarrow i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

RL Circuit

- Inductor (電感) 
  - Usually self-inductance of the rest of the circuit is negligible compared to the inductor
  - $v_L(t) = L\frac{di(t)}{dt}$
- When switch S_1 close and S_2 set to a (at t = 0)

$$\varepsilon - i(t)R - L\frac{di(t)}{dt} = 0$$

$$i(t) = \frac{\varepsilon}{R} - \left(\frac{\varepsilon}{R} - i(o)\right)e^{-Rt/L} = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$$

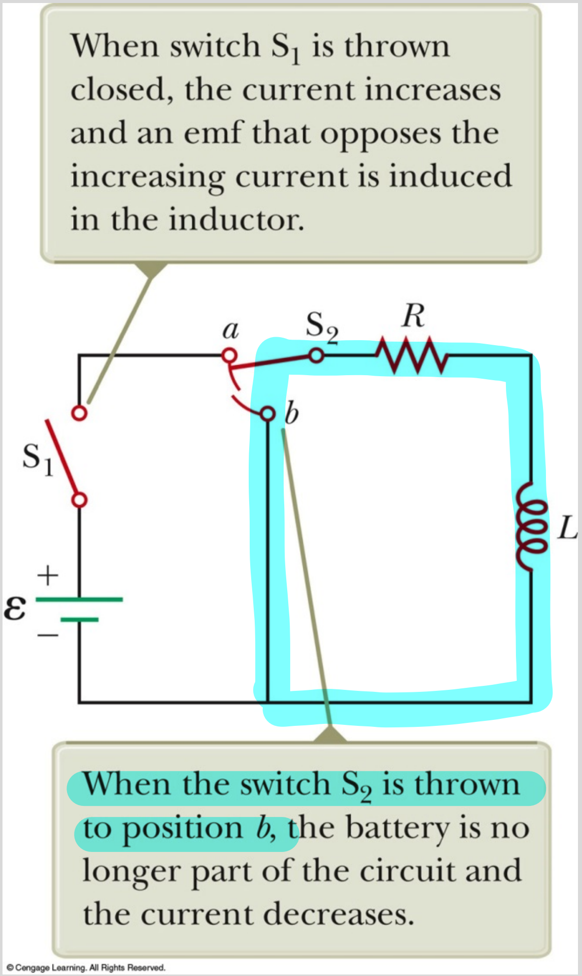
$= \frac{\varepsilon}{R}(1 - e^{-\frac{t}{\tau}})$

- Now set S_2 to position b

$$i(t)R + L\frac{di(t)}{dt} = 0 \Rightarrow i(t) = i(0)e^{-Rt/L} = \frac{\varepsilon}{R}e^{-Rt/L}$$

$= \frac{\varepsilon}{R}e^{-\frac{t}{\tau}}$

- $\tau = L / R$



$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\Rightarrow -\oint \vec{E} \cdot d\vec{l} = \frac{d\Phi_B}{dt}$$

$L = \frac{\Phi_B}{i} \Rightarrow \Phi_B = Li$

$$\Rightarrow -iR = L\frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} = -\frac{R}{L} i \quad \text{(exponential decay equation)}$$

$$\Rightarrow i = i_0 e^{-Rt/L}$$

$t=0$, steady state (DC)

For an inductor L,

$V = L\frac{di}{dt} \Rightarrow V = \frac{di}{dt} = 0$

$\Rightarrow L$ behaves like a short circuit

$\Rightarrow i_0 = \frac{\varepsilon}{R}$

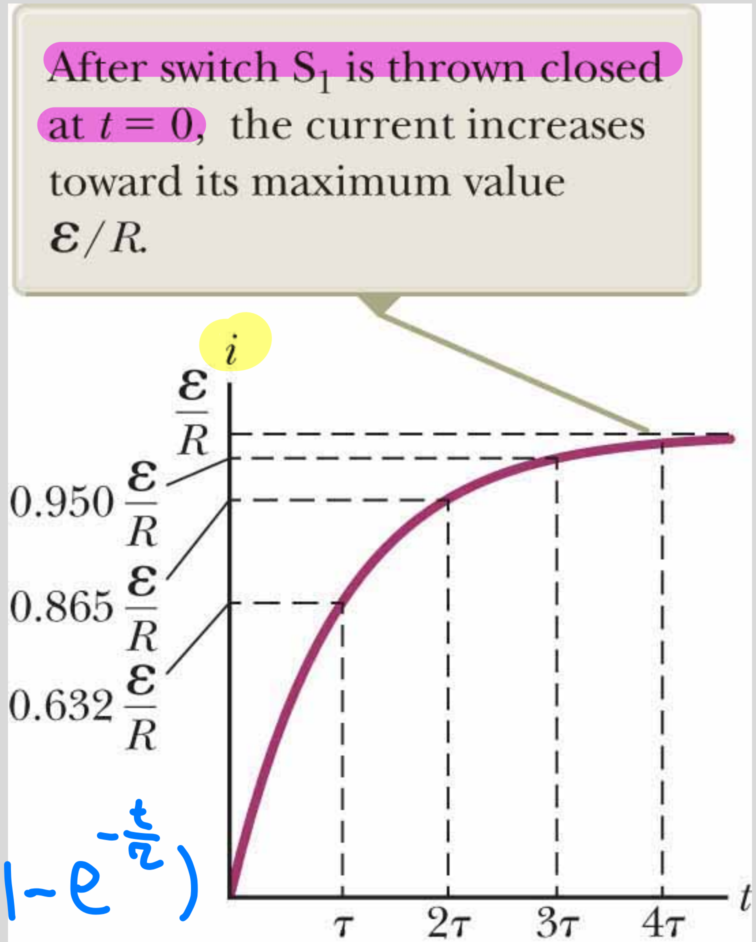
$$\Rightarrow \quad i = \frac{\varepsilon}{R} e^{-Rt/L}$$

τ (time constant) $= \frac{L}{R}$

$$\Rightarrow \quad i = \frac{\varepsilon}{R} e^{-t/\tau}$$

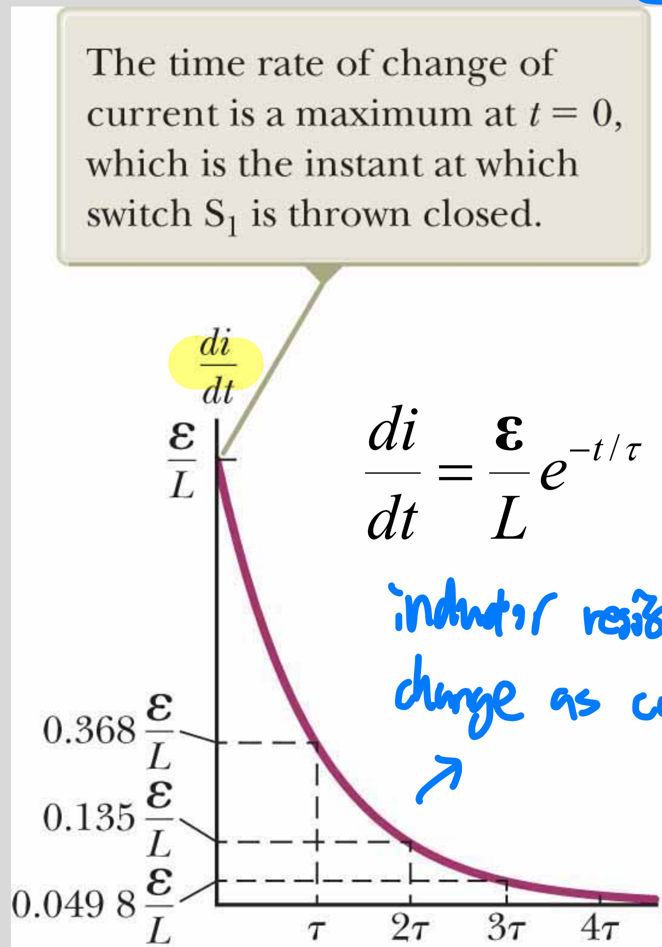
RL Circuit: i-t plot

$$\frac{di(t)}{dt} = \frac{\varepsilon}{R}\left(-e^{-\frac{t}{\tau}}\right)\left(-\frac{R}{L}\right) = \frac{\varepsilon}{L}e^{-\frac{t}{\tau}}$$



$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\left(1 - e^{-1}\right) = 0.632 = 63.2\%$$



$$\frac{di}{dt} = \frac{\varepsilon}{L}e^{-t/\tau}$$

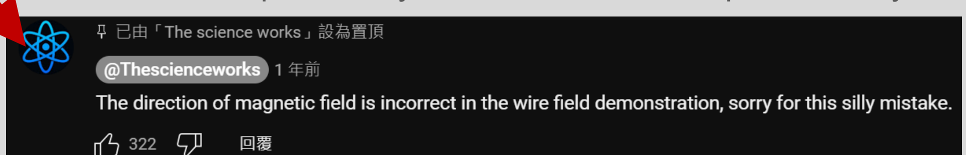
inductor resists further change as current builds

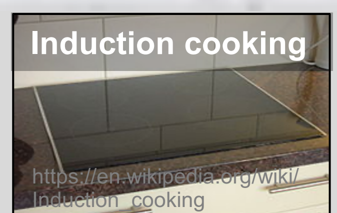
$$i(t) = \frac{\varepsilon}{R}e^{-\frac{t}{\tau}}$$

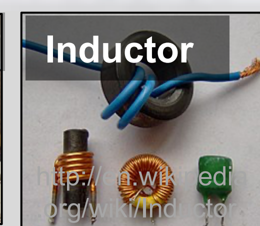
Inductor

網路資訊，同學要記得獨立思考，自主查證

https://www.youtube.com/watch?v=pbwr73R6Ay8







Energy in a Magnetic Field

power P

- For the energy rate in watt (J/s)

$$i\boldsymbol{\varepsilon} = i^2 R + Li\frac{di}{dt}$$

  - Iε : energy supplied by the battery
  - I^2R : energy delivered to the resistor
  - LI (dI/dt) : energy is being stored in the magnetic field

- For the energy (J)

$$\frac{dU_B}{dt} = Li\frac{di}{dt} \qquad U = L\int_0^i i\ di = \frac{1}{2}Li^2$$

ⓐ Energy Stored in an Inductor's Magnetic Field U_B

1° $\varepsilon = -\dfrac{d\Phi_B}{dt}$ (Faraday's Law)

$L = \dfrac{\Phi_B}{i}$

$\Rightarrow \Phi_B = Li$

$\Rightarrow \varepsilon = -L\dfrac{di}{dt}$

$\Rightarrow V_L = L\dfrac{di}{dt}$ (we often drop the minus sign by the passive sign convention)

