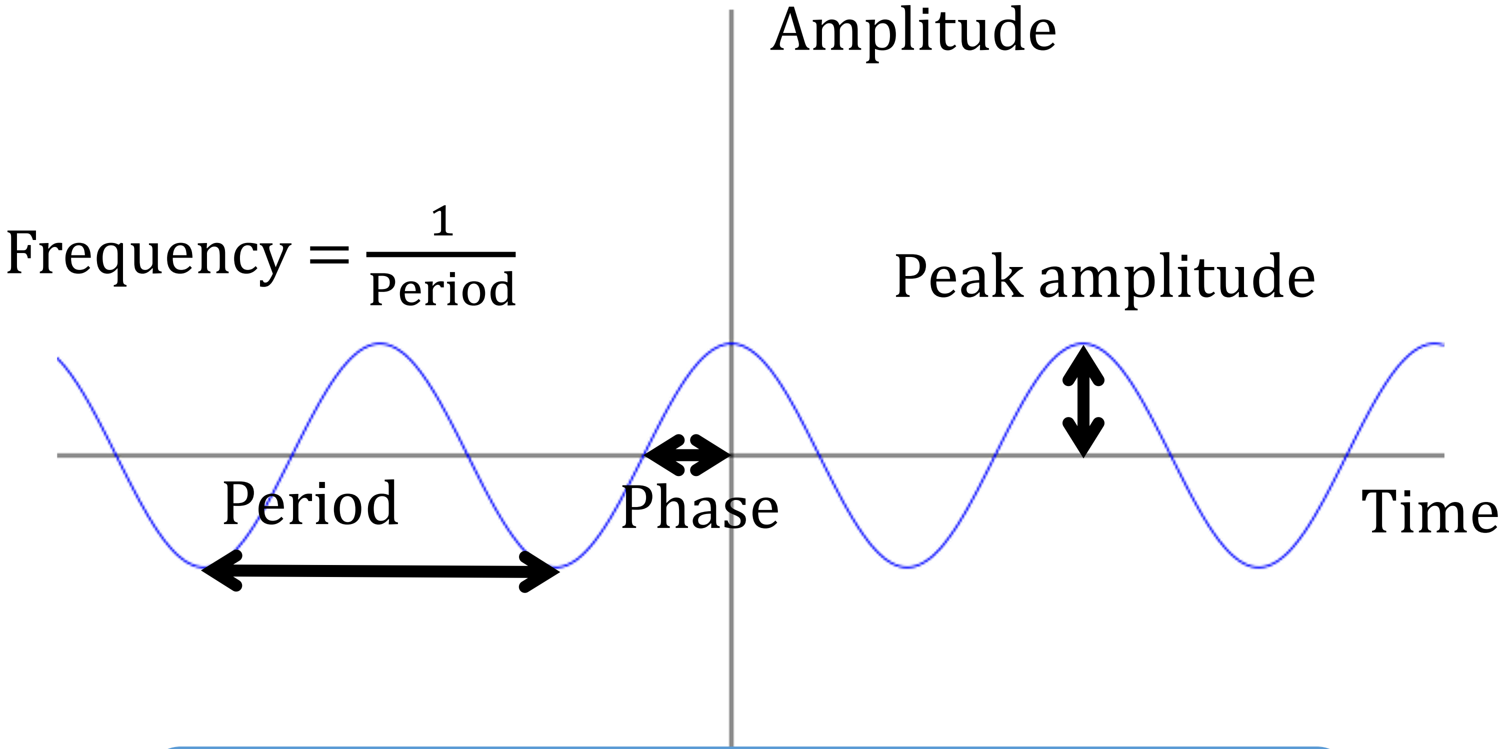
Physical Layer

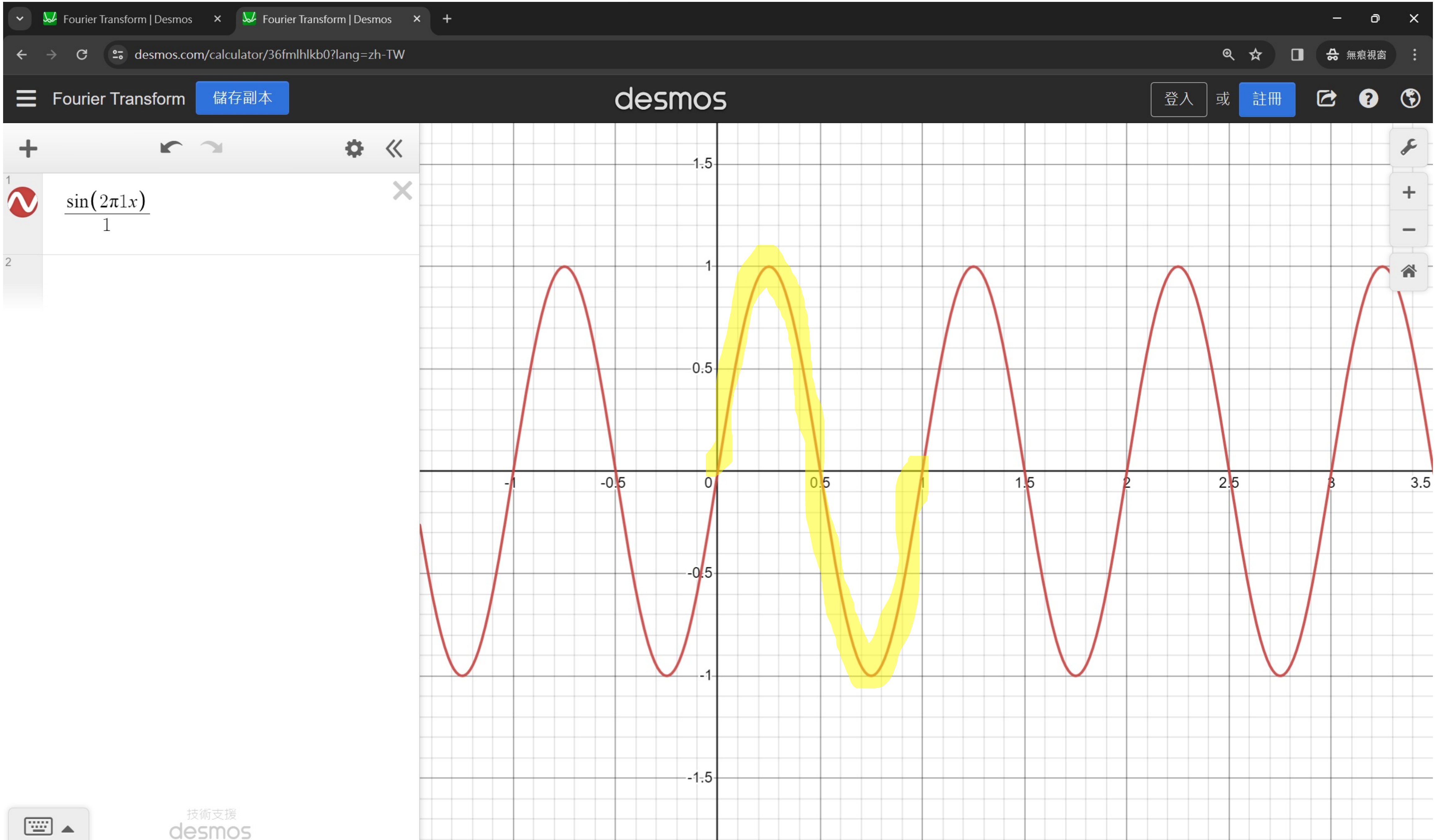
Sine Wave



A sine wave is defined by its peak amplitude, phase, and frequency

