Course Topics

- Intelligent agents (AIMA Ch. 2)
- Search (AIMA Ch. 3, 4, 6, 5)
- Reasoning (AIMA Ch. 7-9, 12-15)
- Machine learning (AIMA Ch. 19-20)
- Deep learning (AIMA Ch. 22)
- Natural language processing (AIMA Ch. 24)
- Generative AI (Optional)

Example: Map Coloring

- Coloring each region either red, green, or blue in such a way that **no two neighboring regions have the same color**

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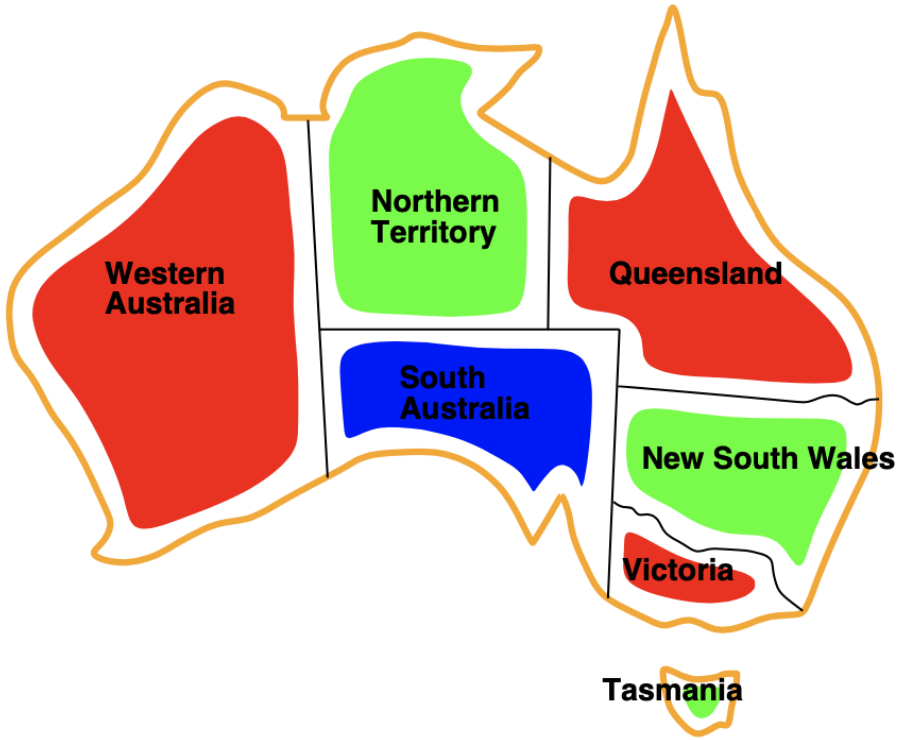


Constraint Satisfaction Problems

L.-Y. Wei

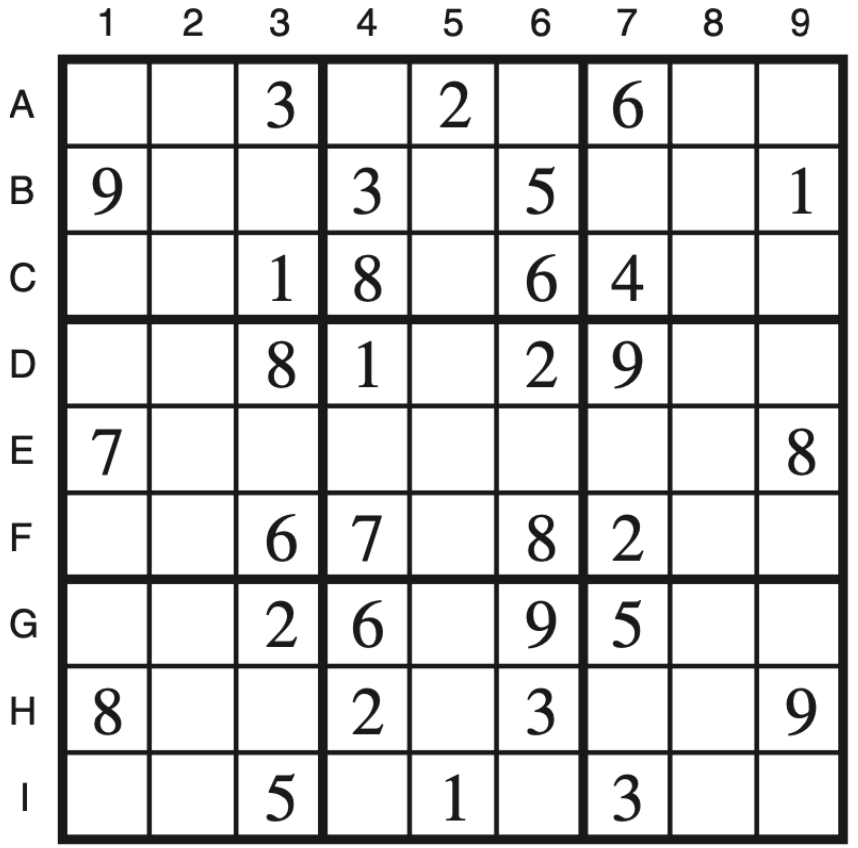
Spring 2024

Constraint Satisfaction Problems (CSPs)

- **State**
  - A set of **variables**, each of which has a **value**
- **Goal test**
  - Each variable has a value that **satisfies all the constraints** on the variable

Example: Sudoku

- A Sudoku board consists of 81 squares, some of which are initially filled with digits from 1 to 9
- Fill in all the remaining squares such that **no digit appears twice in any row, column, or 3x3 box**



	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

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	1	2	3	4	5	6	7	8	9
A	4	8	3	9	2	1	6	5	7
B	9	6	7	3	4	5	8	2	1
C	2	5	1	8	7	6	4	9	3
D	5	4	8	1	3	2	9	7	6
E	7	2	9	5	6	4	1	3	8
F	1	3	6	7	9	8	2	4	5
G	3	7	2	6	8	9	5	1	4
H	8	1	4	2	5	3	7	6	9
I	6	9	5	4	1	7	3	8	2

Example: Job-Shop Scheduling

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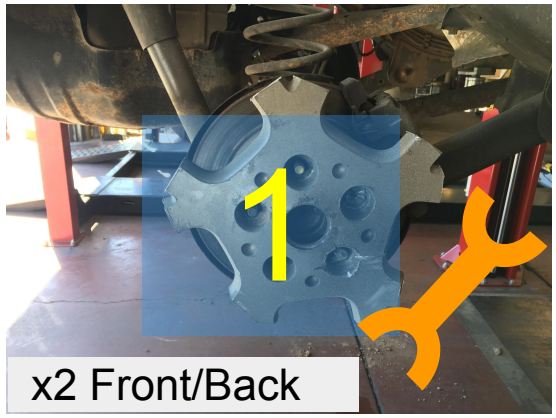

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Example: Cryptarithmetic Puzzle

- Each letter in a cryptarithmetic puzzle represents a different digit

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

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Example: Cryptarithmetic Puzzle

- Each letter in a cryptarithmetic puzzle represents a **different digit**

	T	W	O		7	3	4
$+$	T	W	O		7	3	4
F	O	U	R	1	4	6	8

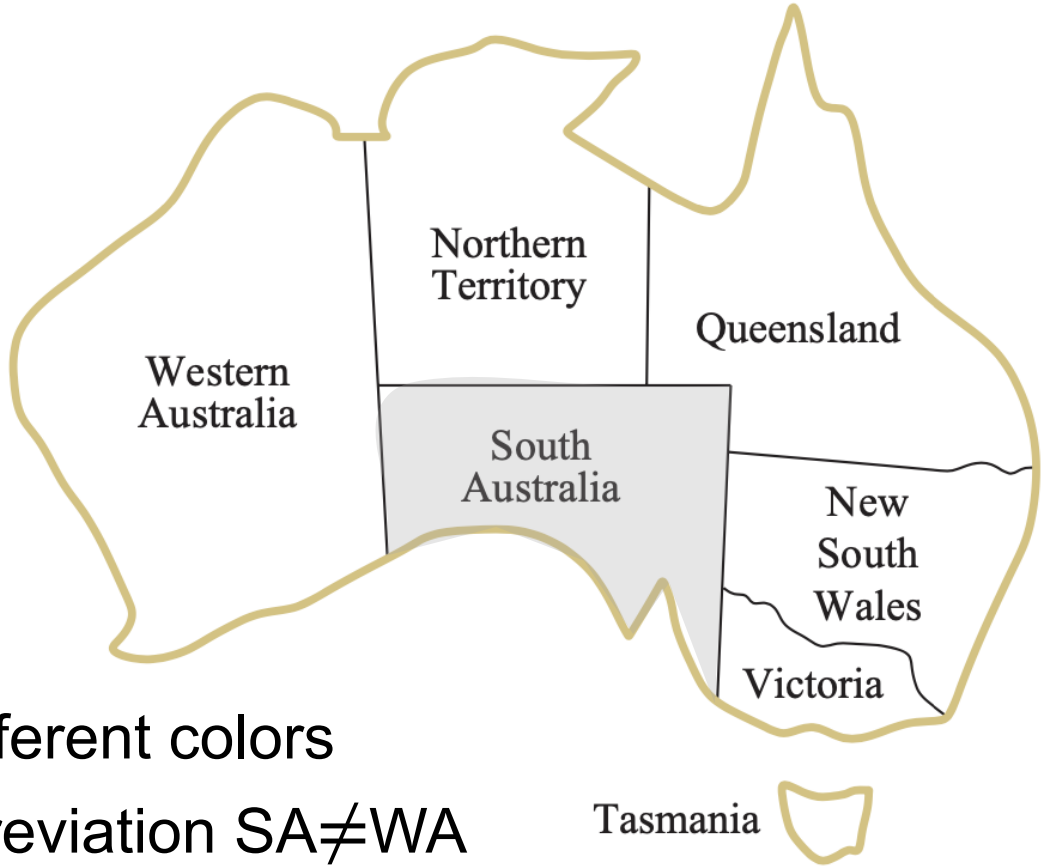
Constraint Satisfaction Problems

- A CSP consists of three components:
  - A set of **variables** $\{X_1, X_2, \ldots, X_n\}$
  - A set of **domains** $\{D_1, D_2, \ldots, D_n\}$
    - A domain, D_i, consists of a set of allowable values, $\{v_1,...,v_k\}$, for variable X_i
  - A set of **constraints**
    - Each constraint C_j consists of a pair <*scope*,*rel*>
      - *scope*: a tuples of variables that participate in the constraint
      - *rel*: a relation that defines the values that those variables can take on

Example: Map Coloring

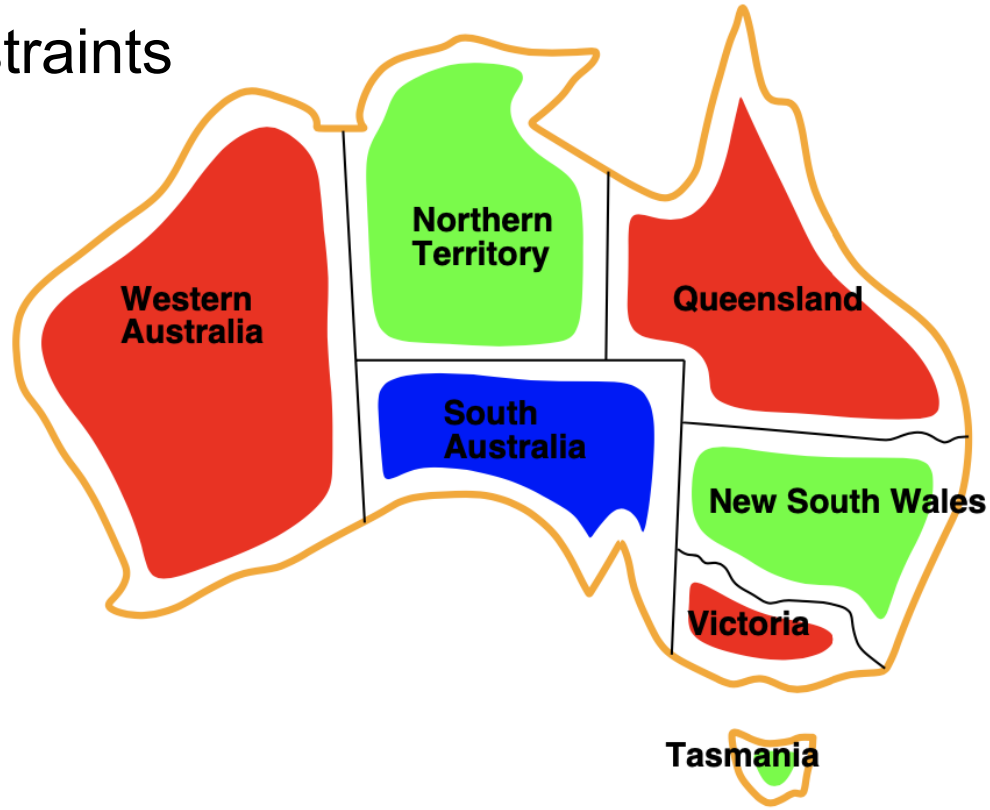
- Variables
  - {WA, NT, SA, Q, NSW, V, T}
- Domains
  - D_i = {red,green,blue}
- Constraints
  - Adjacent regions must have different colors
  - e.g., <(SA,WA),SA≠WA>: abbreviation SA≠WA

$$\{SA \neq WA, SA \neq NT, SA \neq Q, SA \neq NSW, SA \neq V,$$
$$WA \neq NT, NT \neq Q, Q \neq NSW, NSW \neq V\}.$$



Example: Map Coloring

- Solutions
  - Assignments satisfying all constraints
  - e.g., {WA=red,
    NT=green,
    SA=blue,
    Q=red,
    NSW=green,
    V=red,
    T=green}



Example: Sudoku

- Variables
  - {A1,A2,....,I8,I9}
- Domains
  - Empty squares: {1,2,...,9}
  - Pre-filled squares: a single value
- Constraints

rows
$$Alldiff(A1,A2,A3,A4,A5,A6,A7,A8,A9)$$
$$Alldiff(B1,B2,B3,B4,B5,B6,B7,B8,B9)$$
$$\ldots$$

columns
$$Alldiff(A1,B1,C1,D1,E1,F1,G1,H1,I1)$$
$$Alldiff(A2,B2,C2,D2,E2,F2,G2,H2,I2)$$
$$\ldots$$

3x3 boxes
$$Alldiff(A1,A2,A3,B1,B2,B3,C1,C2,C3)$$
$$Alldiff(A4,A5,A6,B4,B5,B6,C4,C5,C6)$$
$$\ldots$$

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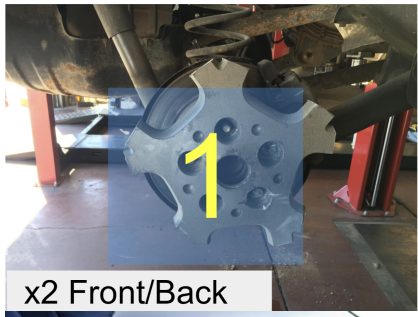
H	8			2		3			9
I			5		1		3		

Example: Job-Shop Scheduling

- Variables

$$\{Axle_F, Axle_B, Wheel_{RF}, Wheel_{LF}, Wheel_{RB}, Wheel_{LB}, Nuts_{RF},$$
$$Nuts_{LF}, Nuts_{RB}, Nuts_{LB}, Cap_{RF}, Cap_{LF}, Cap_{RB}, Cap_{LB}, Inspect\}$$

  where the value of each variable is the time that **the task starts**, expressed as **an integer number of minutes**



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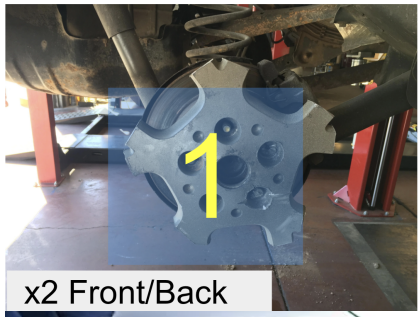
Example: Job-Shop Scheduling

- Constraints
  - Precedence constraints

$$Axle_F + 10 \leq Wheel_{RF};\ Wheel_{RF} + 1 \leq Nuts_{RF};\ Nuts_{RF} + 2 \leq Cap_{RF};$$
$$Axle_F + 10 \leq Wheel_{LF};\ Wheel_{LF} + 1 \leq Nuts_{LF};\ Nuts_{LF} + 2 \leq Cap_{LF};$$
$$Axle_B + 10 \leq Wheel_{RB};\ Wheel_{RB} + 1 \leq Nuts_{RB};\ Nuts_{RB} + 2 \leq Cap_{RB};$$
$$Axle_B + 10 \leq Wheel_{LB}.\ Wheel_{LB} + 1 \leq Nuts_{LB};\ Nuts_{LB} + 2 \leq Cap_{LB}.$$

$X + d_X \leq Inspect$ for every variable X



x2 Front/Back

x4 RF,LF,RB,LB

x4 RF,LF,RB,LB

x4 RF,LF,RB,LB

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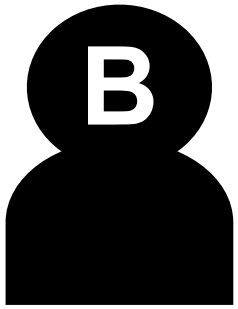
Example: Job-Shop Scheduling

Get the whole assembly done in 30 minutes!