2° $P = IV = i\left(L\dfrac{di}{dt}\right) = Li\dfrac{di}{dt}$

3° Power is the rate of energy

$$\frac{dU_B}{dt} = P = Li\frac{di}{dt}$$

multiply both sides by dt

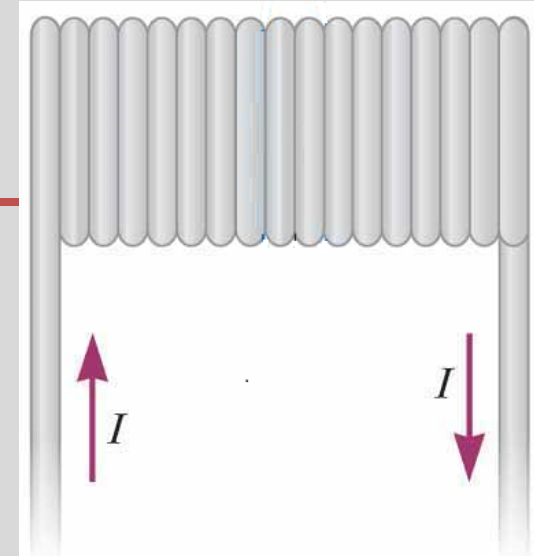
$$\Rightarrow dU_B = Li\,di$$

$$\Rightarrow \int dU_B = \int_0^i Li\,di$$

$$\Rightarrow U_B = L\int_0^i i\,di$$

$$\Rightarrow U_B = \frac{1}{2}Li^2 \quad \#$$

For solenoid



- Energy (J)

$$L = \mu_0 n^2 V \qquad B = \mu_0 n i$$

$$U_B = \frac{1}{2} L i^2$$

Previous slide

Self-Inductance

- When the switch is closed
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- L is called the **inductance (電感)** of the coil

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  - N turns and length ℓ
  - A common circuit element act like capacitor

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$$L = \frac{\Phi_B}{i} = \mu_0 \frac{N^2}{l} A = \mu_0 n^2 V$$

Week 8

Solenoid (電磁閥)

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  - The interior field becomes more uniform
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- Ideal Solenoid
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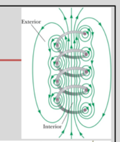
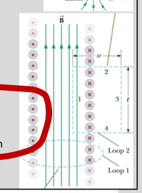
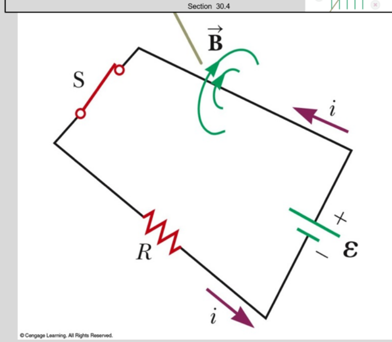
  - $n = N/\ell$ is the number of turns per unit length

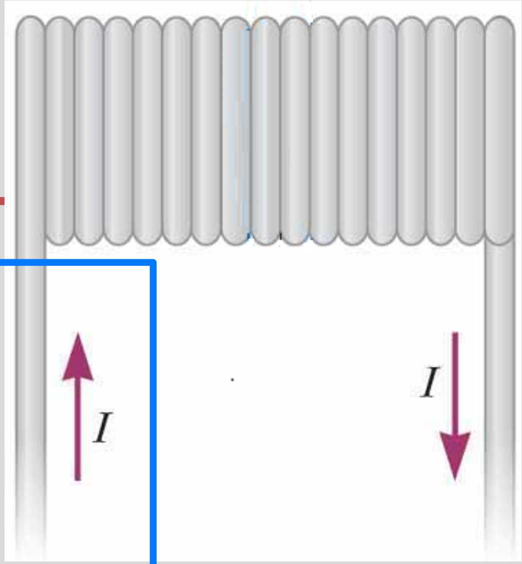




Section 30.4



For solenoid



- Energy (J)

$$L = \mu_0 n^2 V \qquad B = \mu_0 n i$$

$$U_B = \frac{1}{2} L i^2 = \frac{1}{2} \mu_0 n^2 V \left(\frac{B}{\mu_0 n} \right)^2 = \frac{B^2}{2\mu_0} V$$

- Energy density (J/m³)

$$u_B = \frac{U_B}{V} \Rightarrow \quad u_B = \frac{B^2}{2\mu_0}$$

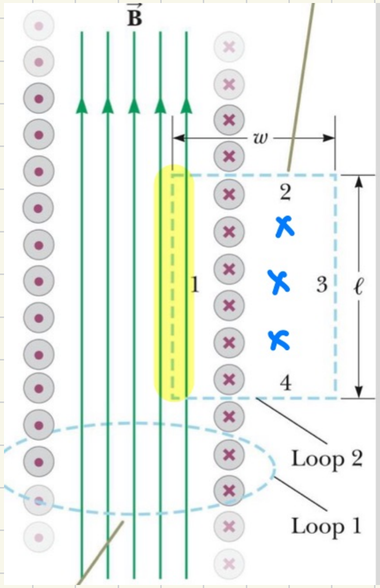
@ Energy and Energy Density

1° Ampere's Law around Loop 2

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

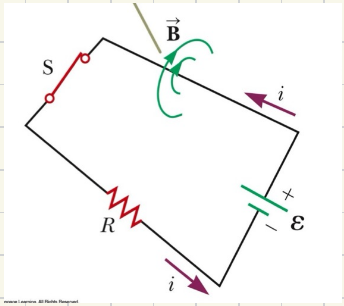
$$\Rightarrow B\ell = \mu_0 N i$$

$$\Rightarrow B = \mu_0 \frac{N}{\ell} i = \mu_0 n i$$



2° $$L = \frac{\phi_B}{i} = \frac{NBA}{i} = \frac{N(\mu_0 n i)A}{i}$$

$$= N\mu_0 n A = \mu_0 \frac{N^2}{\ell^2}(A\ell) = \mu_0 n^2 V$$



3° Energy $U_B = \frac{1}{2} L i^2$

1. From 1°, $B = \mu_0 n i$

$\Rightarrow i = \frac{B}{\mu_0 n}$

2. From 2°, $L = \mu_0 n^2 V$

$= \frac{1}{2}(\mu_0 n^2 V)\left(\frac{B}{\mu_0 n}\right)^2$

$= \frac{B^2}{2\mu_0} V$ #

4° Energy Density $u_B = \frac{U_B}{V} = \frac{B^2}{2\mu_0}$ #

Quick Quiz 31.3

- You are performing an experiment that requires the highest-possible **magnetic energy density** in the interior of a very long current-carrying solenoid. Which of the following adjustments increases the energy density? (More than one choice may be correct.)

(a) increasing the number of turns per unit length on the solenoid $n \uparrow$

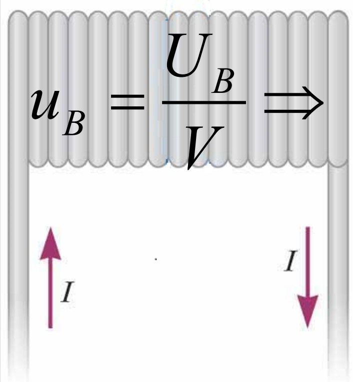
(b) increasing the cross-sectional area of the solenoid

(c) increasing only the length of the solenoid while keeping the number of turns per unit length fixed

(d) increasing the current in the solenoid $i \uparrow$

$$B = \mu_0 n i$$

highest-possible magnetic energy density

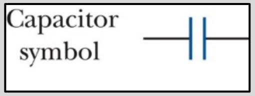


$$u_B = \frac{U_B}{V} \Rightarrow u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0 n^2 i^2}{2}$$

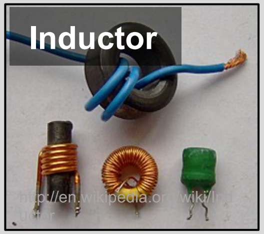
Answers: (a), (d)

Energy Storage Summary

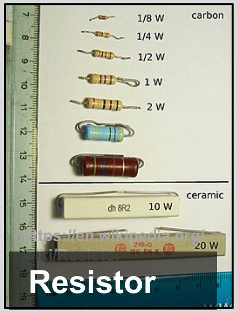
- A resistor, inductor and capacitor all store energy through different mechanisms
- Charged capacitor
  - Stores energy as **electric potential energy** (Electrical field)



- Inductor
  - When it carries a current, stores energy as **magnetic potential energy** (magnetic field)



