Lecture 07 – chapter 30

Source of Magnetic Fields

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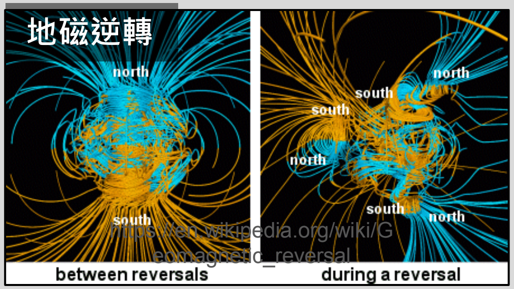
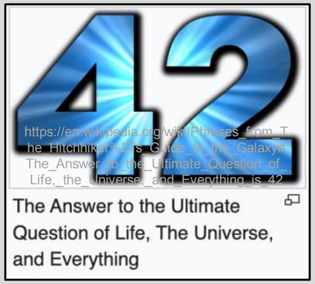
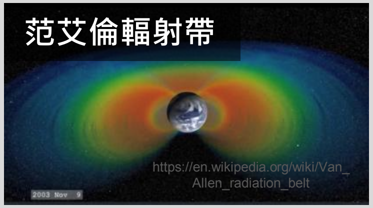
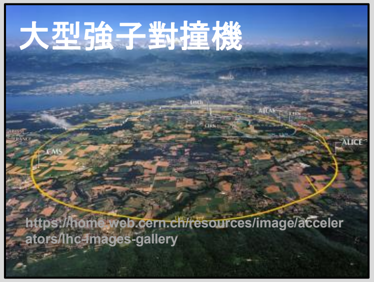
Previous Lecture

- Fascinating Magnetism
  - ○ Earth’s Magnetic Field
  - ○ The issue is what questions to ask
- Definition of Magnetic Field (VS Electric Fields)
- Magnetic Force on a Moving Charged Particle
  - – Motion of this moving charged particle
- Applications
- Magnetic Force on Wire
  - – Magnetic Dipole Moment & Torque
- The Hall effect





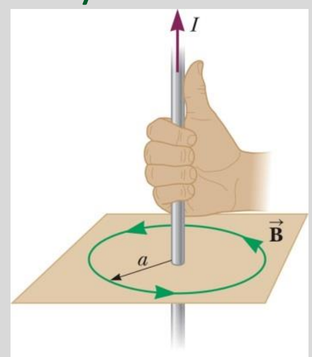


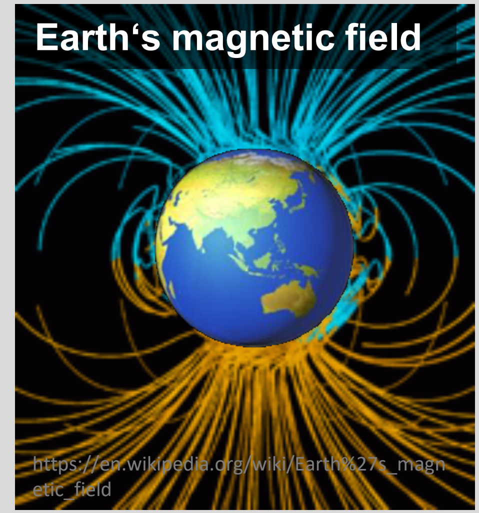


This Lecture

- ○ Some History
- Biot-Savart Law
- Ampere's Law
- Gauss's Law in Magnetism
  - Magnetic Flux
- Magnetism in Matter
  - Atom
  - Substance

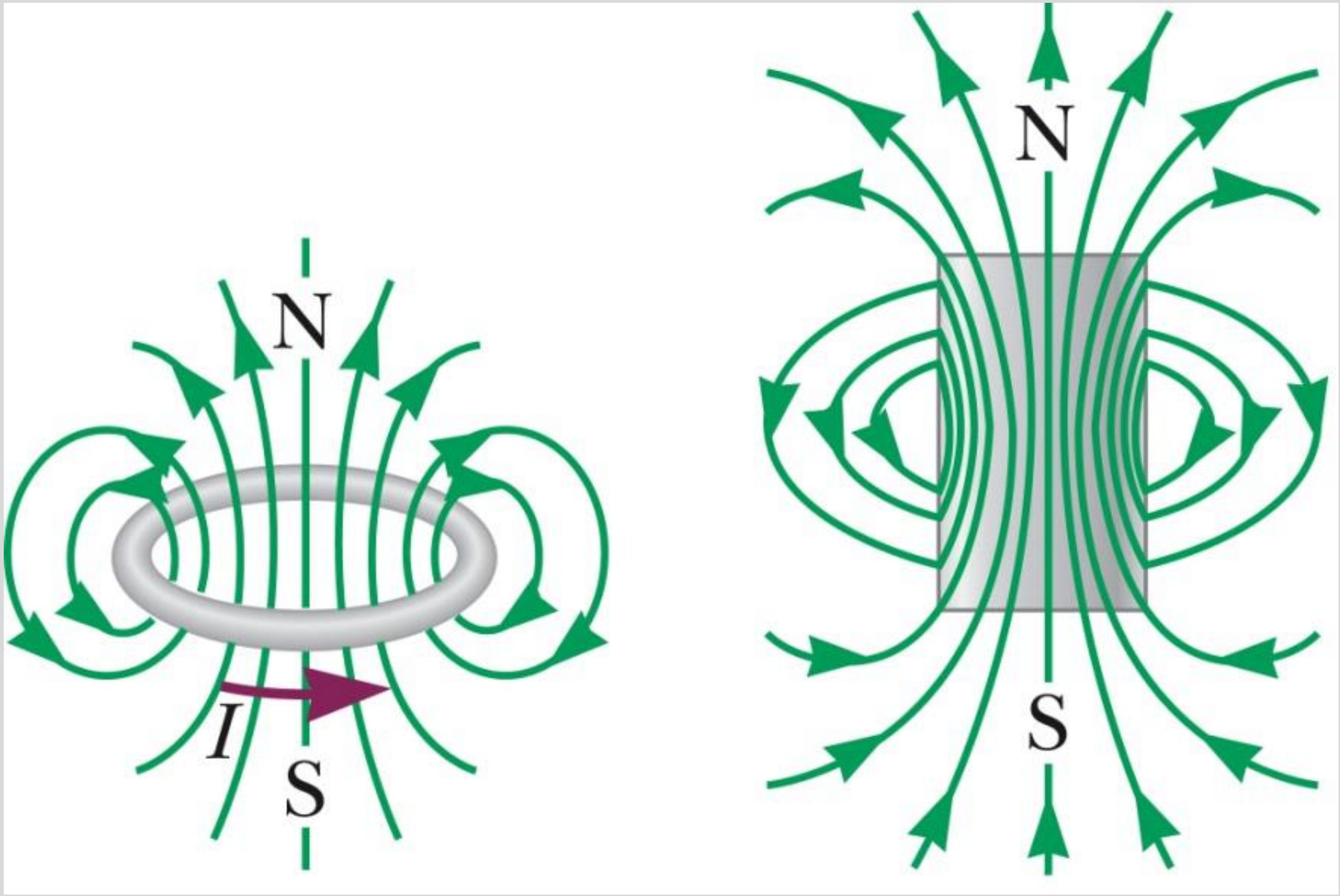
Similar to the use of Gauss's law in calculating $\vec{E}$ of a symmetric configuration





Magnetic Field Lines

Current loop **Bar magnet**

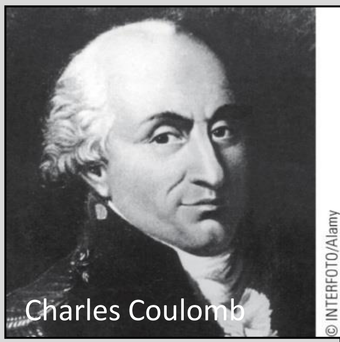


Some History

Not until the early 19th century did scientists establish electricity and magnetism as related phenomena.

- 1785
  - Charles Coulomb confirmed inverse square law form for electric forces

In the upcoming slides

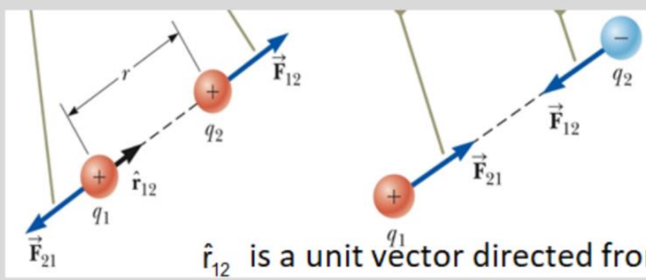


Coulomb's Law (庫倫定律)

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

- Coulomb constant, k_e = 8.9876 x 10^9 N·m²/C² = $1/(4\pi\varepsilon_0)$
- Permittivity (介電常數) of free space, ε_0 = 8.8542 x 10^{-12} C^2 / N·m²

- The force is a **conservative force (保守力)**



$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12}$$

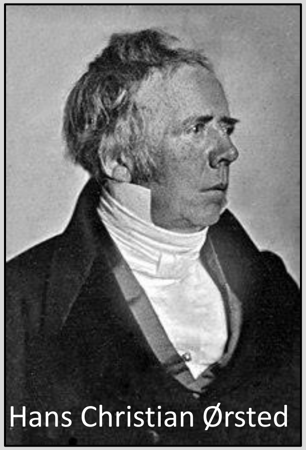
$\hat{r}_{12}$ is a unit vector directed from q_1 to q_2

Week 1

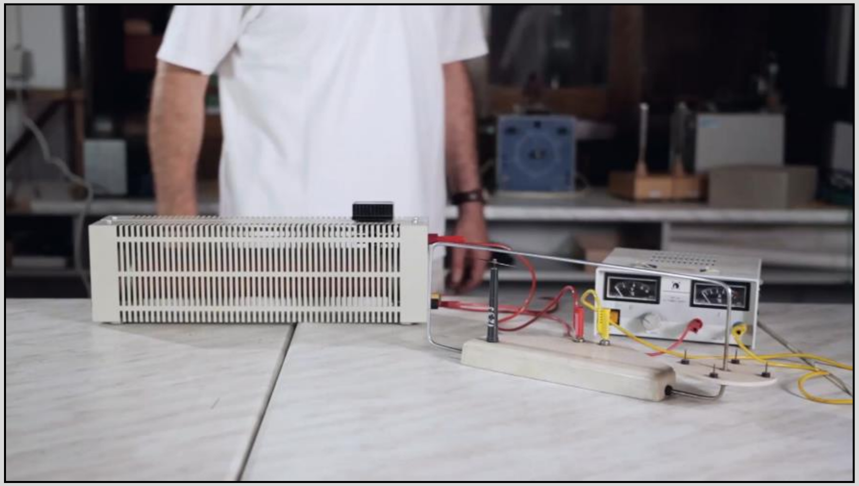
Some History

Not until the early 19th century did scientists establish electricity and magnetism as related phenomena.

- 1785
  - Charles Coulomb confirmed inverse square law form for electric forces
- 1819
  - Hans Oersted found a **compass needle** deflected when near a wire carrying an electric current



Hans Christian Ørsted

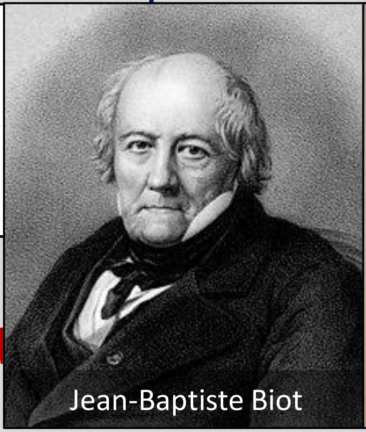


https://en.wikipedia.org/wiki/Hans_Christian_%C3%98rsted

Some History

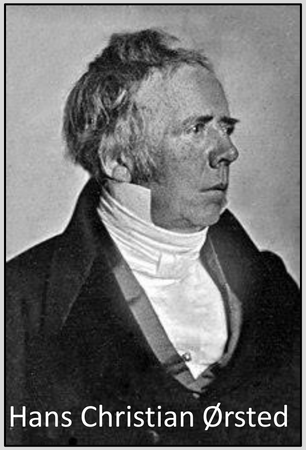
Not until the early 19th century did scientists establish electricity and magnetism as related phenomena.