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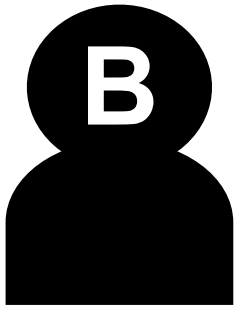
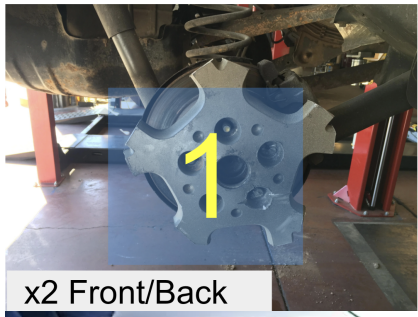


- Domain of all variables
  - $D_i = \{0,1, 2, 3, ..., 30\}$



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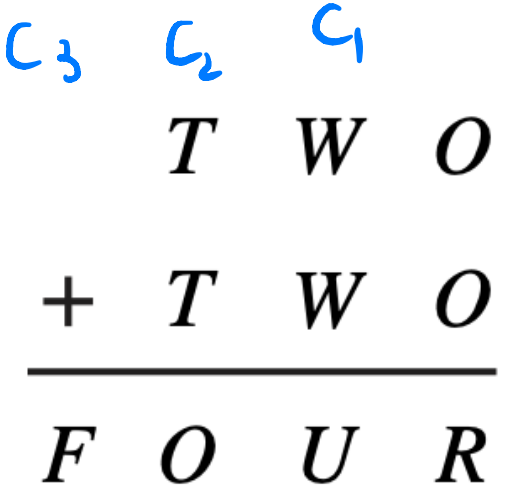
Example: Cryptarithmetic Puzzle

- Variables
  - {F, T, U, W, R, O, C_1, C_2, C_3}
- Domains
  - {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
- Constraints
  - Alldiff(F,T,U,W,R,O)
  - $O + O = R + 10 \cdot C_1$
  - $C_1 + W + W = U + 10 \cdot C_2$
  - $C_2 + T + T = O + 10 \cdot C_3$
  - $C_3 = F$



Varieties of CSPs

- Discrete variables
  - Finite domains
    - e.g., map-coloring problems, scheduling with time limits, and 8-queens problem
  - Infinite domains
    - e.g., a set of integers/strings etc.
    - e.g., job scheduling problem without a deadline
      - Variables are start/end days for each job
      - Implicit constraint: $\text{StartJob}_1 + 5 \leq \text{StartJob}_3$
- Continuous variables
  - e.g., linear programming problems

Varieties of Constraints

- **Unary constraint** restricts the value of a single variable
  - e.g., SA ≠ green
- **Binary constraint** relates two variable
  - e.g., SA ≠ WA
- High-order constraints involve 3 or more variables

Varieties of Constraints (Cont.)

- **Preference constraint**
  - e.g., in a university class-scheduling problem, Prof. M might prefer teaching in the morning, whereas Prof. A prefers teaching in the afternoon
  - Can be encoded as costs on individual variable assignments
    - e.g., assigning an afternoon slot for Prof. M costs 2 points and assigning a morning slot costs 1.
    - CSPs with preferences can be solved with optimization search methods
      (**constrained optimization problems**, COPs)

Node Consistency

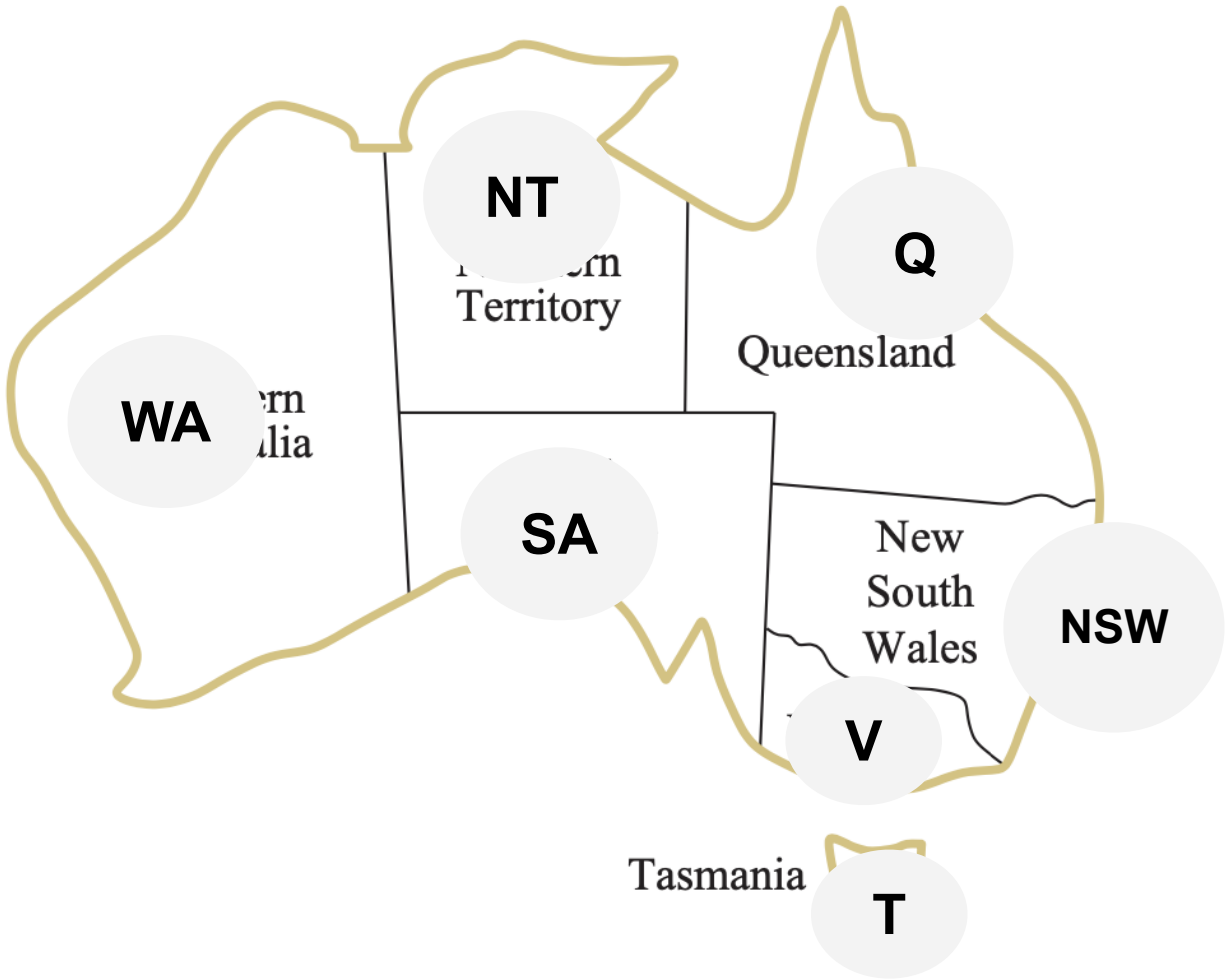
- A single variable (corresponding to a node in the CSP graph) is **node-consistent** if all the values in the variable's domain satisfy the variable's unary constraints
- Example
  - The variable SA starts with domain {red, green, blue} but a constraint {SA ≠ green}
  - Make it node-consistent by eliminating "green", i.e., SA with the reduced domain {red, blue}
- A graph is node-consistent if every variable in the graph is node-consistent

Binary CSP

- Constraint types of binary CSPs
  - Unary constraints
  - Binary constraints
- Binary CSP can be represented as a constraint graph
- **Constraint graph**
  - Nodes are variables of the CSP
  - Edges show constraints
    - Edges connect any two variables that participate in a constraint

Constraint Graph of Map Coloring Problem

Constraint Graph of Map Coloring Problem



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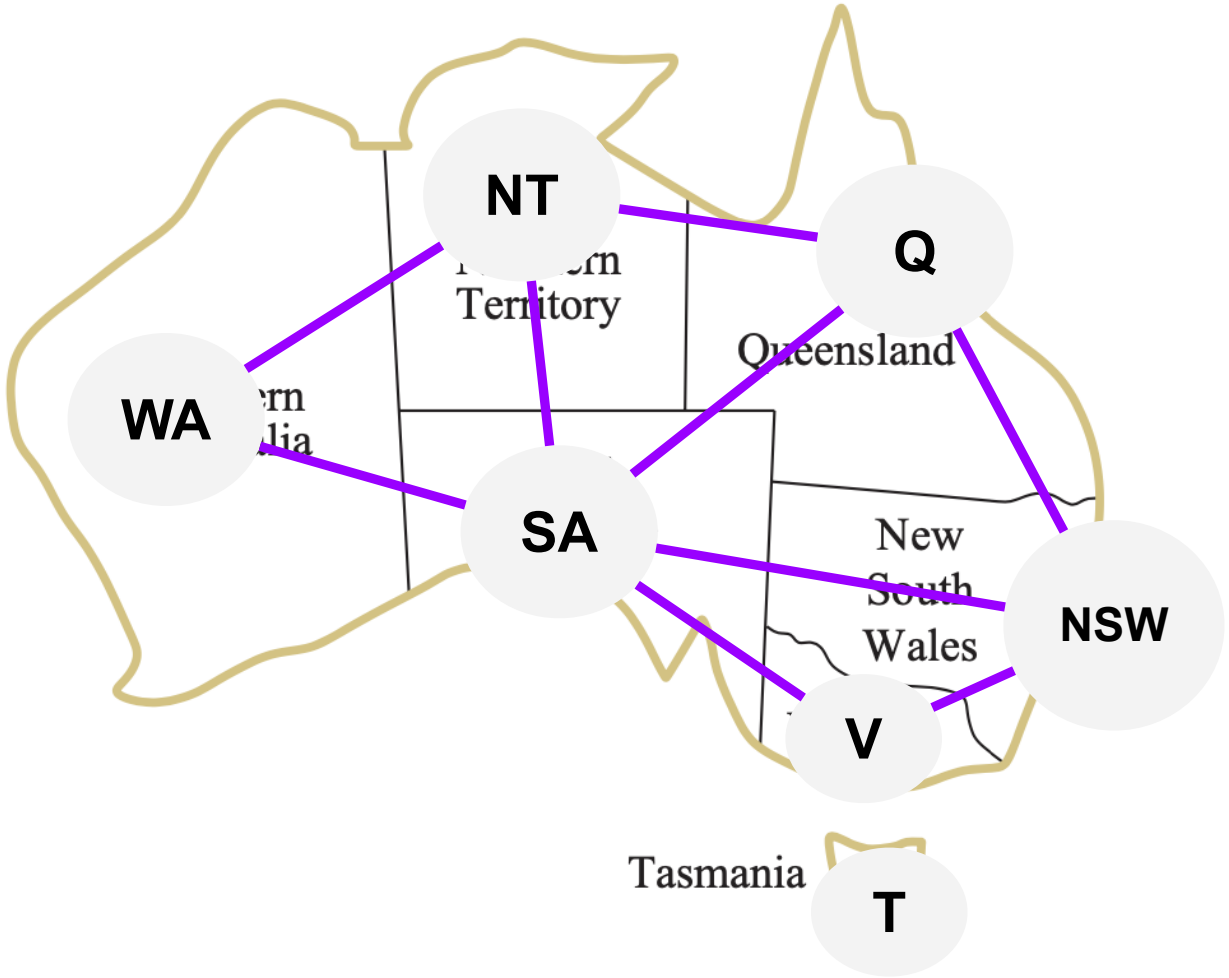

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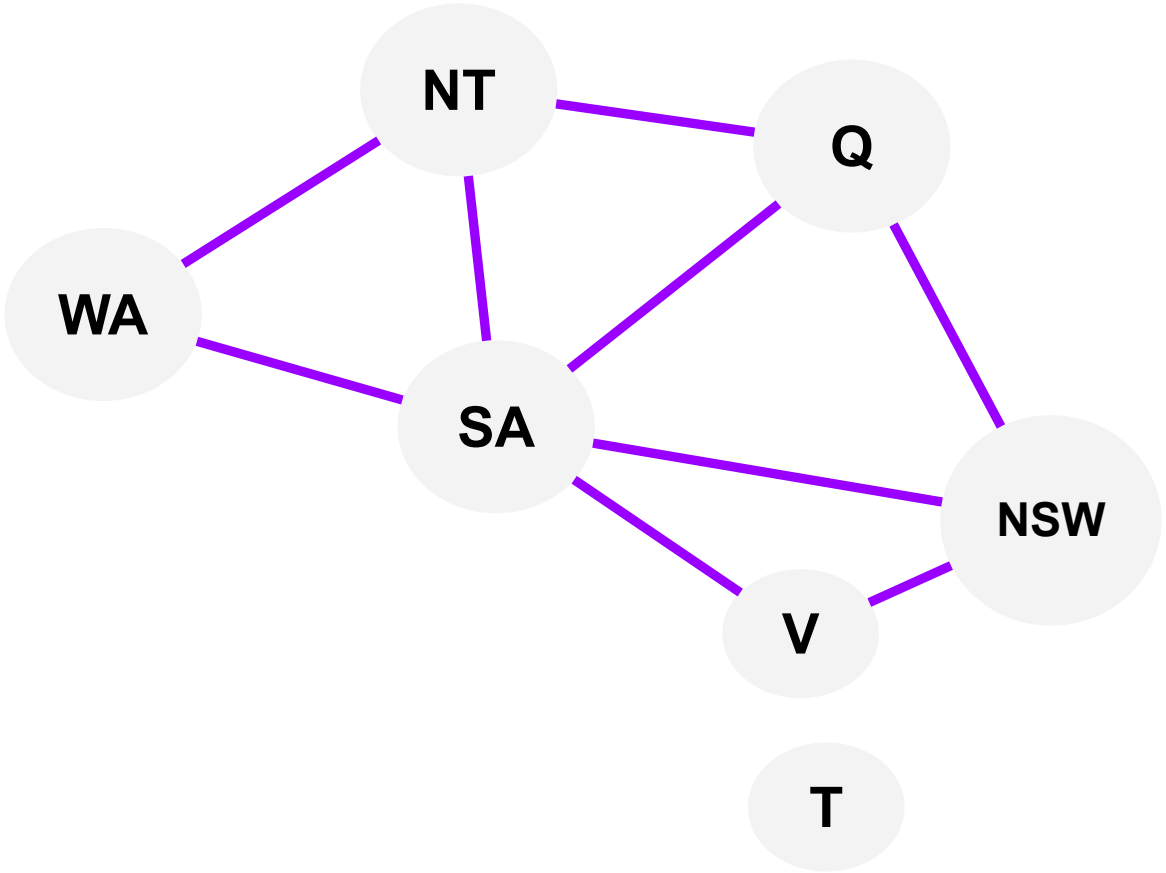

Constraint Hypergraph of Cryptarithmetic Puzzle Problem

- Variables
  - {F, T, U, W, R, O, C_1, C_2, C_3}
- Domains
  - {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
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  - $C_3 = F$

high-order constraints

Ordinary Nodes

Constraint Hypergraph of Cryptarithmetic Puzzle Problem

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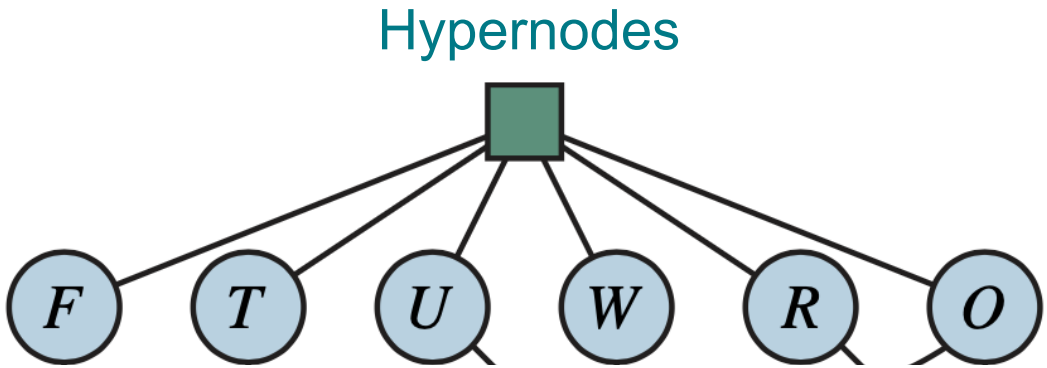
  - $C_3 = F$