An Example

- $x(t) = \mathrm{A}\cos(t)$
- What is the amplitude and frequency?
- Amplitude: A
- Frequency: $\frac{1}{2\pi}$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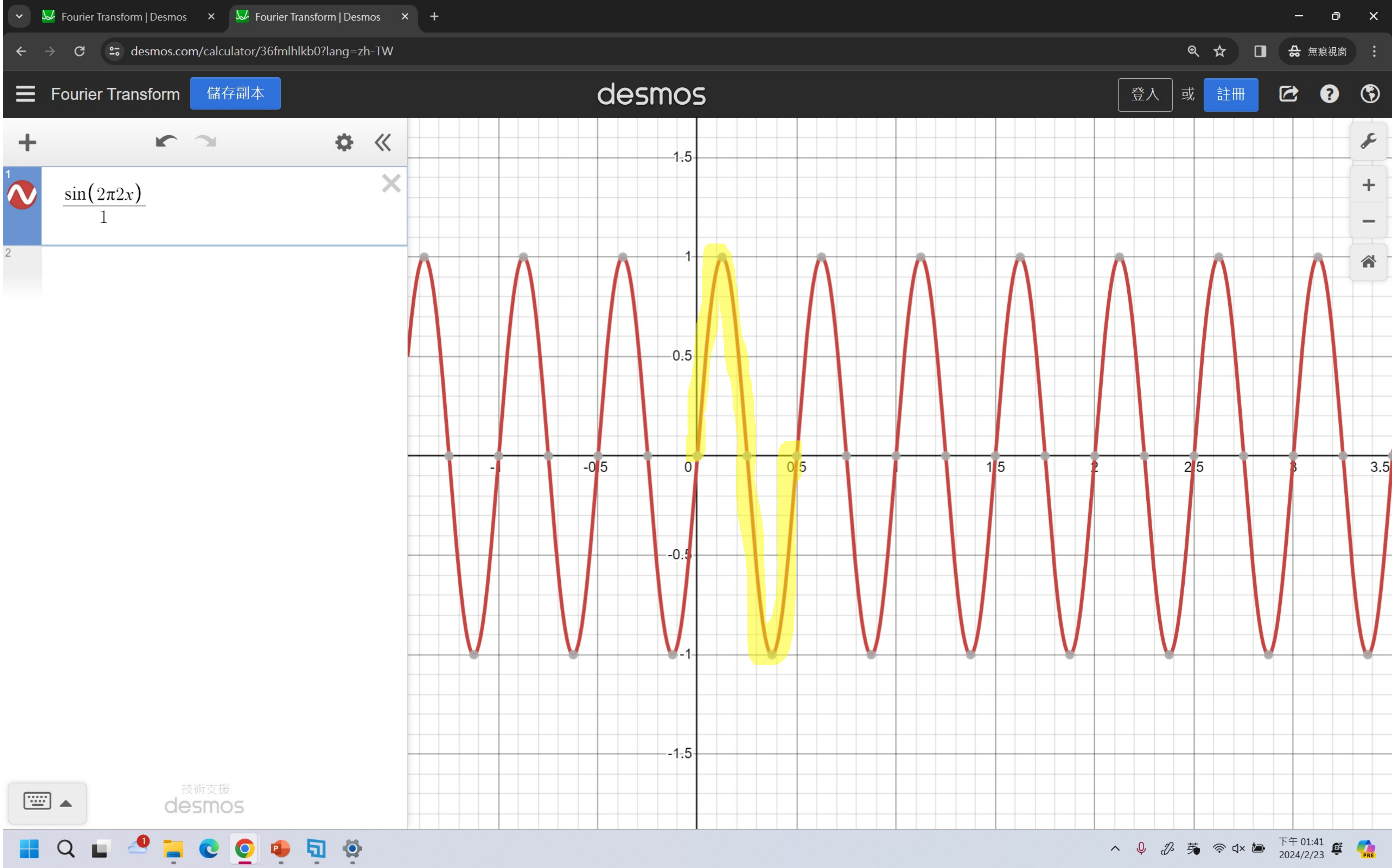
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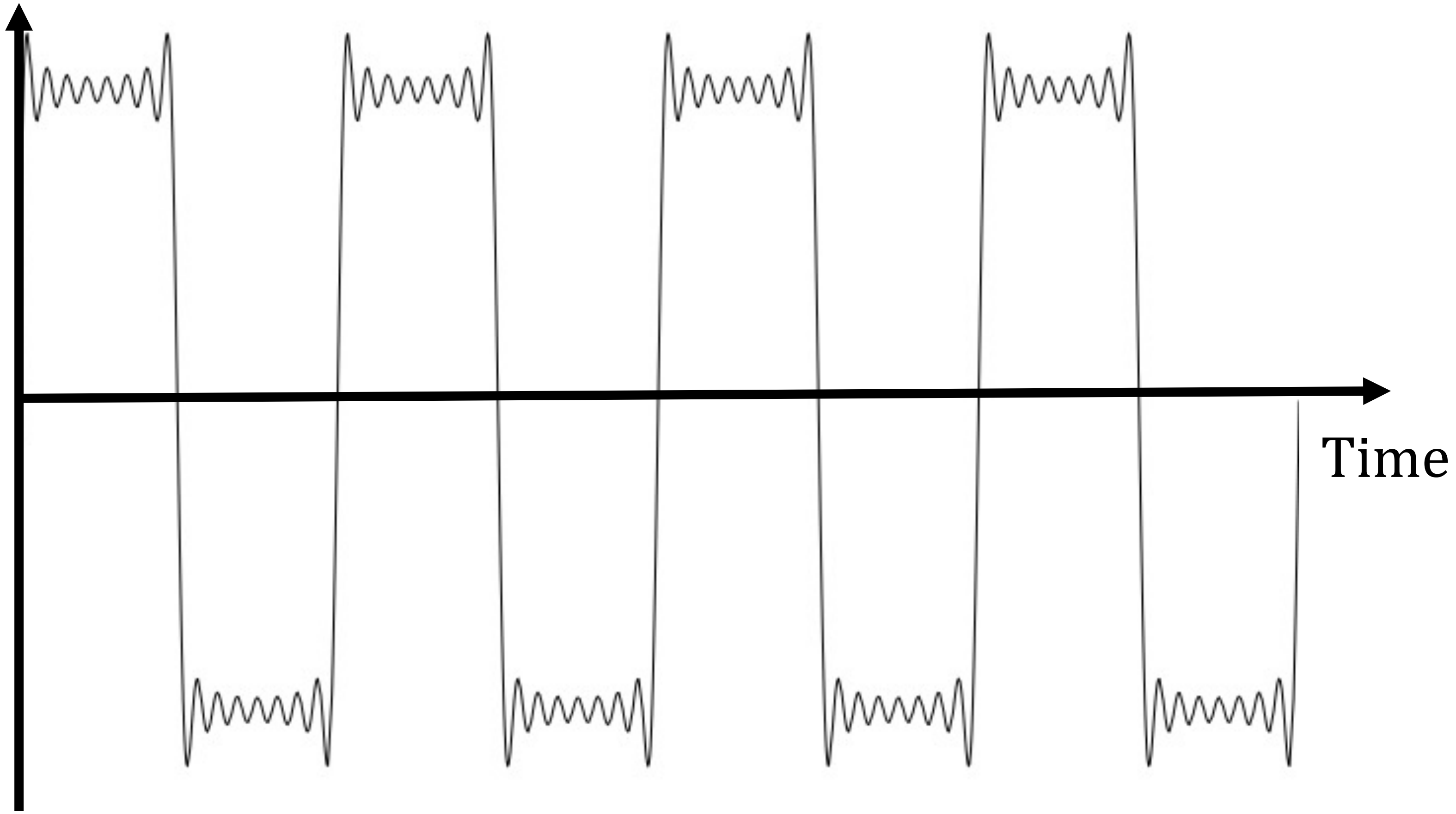
General Form of Sine Wave