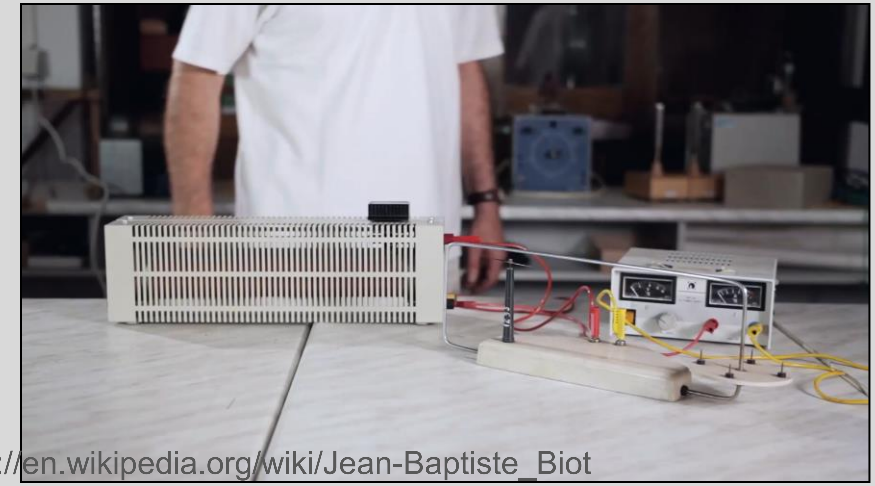
- Shortly after Oersted's discovery,
  - Biot-Savart Law $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I d\vec{S} \times \hat{r}}{r^2}$


Jean-Baptiste Biot

Félix Savart

- 1819
  - Hans Oersted found a **compass need**
    carrying an electric current


Hans Christian Ørsted



https://en.wikipedia.org/wiki/Jean-Baptiste_Biot
https://en.wikipedia.org/wiki/F%C3%A9lix_Savart
https://en.wikipedia.org/wiki/Hans_Christian_%C3%98rsted

Biot-Savart Law

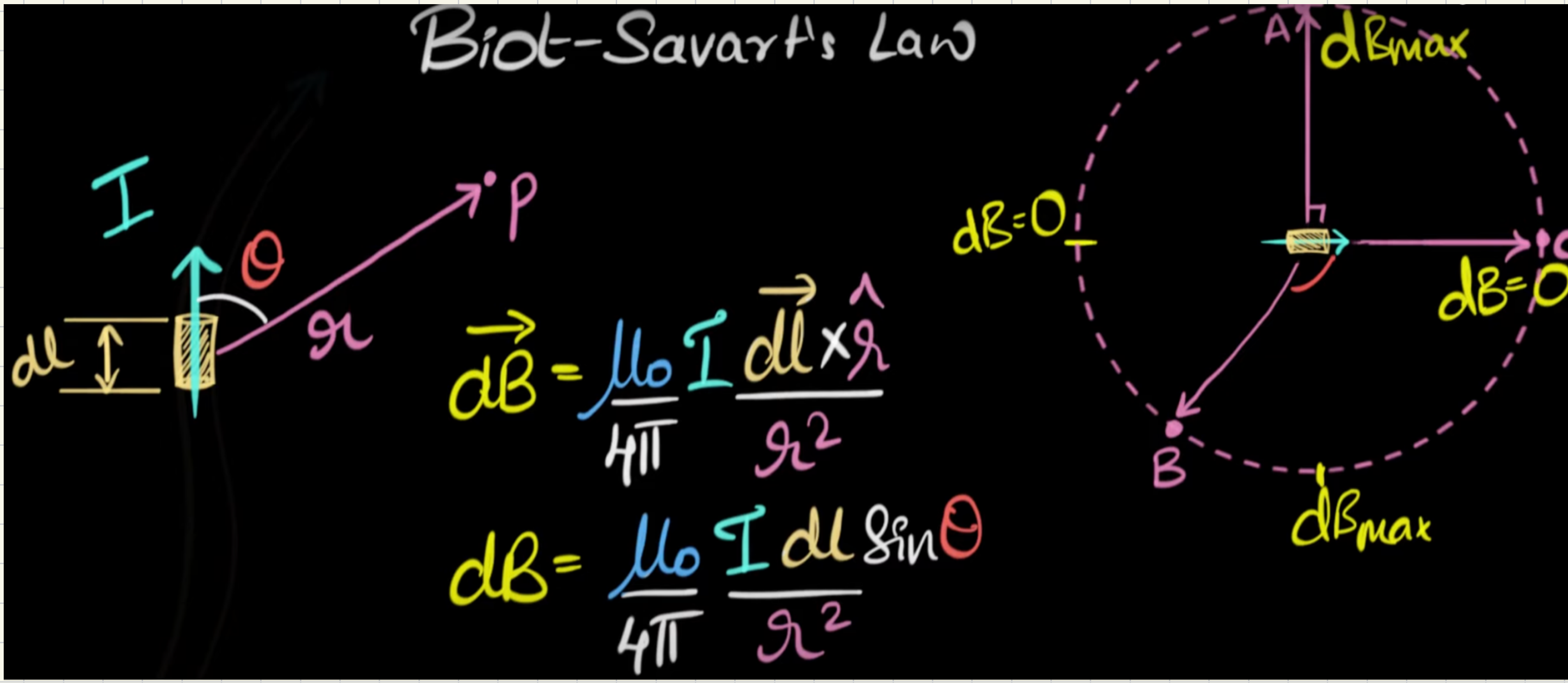
- The origin of the magnetic field is **moving charges** (电动生磁)
  - The magnetic field due to various current

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{S} \times \hat{r}}{r^2}$$

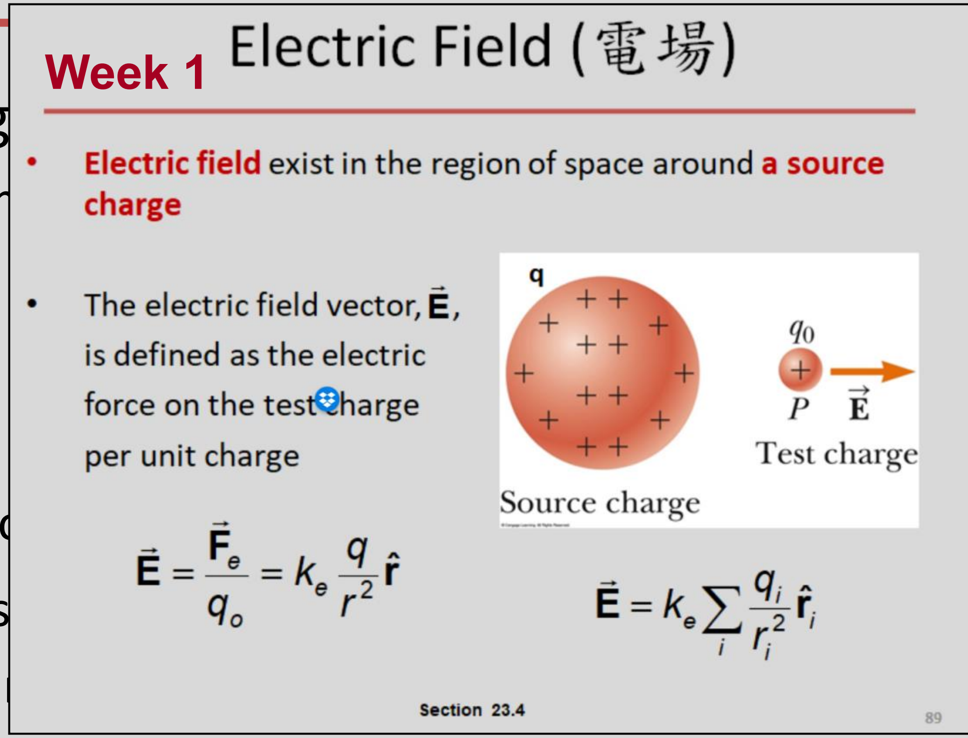
导磁率

  - The constant $\mu_0 = 4\pi \times 10^{-7}$ T· m/A is **permeability of free space**
  - $d\vec{B}$ is the field created by the current in the length segment $d\vec{S}$
  - The unit vector $\hat{r}$ directed from $d\vec{S}$ toward *P*
- The total field from all the current elements

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{S} \times \hat{r}}{r^2}}$$


Biot-Savart's Law
I
θ
P
dl
r
$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,\vec{dl} \times \hat{r}}{r^2}$
$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\,\sin\theta}{r^2}$
A
dBmax
dB=0
C
dB=0
B
dBmax

Biot-Savart Law

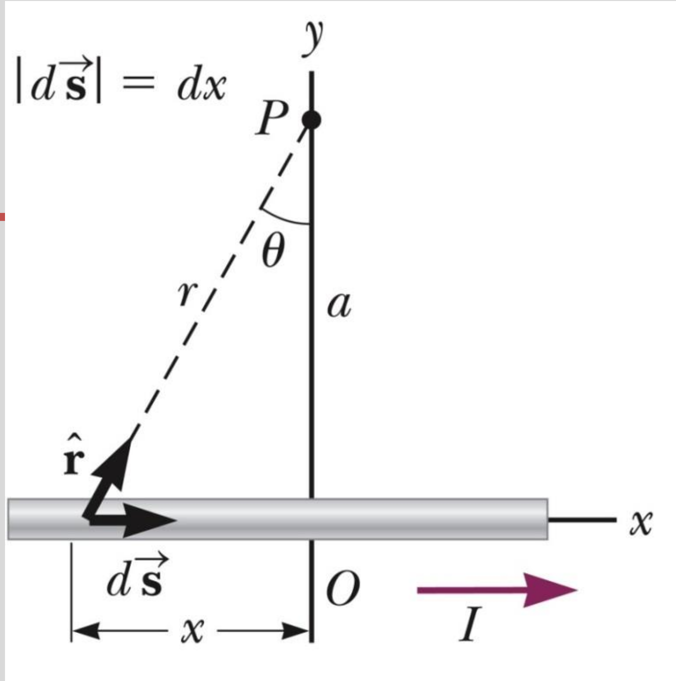
- The orig rges
  - The m
  

  導磁性;透磁率
  - The c of free space
  - $d\vec{B}$ is segment $d\vec{S}$
  - The u
- The total field from all the current elements

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{S} \times \hat{r}}{r^2}$$

A Long, Straight Conductor

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = \left| d\vec{\mathbf{s}} \times \hat{\mathbf{r}} \right| \hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right] \hat{\mathbf{k}} = (dx\cos\theta)\hat{\mathbf{k}}$$

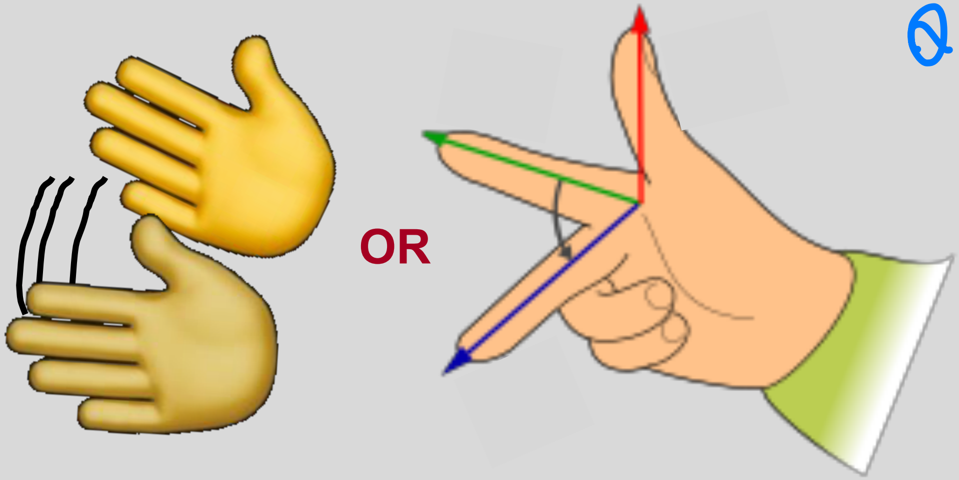


· Recall : cross product

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta$$

Biot-Savart Law

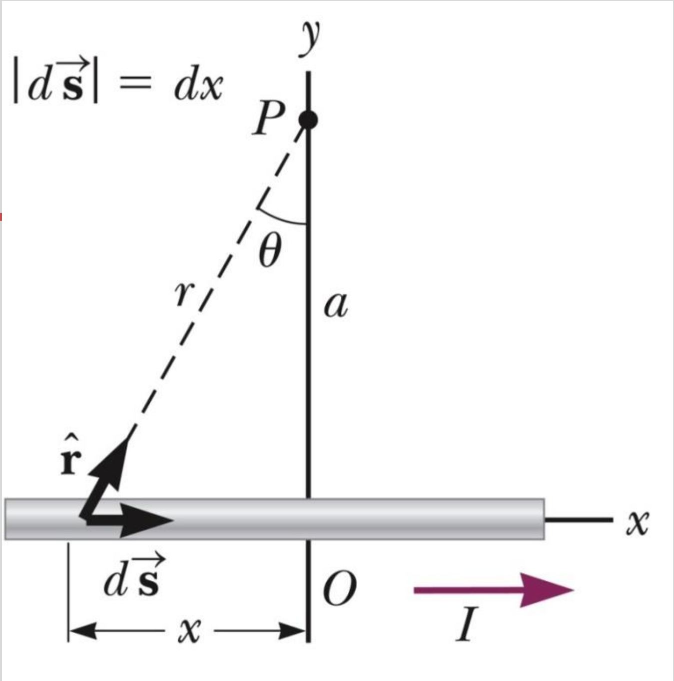
$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{S} \times \hat{r}}{r^2}}$$



Q: Work out the magnetic field B.

A Long, Straight Conductor

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = \left| d\vec{\mathbf{s}} \times \hat{\mathbf{r}} \right| \hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta \right) \right] \hat{\mathbf{k}} = (dx \cos\theta) \hat{\mathbf{k}}$$



$$(1) \quad \mathrm{d}\vec{\mathbf{B}} = (\mathrm{d}B)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi} \frac{dx \cos\theta}{r^2} \hat{\mathbf{k}}$$

Biot-Savart Law

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{S} \times \hat{r}}{r^2}}$$

$$(2) \quad r = \frac{a}{\cos\theta}$$

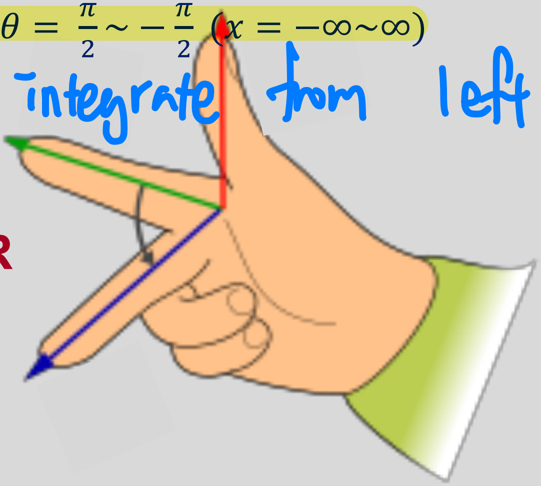
$$x = -a \tan\theta$$

O 是原點，所以這邊是負，

無限長導線，等下會積分從$\theta = \frac{\pi}{2} \sim -\frac{\pi}{2}$ $(x = -\infty \sim \infty)$

