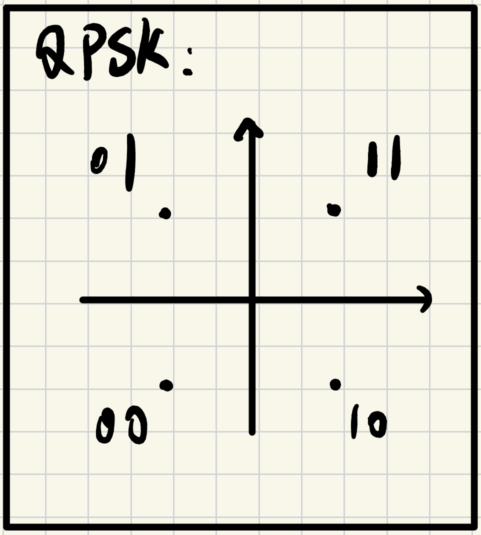
1. (a) $11\ 10\ 10\ 01$

$$\begin{cases} \text{subchannel 1: } 11 \\ \text{subchannel 2: } 10 \\ \text{subchannel 3: } 10 \\ \text{subchannel 4: } 01 \end{cases}$$

#



(b)

$$a_0 = 1 + i$$

$$a_1 = 1 - i$$

$$a_2 = 1 - i$$

$$a_3 = -1 + i$$

#

(c)

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \begin{pmatrix} 1+i \\ 1-i \\ 1-i \\ -1+i \end{pmatrix} = \begin{pmatrix} 2 \\ 2+4i \\ 2 \\ -2 \end{pmatrix}$$

#

①

$$1+i + i(1-i) - (1-i) - i(-1+i)$$

$$= 1+i+i+1-1+i+i+1$$

$$= 2+4i$$

② $1+i-(1-i)+(1-i)-(-1+i)$
$= 2$

③ $1+i-i(1-i)-(1-i)+i(-1+i)$
$= 1+i-i-1-1+i-i-1$
$= -2$

(d) $S_{Re} = [2, 2, 2, -2]$
$h = [1, 0.5, 0.25, 0]$

$S_{Re} * h =$

2	→	2	1	0.5	0			
2	→		2	1	0.5	0		
2	→			2	1	0.5	0	
+) −2	→				−2	−1	−0.5	0
		2	3	3.5	−0.5	−0.5	−0.5	0

$\Rightarrow [2, 3, 3.5, -0.5, -0.5, -0.5, 0]$ #

(e) $S_{Re} = [2, 2, 2, -2]$

$h = [1, 0.5, 0.25, 0]$

$S_{Re} \circledast h =$

2	$\rightarrow$	2	1	0.5	0
2	$\rightarrow$	0	2	1	0.5
2	$\rightarrow$	0.5	0	2	1
+) −2	$\rightarrow$	−1	−0.5	0	−2
		1.5	2.5	3.5	−0.5

$\Rightarrow [1.5, 2.5, 3.5, -0.5]$ #

(f)

$$\boxed{\begin{array}{l}\text{DFT:} \\ a_j = \dfrac{s \cdot b_j}{n}\end{array}}$$

① DFT(S_{Re})

$$a_0 = \frac{(2, 2, 2, -2) \cdot (1, 1, 1, 1)}{4} = 1$$

$$a_1 = \frac{(2, 2, 2, -2) \cdot (1, -i, -1, i)}{4} = -i$$

$$a_2 = \frac{(2,2,2,-2)\cdot(1,-1,1,-1)}{4} = 1$$

$$a_3 = \frac{(2,2,2,-2)\cdot(1,i,-1,-i)}{4} = i$$

$$\mathrm{DFT}(S_{Re}) = (1, -i, 1, i)$$

② DFT(h)

$$a_0 = \frac{(1, 0.5, 0.25, 0)\cdot(1,1,1,1)}{4}$$

$$= 0.4375$$

$$a_1 = \frac{(1, 0.5, 0.25, 0)\cdot(1,-i,-1,i)}{4}$$

$$= 0.1875 - 0.125i$$

$$a_2 = \frac{(1, 0.5, 0.25, 0)\cdot(1,-1,1,-1)}{4}$$

$$= 0.1875$$

$$a_3 = \frac{(1, 0.5, 0.25, 0) \cdot (1, i, -1, -i)}{4}$$

$$= 0.1875 + 0.125i$$

$$DFT(h) = (0.4375, 0.1875 - 0.125i, 0.1875, 0.1875 + 0.125i)$$

③ $DFT(S_{Re} \circledast h)$

$$a_0 = \frac{(1.5, 2.5, 3.5, -0.5) \cdot (1, 1, 1, 1)}{4}$$

$$= 1.75$$

$$a_1 = \frac{(1.5, 2.5, 3.5, -0.5) \cdot (1, -i, -1, i)}{4}$$

$$= -0.5 - 0.75i$$

$$a_2 = \frac{(1.5, 2.5, 3.5, -0.5) \cdot (1, -1, 1, -1)}{4}$$

$$= 0.75$$

$$a_3 = \frac{(1.5, 2.5, 3.5, -0.5) \cdot (1, i, -1, -i)}{4}$$

$$= -0.5 + 0.75i$$

$$DFT(S_{Re} \circledast h) = (1.75, -0.5-0.75i, 0.75, -0.5+0.75i)$$

$$DFT(S_{Re}) = (1, -i, 1, i)$$

$$DFT(h) = (0.4375, 0.1875-0.125i, 0.1875, 0.1875+0.125i)$$

$$DFT(S_{Re} \circledast h) = (1.75, -0.5-0.75i, 0.75, -0.5+0.75i)$$

They follow

$$DFT(S_{Re})[j] = \frac{DFT(S \circledast Re)[j]}{N \cdot DFT(h)[j]}$$

(g) ① $h = [1, 0.5, 0.25]$

$\Rightarrow$ delay spread $= 2$ #

② $CP = (2, -2)$ #

2. ① The frequency of the resulting wave might not match the channel frequency since $f \ll f_c$.

② The real part and imaginary parts of the result of IFFT will interfere, and we can't discern between the two.

③ Bandwidth might be too small, and therefore results in inefficient use of the spectrum considering the guard time

⋮