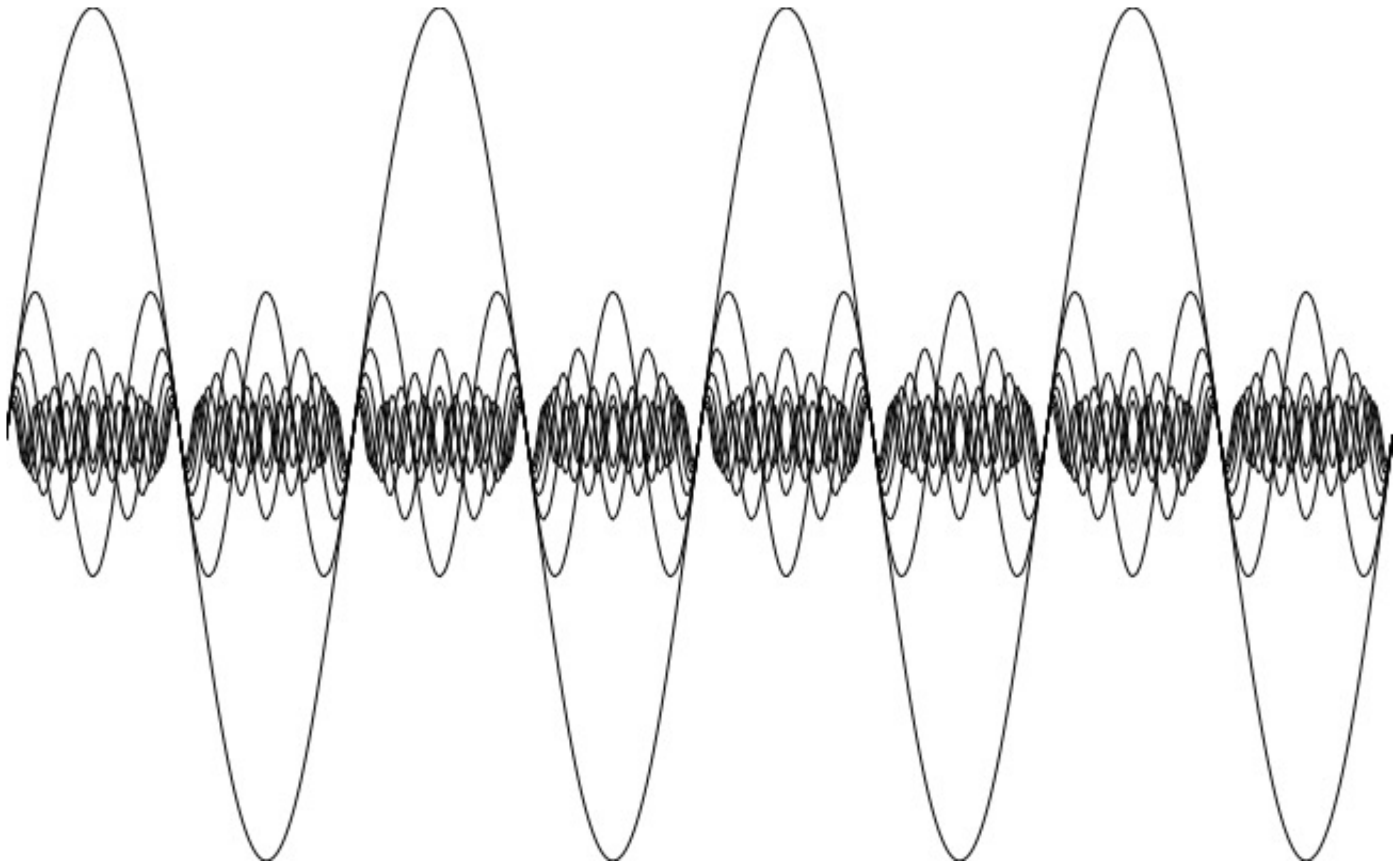
- How to express a sine wave $x(t)$ whose
  - Peak amplitude = A
  - Frequency = f
- $x(t) = A\cos(2\pi f t + \theta)$
  - Phase = θ