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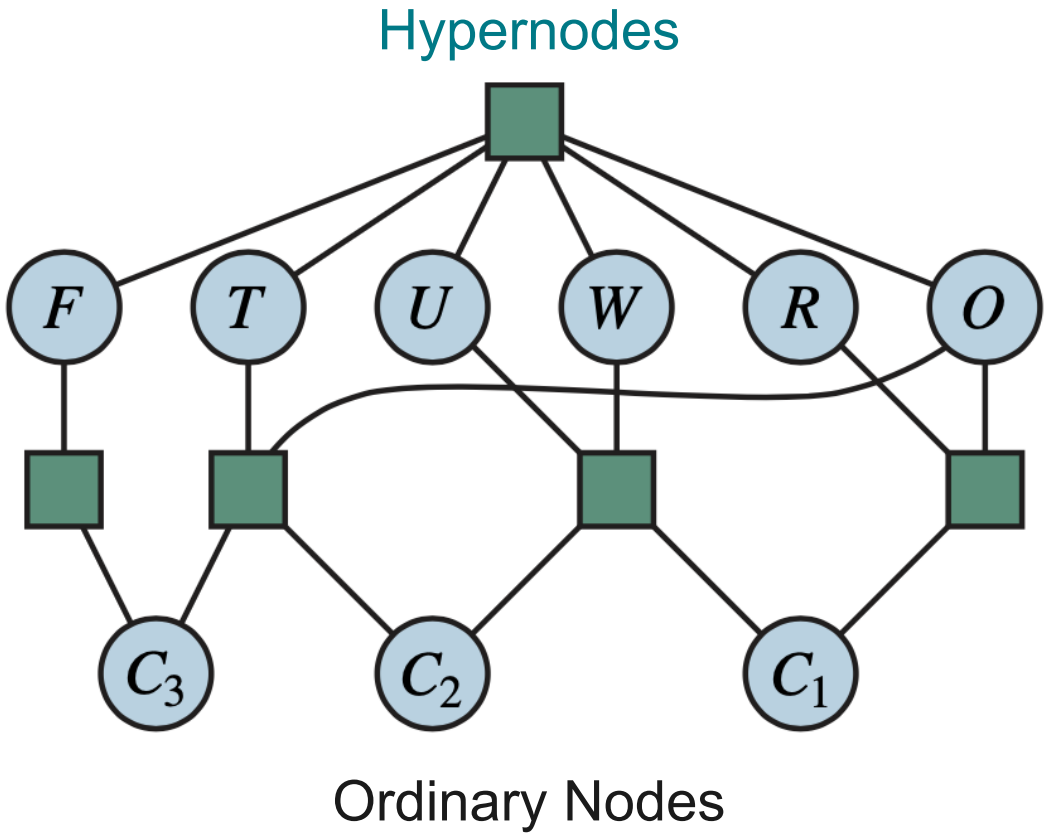

Real-World CSPs

- Scheduling problems
  - e.g., when can we all meet?
- Timetabling problem
  - e.g., which class is offered when and where?
- Assignment problems
  - e.g., who teaches what class
- Transportation scheduling
- Circuit layout
- Factory scheduling
- …

Search Problem

State space

Goal

Actions

Transition model

Path cost

CSPs as Search Problems

- Initial state
  - The empty assignment, {}
- States
  - Defined by the values assigned so far (partial assignments, i.e., some variables unassigned)
- Actions
  - Add a {variable = value} to assignment
- Transition model
  - Show how adding an assignment changes the assignment
- Goal test
  - Check the current assignment is **complete** and **consistent**
    - A complete assignment is one in which every variable is assigned a value
    - An assignment that does not violate any constraints is called a consistent or legal assignment
- Path cost
  - All paths have same cost

Example: Map Coloring

- Variables
  - {WA, NT, SA, Q, NSW, V, T}
- Domains
  - D_i = {red,green,blue}
- Constraints
  - Adjacent regions must have different colors
  - e.g., <(SA,WA),SA≠WA>: abbreviation SA≠WA

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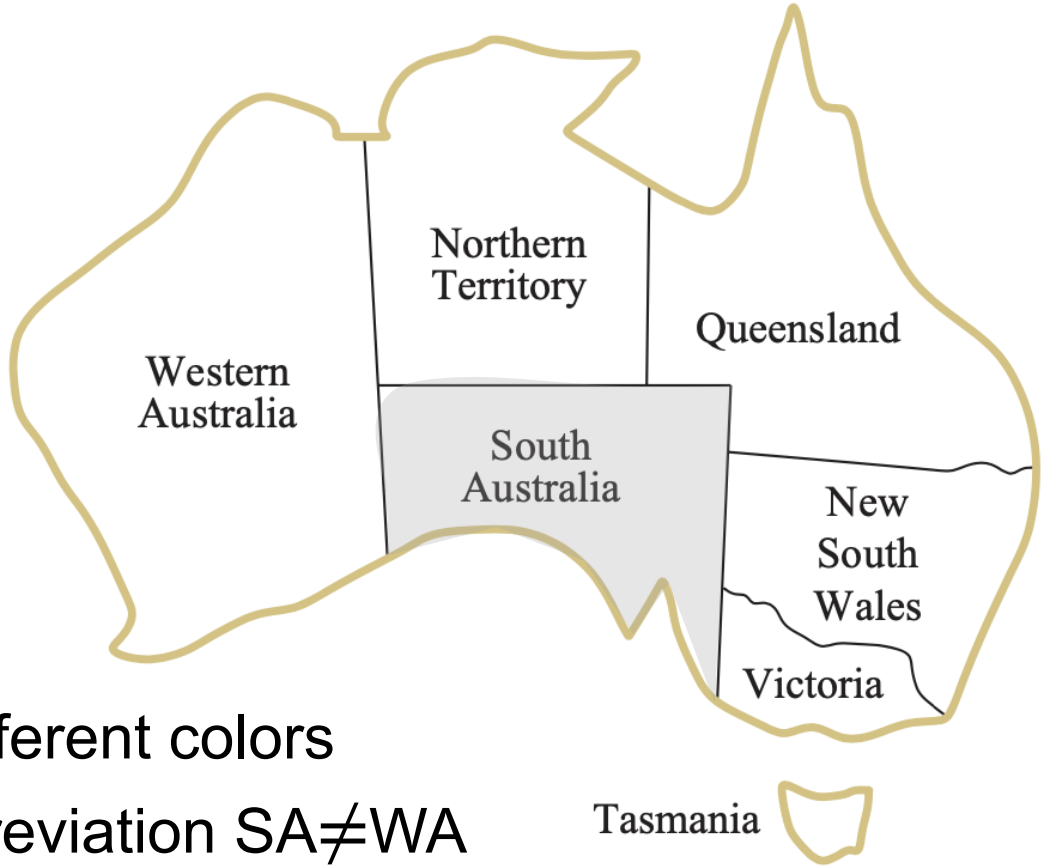


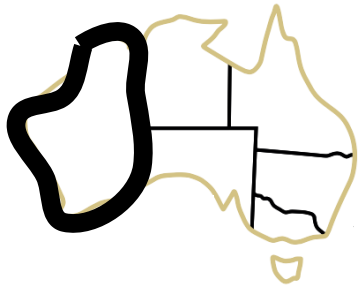
Search Methods

- What would BFS do?
- What would DFS do?

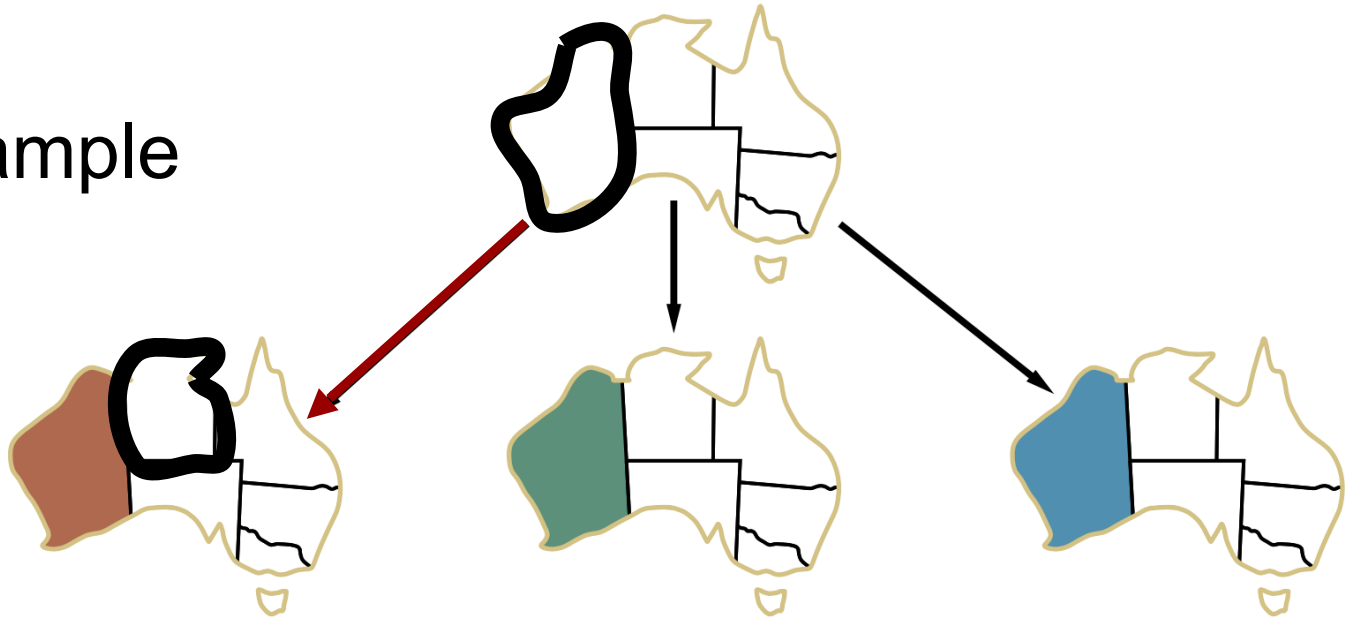
Backtracking Search for CSPs

- Backtracking search is the basic uninformed algorithm for solving CSPs
- Idea 1: One variable at a time
  - CSP is commutative
    - If a problem is **commutative** if the order of application of any given set of actions does not matter
    - e.g., [WA=red then NT=green] same as [NT=green and WA=red]
  - Consider assignments to a single variable at each step
- Idea 2: Check constraints as you go
  - Consider only values which do not conflict with previous assignments
  - Might have to do some computation to check the constraints
- Depth-first search with these two improvements is called **backtracking search**

Backtracking Example



Backtracking Example



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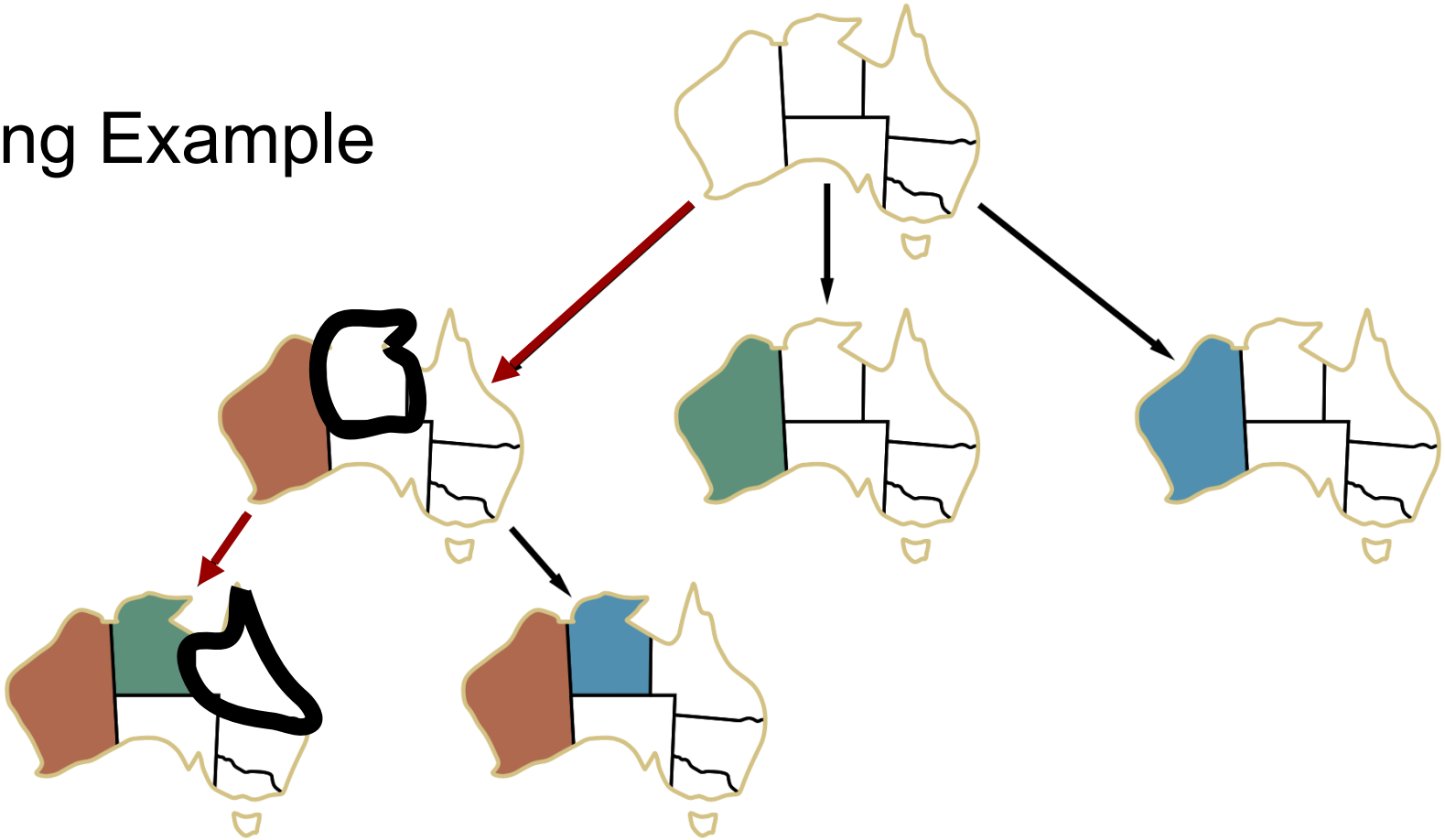

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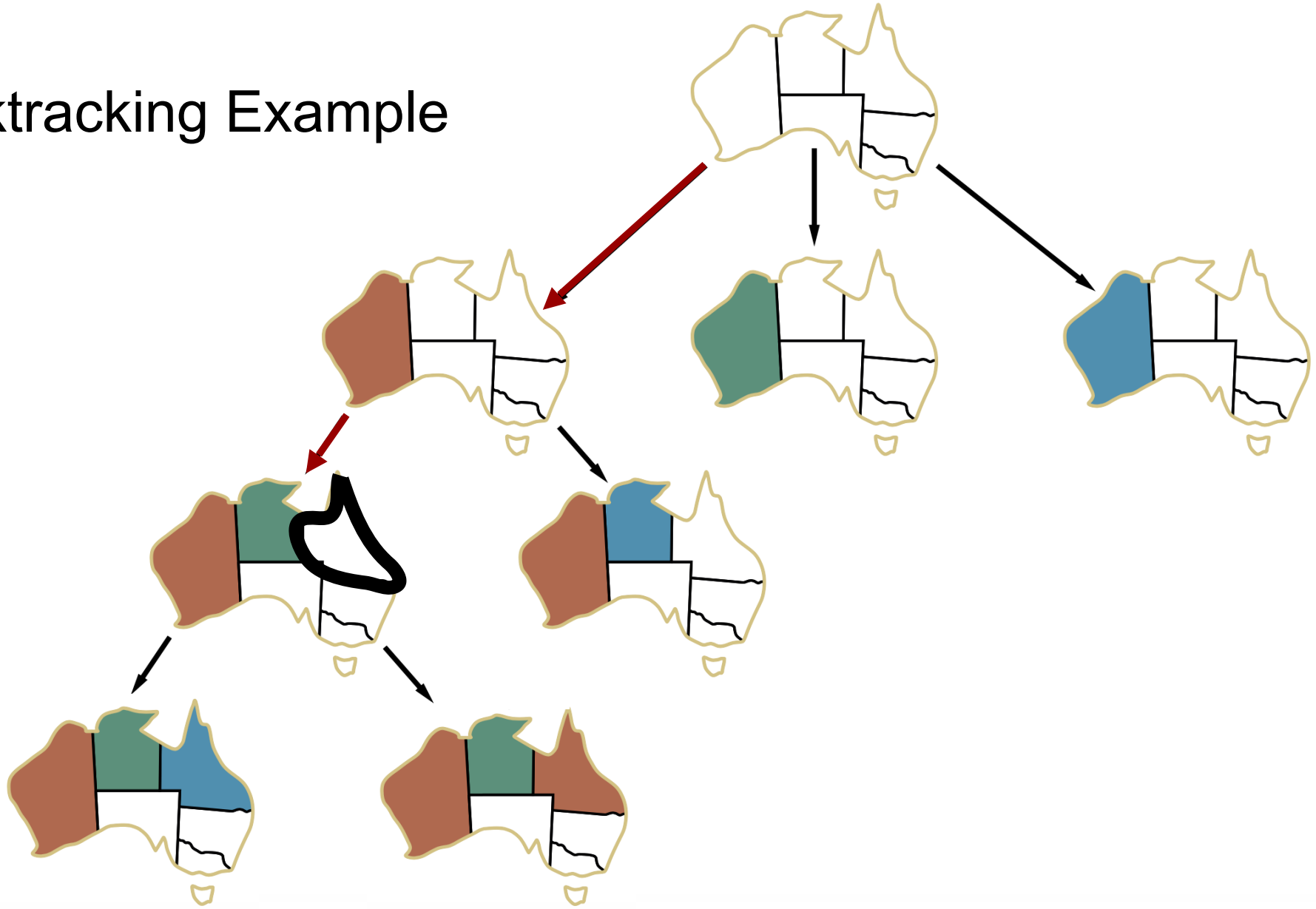