We integrate from left to right

OR



Section 30.1

A Long, Straight Conductor

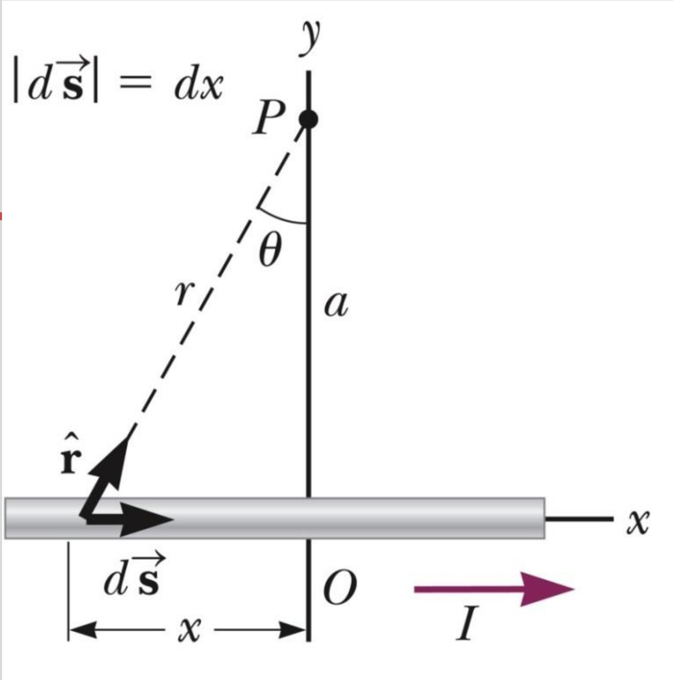
$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = \left| d\vec{\mathbf{s}} \times \hat{\mathbf{r}} \right| \hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta \right) \right] \hat{\mathbf{k}} = (dx \cos\theta) \hat{\mathbf{k}}$$

$$(1) \quad \mathrm{d}\vec{\mathbf{B}} = (\mathrm{d}B)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi} \frac{dx \cos\theta}{r^2} \hat{\mathbf{k}}$$

$$(2) \quad r = \frac{a}{\cos\theta}$$

$$x = -a \tan\theta$$

$$(3) \quad dx = -a \sec^2\theta \, d\theta = -\frac{a \, d\theta}{\cos^2\theta}$$



Biot-Savart Law

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{S} \times \hat{r}}{r^2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{d}{d\theta}\left(\frac{\sin\theta}{\cos\theta}\right) = \frac{\cos\theta \cdot \frac{d}{d\theta}(\sin\theta) - \sin\theta \cdot \frac{d}{d\theta}(\cos\theta)}{\cos^2\theta}$$

$$= \frac{\cos\theta \cdot \cos\theta - \sin\theta \cdot (-\sin\theta)}{\cos^2\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$$

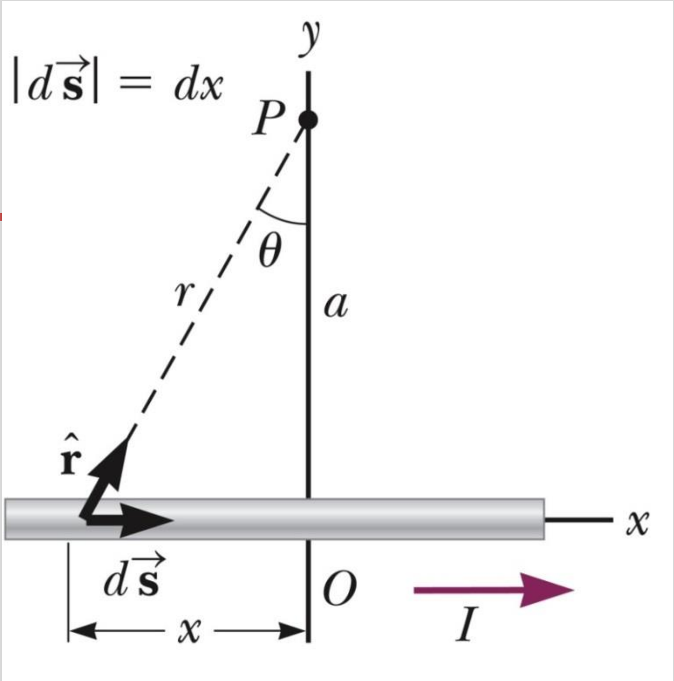
$$= \frac{1}{\cos^2\theta} = \sec^2\theta$$

memorize:

$$\frac{d \tan\theta}{d\theta} = \sec^2\theta$$

A Long, Straight Conductor

$$d\vec{\mathbf{s}}\times\hat{\mathbf{r}} = \left|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}\right|\hat{\mathbf{k}} = \left[dx\sin\left(\frac{\pi}{2}-\theta\right)\right]\hat{\mathbf{k}} = (dx\cos\theta)\hat{\mathbf{k}}$$



(1) $$\mathrm{d}\vec{\mathbf{B}} = (\mathrm{d}B)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$

Biot-Savart Law

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{S}\times\hat{r}}{r^2}}$$

(2) $$r = \frac{a}{\cos\theta}$$

$$x = -a\tan\theta$$

(3) $$dx = -a\sec^2\theta\, d\theta = -\frac{a\,d\theta}{\cos^2\theta}$$

(4) $$dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\underline{\left(\frac{\cos^2\theta}{a^2}\right)}\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\,d\theta$$

$r = \dfrac{a}{\cos\theta}$ ... (2)

A Long, Straight Conductor

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = \left| d\vec{\mathbf{s}} \times \hat{\mathbf{r}} \right| \hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta \right) \right] \hat{\mathbf{k}} = (dx \cos\theta) \hat{\mathbf{k}}$$

$$(1) \quad \mathrm{d}\vec{\mathbf{B}} = (\mathrm{d}B)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi} \frac{dx \cos\theta}{r^2} \hat{\mathbf{k}}$$

$$(2) \quad r = \frac{a}{\cos\theta}$$

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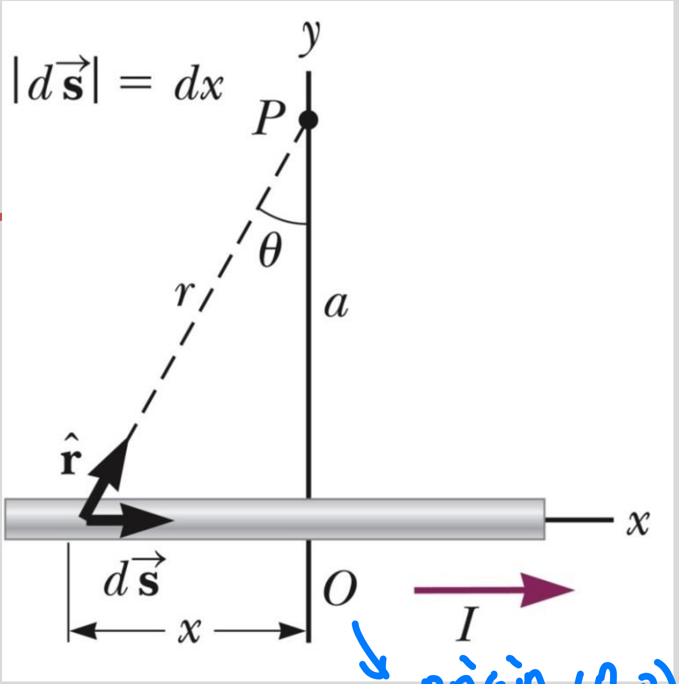
$$(4) \quad dB = -\frac{\mu_0 I}{4\pi} \left(\frac{a \, d\theta}{\cos^2\theta} \right) \left(\frac{\cos^2\theta}{a^2} \right) \cos\theta = -\frac{\mu_0 I}{4\pi a} \cos\theta \, d\theta$$

$$B = -\frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \cos\theta d\theta = \frac{\mu_0 I}{4\pi a} (\sin\theta_1 - \sin\theta_2) \qquad \textbf{(30.4)}$$



origin (0,0)

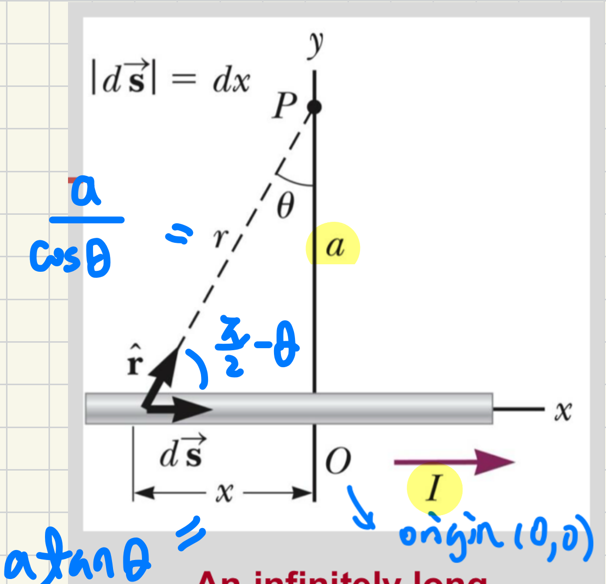
An infinitely long, straight wire, $\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$

$$B = \frac{\mu_0 I}{2\pi a}$$ #

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dx \sin\left(\frac{\pi}{2} - \theta\right)}{(a/\cos\theta)^2} \hat{k}$$

$$\left(\begin{aligned} x &= -a \tan\theta \\ \Rightarrow dx &= -a \sec^2\theta \, d\theta \end{aligned} \right)$$



An infinitely long, straight wire, $\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$

(a, I: given)

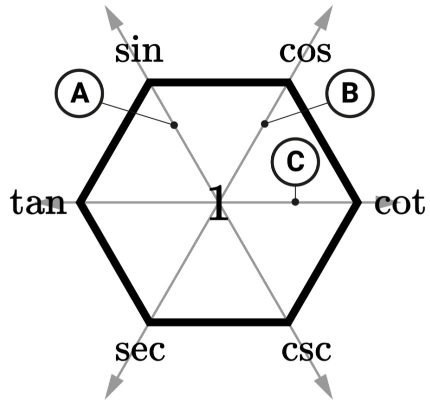
$$= \frac{\mu_0 I}{4\pi} \int_{\pi/2}^{-\pi/2} \frac{-a \cancel{\sec^2\theta} \, d\theta \sin\left(\frac{\pi}{2} - \theta\right)}{(a/\cancel{\cos\theta})^2} \hat{k}$$

$$= -\frac{\mu_0 I}{4\pi a} \int_{\pi/2}^{-\pi/2} \cos\theta \, d\theta \, \hat{k} = -\frac{\mu_0 I}{4\pi a} \left[\sin\theta \right]_{\pi/2}^{-\pi/2} \hat{k}$$

$$= \frac{\mu_0 I}{2\pi a} \hat{k} \quad \#$$

✱ Recall : Trig. Function Identities

Derivatives or Differentiation Formulas	Antiderivatives or Integration Formulas
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x dx = -\cot x + C$
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x dx = -\csc x + C$



Reciprocal Identities
Identities are opposite of the lines shown in the figure

A $\sin x = \frac{1}{\csc x}$ $\csc x = \frac{1}{\sin x}$

B $\cos x = \frac{1}{\sec x}$ $\sec x = \frac{1}{\cos x}$

C $\tan x = \frac{1}{\cot x}$ $\cot x = \frac{1}{\tan x}$

A Long, Straight Conductor

$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = \left| d\vec{\mathbf{s}} \times \hat{\mathbf{r}} \right| \hat{\mathbf{k}}$ ...

(1) $\mathrm{d}\vec{\mathbf{B}} = (\mathrm{d}B)\hat{\mathbf{k}}$ ...

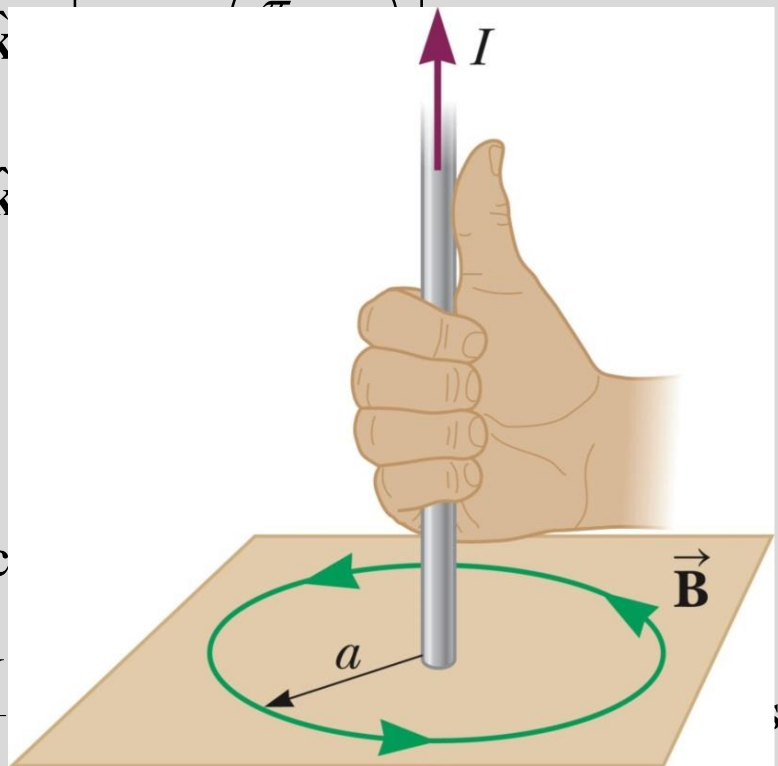
(2) $r = \dfrac{a}{\cos\theta}$

$x = -a\tan\theta$

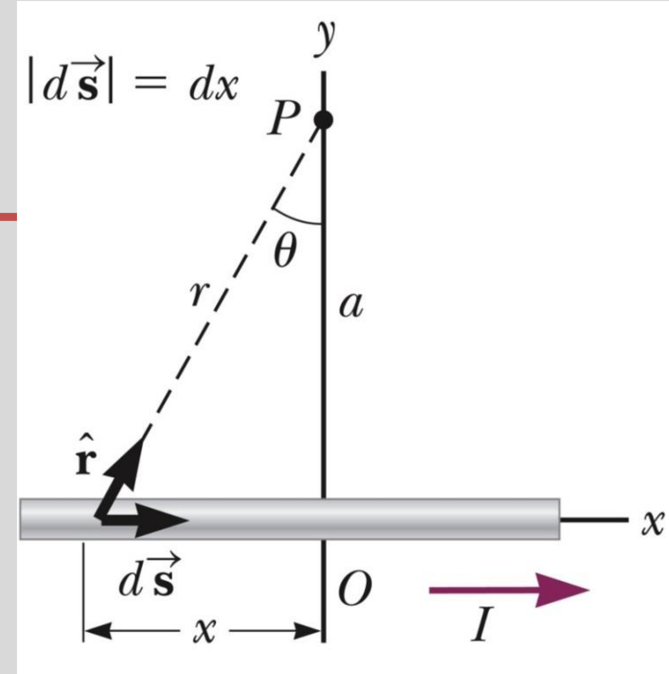
(3) $dx = -a\sec$ ...