- Resistor
  - Energy delivered is transformed into **internal energy**

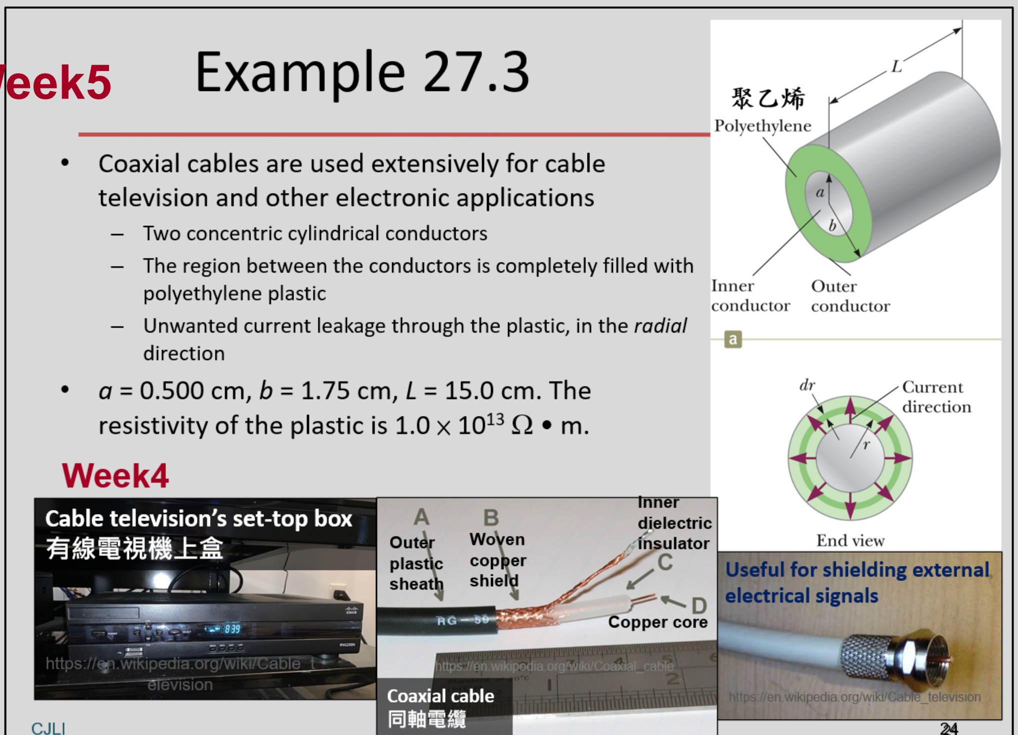


the heat increases the random motion of atoms in a resistor

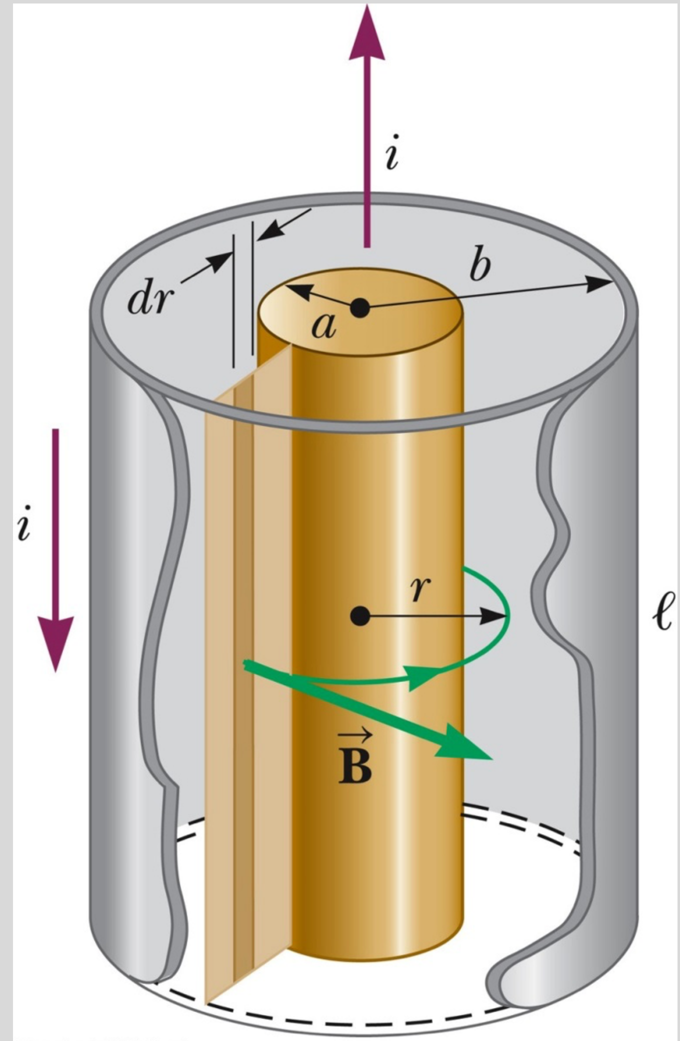
Example: The Coaxial Cable

- Coaxial cables are often used to connect electrical devices, such as your video system, and in receiving signals in television cable systems

Week5



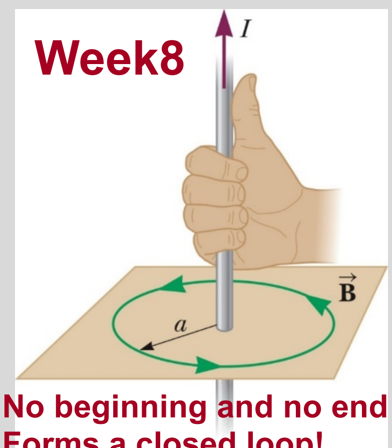
Section 32.3

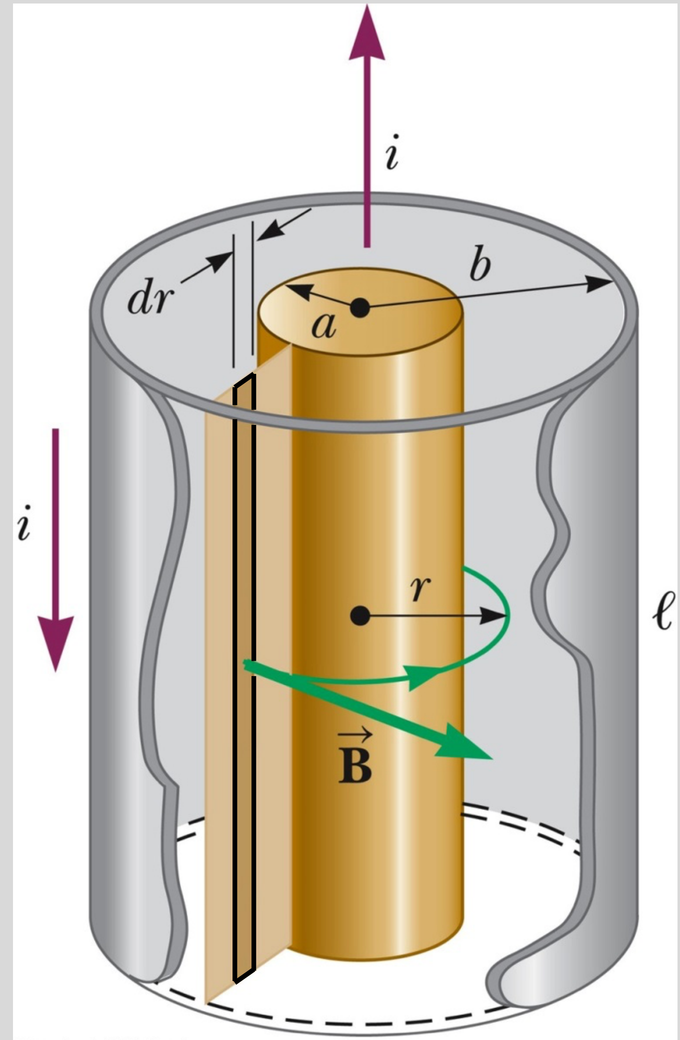


Example: The Coaxial Cable

- Coaxial cables are often used to connect electrical devices, such as your video system, and in receiving signals in television cable systems
- Model a long coaxial cable as a thin, cylindrical conducting shell of radius *b* concentric with a solid cylinder of radius *a*

$$d\Phi_B = BdA = B\ell dr$$





Section 32.3

Example: The Coaxial Cable

Week8 A Long, Straight Conductor

$$d\vec{s}\times\hat{r} = |d\vec{s}\times\hat{r}|\hat{k} = \left[dx\sin\left(\frac{\pi}{2}-\theta\right)\right]\hat{k} = (dx\cos\theta)\hat{k}$$

(1) $d\vec{B} = (dB)\hat{k} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{k}$

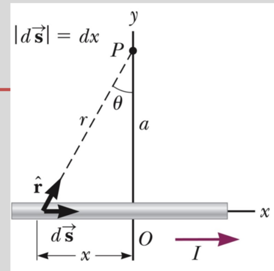
(2) $r = \frac{a}{\cos\theta}$

$x = -a\tan\theta$

(3) $dx = -a\sec^2\theta\, d\theta = -\frac{a\,d\theta}{\cos^2\theta}$

(4) $dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\, d\theta$

$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$ **(30.4)**



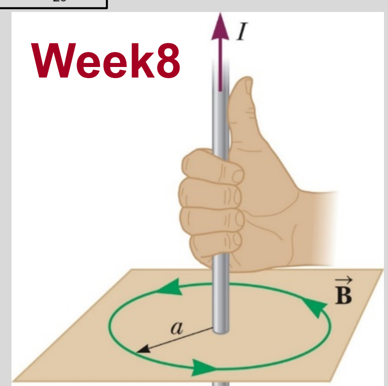
An infinitely long, straight wire, $\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$

$$B = \frac{\mu_0 I}{2\pi a}$$

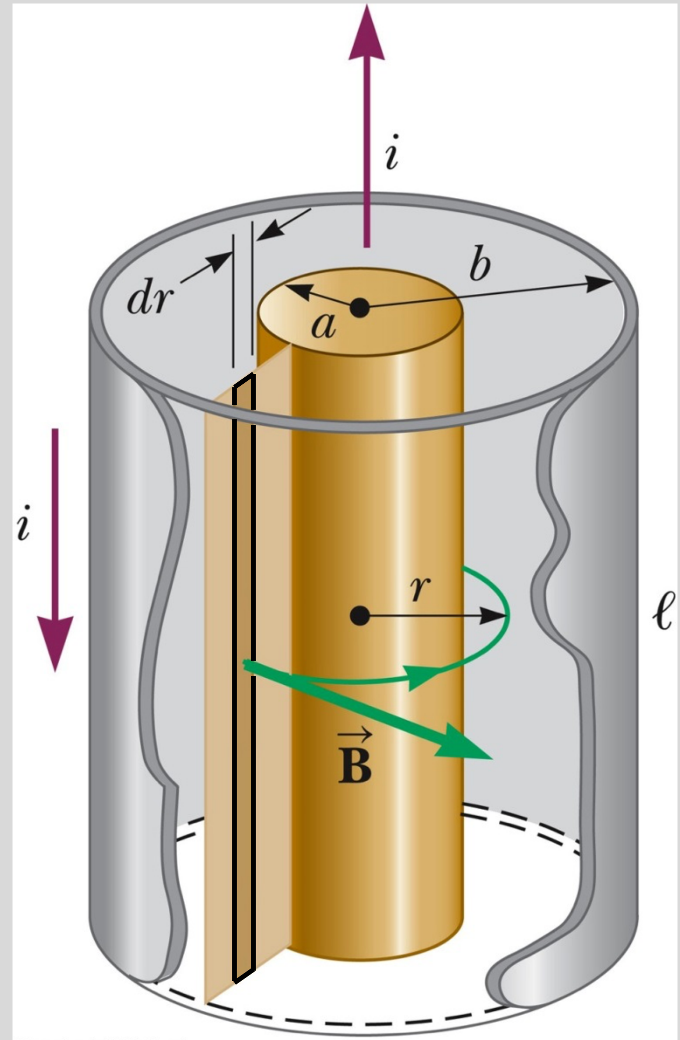
Section 30.1 29

- … connect … ideo … n television
- … hin. … f radius *a*

Same magnitude at a distance of a



No beginning and no end
Forms a closed loop!