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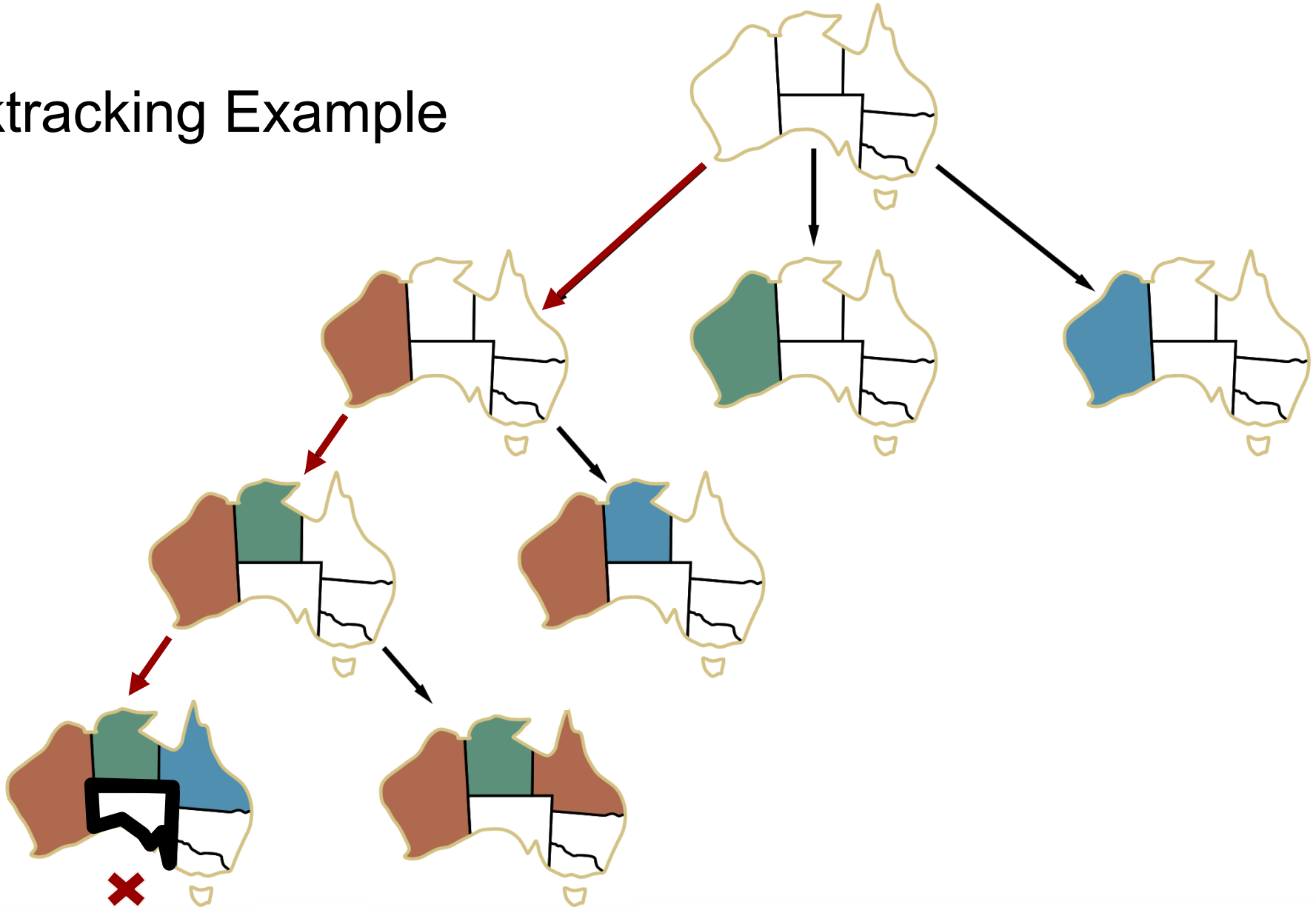
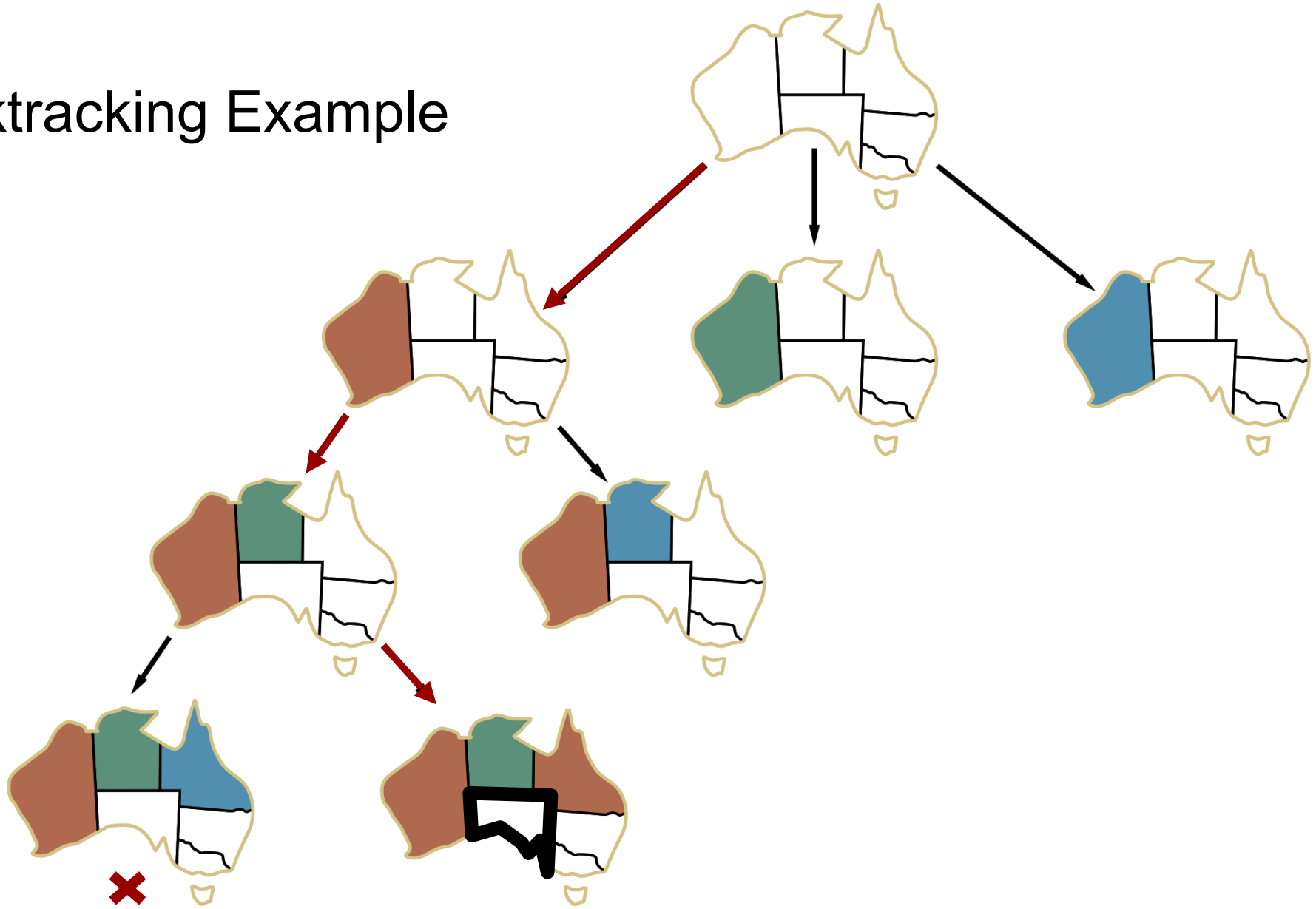
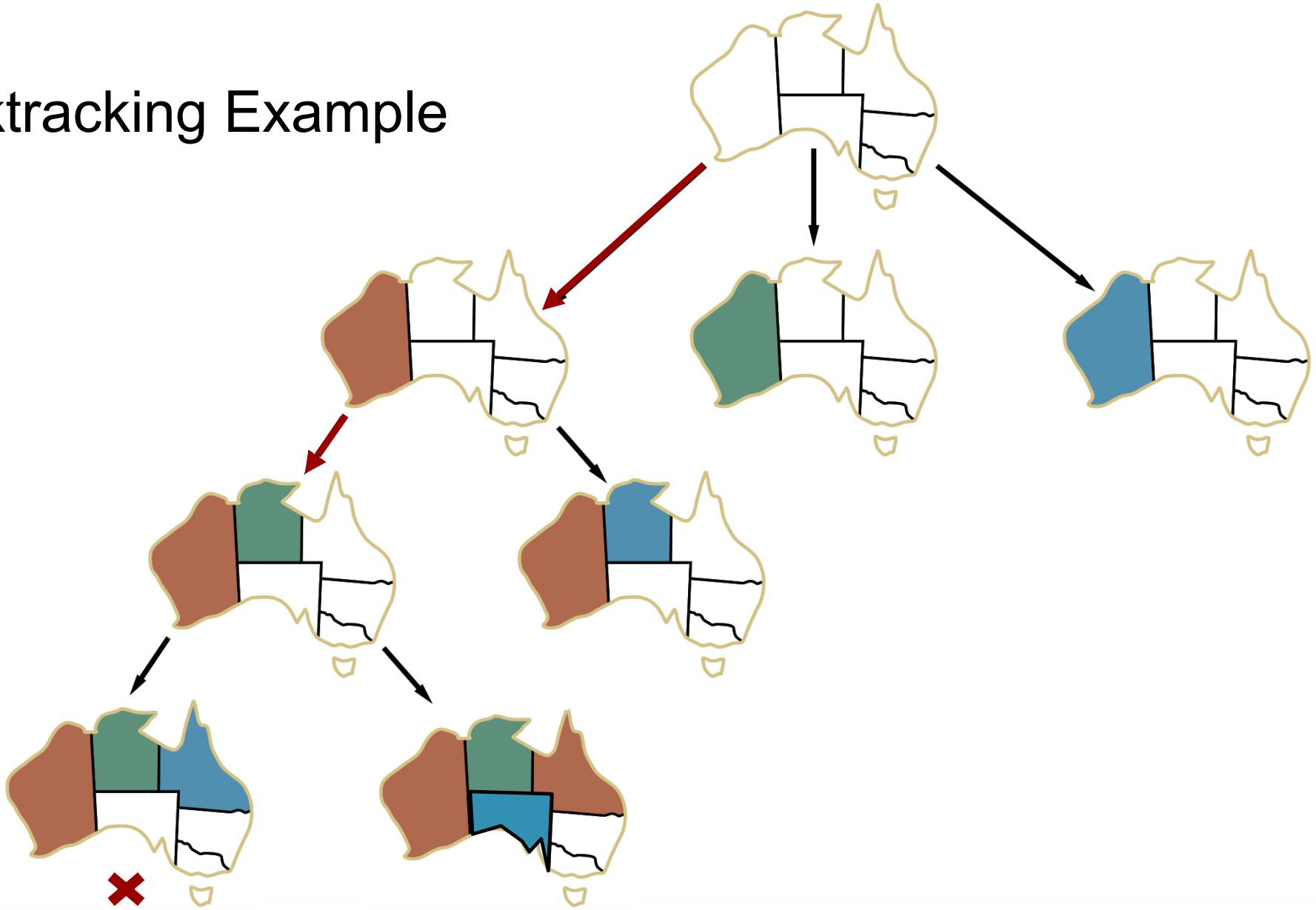


Backtracking Example



Backtracking Example



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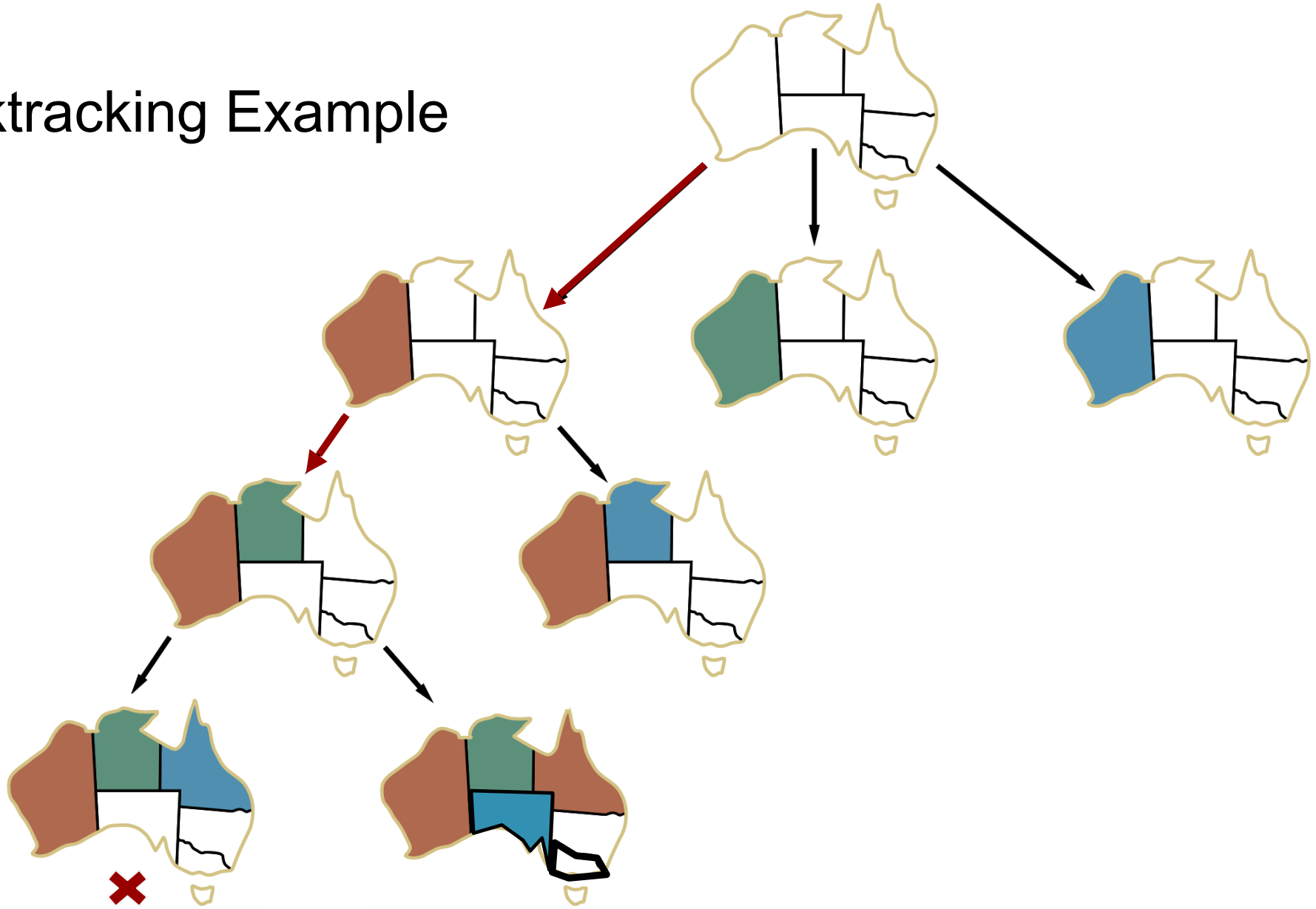