Composite Signal



Time and Frequency Domain

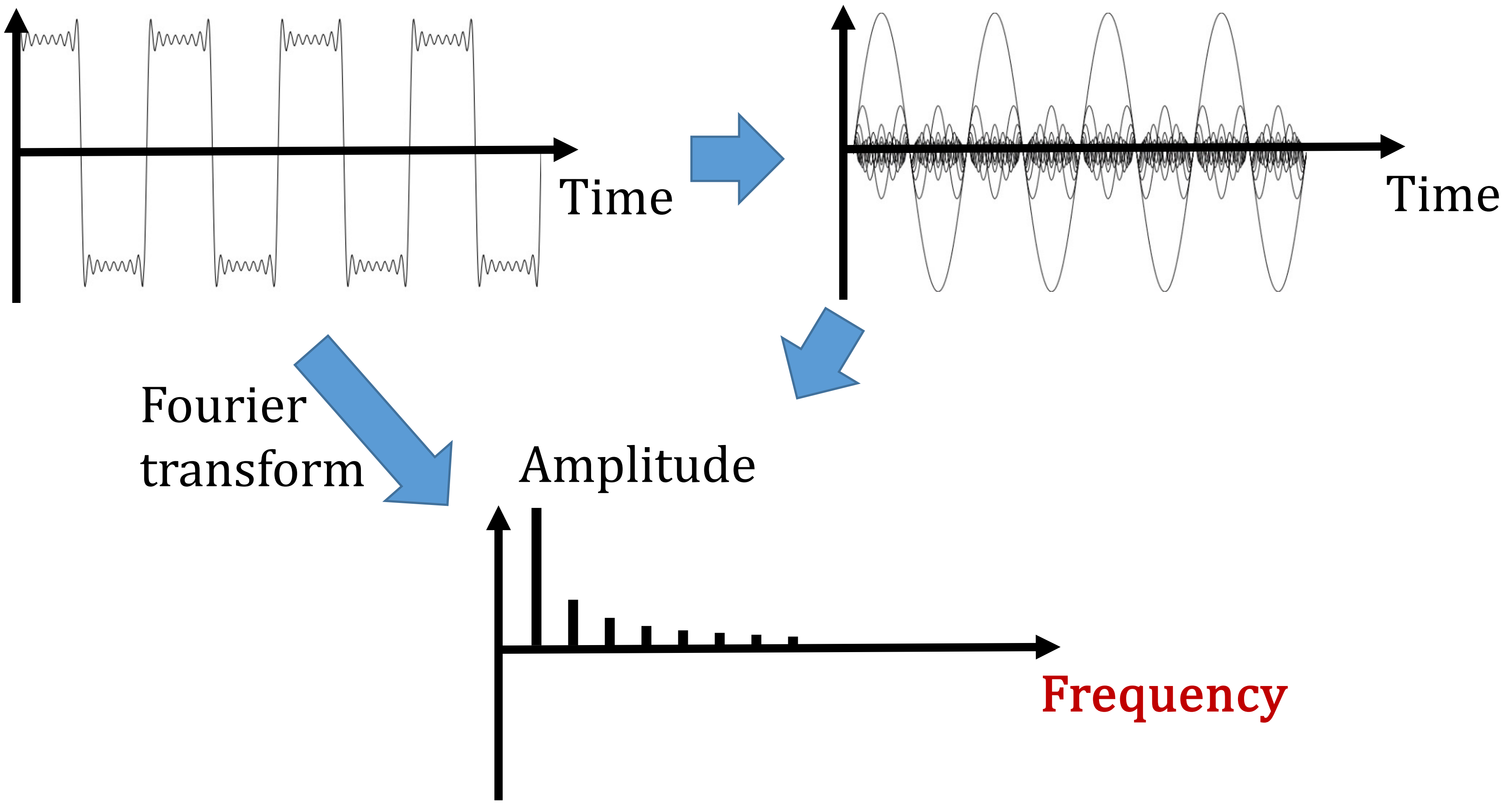
- The above signal can be obtained by combining 8 sine waves