$$d\Phi_B = BdA = B\ell dr$$

$$\Phi_B = \int_a^b \frac{\mu_0 i}{2\pi r}\ell dr$$

Example: The Coaxial Cable

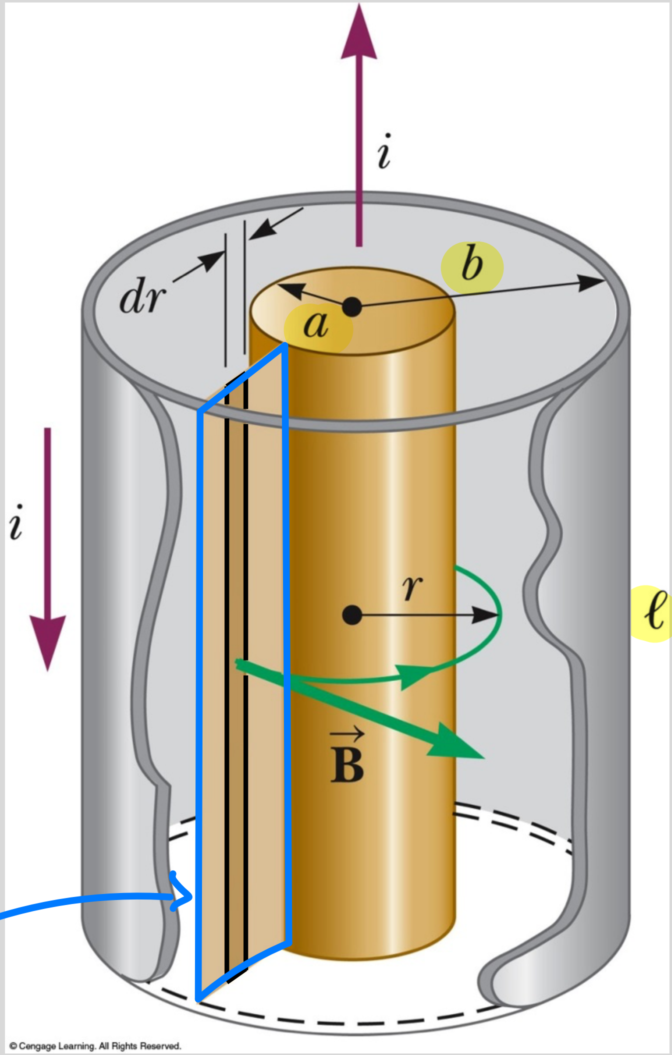
- Coaxial cables are often used to connect electrical devices, such as your video system, and in receiving signals in television cable systems
- Model a long coaxial cable as a thin, cylindrical conducting shell of radius *b* concentric with a solid cylinder of radius *a*

$$d\Phi_B = B\,dA = B\ell\,dr$$

$$\Phi_B = \int_a^b \frac{\mu_0 i}{2\pi r}\ell\,dr = \frac{\mu_0 i\ell}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i\ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$L = \frac{\Phi_B}{i} = \frac{\mu_0 \ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

chosen surface



Section 32.3

@ Inductance in a Coaxial Cable

$$L = \frac{\phi_B}{i}$$

$$= \frac{\int B \cdot dA}{i}$$

1. $dA = \ell \, dr$

2. Obtain B using Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \Rightarrow B = \frac{\mu_0 i}{2\pi r}$$

$$= \frac{\int_a^b \frac{\mu_0 i}{2\pi r}\, l\, dr}{i}$$

$$= \int_a^b \frac{\mu_0 l}{2\pi r}\, dr$$

$$= \frac{\mu_0 l}{2\pi} \int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 l}{2\pi} \ln \frac{b}{a} \quad \#$$

1. Why do the currents flow in opposite directions in the inner and outer conductors?

This is **intentional and necessary** for coaxial cables.

- The **inner wire** carries current $+i$ (say, upward).
- The **outer cylindrical shell** carries **return current** $-i$ (downward).

Why this matters:

- **Net external magnetic field = 0** outside the coaxial cable.
- The opposing currents **cancel each other's magnetic field** outside the cable.
- This makes coaxial cables highly resistant to **EM interference**, **signal radiation**, and **cross-talk**.

> ✓ This current configuration confines the magnetic field **between** the conductors — where inductance is calculated.

Mutual Inductance

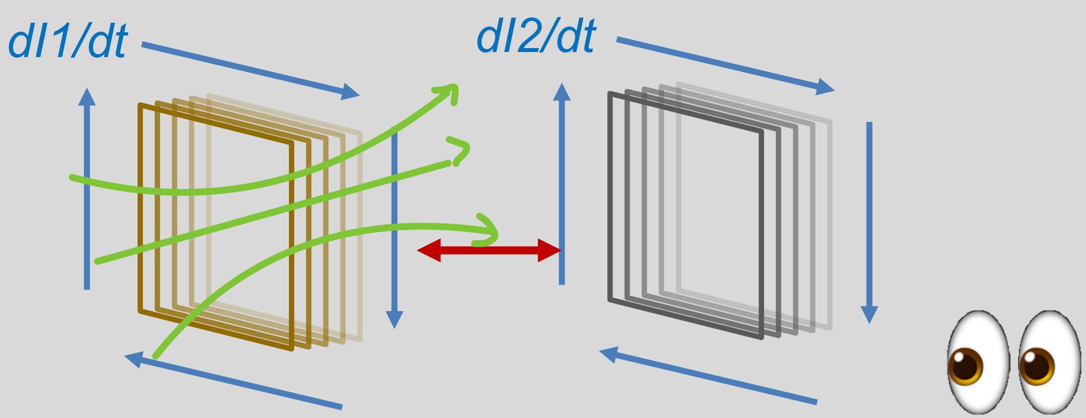
互感

- If current I_1 varies with time, the emf induced

$$\varepsilon_2 = -N_2 \frac{d\Phi_{12}}{dt} = -M_{12}\frac{di_1}{dt} \qquad \textbf{(32.15)}$$

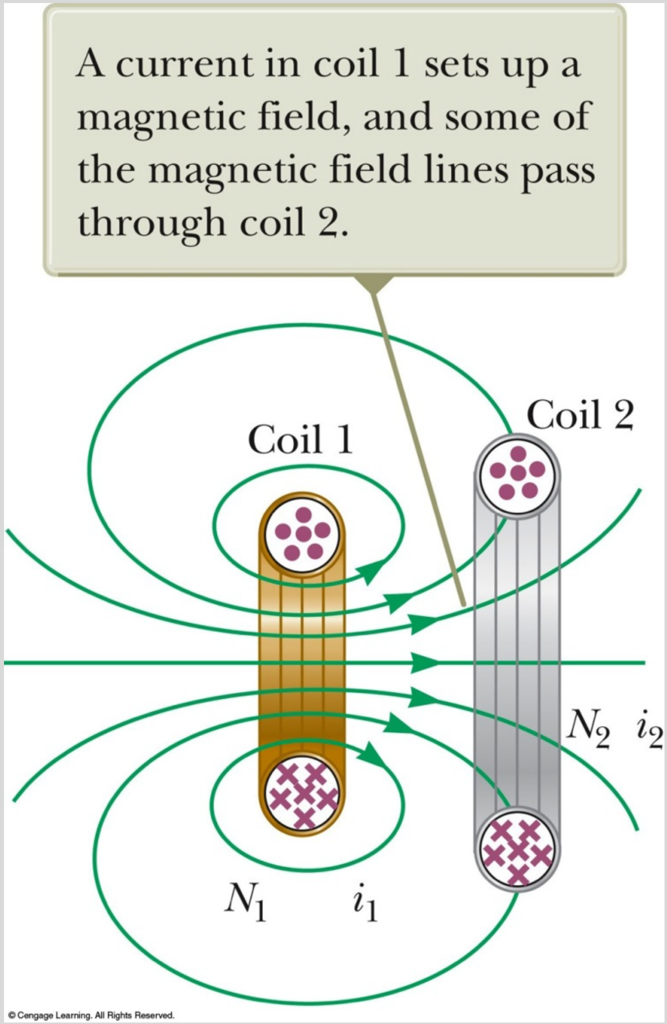
  - **Mutual inductance M_{12}** of coil 2 with respect to coil 1

$$M_{12} = \frac{N_2 \Phi_{12}}{i_1} \qquad \textbf{(32.16)}$$



As the circuit separation distance increases, the mutual inductance decreases because the magnetic flux through the 2nd circuit decreases.

Section 32.4



Mutual Inductance

- If current I_1 varies with time, the emf induced

$$\boldsymbol{\varepsilon}_2 = -N_2 \frac{d\Phi_{12}}{dt} = -M_{12}\frac{di_1}{dt} \qquad \textbf{(32.15)}$$

  - **Mutual inductance M_{12}** of coil 2 with respect to coil 1

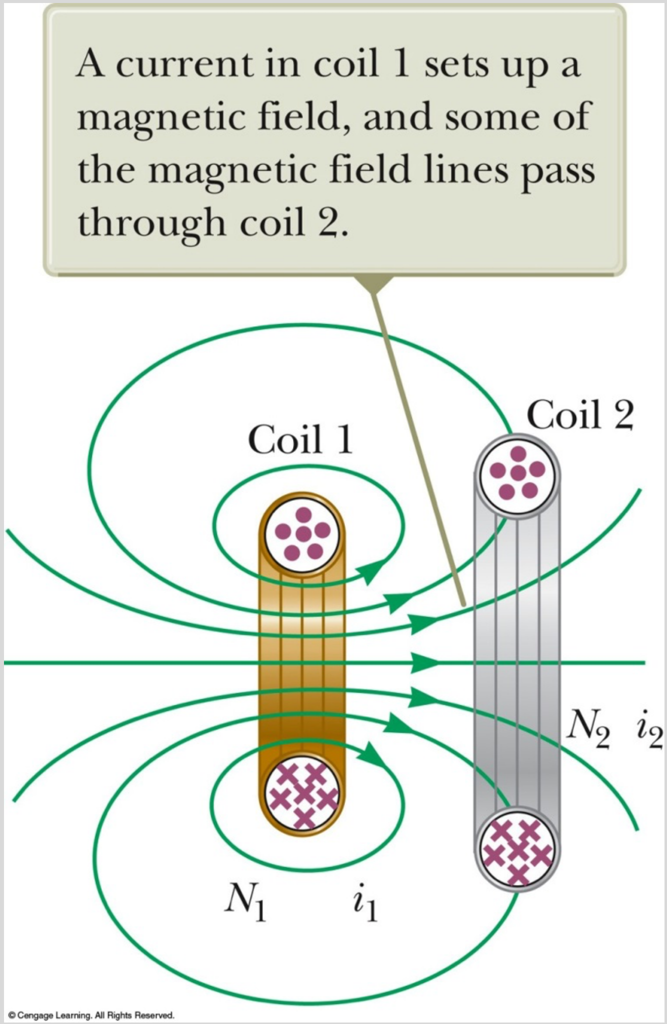
$$M_{12} = \frac{N_2 \Phi_{12}}{i_1} \qquad \textbf{(32.16)}$$

- If current 2 varies with time, the emf induced

$$\boldsymbol{\varepsilon}_1 = -M_{21}\frac{di_2}{dt}$$

- **$M_{12} = M_{21} = M$** (mutual inductance)

$$\boldsymbol{\varepsilon}_1 = -M\frac{di_2}{dt} \quad \text{and} \quad \boldsymbol{\varepsilon}_2 = -M\frac{di_1}{dt}$$



Section 32.4

① i = constant

$$N_2 \, \phi_{B_2} = M i_1 \Rightarrow M = \frac{N_2 \phi_{B_2}}{i_1}$$

differentiate both sides

② $i \neq$ constant

$$\begin{cases} N_2 \dfrac{d\phi_{B_2}}{dt} = M \dfrac{di_1}{dt} \\ \varepsilon_2 = -N_2 \dfrac{d\phi_{B_2}}{dt} \end{cases}$$

(Faraday's Law of Induction)

$$\Rightarrow \varepsilon_2 = -M \frac{di_1}{dt}$$

🔍 What is this formula actually doing?

This equation **does not imply that mutual inductance depends on** i_1 in real time.