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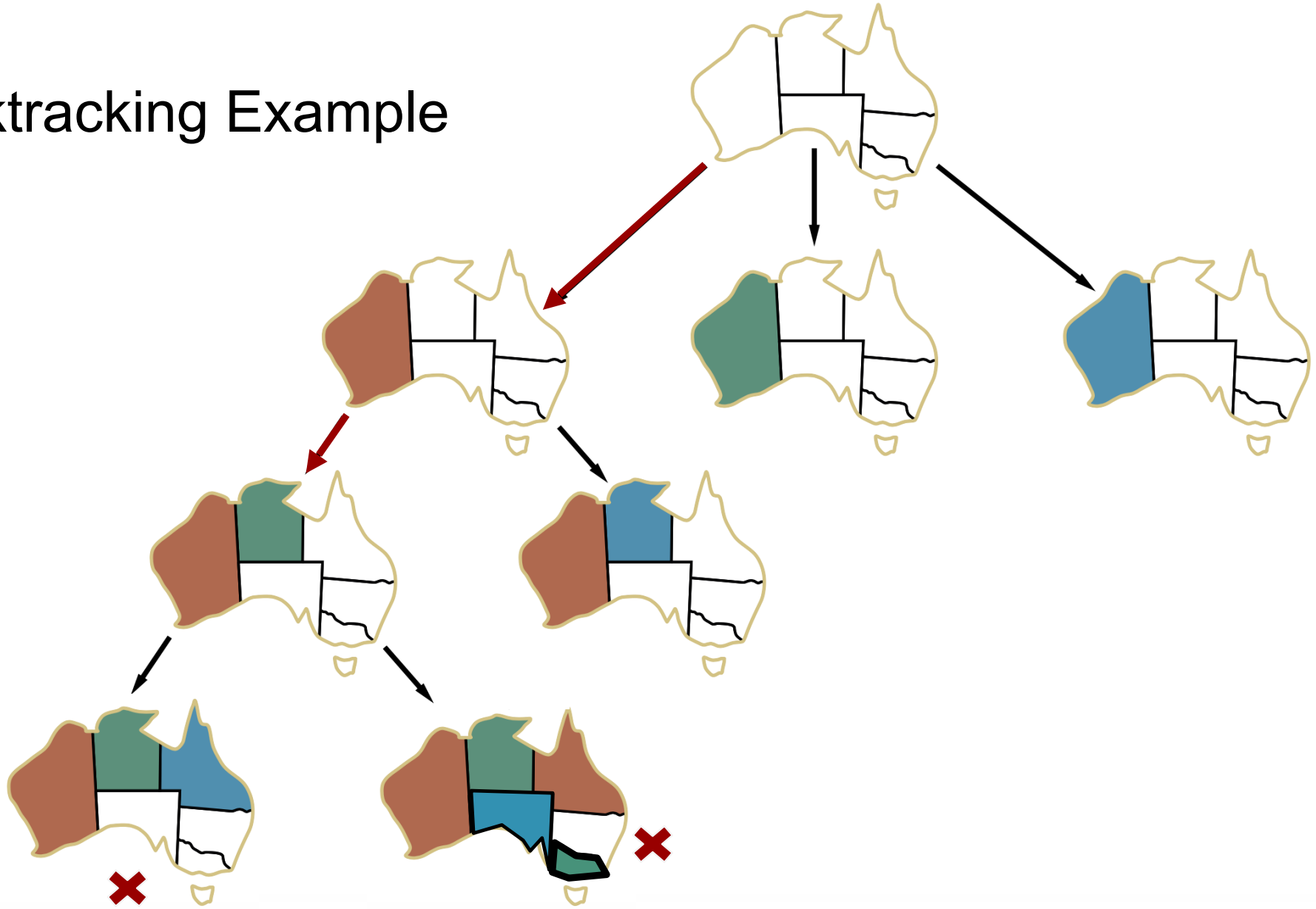

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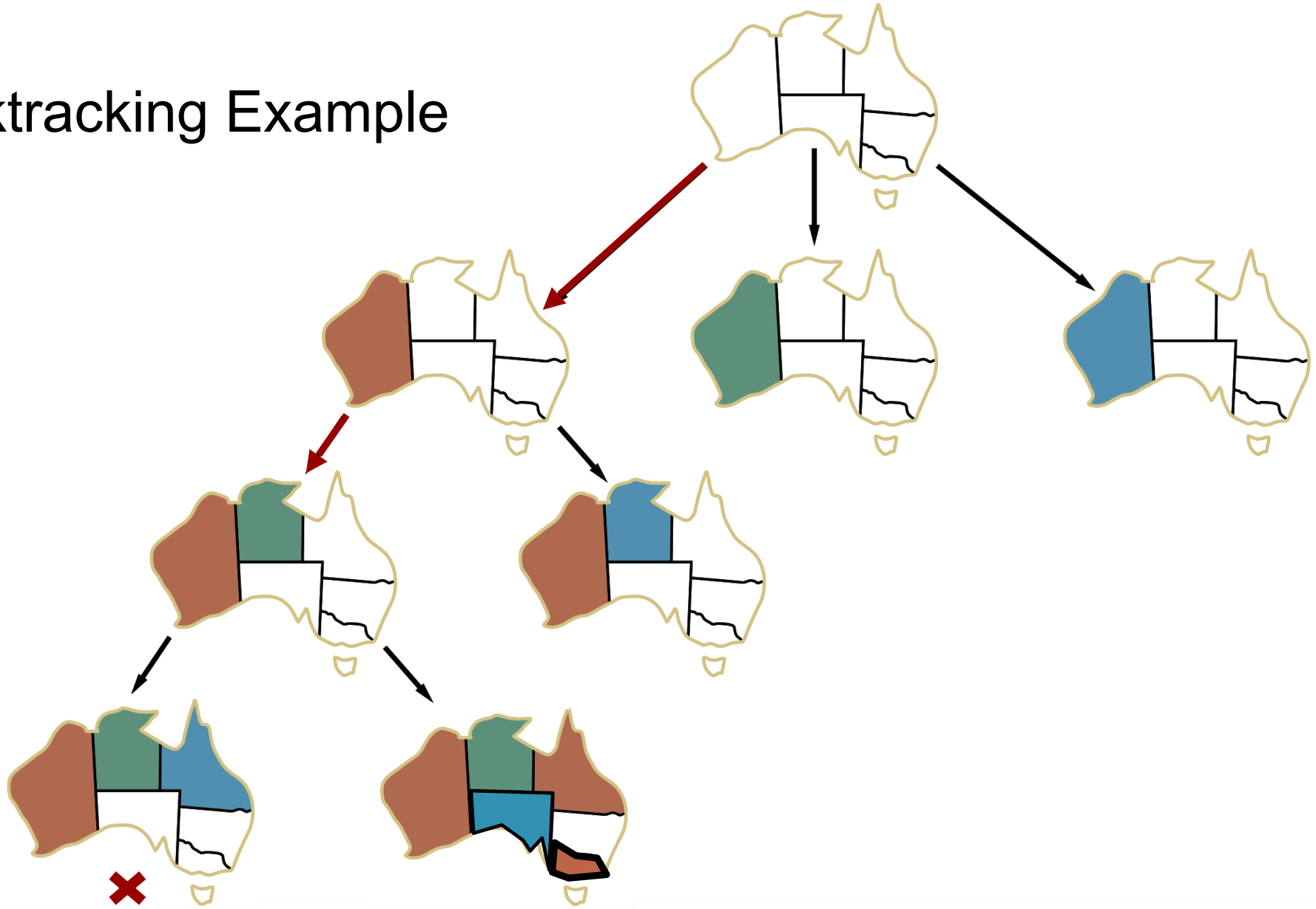

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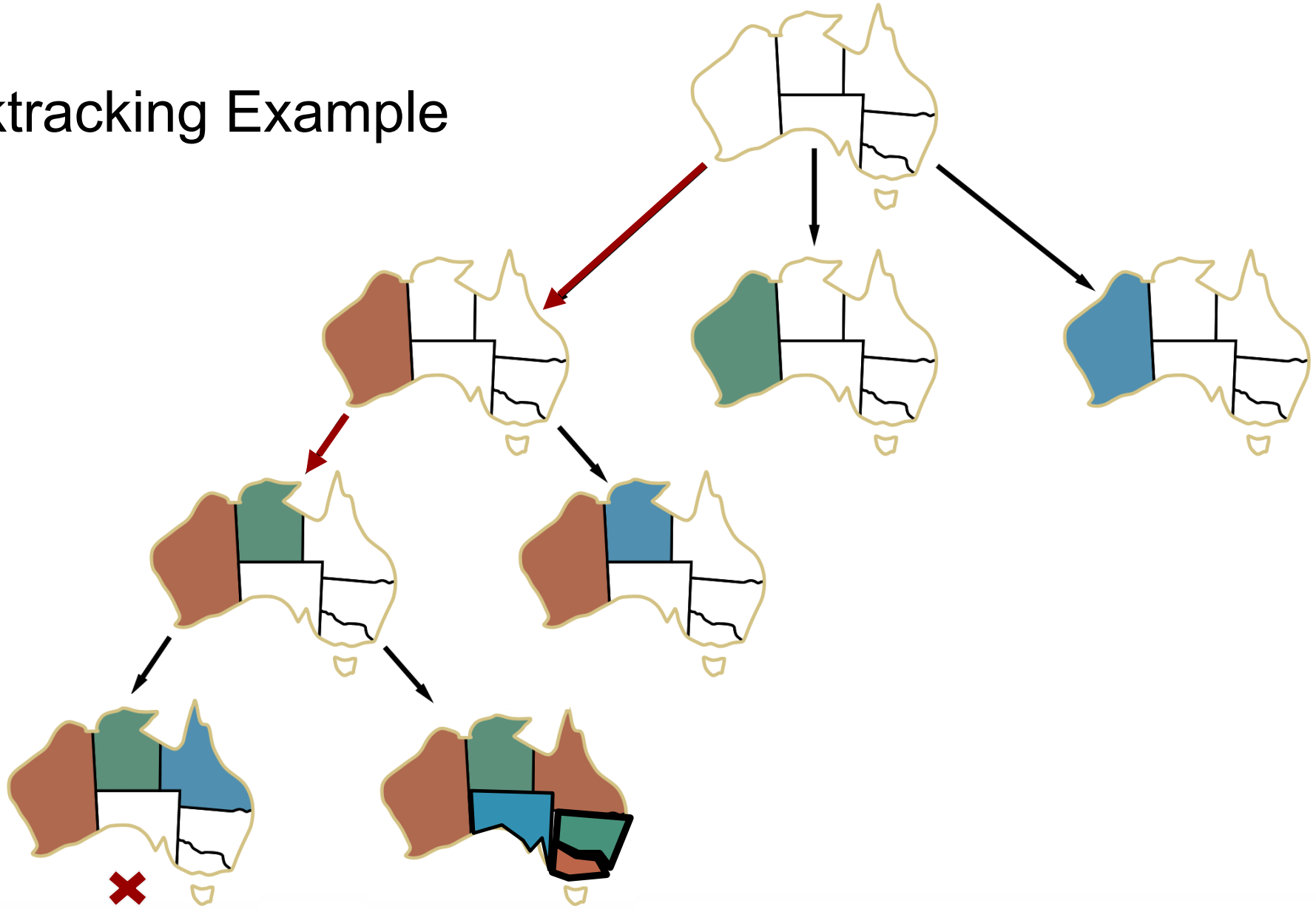

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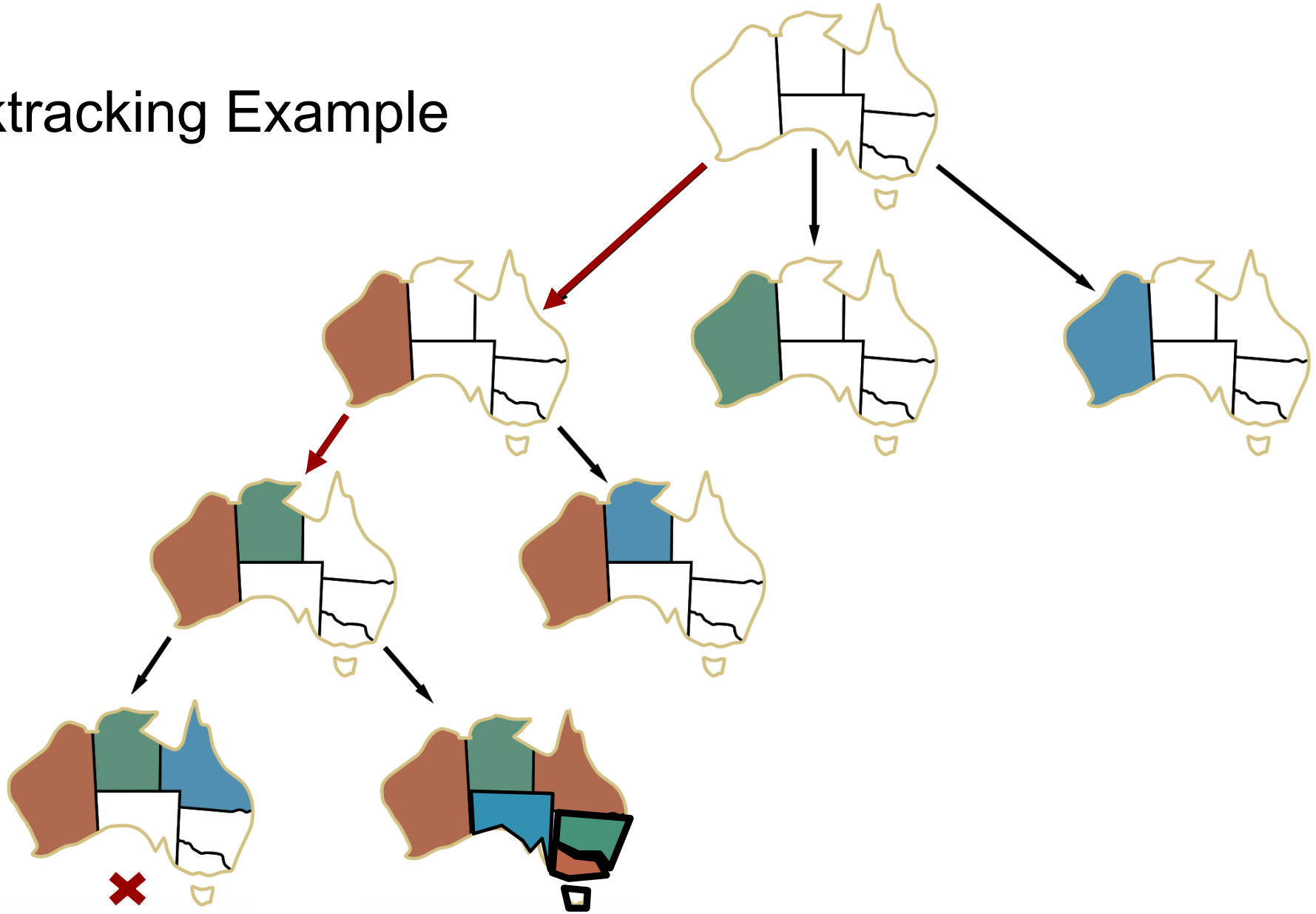