(4) $dB = -\dfrac{\mu_0 I}{4\pi}$ ... $\cos\theta\, d\theta$

$B = -\dfrac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \mathrm{co}$ ...



No beginning and no end
Forms a closed loop!

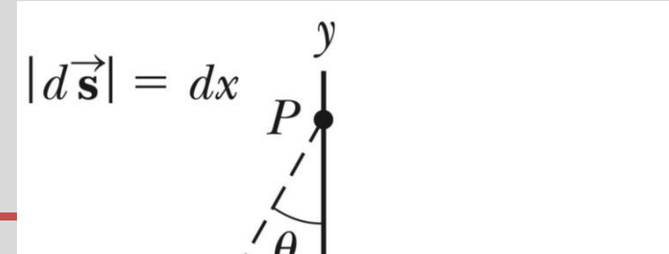


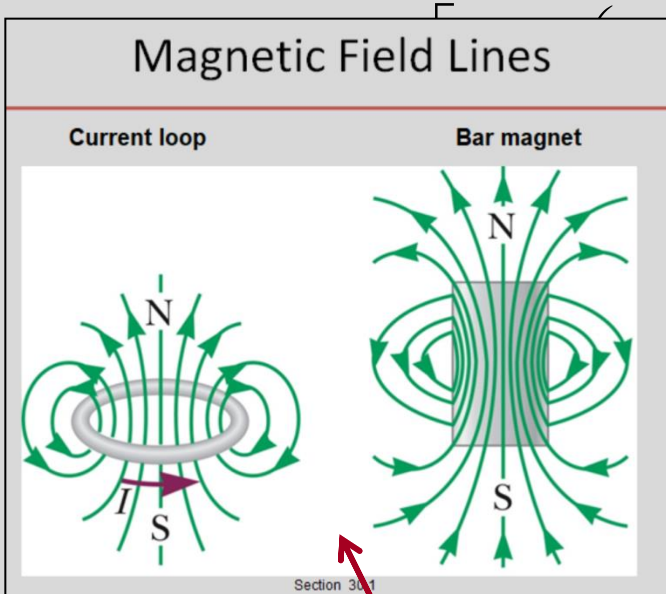
An infinitely long, straight wire,
$\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$

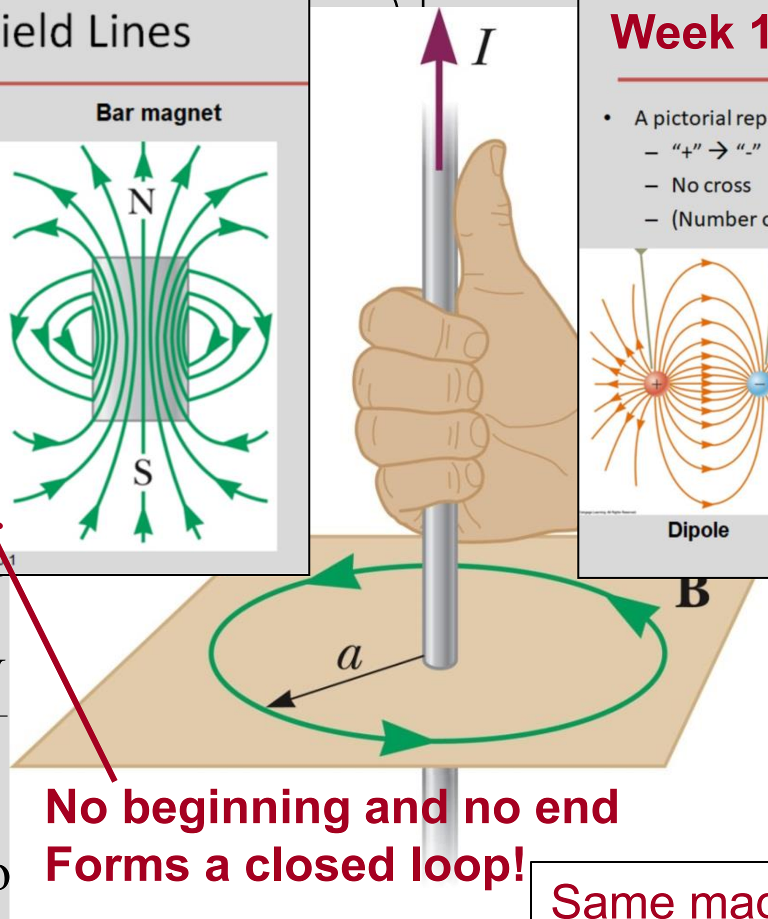
$$B = \frac{\mu_0 I}{2\pi a}$$

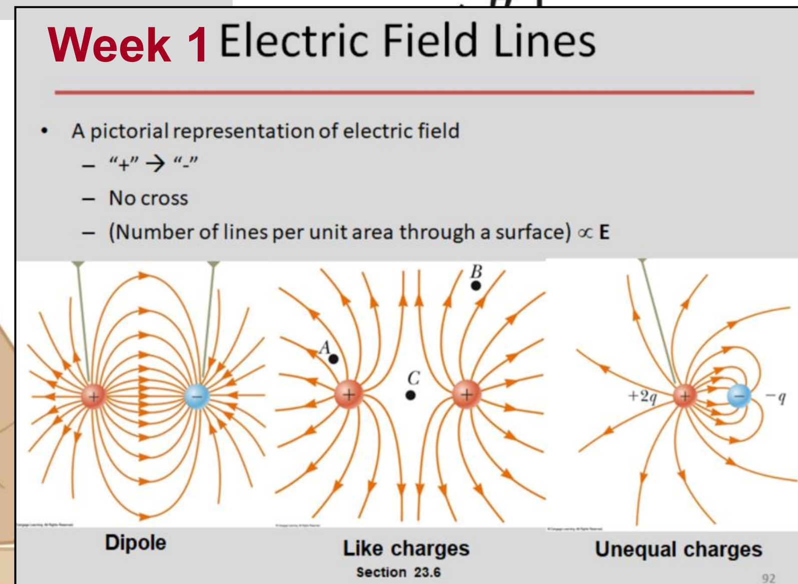
Same magnitude at a distance of a

A Long, Straight Conductor









$\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$

$$(4) \quad dB = -\frac{\mu_0 I}{4\pi} \ldots \cos\theta \, d\theta$$

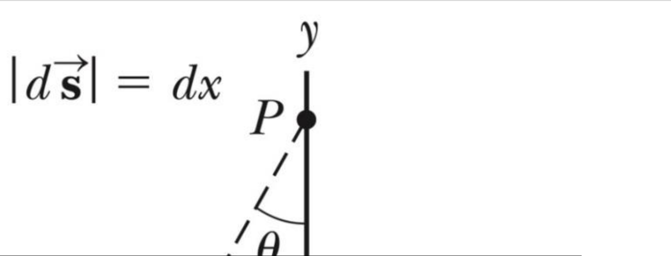
$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \text{co}\ldots$$

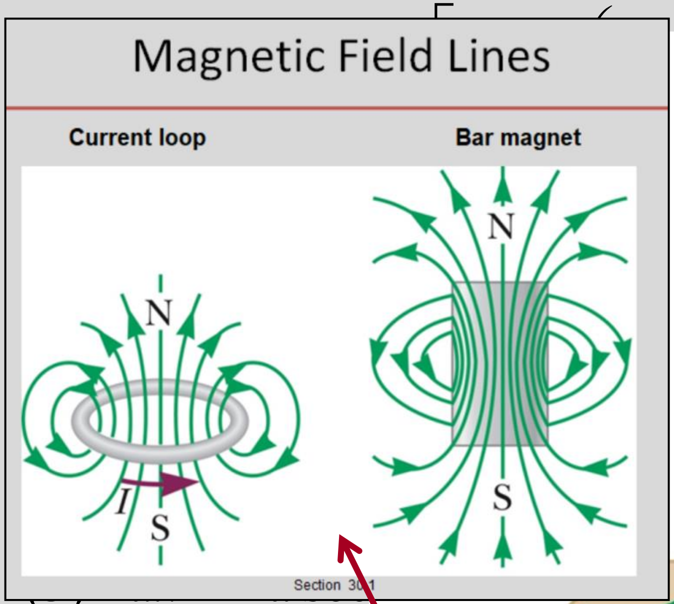
$$B = \frac{\mu_0 I}{2\pi a}$$

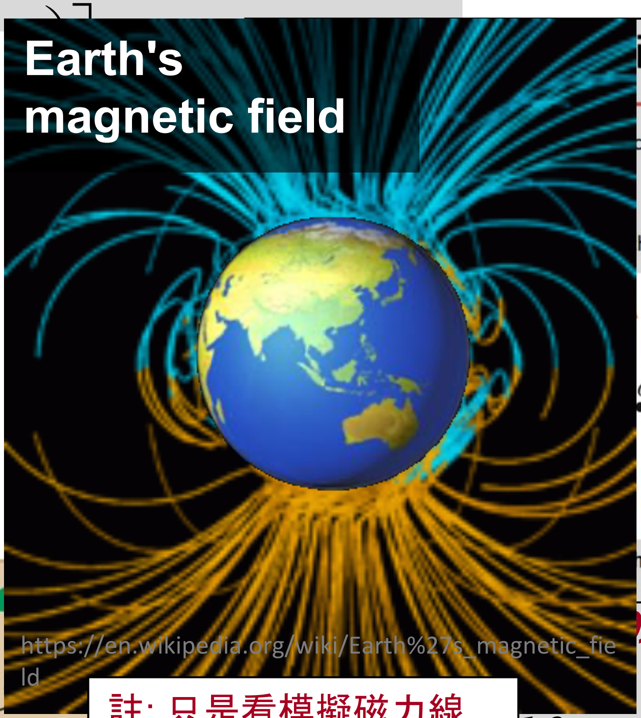
No beginning and no end
Forms a closed loop!

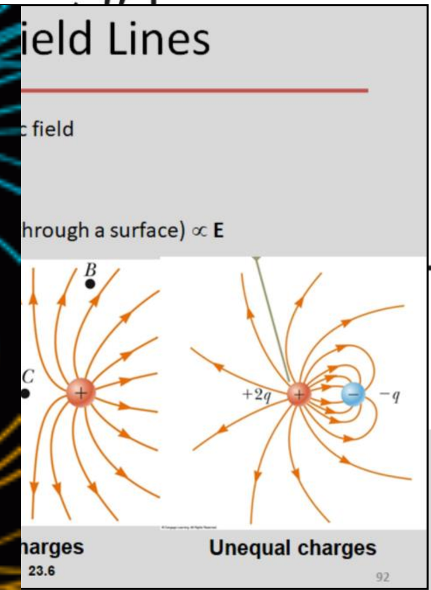
Same magnitude at a distance of a

A Long, Straight Conductor

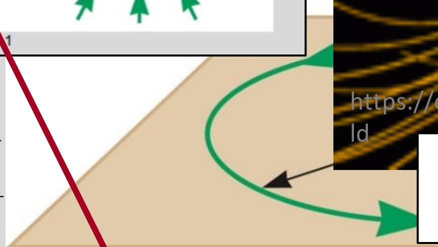








$\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$



註: 只是看模擬磁力線,
實際沒左圖這麼簡單

(4) $dB = -\frac{\mu_0 I}{4\pi}$ ... $d\theta$

$$B = \frac{\mu_0 I}{2\pi a}$$

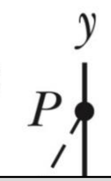
No beginning and no end
Forms a closed loop!

$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \text{co}$

Same magnitude at a distance of a

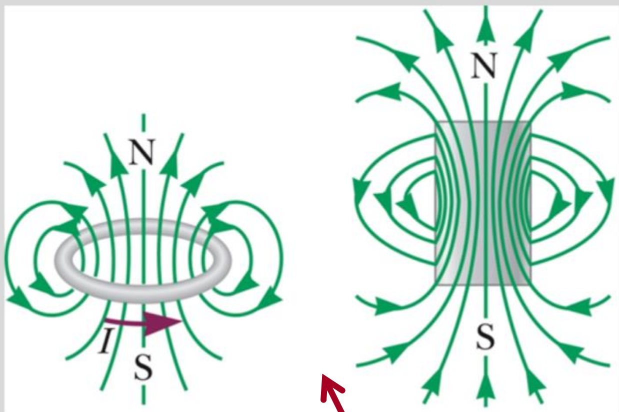
A Long, Straight Conductor

$|d\vec{\mathbf{s}}| = dx$

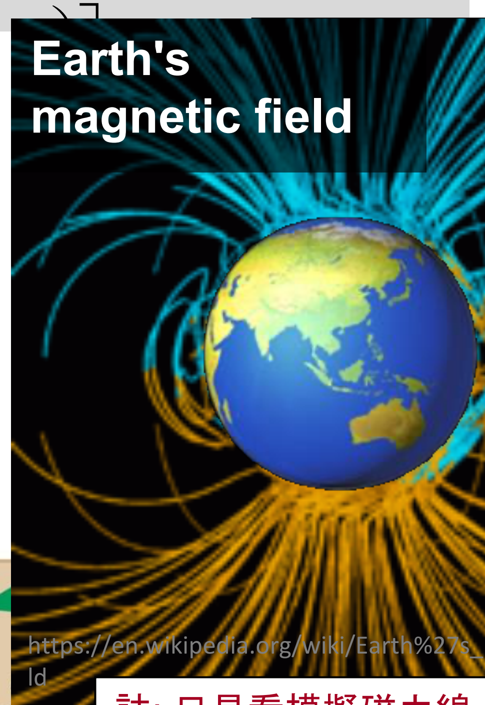


Magnetic Field Lines

Current loop **Bar magnet**



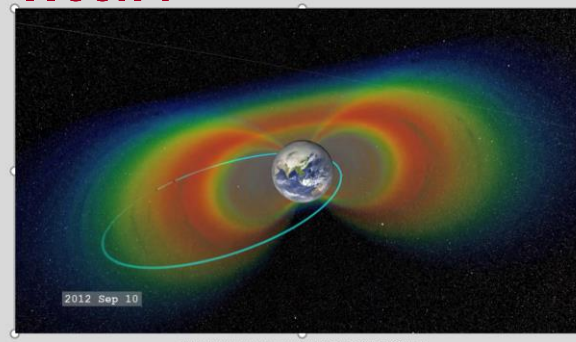
Section 30.1



註: 只是看模擬磁力線,
實際沒左圖這麼簡單

Van Allen Radiation Belts (幅射帶)

Week 7



https://www.youtube.com/watch?v=CKUNT2Qshk4

Week 7 ATOMIC EXCITATION

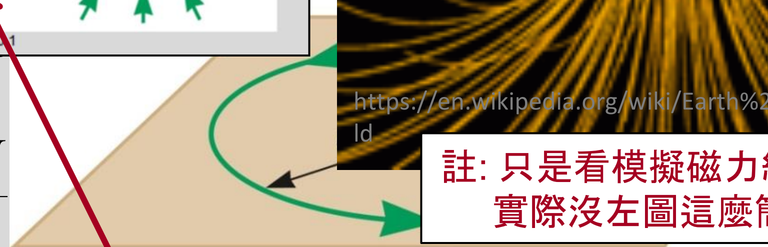
How atoms absorb and emit energy

Auroras are caused by fast-moving protons and electrons from the sun exciting atoms in the upper atmosphere. Green hues come from oxygen, while reds come from both oxygen and nitrogen.