Instead, it's a **definition** that calculates M_{12} *based on a test value* of current i_1 and the resulting flux Φ_{12} through coil 2:

$$M_{12} = \text{Flux through coil 2 caused by unit current in coil 1}$$

⚠️ Important clarification:

- **Mutual inductance** M is a **property of the geometry** and **material** (e.g., number of turns, coil spacing, area, core permeability).
- It is **constant** for a fixed setup.

So why is $M \propto \frac{1}{i_1}$ in this equation?

Because you're using this as a **one-time measurement**:

- You apply current i_1,
- You **measure** the resulting flux linkage $N_2\Phi_{12}$,
- You calculate:

$$M_{12} = \frac{N_2\Phi_{12}}{i_1}$$

If you **double** i_1, the flux Φ_{12} will also double → and the ratio stays **constant**.

🧠 In short:

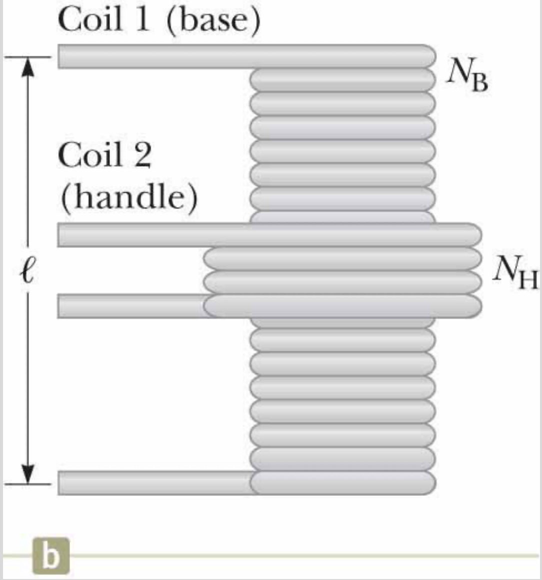
> The formula looks like $M \propto \frac{1}{i_1}$, but it's not actually a variable dependence — it's just a way to **define** M from one instance of i_1 and the resulting flux.

Example: "Wireless" Battery Charger

- An electric toothbrush has a base designed to hold the toothbrush handle when not in use. As shown in the photo, the handle has a cylindrical hole that fits loosely over a matching cylinder on the base. When the handle is placed on the base, a changing current in a solenoid inside the base cylinder induces a current in a coil inside the handle. This induced current charges the battery in the handle.

$$B = \mu_0 \frac{N_B}{\ell} i$$

$$M = \frac{N_H \Phi_{BH}}{i} = \frac{N_H B A}{i} = \boxed{\mu_0 \frac{N_B N_H}{\ell} A}$$



(i : current in the base
A : cross-sectional area of the base coil)

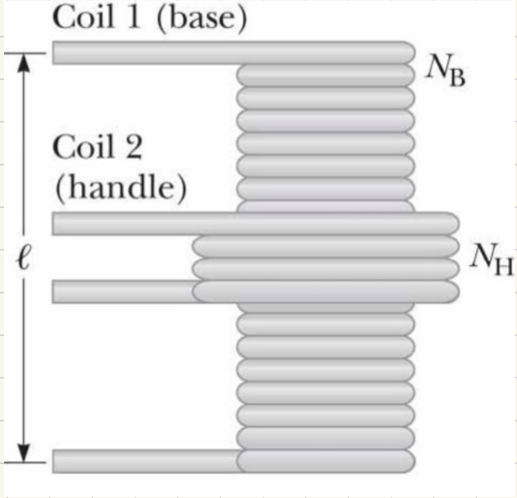
@ Mutual Inductance of a "Wireless" Battery Charger

$$M = \frac{N_H \Phi_{BH}}{i}$$

$$\Phi_{BH} = BA\cos 0$$

$$= BA$$

$$= \frac{N_H B A}{i}$$

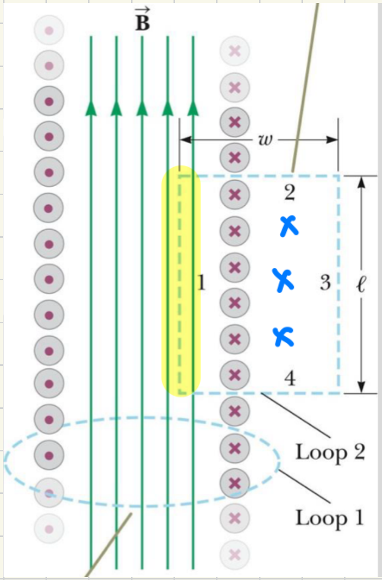


Obtain B using Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$\Rightarrow B\ell = \mu_0 N i$$

$$\Rightarrow B = \mu_0 \frac{N}{\ell} i = \mu_0 n i$$



$$= \frac{N_H\,(\mu_0 n i)\,A}{i}$$

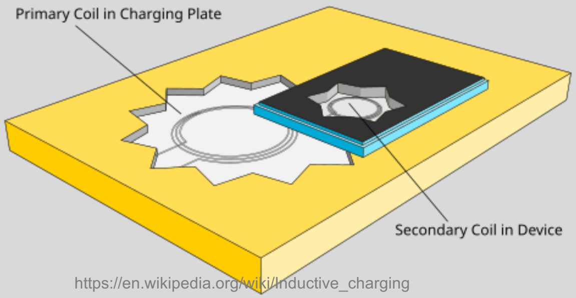
$$= \mu_0 N_H n A$$

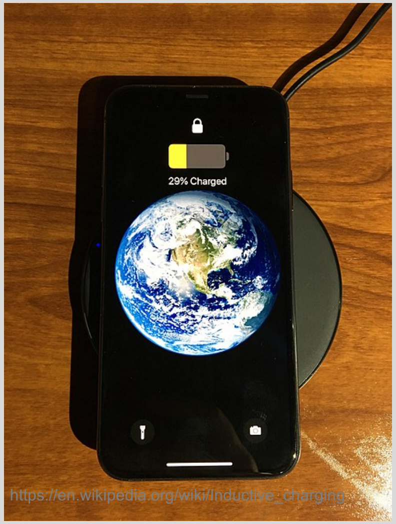
$$n = \frac{N_B}{l}$$

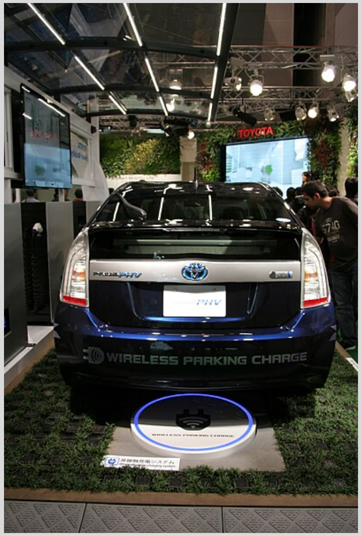
$$= \mu_0 \frac{N_B N_H}{l} A$$

#

Inductive charging







The primary coil in the charger induces a current in the secondary coil of the device being charged.

https://en.wikipedia.org/wiki/Inductive_charging

LC Circuits

- Assume the capacitor is initially charged and then the switch is closed

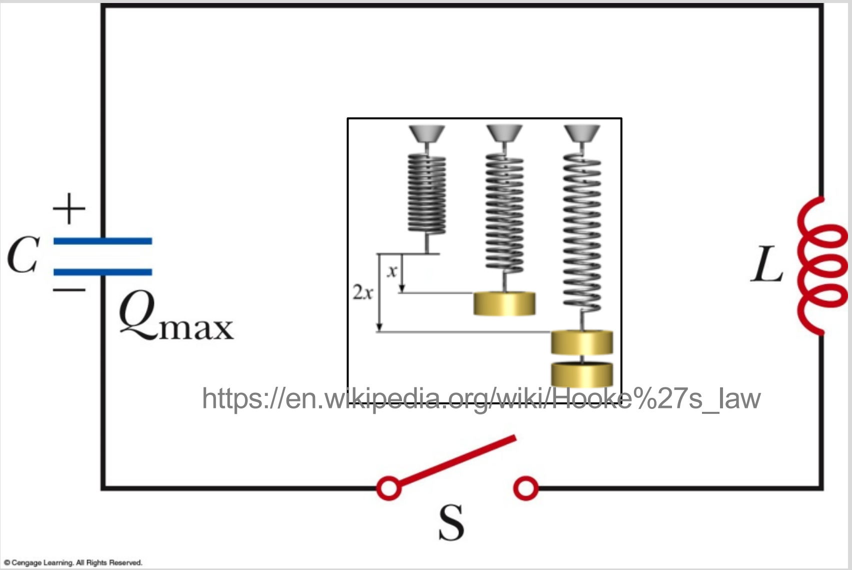
$$\boxed{U = U_E + U_B = \frac{q^2}{2C} + \tfrac{1}{2}Li^2 = \frac{Q_{max}^2}{2C}}$$

energy conservation

KVL: $v_C + v_L = 0 \rightarrow$

$$\frac{q}{C} = -L\frac{di}{dt} = -L\frac{d^2q}{dt^2}$$

$$\boxed{\frac{d^2q}{dt^2} = -\frac{1}{LC}q = -w^2q}$$



Hooke's Law, k: spring constant

A spring system move as SHM

$$F = -kx = ma$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -w^2x$$

Time Functions of an *LC* Circuit

- The function of charge (on the capacitor)

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q = -w^2q$$

$$q = Q_{max} \cos(\omega t + \phi)$$

(When = 0, q=Qmax) **(32.21)**

- The function of current

$$i = \frac{dq}{dt} = -\omega Q_{max} \sin(\omega t + \phi) = -I_{max} \sin(\omega t + \phi)$$ **(32.23)**

- The function of total energy

$$\boxed{U = U_E + U_B = \frac{q^2}{2C} + \tfrac{1}{2}Li^2 = \frac{Q_{max}^2}{2C}}$$

$$U = U_E + U_B = \frac{Q_{max}^2}{2c}\cos^2 \omega t + \tfrac{1}{2}LI_{max}^2 \sin^2 \omega t \rightarrow$$ **(32.26)**

Omitting ϕ assumes the system starts at maximum charge, $\phi = 0$.