Simple Backtracking Algorithm for CSPs

```
function BACKTRACKING-SEARCH(csp) returns a solution or failure
    return BACKTRACK(csp, { })

function BACKTRACK(csp, assignment) returns a solution or failure
    if assignment is complete then return assignment
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recursion

Efficiency Improvement for Backtracking Search

- Which variable should be assigned next?
- In what order should its values be tried?
- Can we detect inevitable failure early?

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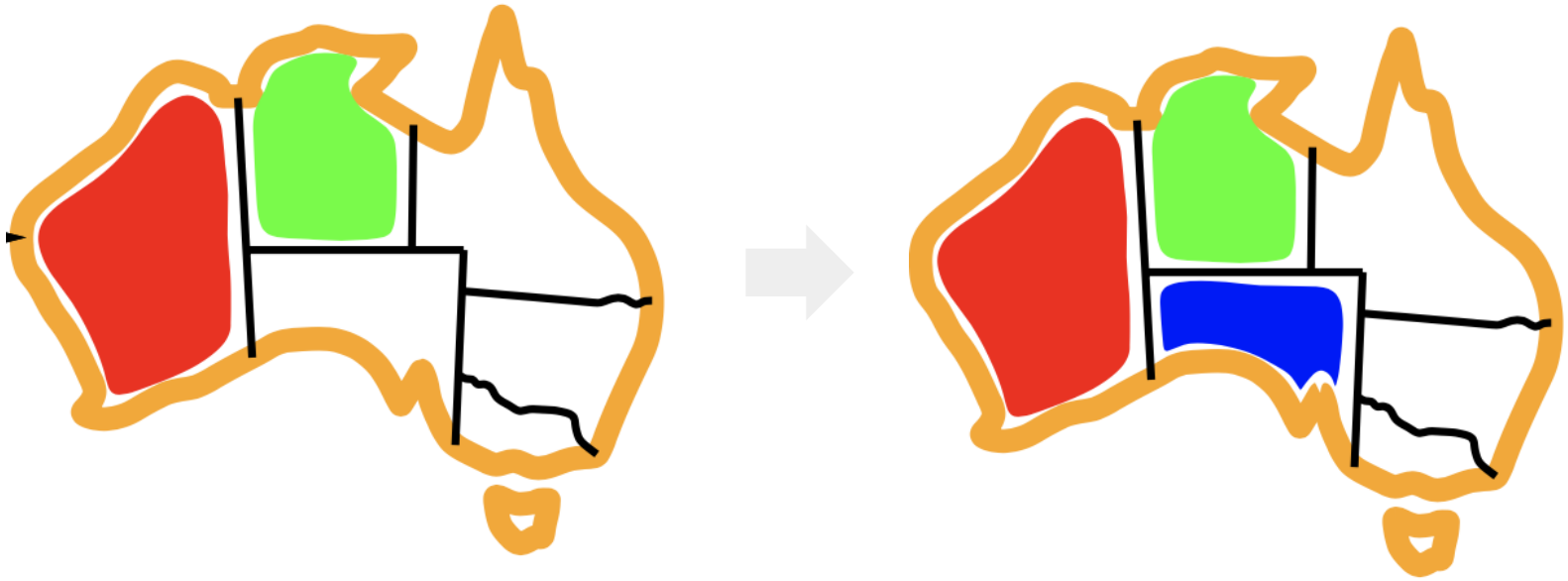
Select-Unassigned-Variable

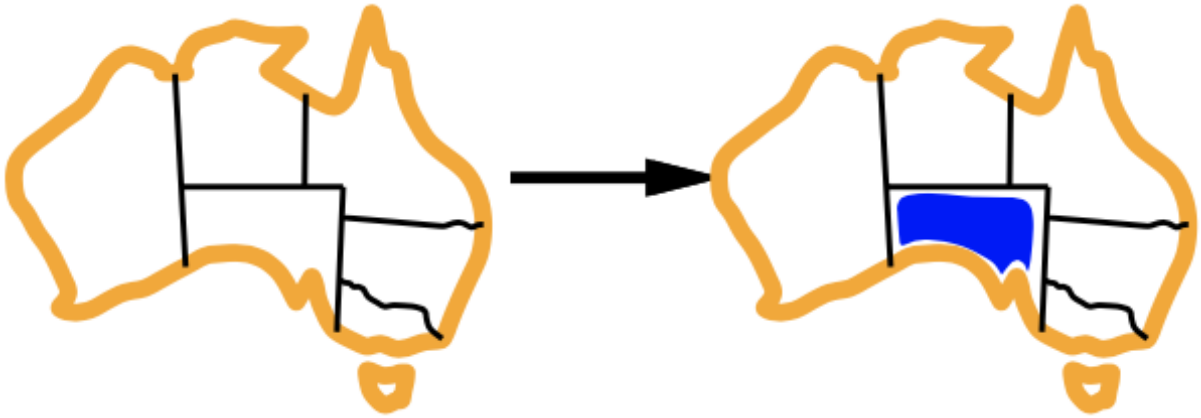
- Minimum-remaining-values (MRV) heuristic
  - Select the variable with the fewest legal values

    (most constrained variable or fail-first heuristic)



Select-Unassigned-Variable (Cont.)

- Degree heuristic
  - Idea: Reduce the branching factor on future choices
  - Select the variable that is involved in the largest number of constraints on other unassigned variables
    i.e., Select the variable that has the highest degree



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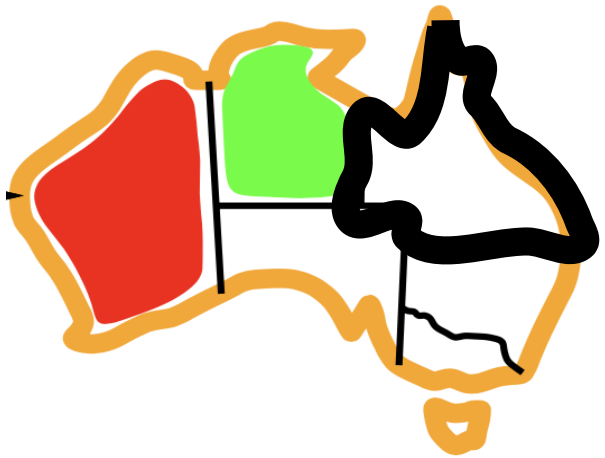
Domain-Values

- Least-constraining-value heuristic
  - Idea: Leave the maximum flexibility for subsequent variable assignments
  - It prefers the value that rules out the fewest values in the remaining variables



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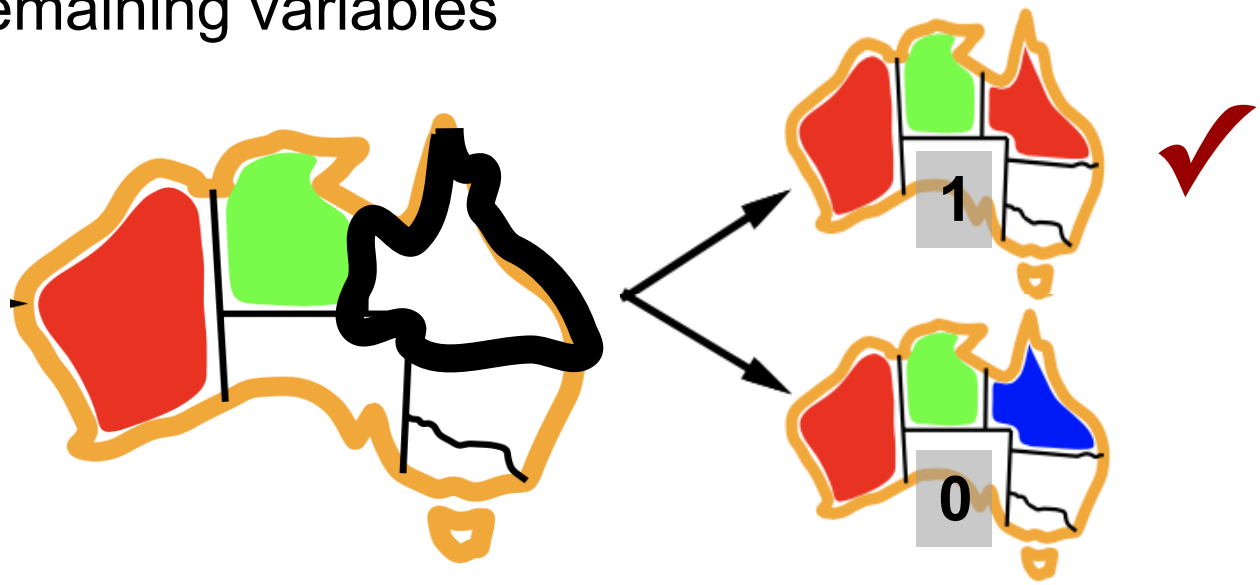

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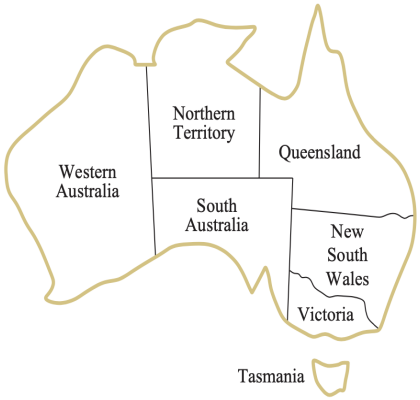
Inference by Forward Checking

- Filtering
  - Keep track of remaining legal values for unassigned variables and cross off bad options
  - Terminate search when any variable has no legal values
- Forward checking
  - Propagate information from assigned to unassigned variables