127/121

$$(4)\quad dB = -\frac{\mu_0 I}{4\pi}$$

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \text{co}$$



No beginning and no end
Forms a closed loop!

Same magnitude at a distance of a

Quick Quiz 30.2

closed, turned on, activated

- A loose spiral spring carrying no current is hung from a ceiling. When a switch is thrown so that a current exists in the spring, do the coils

~~(a)~~ move closer together,

(b) move farther apart, or

(c) not move at all?

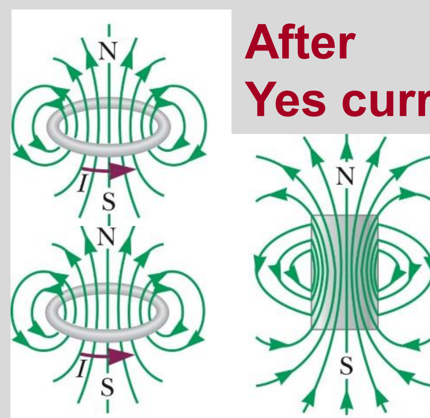
- **Answer: (a)**

https://en.wikipedia.org/wiki/Hooke%27s_law

Before
No current

each coil acts like a magnet

After
Yes current



https://en.wikipedia.org/wiki/Hooke%27s_law

Ampere's Law

permeability of free space

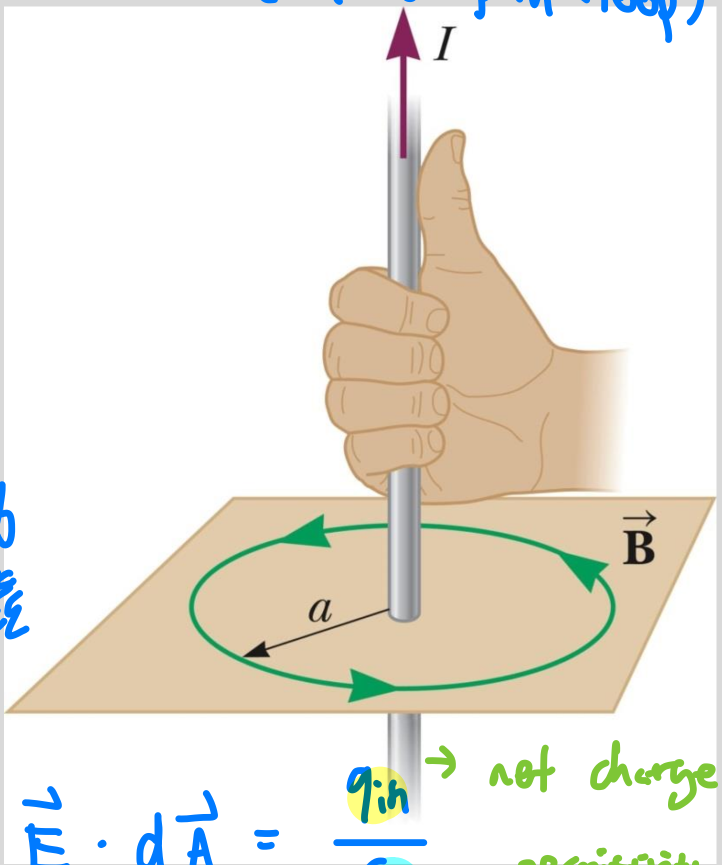
$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 I} \quad (30.13)$$

net current

$d\vec{s}$: a tiny element along the closed path (loop)

- I is the total steady current passing through any surface bounded by **the closed path**
- Useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current

电动生磁



* Similar to using Gauss's Law $\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$ → net charge, → permittivity of free space

in calculating $\vec{E}$ of a symmetric configuration

Aspect	Gauss's Law (Electric Fields)	Ampère's Law (Magnetic Fields)
Mathematical Form	$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0}$	$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{in}}$
Source Quantity	Enclosed **charge** q_{in}	Enclosed **current** I_{in}
Field Type	**Electric field** $\vec{E}$	**Magnetic field** $\vec{B}$
Closed Geometry	**Closed surface** $d\vec{A}$	**Closed loop/path** $d\vec{s}$
Field Behavior	Field **radiates** outward/inward from charge	Field **circulates** around current
Symmetry Used	Spherical, cylindrical, or planar **(radial symmetry)**	Cylindrical or circular **(azimuthal symmetry)**
Usefulness	Finds $\vec{E}$ from symmetric charge distributions	Finds $\vec{B}$ from symmetric current distributions

🧠 Intuition:

- Gauss's Law tells you: **"How much field is coming out of a surface? Depends on the charge inside."**
- Ampère's Law tells you: **"How much field wraps around a loop? Depends on the current passing through it."**

Ampere's Law : A Long Straight Wire

- A wire carrying a steady and uniformly distributed current *I*.
- For $r \geq R$ (outside the wire)

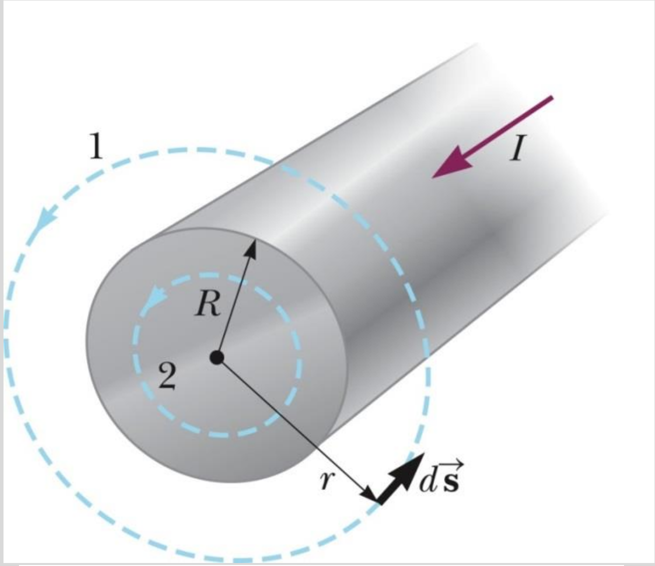
$$\oint \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 I \;\Rightarrow\; B = \frac{\mu_0 I}{2\pi r}$$

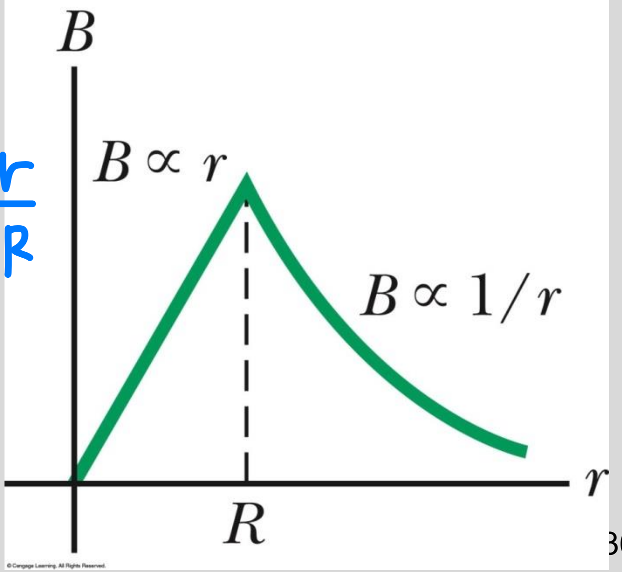
- For $r < R$ (inside the wire)

$$\oint \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 I'$$

$$= \frac{\mu_0 I}{2\pi R} \frac{r}{R}$$

$$I' = I\frac{\pi r^2}{\pi R^2} \;\Rightarrow\; B = \frac{\mu_0 I'}{2\pi r} = \frac{\mu_0 I}{2\pi R^2} r$$





Section 30.3

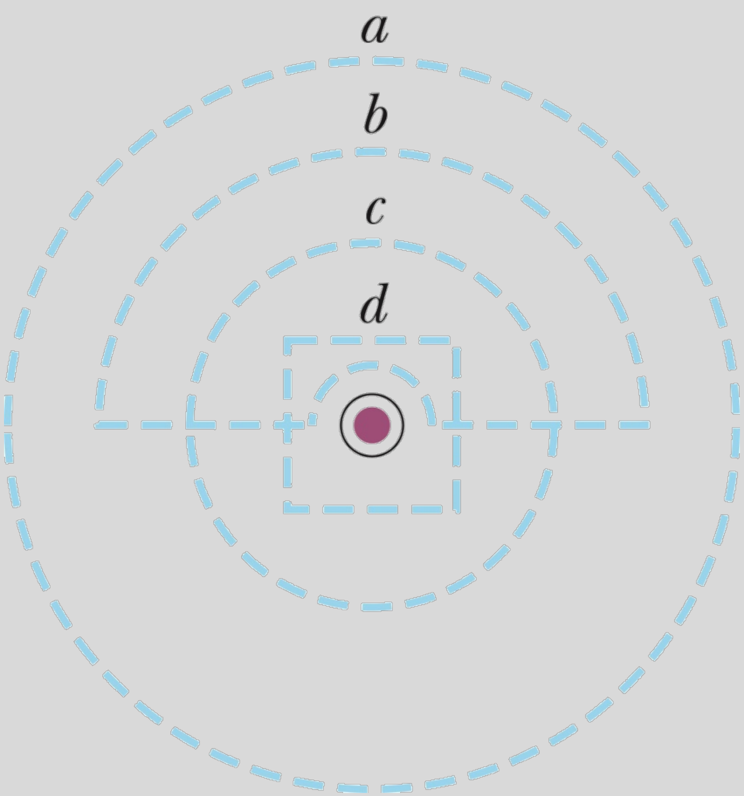
Quick Quiz 30.4

- Rank the magnitudes of $\oint \vec{B} \cdot d\vec{s}$ for the closed paths *a* through *d* in Figure 30.12 from greatest to least.

1. b doesn't include the wire

   $\Rightarrow b = 0$

2. $a = c = d$



- **Answer:** $a = c = d > b = 0$

Solenoid (電磁閥)

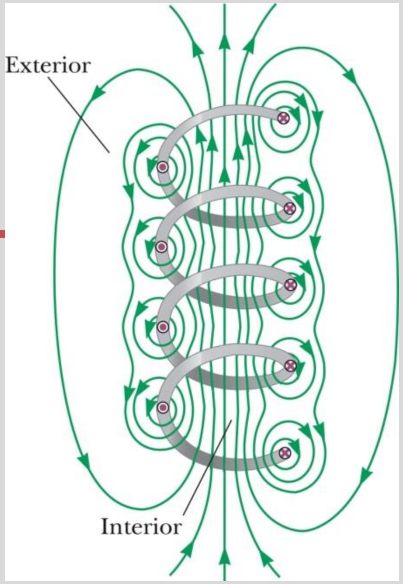
→ Solenoid valve

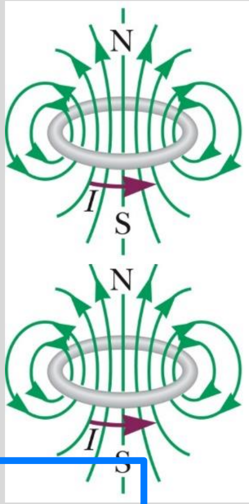
螺線管

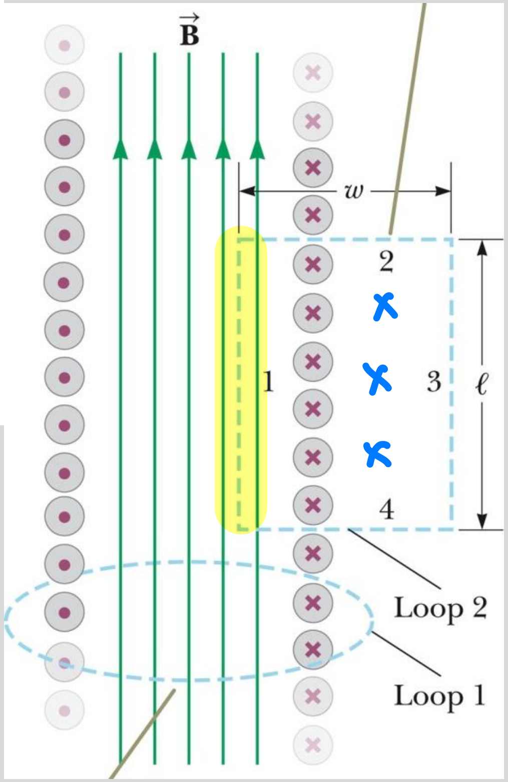
- As the length increases
  - The interior field becomes more uniform
  - The exterior field becomes weaker
- Ideal Solenoid
  - The turns are closely spaced
  - The length >> radius of the turns
- Ampere's Law for loop 2

$$\oint \vec{B}\cdot d\vec{s} = B\ell = \mu_0 NI \quad \Rightarrow \boxed{B = \mu_0 \frac{N}{\ell} I = \mu_0 nI}$$