- The angular frequency

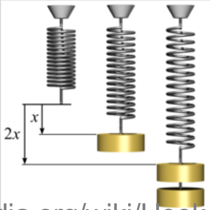
$$\omega = 1 / \sqrt{LC}$$

A spring system move as SHM

$$F = -kx = ma$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -w^2x$$ **(32.22)**

  - It is the natural frequency of oscillation of the circuit.



https://en.wikipedia.org/wiki/Hooke%27s_law

The function of charge on the capacitor C

$$q = Q_{max} \cos(\omega t + \phi)$$

$$\Rightarrow \frac{dq}{dt} = i = -\omega Q_{max} \sin(\omega t + \phi)$$

$$\Rightarrow \frac{d^2q}{dt^2} = -\omega^2 Q_{max} \cos(\omega t + \phi)$$

$$= -\omega^2 q$$

$$\omega = \frac{1}{\sqrt{LC}}$$

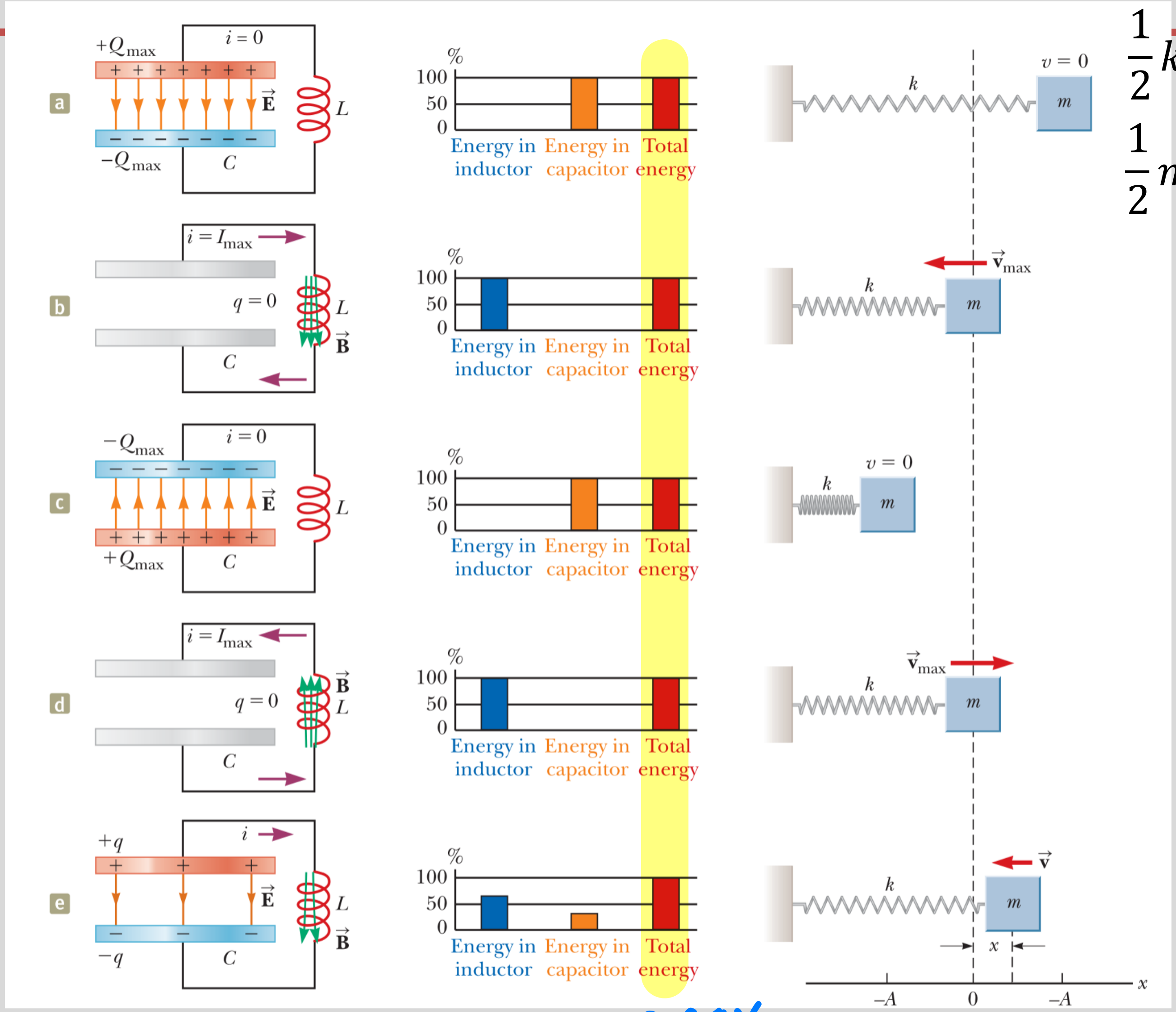
$$= -\frac{q}{LC}$$

LC Circuits

Spring LC-Circuit

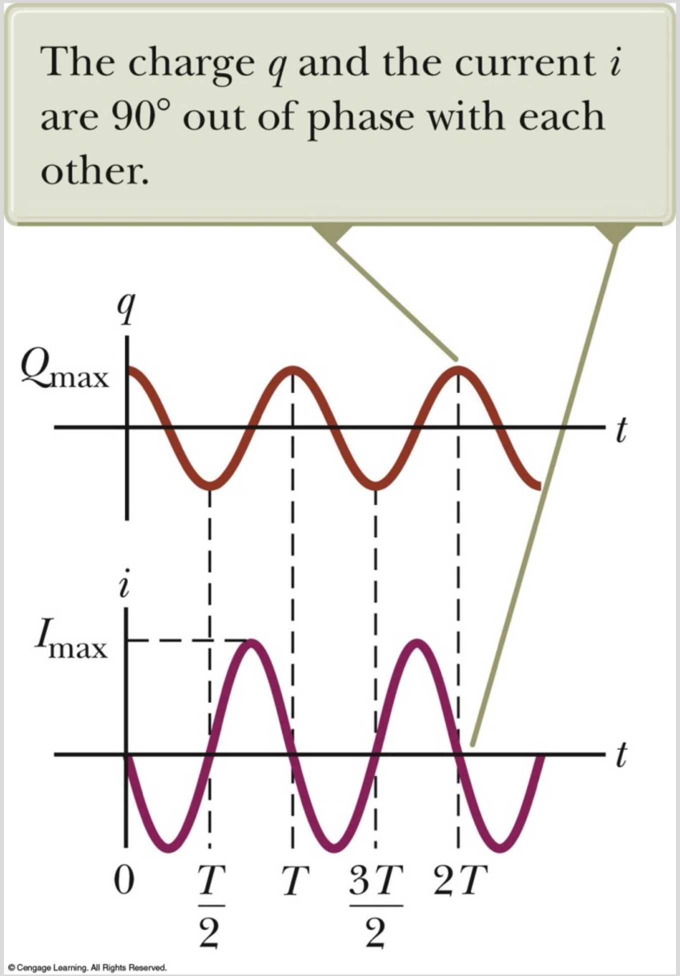
$$\frac{1}{2}kx^2 \Leftrightarrow \frac{Q^2}{2C}$$

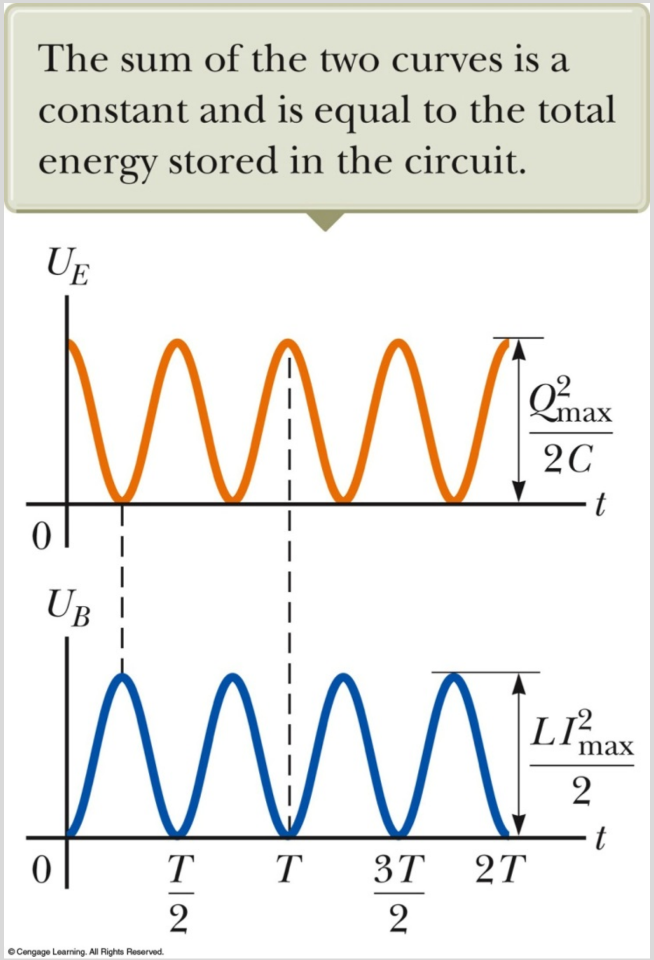
$$\frac{1}{2}mv^2 \Leftrightarrow \frac{1}{2}Li^2$$