  - Cross off values that violate a constraint when added to the existing assignment
  - When a variable X is assigned, the forward-checking process establishes **arc consistency** for it
    - For each unassigned variable Y that is connected to X by a constraint, delete from Y's domain any value that is inconsistent with the value chosen for X

Example: Forward Checking



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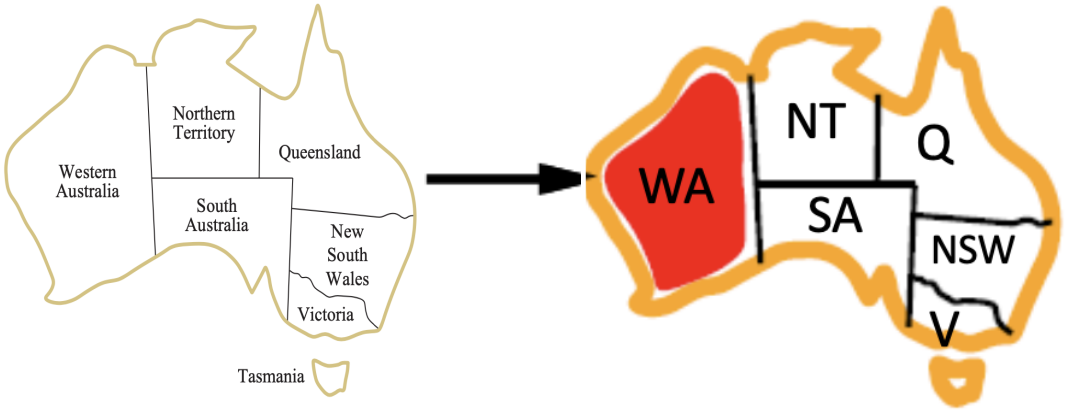

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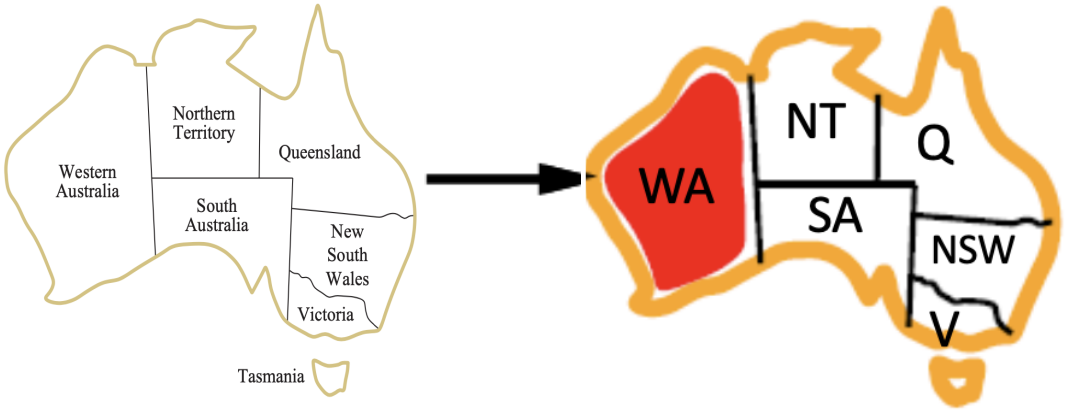

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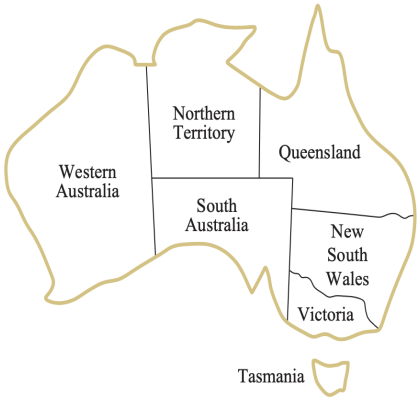

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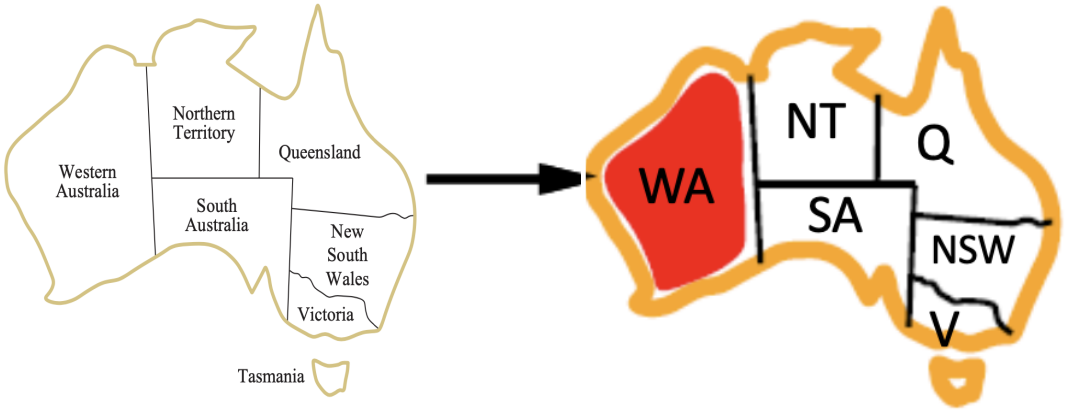

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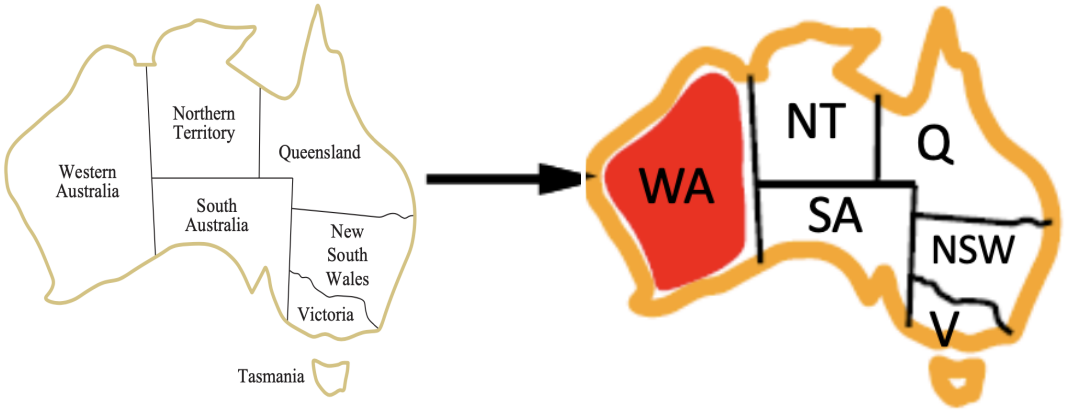

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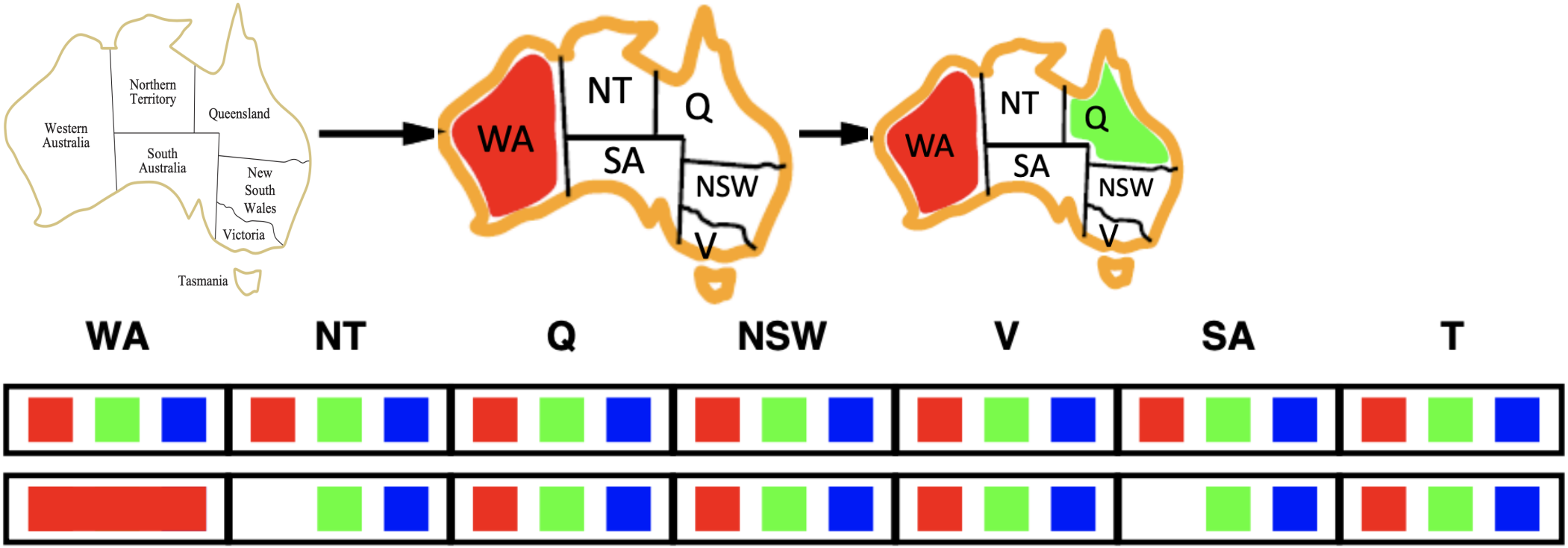

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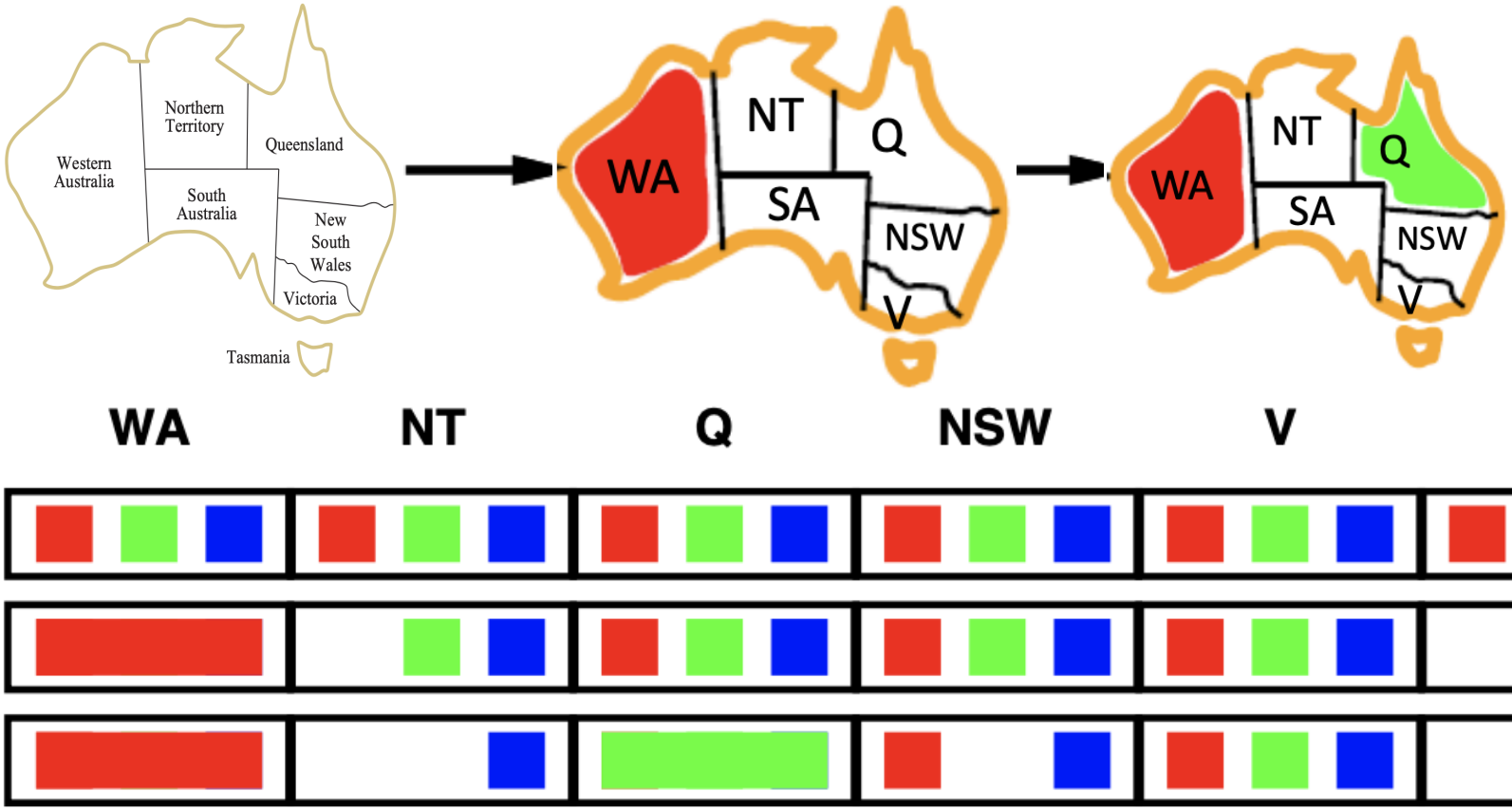

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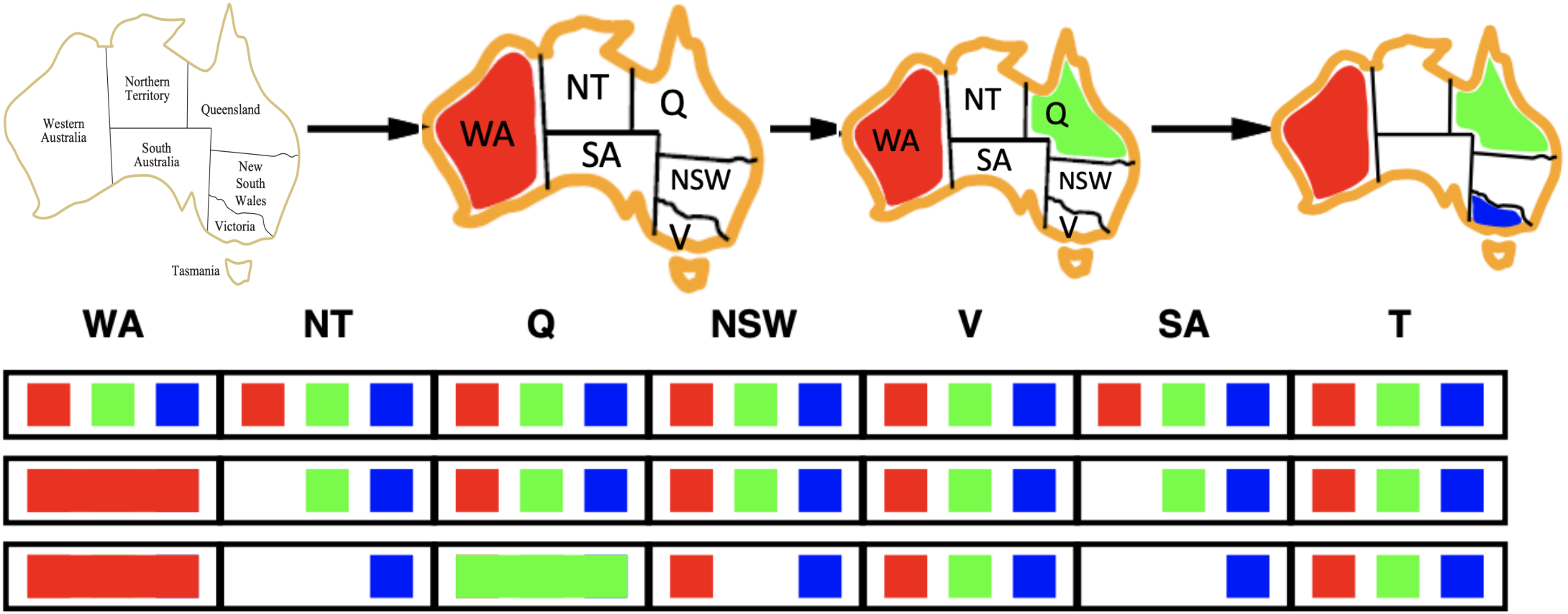
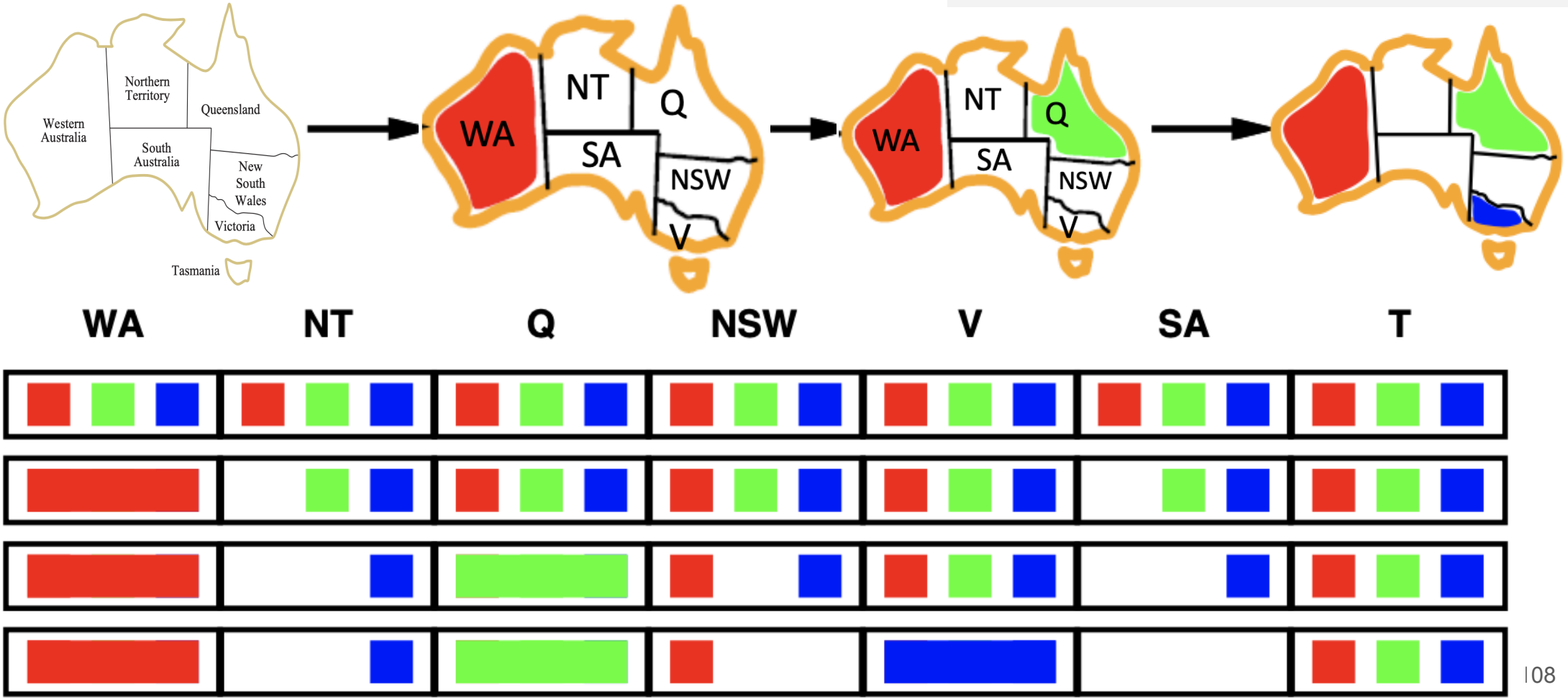


Example: Forward Checking

{WA=red, Q=green,V=blue}
SA: no legal values!



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            remove {var = value} from assignment
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```
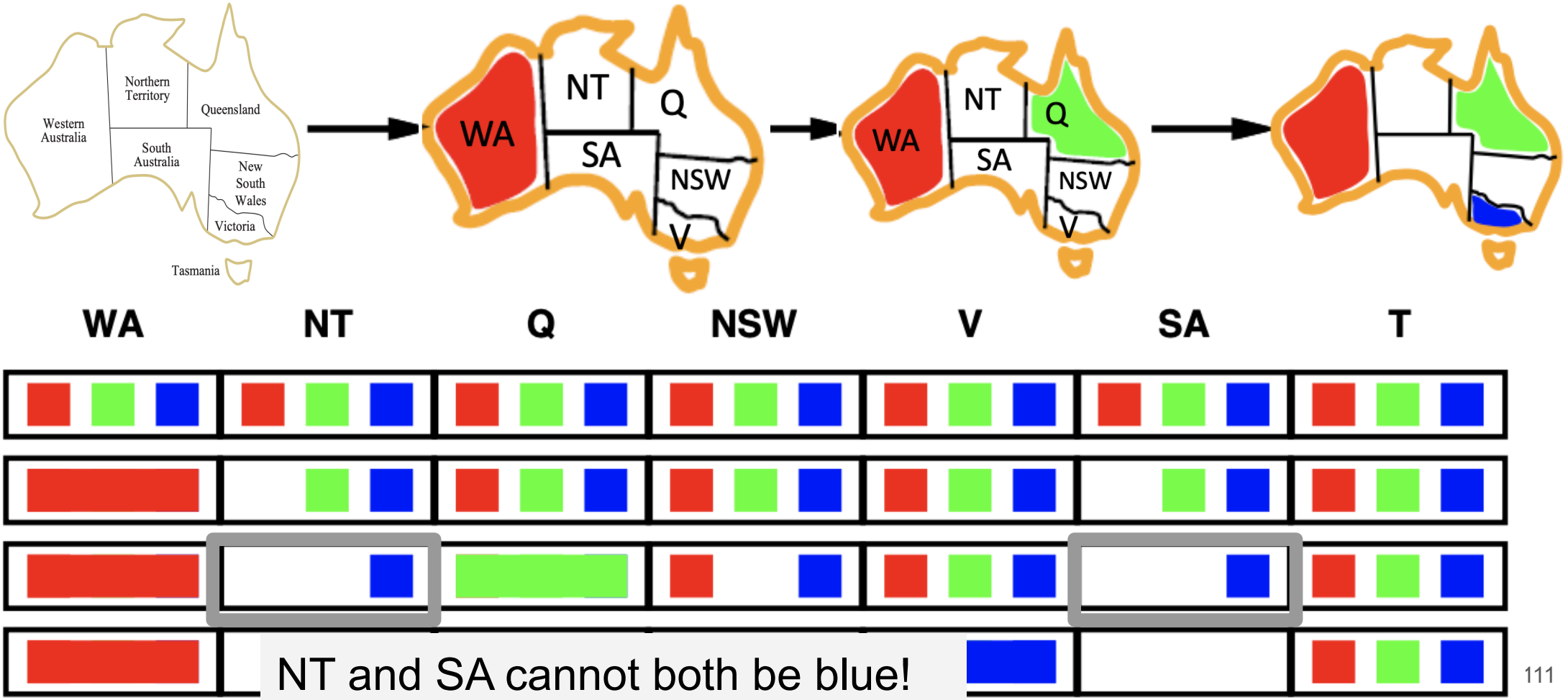

Weakness of Forward Checking

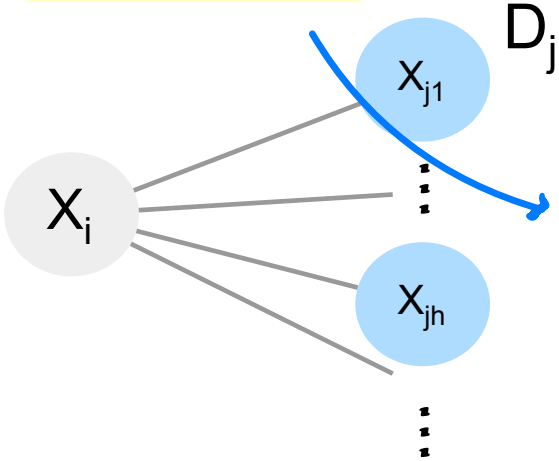
- Forward checking propagates information from assigned to unassigned variables, but does not provide **early** detection for all failures

Example: Forward Checking



MAC (Maintaining Arc Consistency) Algorithm

- After a variable X_i is assigned a value, the INFERENCE procedure calls AC-3 to detect inconsistencies