  - $n = N/\ell$ is the number of turns per unit length







For 2, 4 : $\vec{B}$ and $d\vec{s}$ are perpendicular $\Rightarrow \vec{B}\cdot d\vec{s} = 0$

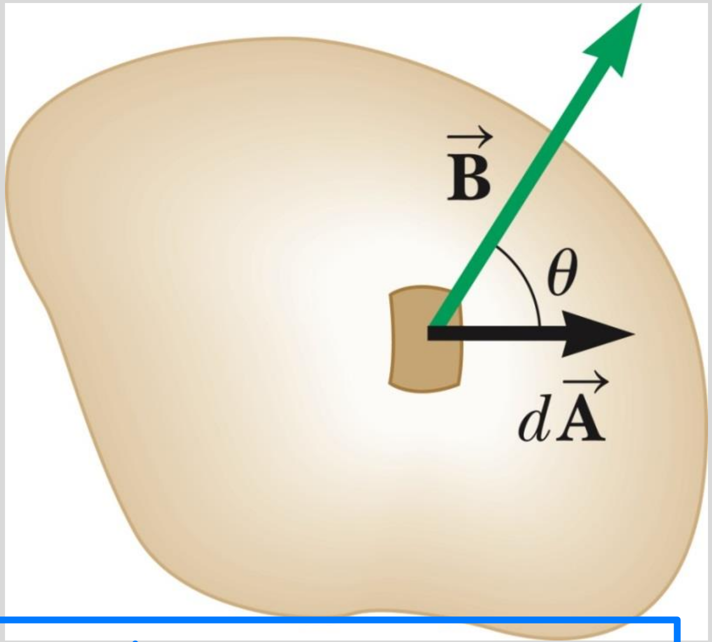
For 3 : B outside the solenoid is very weak

Magnetic Flux (磁通量)

- Consider an area element on an arbitrarily shaped surface
  - The magnetic flux (Φ_B)

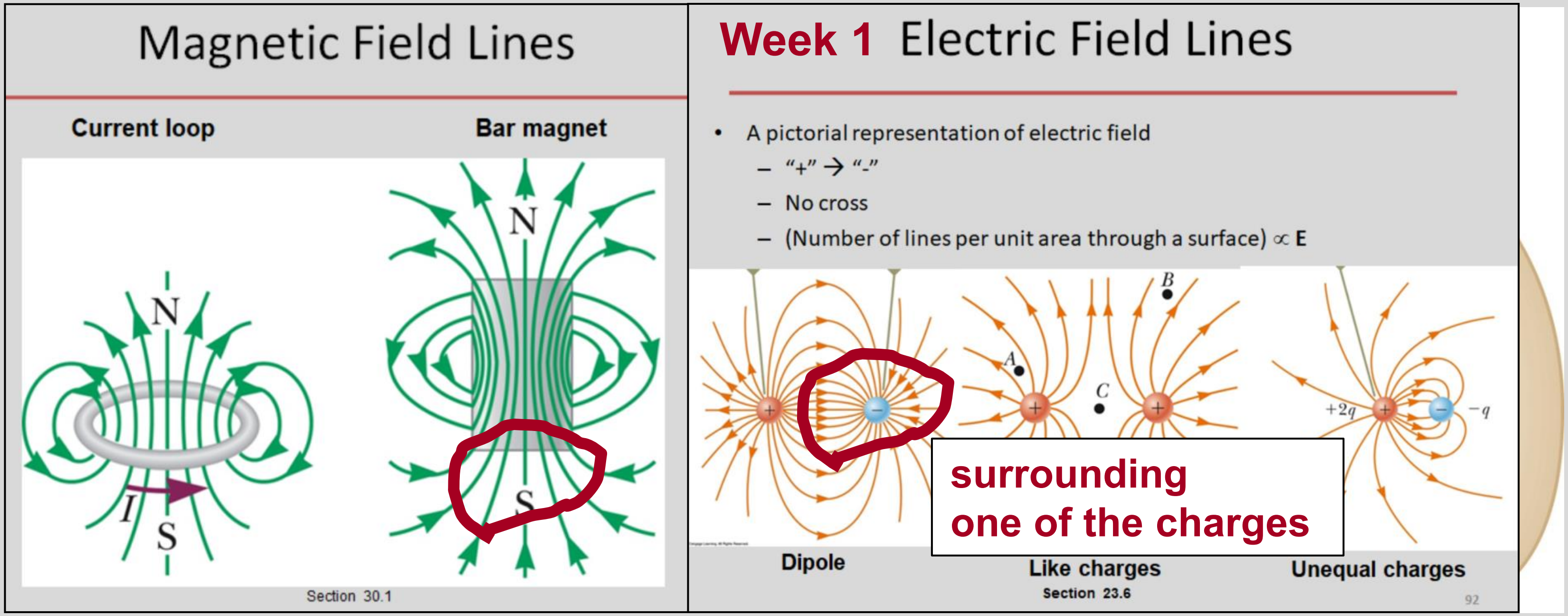
$$\boxed{\boldsymbol{\Phi_B} = \int \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}} \quad \textbf{(30.18)}$$

  - $d\vec{A}$ is perpendicular to the surface
  - Unit : T· m^2 = Wb (*weber*)



- Gauss' Law in Magnetism ↗ different than electric field lines
  - **Magnetic field lines** are continuous and form **closed loops**
  - The magnetic flux through any **closed surface** : $\boldsymbol{\Phi_B} = \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = \mathbf{0}$
  - Note : Isolated magnetic poles (monopoles) have never been detected or perhaps they do not exist 磁單極

Magnetic Flux (磁通量)

- 

- Gauss' Law in Magnetism
  - **Magnetic field lines** are continuous and form **closed loops**
  - The magnetic flux through any **closed surface** : $\boldsymbol{\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0}$
  - Note : Isolated magnetic poles (monopoles) have never been detected or perhaps they do not exist

Section 30.4

🧲 Gauss’s Law in Magnetism

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

- **Interpretation**: The **net magnetic flux through any closed surface is zero**.
- Why? Because **magnetic field lines are closed loops** — every line that enters a surface also exits it.
- 🔁 No magnetic monopoles (i.e., isolated north/south poles) have been found.

⚡ Gauss’s Law in Electrostatics

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0}$$

- **Interpretation**: The **net electric flux through a closed surface is proportional to the enclosed charge**.
- Why? Because **electric field lines start and end on charges** — they can originate from positive charges and terminate on negative ones.
- ➕ Isolated charges (unlike magnetic poles) **do exist**, so the flux can be nonzero.

Example 30.7

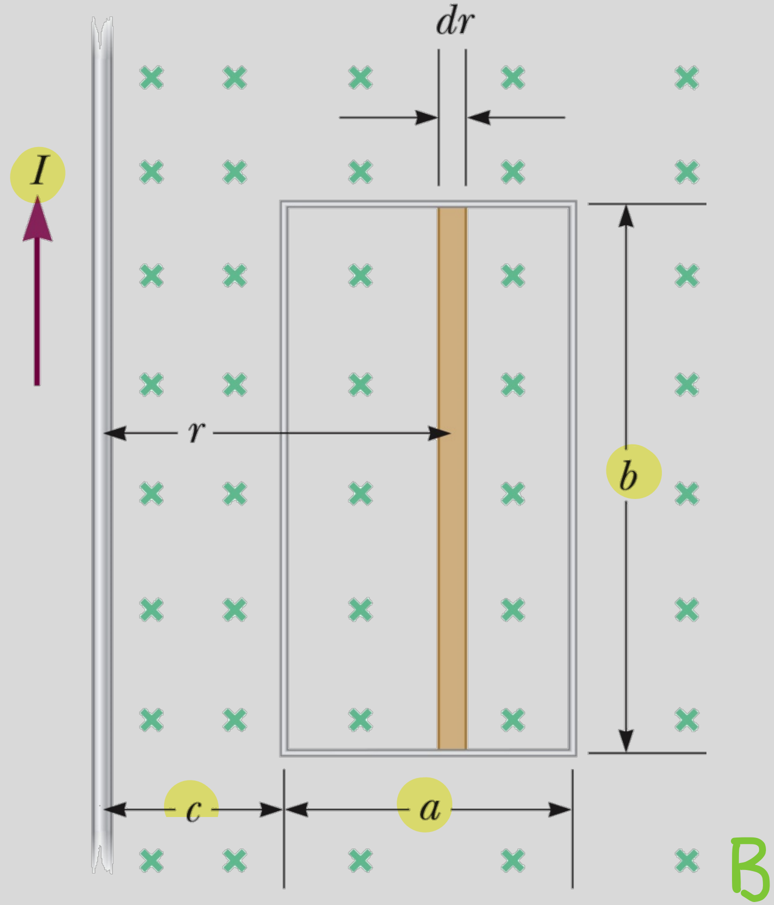
- A rectangular loop of width *a* and length *b* is located near a long wire carrying a current *I*
- The distance between the wire and the closest side of the loop is *c*.
- Find the total magnetic flux through the loop due to the current in the wire.

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int \frac{\mu_0 I}{2\pi r}\,dA$$

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r}\,b\,dr = \frac{\mu_0 I b}{2\pi} \int \frac{dr}{r}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi} \ln r \Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right) = \frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c}\right)$$



1° Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$\Rightarrow B(2\pi r) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

2°

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$\vec{B}$ and $d\vec{A}$ are parallel

$$= \int B \, dA$$

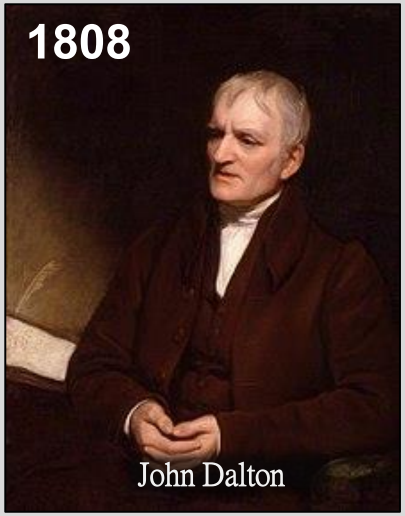
$$= \int_c^{a+c} \left(\frac{\mu_0 I}{2\pi r}\right)(b\,dr)$$

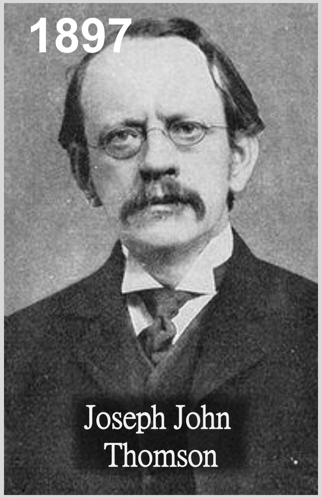
$$= \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r}$$

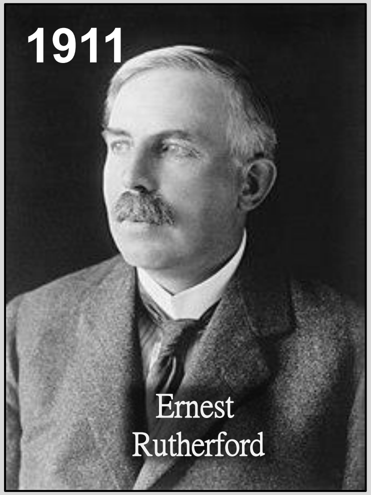
$$= \frac{\mu_0 I b}{2\pi} \Big[\ln |r| \Big]_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi} \ln \left(\frac{a+c}{c} \right) \quad \#$$

Source of Magnetic Effects in Atomic Level





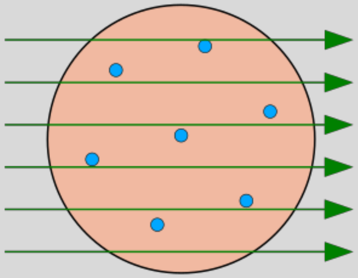


ATOMIC THEORY

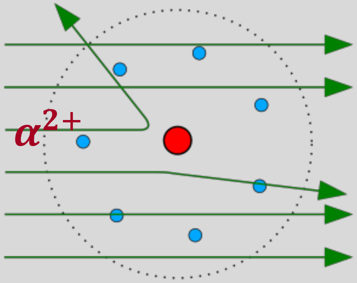
Discovery of the electron, the first subatomic particle to be found.