energy conservation

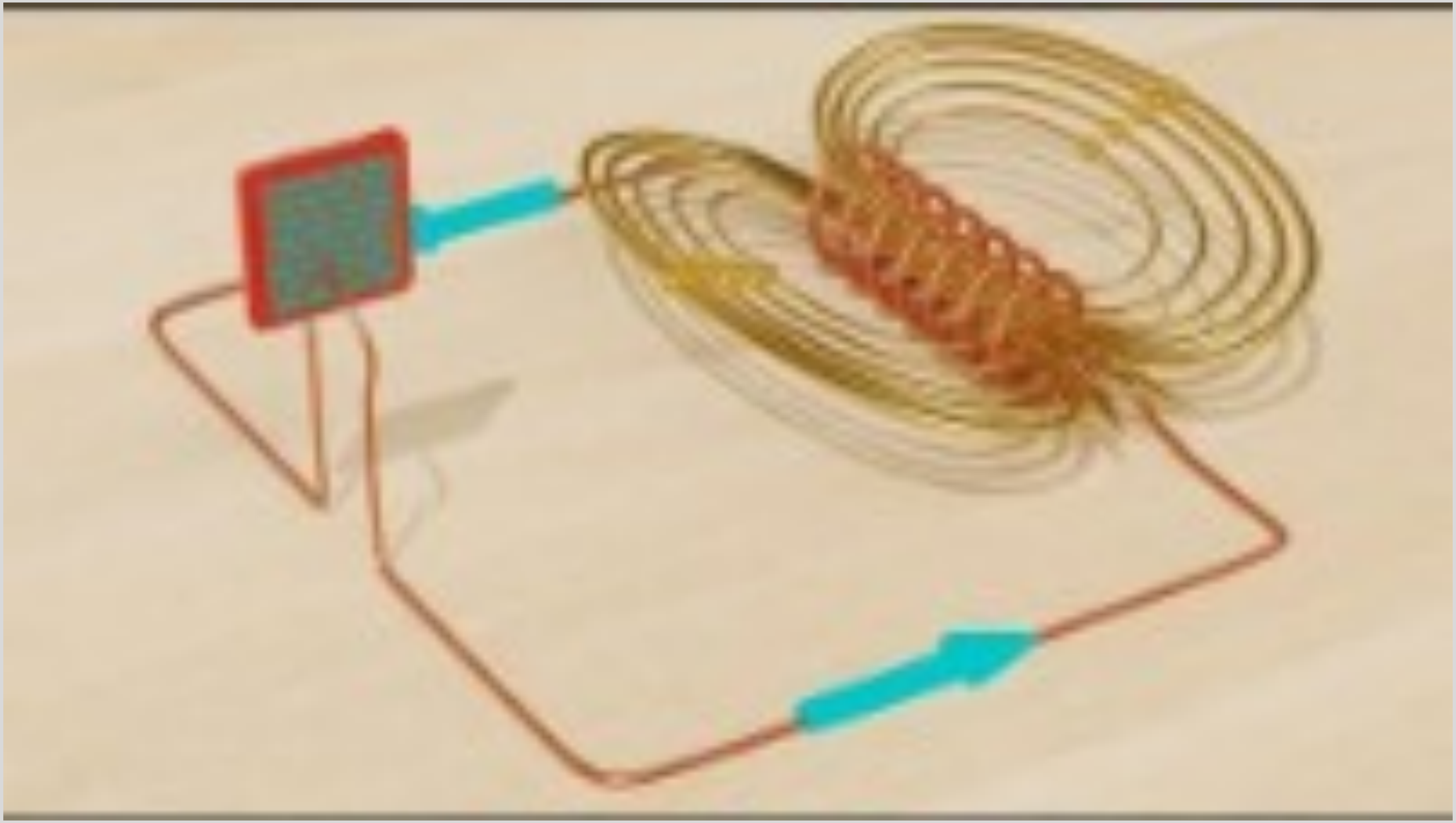
Charge, Current, Energy





LC Circuits

網路資訊，同學要記得獨立思考，自主查證



https://www.youtube.com/watch?v=2_y_3_3V-so

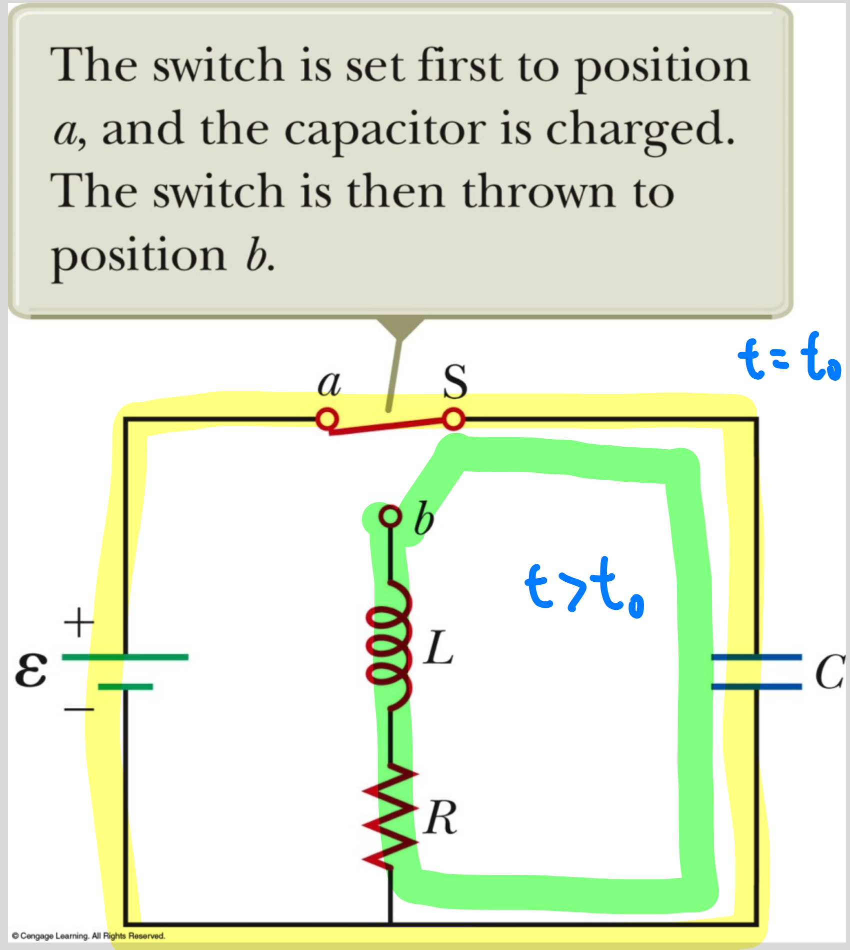
2nd-order linear differential equation

The *RLC* Circuit

- The total energy is not constant
  - Transformation to internal energy in the resistor at the rate of $dU/dt = -I^2R$
- Kirchhoff's loop laws

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

- The *RLC* circuit is analogous to a damped harmonic oscillator
  - Small R
  - Large R
  - R=Rc



Damped *RLC* Circuit

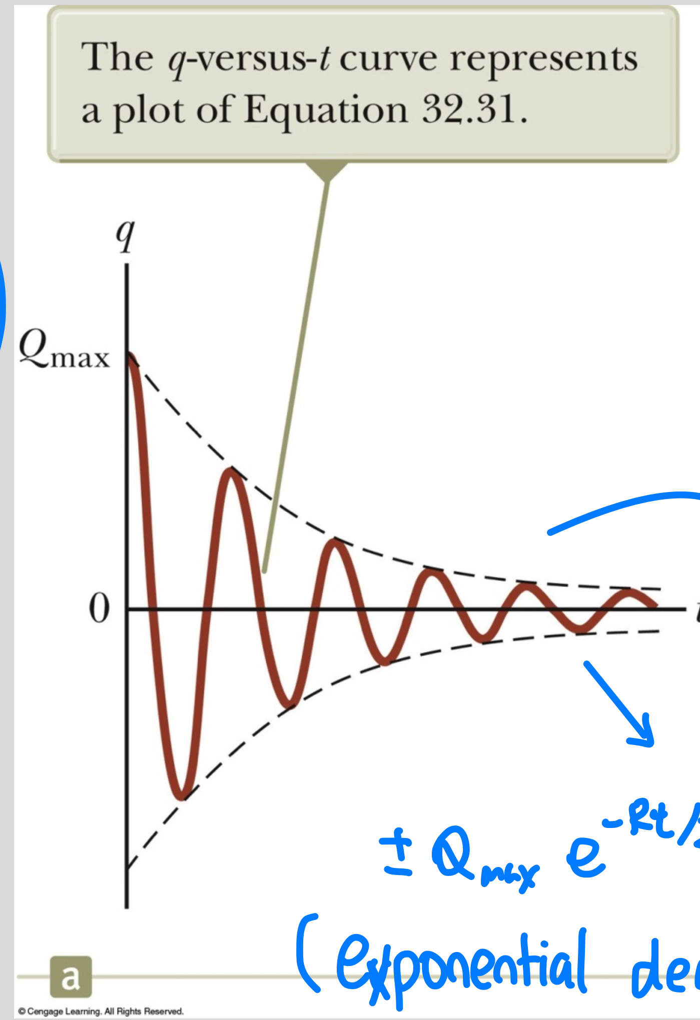
causes the amplitude to shrink over time

- $q = Q_{max}\, e^{-Rt/2L} \cos \omega_d t$

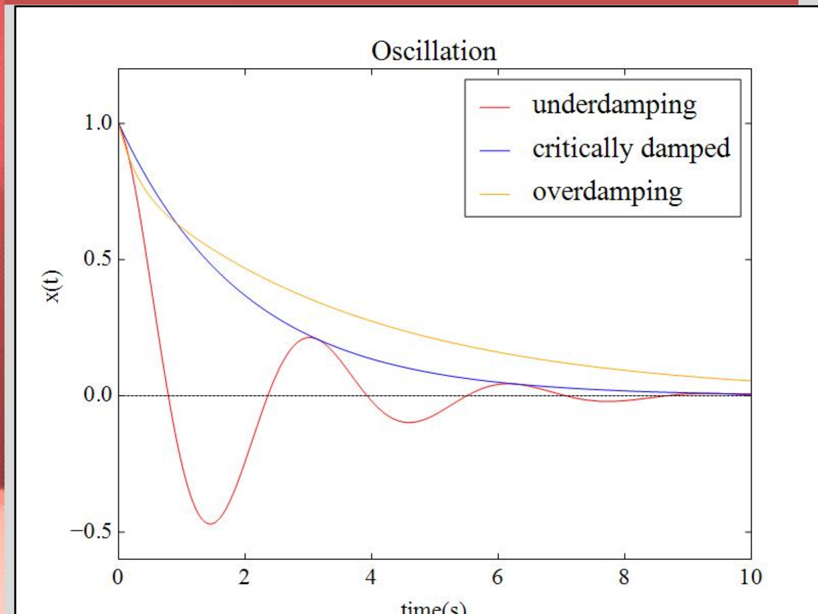
$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L} \right)^2 \right]^{1/2}$$ (damped angular velocity)

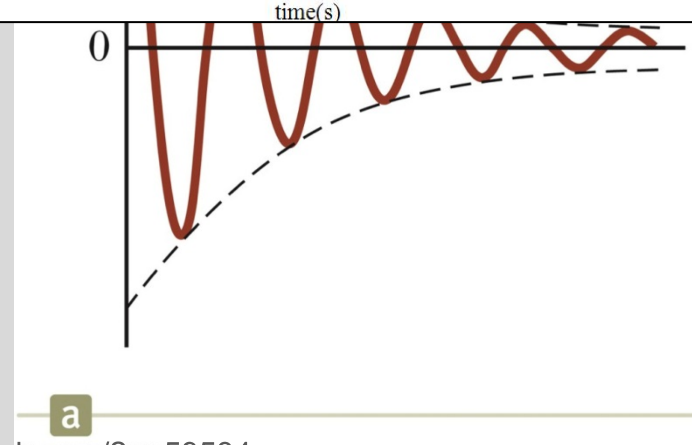
$$R_C = \sqrt{4L/C}$$ (critical resistance)

  - $R < R_C$: damped oscillation
  - $R = R_C$: critically damped
  - $R > R_C$: overdamped



Damped *RLC* Circuit





- $R < R_C$: damped oscillation
- $R = R_C$: critically damped
- $R > R_C$: overdamped