  - Idea: we start with only the arcs (X_j, X_i) for all X_j that are unassigned variables that are neighbors of X_i, then do **constraint propagation**



revised!

D_{j1}:{v_1,v_2,✖. }

When the domain of a variable is updated, the information is propagated throughout the system, leading to further domain reductions for other variables.

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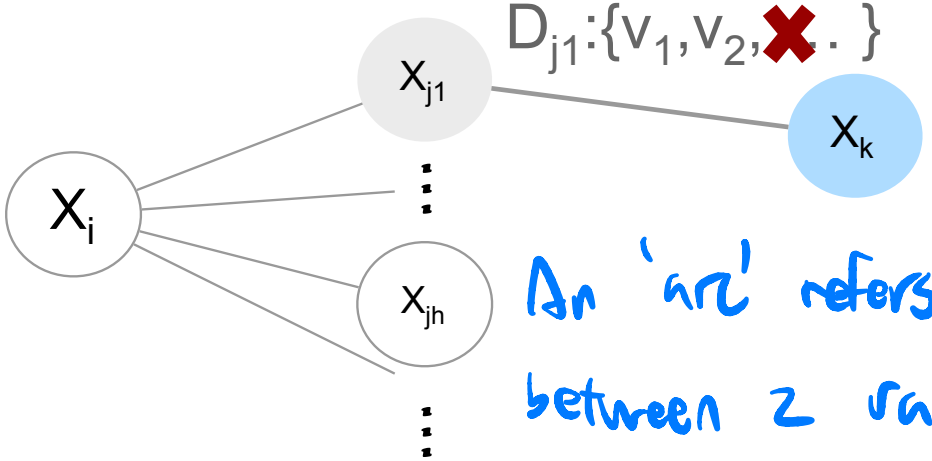

$A \neq B$ becomes $A \neq B$ & $B \neq A$

An 'arc' refers to a directional relationship between 2 variables that are subject to a constraint. e.g. An arc from X_i to X_j would represent the value of X_i must be compatible to X_j

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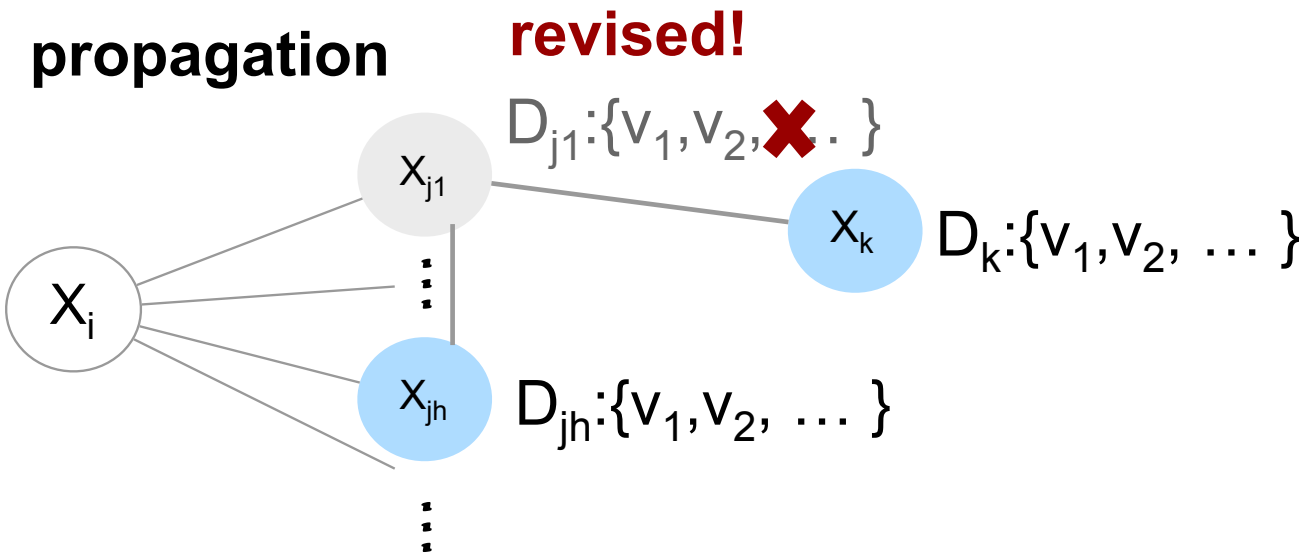
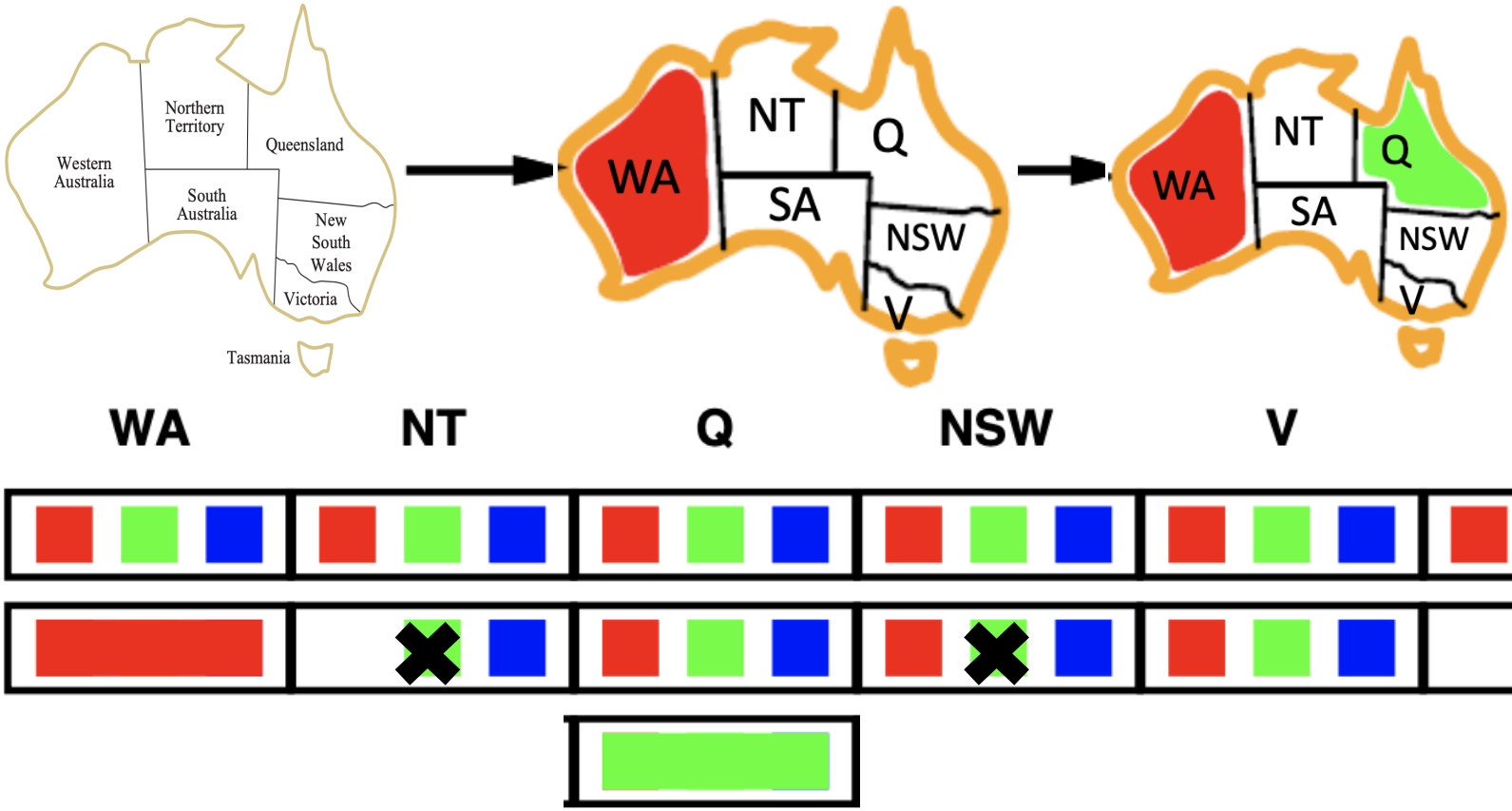


Example: Forward Checking



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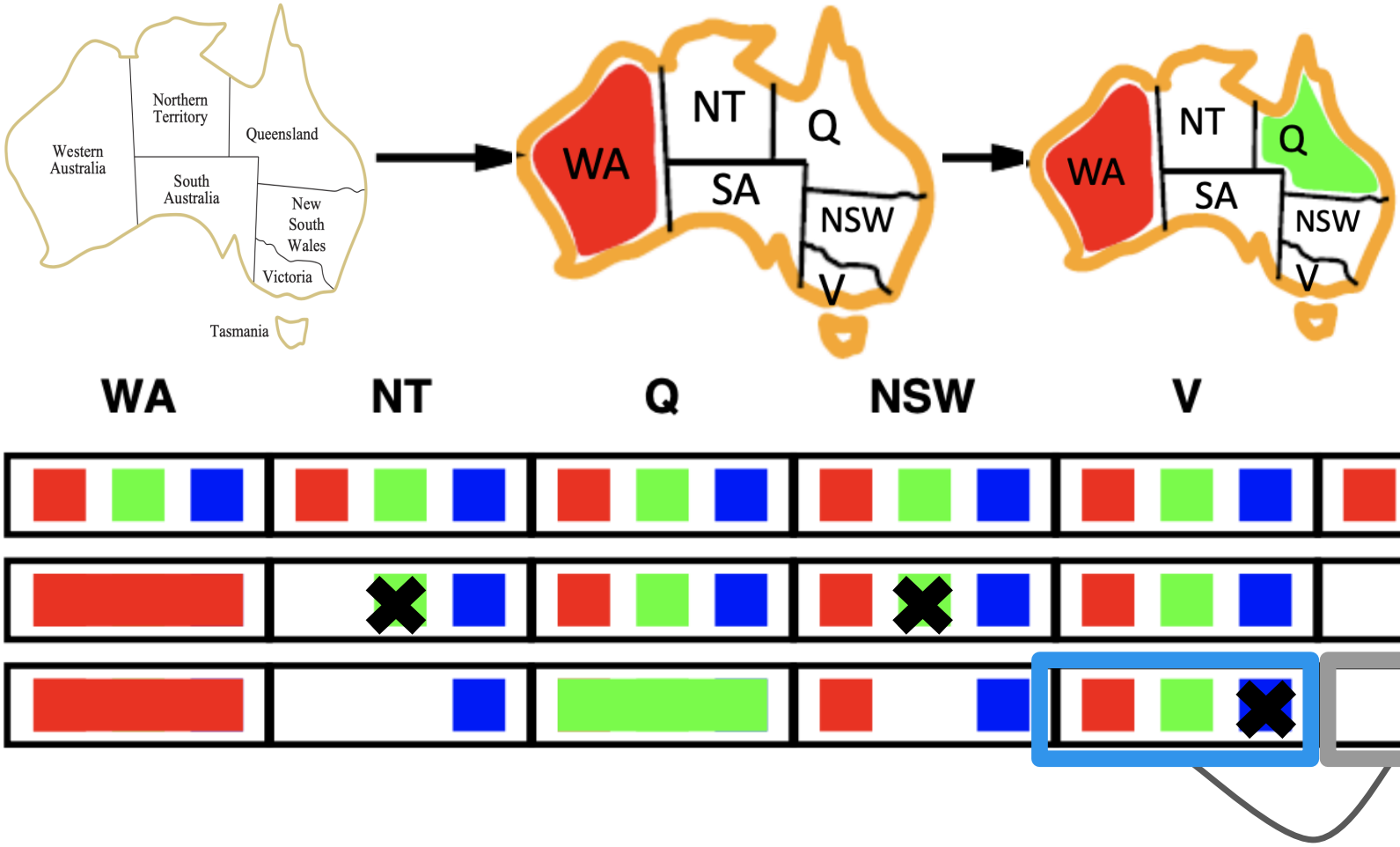

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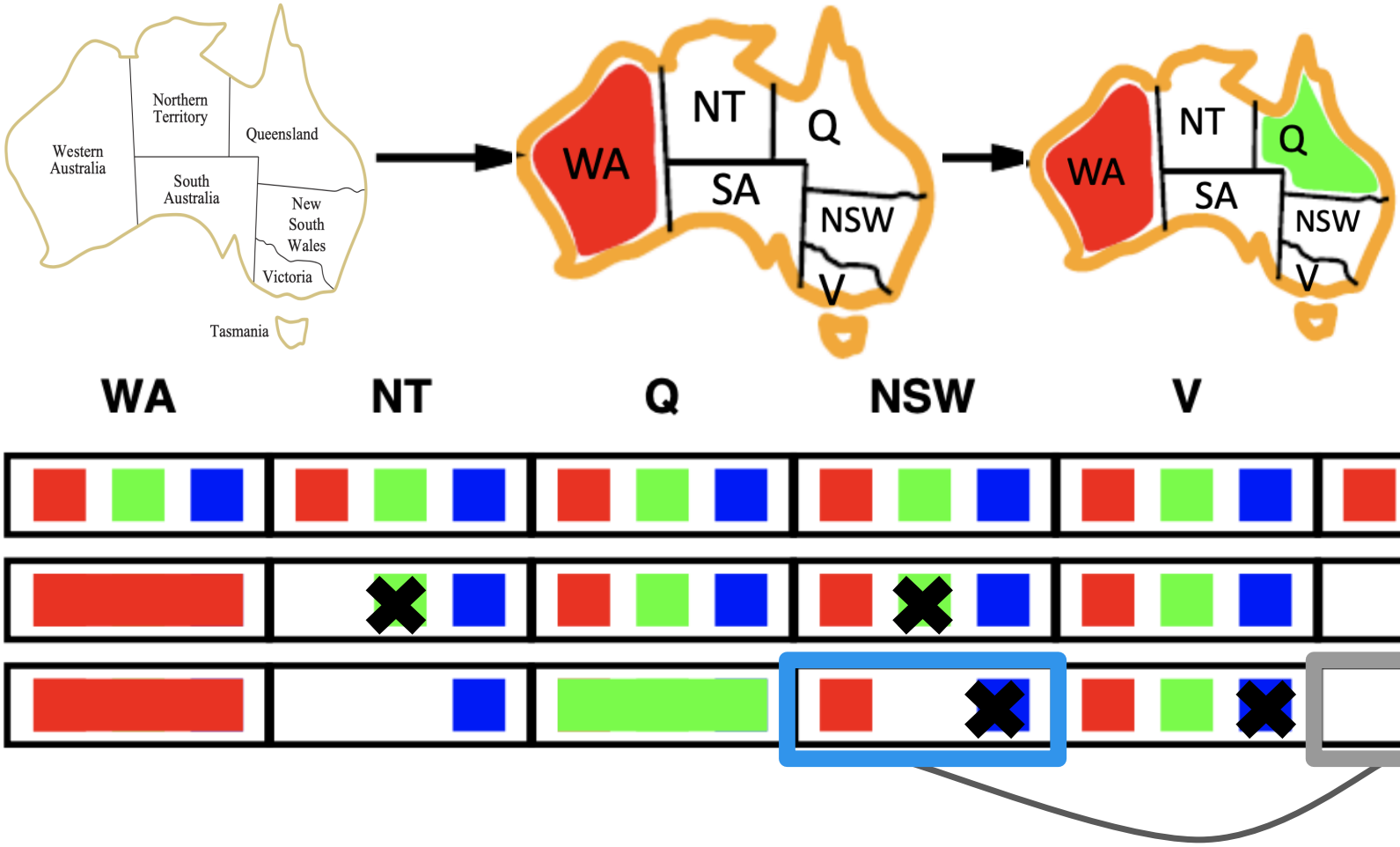

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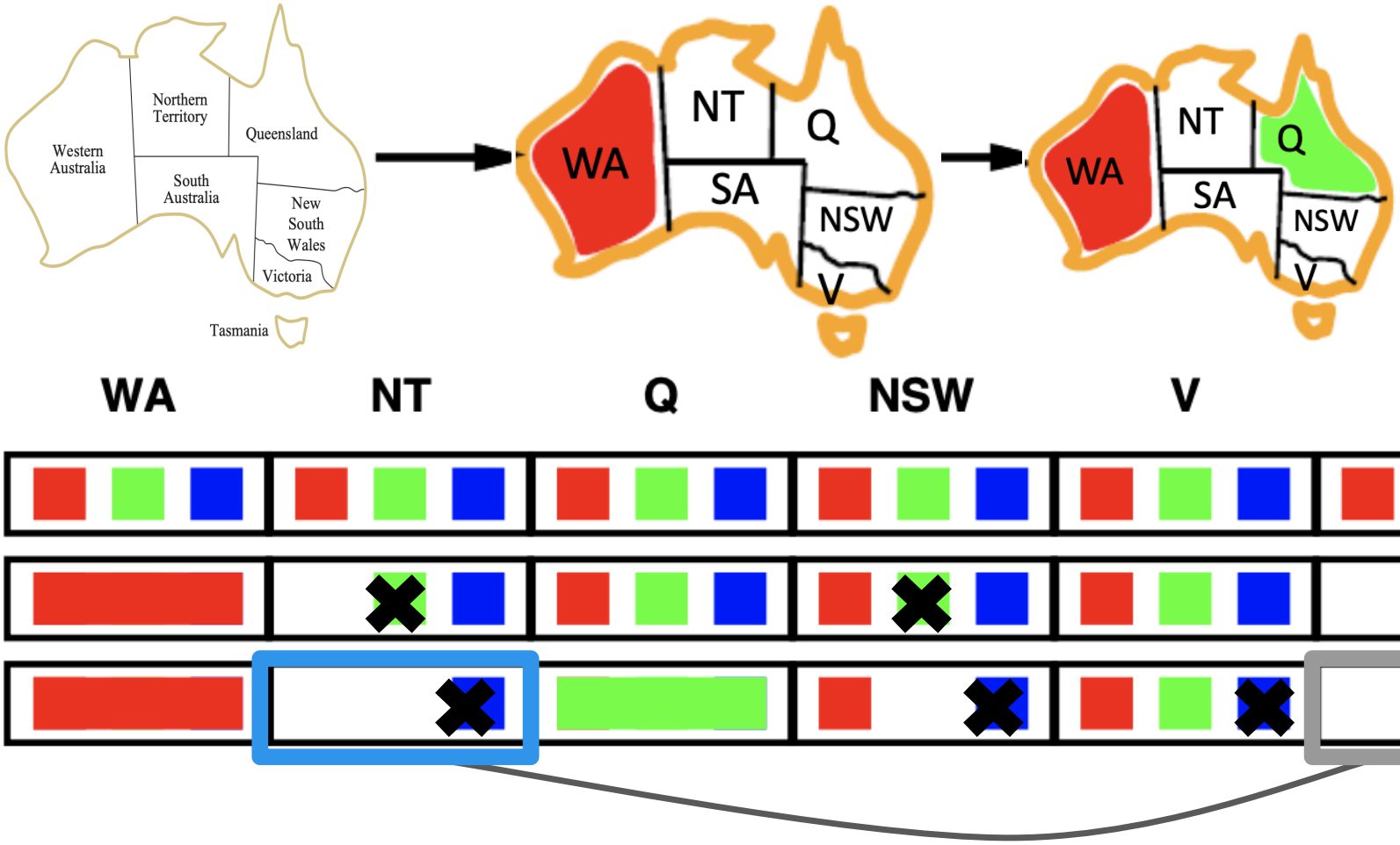

Arc Consistency Algorithm

```
function AC-3(csp) returns false if an inconsistency is found and true otherwise
    queue ← a queue of arcs, initially all the arcs in csp

    while queue is not empty do
        (X_i, X_j) ← POP(queue)
        if REVISE(csp, X_i, X_j) then
            if size of D_i = 0 then return false
            for each X_k in X_i.NEIGHBORS - {X_j} do
                add (X_k, X_i) to queue
    return true
```

inconsistency ⇒ no solution exists

```
function REVISE(csp, X_i, X_j) returns true iff we revise the domain of X_i
    revised ← false
    for each x in D_i do
        if no value y in D_j allows (x,y) to satisfy the constraint between X_i and X_j then
            delete x from D_i
            revised ← true
    return revised
```