→ quantum mechanics

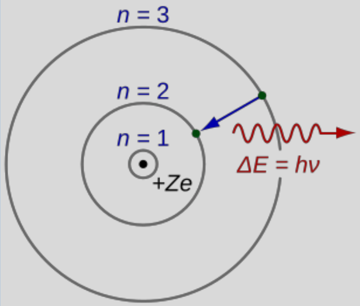
Thomson model



Rutherford model



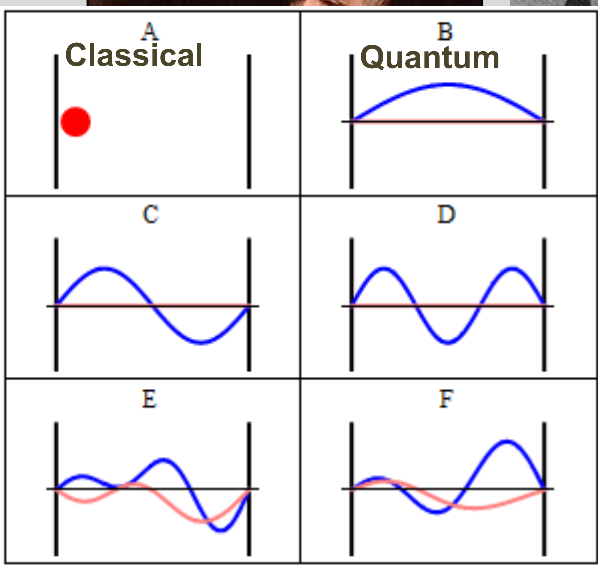
Bohr model



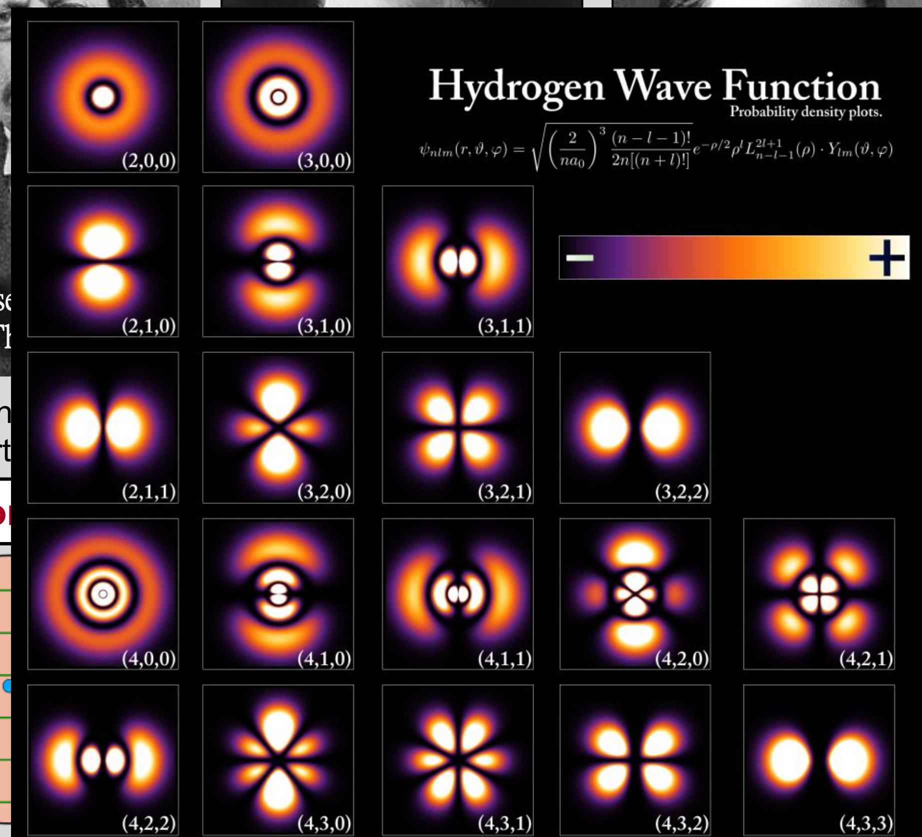
Energy levels (quantized orbits)

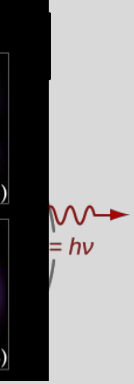
https://en.wikipedia.org/wiki/John_Dalton
https://en.wikipedia.org/wiki/J._J._Thomson
https://en.wikipedia.org/wiki/Ernest_Rutherford
https://en.wikipedia.org/wiki/Niels_Bohr
https://en.wikipedia.org/wiki/History_of_atomic_theory

Source of Magnetic Effects in Atomic Level



https://en.wikipedia.org/wiki/Particle_in_a_box





114-1 Modern Physics

https://en.wikipedia.org/wiki/Atomic_orbital

Energy levels (quantized orbits)

Quantum Mechanics: no definite orbits

· atomic orbital : 軌域

Source of Magnetic Effects in Atomic Level

- The electrons move in **circular orbits** having current

elementary charge $= -1.6\times10^{-19}$ (C)

orbital period $T = \dfrac{2\pi r}{v}$

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

- **The magnetic moment (磁矩)**

$$\mu = IA = \tfrac{1}{2}evr \qquad \textbf{(30.21)}$$

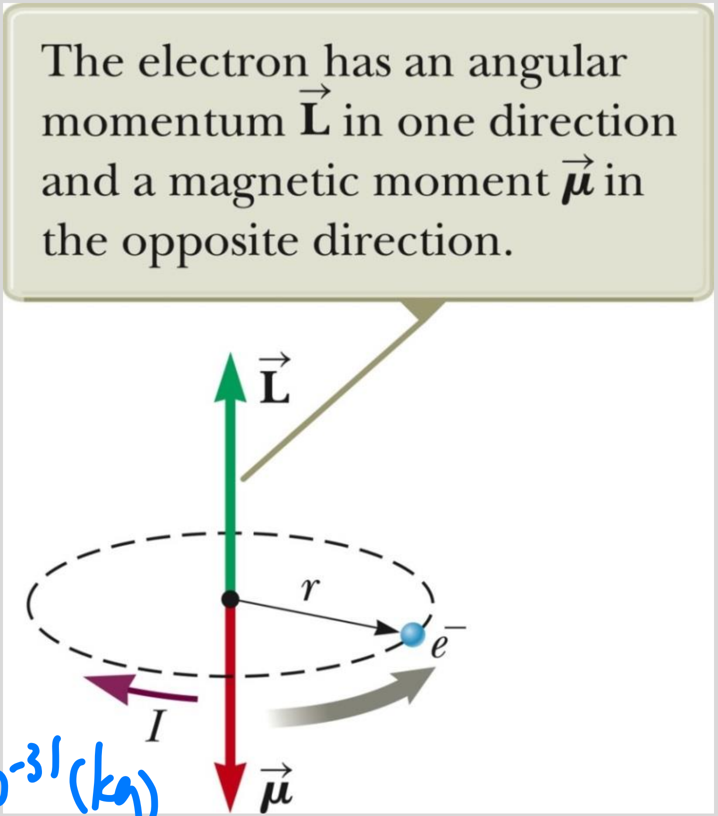
$$\mu = \left(\frac{e}{2m_e}\right)L \qquad \textbf{(30.22)}$$

electron mass $= 9.1\times10^{-31}$ (kg)

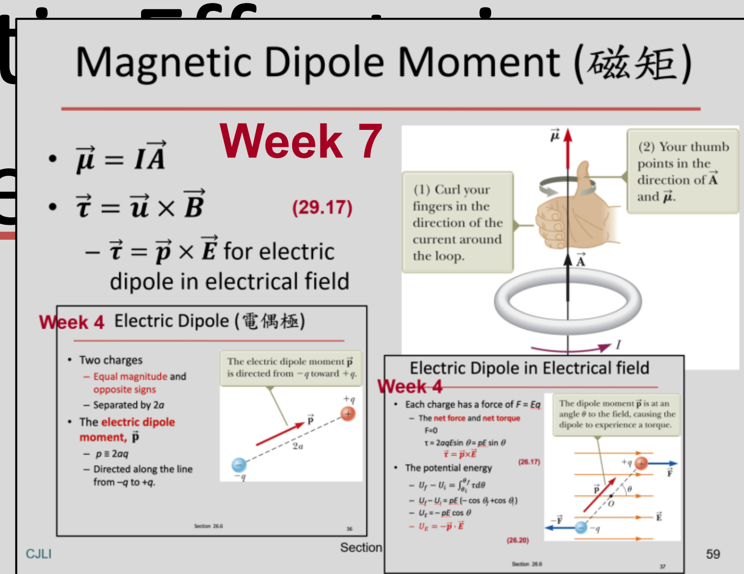
- The angular momentum, L= mvr
  - Quantum physics: L is quantized

quantization of angular velocity

- The vectors $\vec{L}$ and $\vec{u}$ point in *opposite* directions



Source of Magnet… Atomic Le…



- The electrons move in **circular orbits** having current

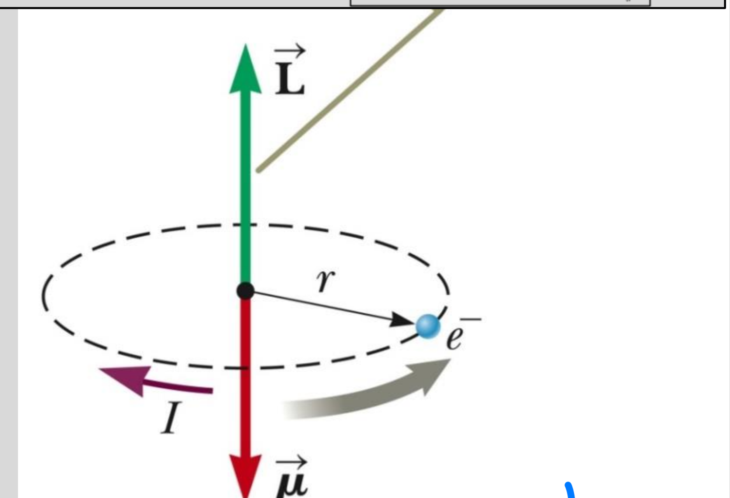
$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

- **The magnetic moment (磁矩)**

$$\mu = IA = \tfrac{1}{2} evr \qquad \textbf{(30.21)}$$

$$\mu = \left(\frac{e}{2m_e}\right) L \qquad \textbf{(30.22)}$$

- The angular momentum, L= mvr
  - Quantum physics: L is quantized
- The vectors $\vec{L}$ and $\vec{u}$ point in *opposite* directions



Bohr model : $L = \frac{nh}{2\pi}$

(L: angular momentum, n: principal quantum number, h: Planck's constant)

- $I = \dfrac{e}{T}$ orbital period $T = \dfrac{2\pi r}{v}$

$= \dfrac{ev}{2\pi r}$

- magnetic moment $\mu = IA = \dfrac{ev}{2\pi r} \cdot \pi r^2$

$= \dfrac{1}{2} evr$ angular momentum

$= \left(\dfrac{e}{2m_e}\right) L$ $L = m_e v r$

🔍 Comparison Table

Quantity	Linear Momentum ($\vec{p}$)	Angular Momentum ($\vec{L}$)
Formula	$\vec{p} = m\vec{v}$	$\vec{L} = \vec{r} \times m\vec{v}$ or $L = mvr$
Direction	Along the direction of motion	Perpendicular to the plane of rotation
Conserved?	Yes (if no external force)	Yes (if no external torque)
Unit	kg·m/s	kg·m²/s
Describes	Straight-line motion	Rotational motion

✨ Intuition:

- **Linear momentum** tells you how “hard” something is moving straight.
- **Angular momentum** tells you how “hard” something is spinning around a point.

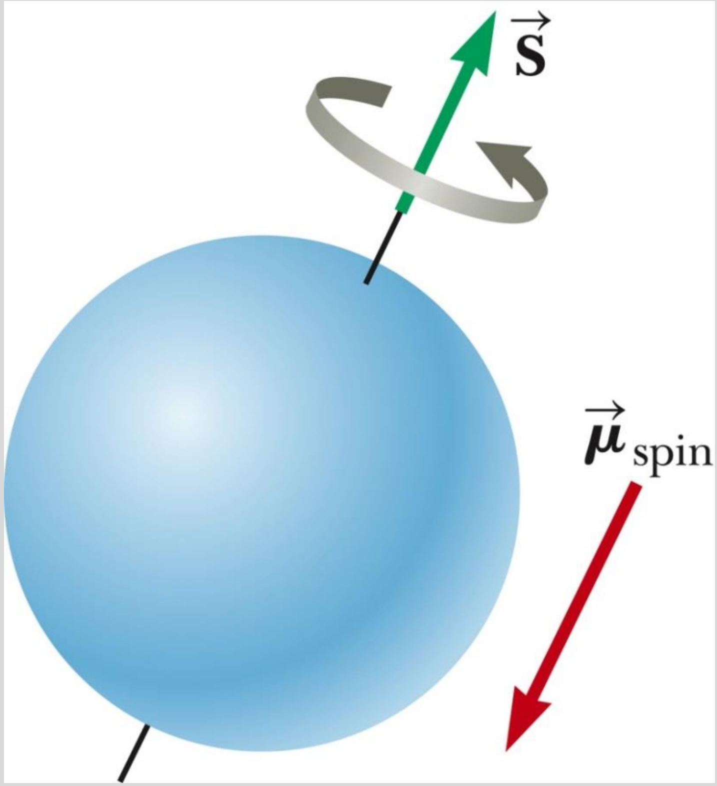
Spin of electron or other particle
电子自旋

- Electrons (and other particles) have an intrinsic property called **spin**
  - The electron is not physically spinning
  - It has an **intrinsic angular momentum** as if it were spinning
- The angular momentum