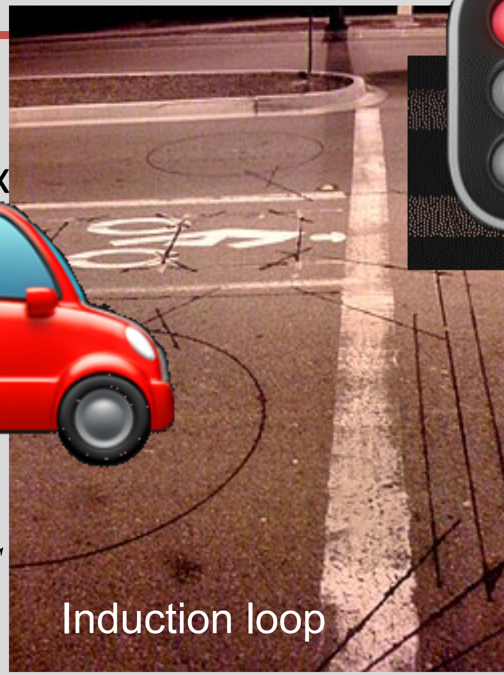
油壓關門裝置，避免撞擊噪音，
但也希望它能儘快回到平衡位置

https://highscope.ch.ntu.edu.tw/wordpress/?p=59534

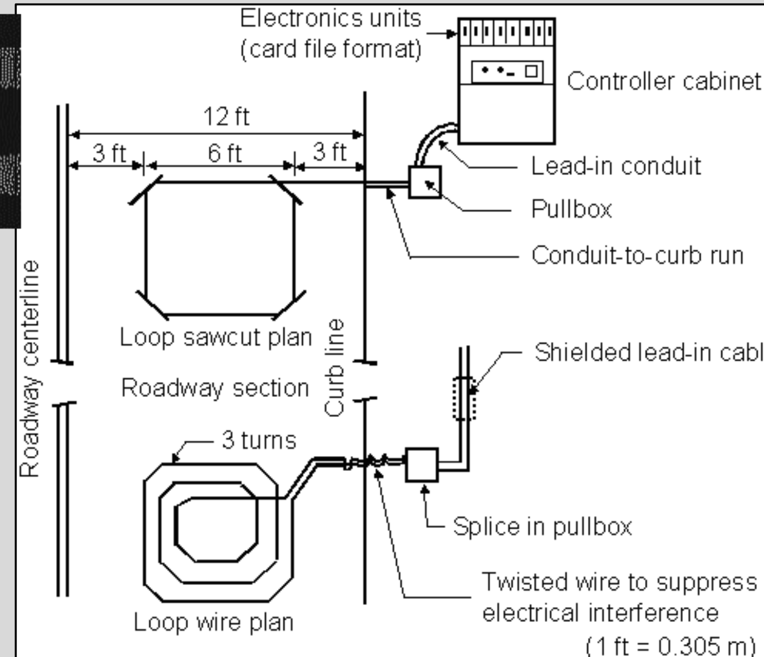
Section 32.6

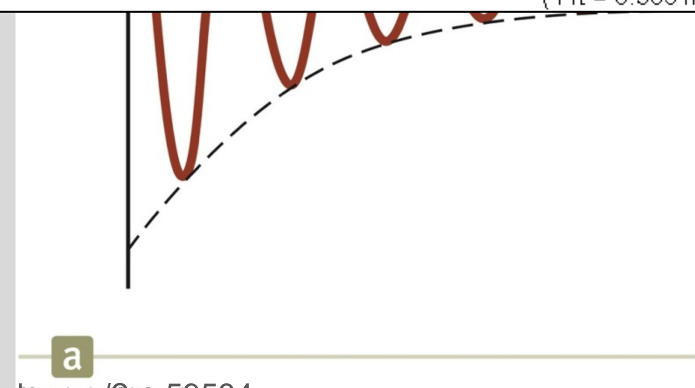
Damped *RLC* Circuit

- $q = Q_{\max}$



R_C





  - $R < R_C$: damped oscillation
  - $R = R_C$: critically damped
  - $R > R_C$: overdamped

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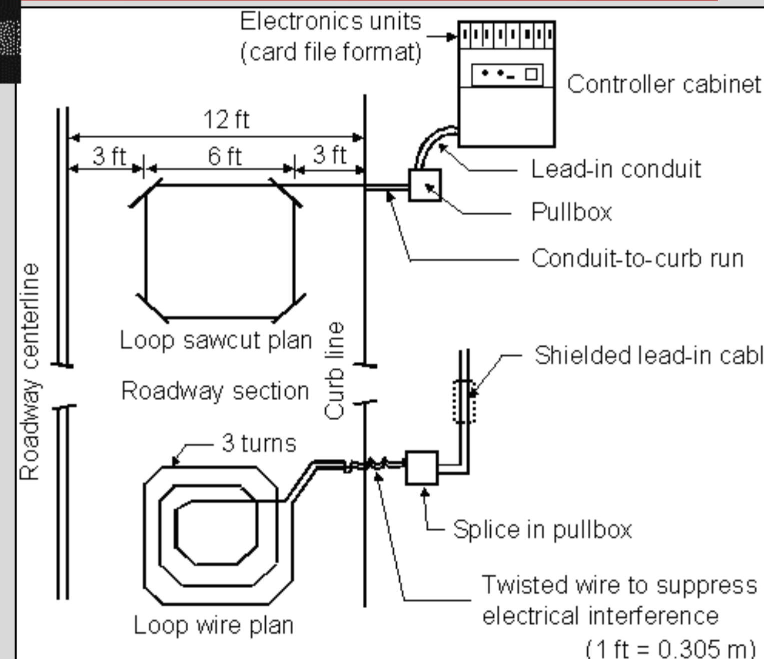
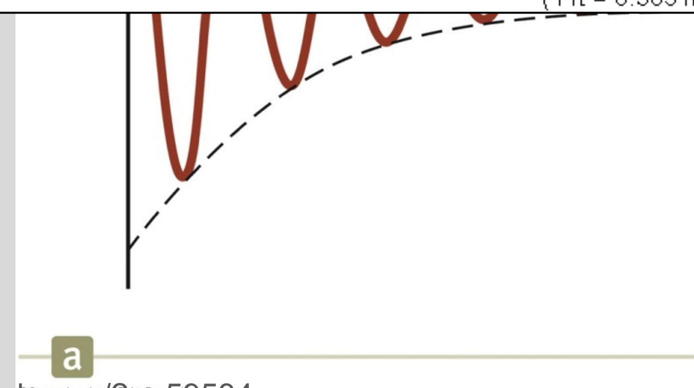
Damped *RLC* Circuit

- $q = Q_{max}$

R_C





  - $R < R_C$: damped oscillation
  - $R = R_C$: critically damped
  - $R > R_C$: overdamped

https://highscope.ch.ntu.edu.tw/wordpress/?p=59534

Section 32.6

Summary: Analogies Between Electrical and Mechanic Systems

Table 32.1 Analogies Between the *RLC* Circuit and the Particle in Simple Harmonic Motion

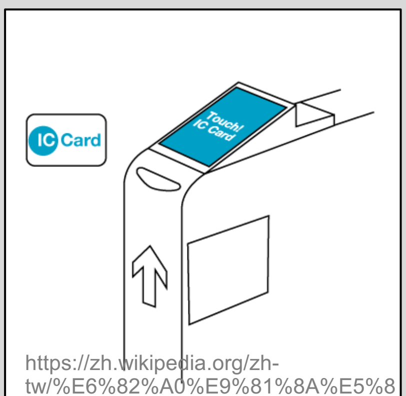
RLC Circuit		One-Dimensional Particle in Simple Harmonic Motion
Charge	$q \leftrightarrow x$	Position
Current	$i \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$i = \frac{dq}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{di}{dt} = \frac{d^2q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$\boxed{U_B = \frac{1}{2}Li^2 \leftrightarrow K = \frac{1}{2}mv^2}$	Kinetic energy of moving object
Energy in capacitor	$\boxed{U_E = \frac{1}{2}\frac{q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2}$	Potential energy stored in a spring
Rate of energy loss due to resistance	$i^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring

法拉第電磁感應定律

→ radio frequency identification

無線射頻辨識系統 RFID:
悠遊卡、信用卡感應支付、停車票卡、國道ETC

每張悠遊卡皆內建獨立的電子標籤，當卡片進入磁場感應範圍內，透過電磁感應，電子標籤內的線圈產生感應電流，以供應電子標籤運作，並將資訊傳送至讀卡機，以解讀晶片資料。



讀卡機 reader、電子標籤 tag

近場通訊 NFC: 手機支付

https://pansci.asia/archives/359263

Summary

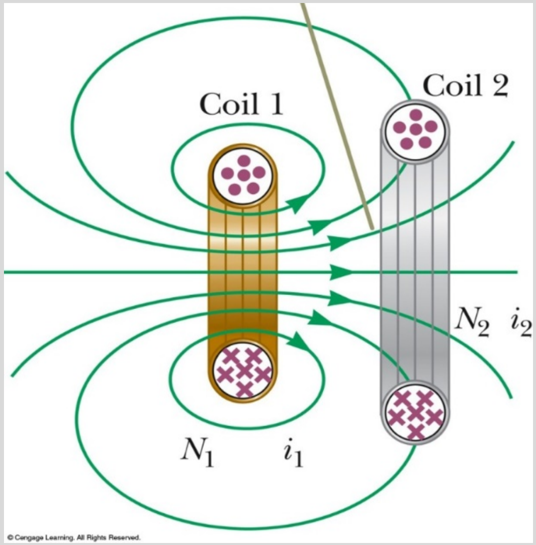
- Inductance (L)

$$\boldsymbol{\varepsilon}_L = -L\frac{di}{dt}$$

- Mutual inductance

$$M = \frac{N_2\Phi_{12}}{i_1} = \frac{N_1\Phi_{21}}{i_2}$$

$$\boldsymbol{\varepsilon}_1 = -M\frac{di_2}{dt} \quad \text{and} \quad \boldsymbol{\varepsilon}_2 = -M\frac{di_1}{dt}$$

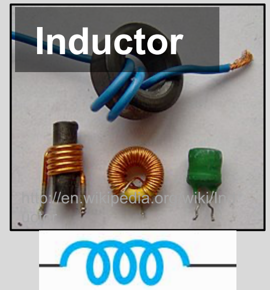


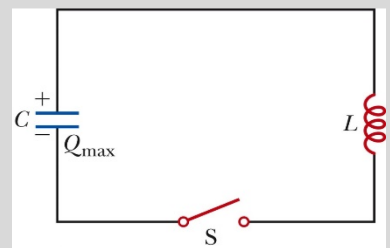
- Inductor 

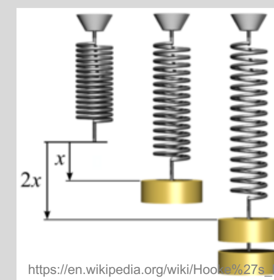
$$v_L(t) = L\frac{di(t)}{dt}$$

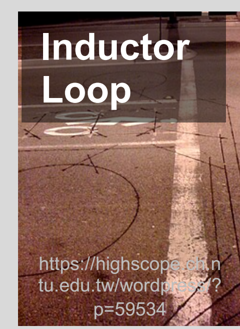
This Lecture

- Inductance
  - Self-Inductance
- *RL* Circuit
  - Energy in a Magnetic Field
- Energy Storage Summary
  - Charged capacitor, Inductor, Resistor
- Mutual Inductance
- *LC* Circuits
- *RLC* Circuit









Announcement

- Homework W11 has been assigned on Moodle.
  - One-week submission: 100% credit.
  - Afterwards: 70% credit.