$$\text{spin} \quad S = \frac{\sqrt{3}}{2}\hbar$$

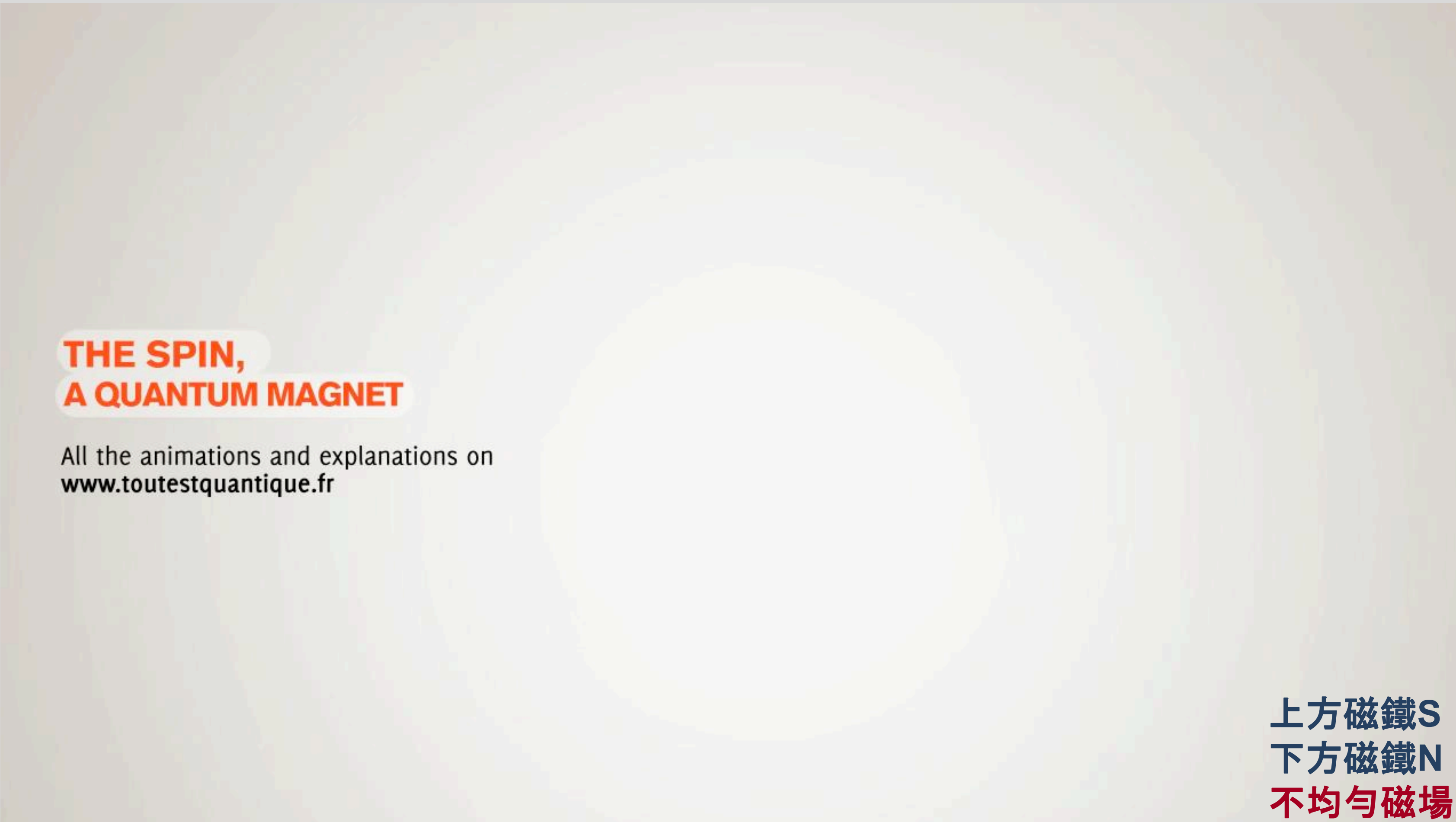
  - $\hbar$ is Planck's constant
- **The magnetic moment**
  - $$\mu_{\text{spin}} = \frac{e\hbar}{2m_e} \qquad \textbf{(30.24)}$$
  - This combination of constants is called the **Bohr magneton** μ_B = 9.27 x 10^{-24} J/T

玻尔磁子



$\hbar$: Planck's Constant

1922 The stern-gerlach experiment



上方磁鐵S
下方磁鐵N
不均勻磁場

Quantized nature of particle spin

https://en.wikipedia.org/wiki/Stern%E2%80%93Gerlach_experiment
https://toutestquantique.fr/en/credits/

Quantum entanglement

114-1 Modern Physics

Electron Magnetic Moment

↗ total magnetic moment

- The **total *u*** is the **vector sum** of the **orbital** and **spin** magnetic moments
  - ***u*** of a proton or neutron is usually neglected (<< ***u*** of an electron)
    - Proton : 14.11 x 10^{-27} J/T
    - Neutron: 9.66 x 10^{-27} J/T

$$\mu_{total} = \mu_{orbital} + \mu_{spin}$$

- In most substances,
  - Electrons **orbiting in the opposite** direction cancel each other ($u_{orbital}$ close to 0)
  - Pair up electrons with **spins opposite** cancel each other unless there is **odd number of electrons** (u_{spin} pair up)

Table 30.1 Magnetic Moments of Some Atoms and Ions

Atom or Ion	Magnetic Moment (10^{-24} J/T)
H	9.27
He	0
Ne	0
Ce^{3+}	19.8
Yb^{3+}	37.1

Substance under magnetic field

- Diamagnetic (反磁性) substances
  - A weak magnetic moment is induced opposite to the external magnetic field
  - Weakly repelled by a magnet
- Paramagnetic (順磁性) substances
  - Small but positive magnetism from atoms tend to line up with the external magnetic field
  - Thermal motion tend to randomize the orientation
- Ferromagnetism (鐵磁性)
  - Strong magnetic effects such as Iron (Fe, 鐵)、Cobalt (Co, 鈷)、Nickel (Ni, 鎳)
  - Materials are made up of microscopic regions called domains 磁疇ㄔㄡˊ磁域
  - Within domain, all magnetic moments are aligned parallel to each other even in weak external magnetic field
  - The boundaries are called domain walls

🧲 1. Diamagnetism (反磁性)

冷次定律

- **Cause**: Arises from **induced currents** that oppose changes in the magnetic field (Lenz's Law).
- Even atoms **with no unpaired electrons** show this effect.
- Electrons slightly adjust their orbits, generating a **magnetic moment opposite** to the applied field.

🔷 **Related to electron orbital motion**, but **not spin**.

🧲 2. Paramagnetism (順磁性)

- **Cause**: Atoms or ions with **unpaired electrons** (i.e., non-zero electron magnetic moments).
- These unpaired electron spins tend to **align with an external magnetic field**, creating a **small net magnetization**.
- However, **thermal motion** tends to randomize the spin directions when the field is removed.

🔷 **Directly caused by unpaired electron spin magnetic moments**.

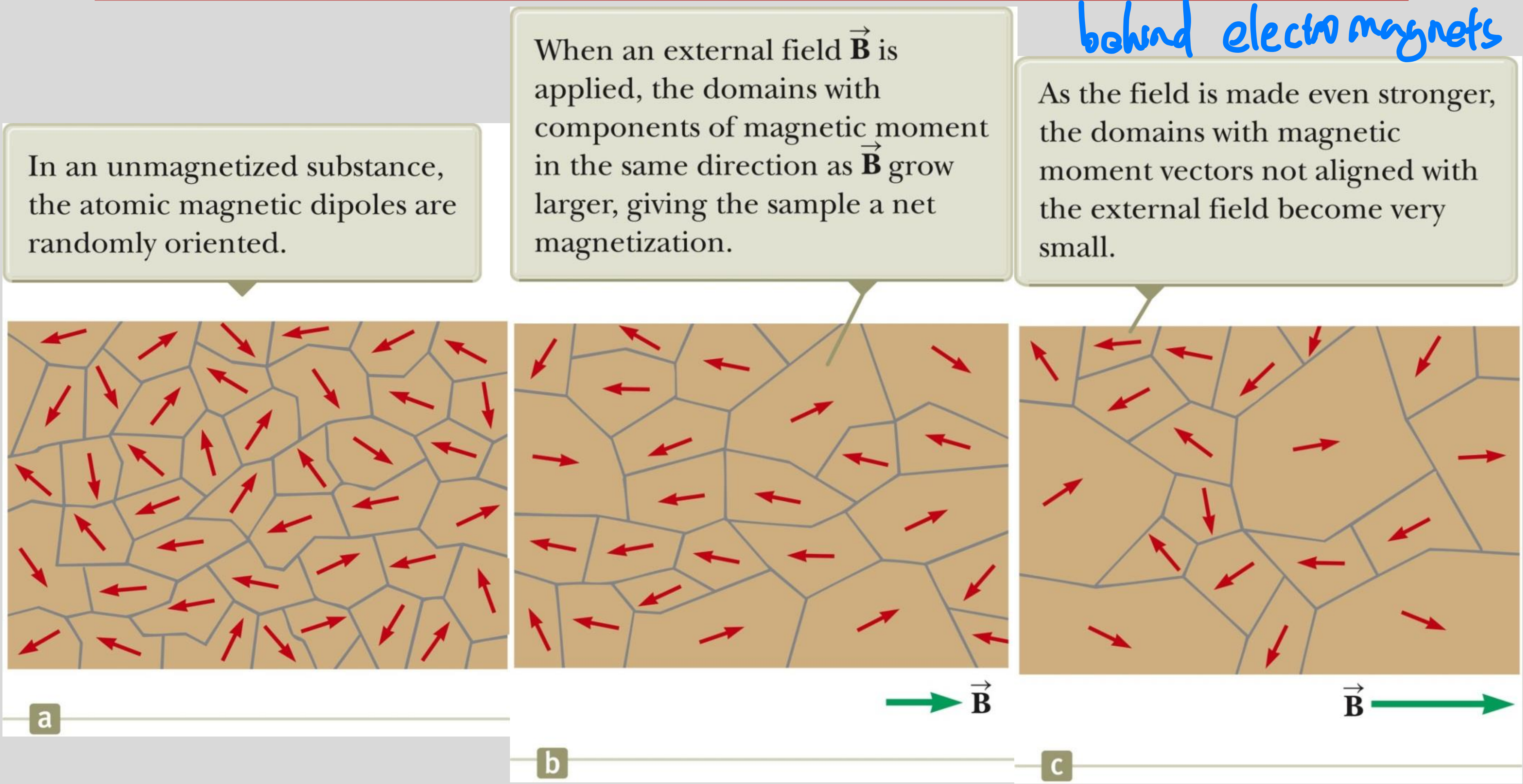
🧲 3. Ferromagnetism (鐵磁性)

- **Cause**: Materials like iron, cobalt, and nickel contain **many unpaired electrons**, whose spins **spontaneously align** in regions called **magnetic domains**.
- Within each domain, the **electron magnetic moments (from spin)** are **parallel**, reinforcing each other.
- The material can retain magnetization even after the external field is removed — this is **permanent magnetism**.

🔷 **Strongly dependent on aligned spin magnetic moments of electrons**.

Domains with external field

the core principle behind electromagnets



When the external field is removed, the material may retain a net magnetization in the direction of the original field (memory)

↘ magnetic hysteresis, used in making permanent magnets

Curie Temperature

- The critical temperature above which a ferromagnetic material loses its residual magnetism
  - The material will become paramagnetic
- Above the Curie temperature, the thermal agitation is great enough to cause a random orientation of the moments

Table 30.2 Curie Temperatures for Several Ferromagnetic Substances

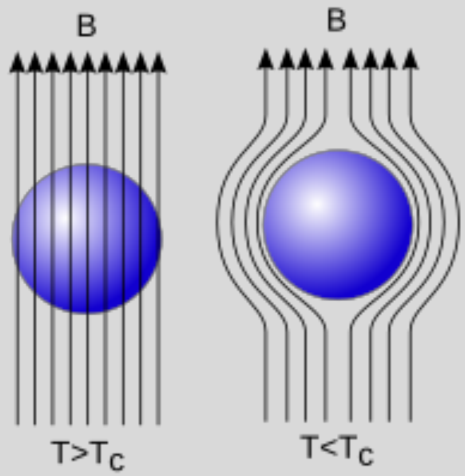
Substance	T_{Curie} (K)
Iron	1 043
Cobalt	1 394
Nickel	631
Gadolinium	317
Fe_2O_3	893

Section 30.6

Superconductor

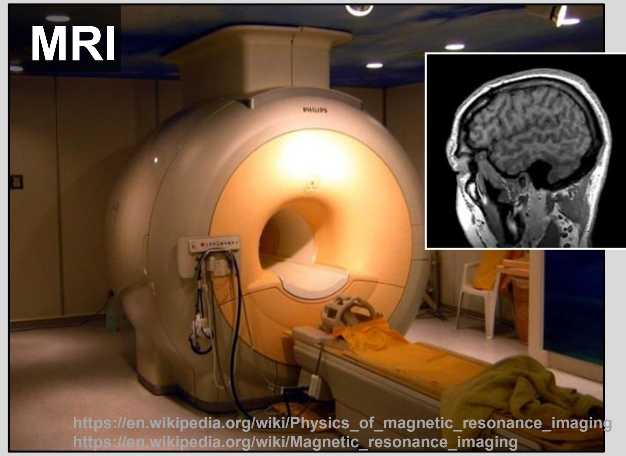
Week 5

Meissner effect



Diamagnetism in the superconducting state

https://en.wikipedia.org/wiki/Meissner_effect



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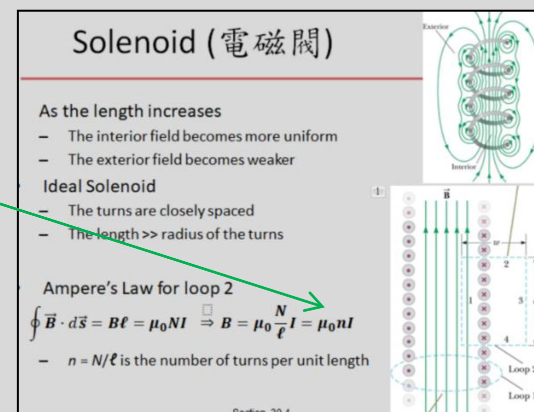
The resistance of the superconducting wires in the MRI's solenoid is ~zero, a very high current is possible.

$\Rightarrow$ **Strong B**



Solenoid (電磁閥)

As the length increases
- The interior field becomes more uniform
- The exterior field becomes weaker

Ideal Solenoid
- The turns are closely spaced
- The length >> radius of the turns

Ampere's Law for loop 2

$$\oint \vec{B}\cdot d\vec{s} = B\ell = \mu_0 NI \Rightarrow B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$

- $n = N/\ell$ is the number of turns per unit length

Section 30.4

Summary

- Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{S} \times \hat{r}}{r^2}$
- Ampere's Law $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$

电动
生磁

- Magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$
- Gauss's law of magnetism $\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$

This Lecture

- ○ Some History
- Biot-Savart Law
- Ampere's Law
- Gauss's Law in Magnetism
  - – Magnetic Flux
- Magnetism in Matter
  - – Atom
  - – Substance

Similar to the use of Gauss's law in calculating $\vec{E}$ of a symmetric configuration