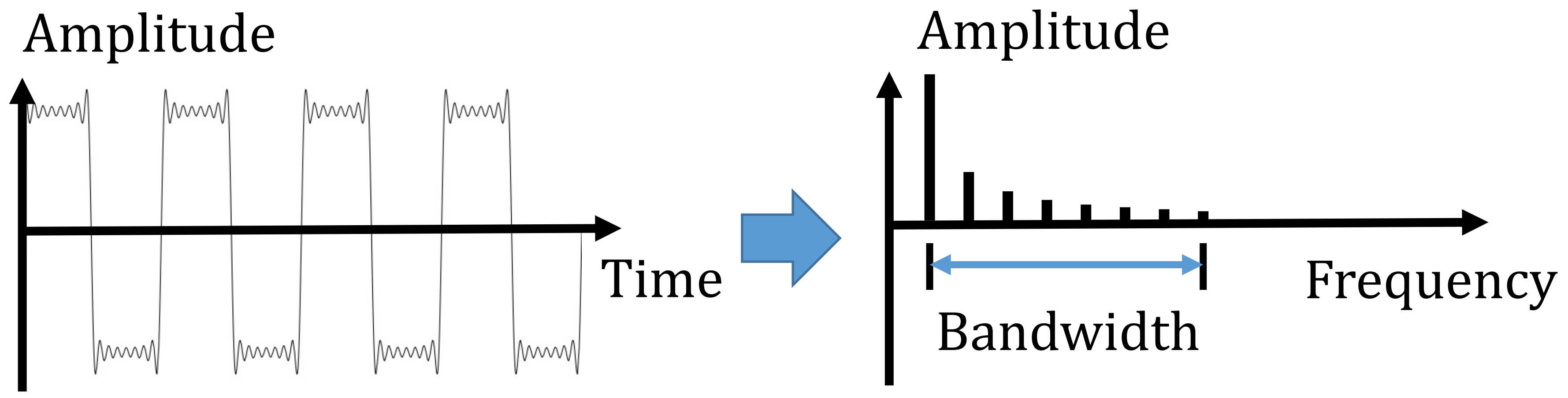
Time and Frequency Domain

- According to Fourier analysis, any wave can be decomposed into sine waves
  - $A_1 \cos(2\pi f_1 t + \theta_1) + A_2 \cos(2\pi f_2 t + \theta_2) + A_3 \cos(2\pi f_3 t + \theta_3) + \cdots$
- f_1, f_2, …, are called the **frequency components** of the original wave

Time and Frequency Domain



Bandwidth



- Unlike the usual bits/second, the unit of bandwidth is Hertz
- Bandwidth is determined by the signal, not the medium

Bandwidth

$$s(t) = \sin(2\pi \times 1 \times t) + \frac{\sin(2\pi \times 3 \times t)}{3} + \frac{\sin(2\pi \cdot 5 \cdot t)}{5}$$

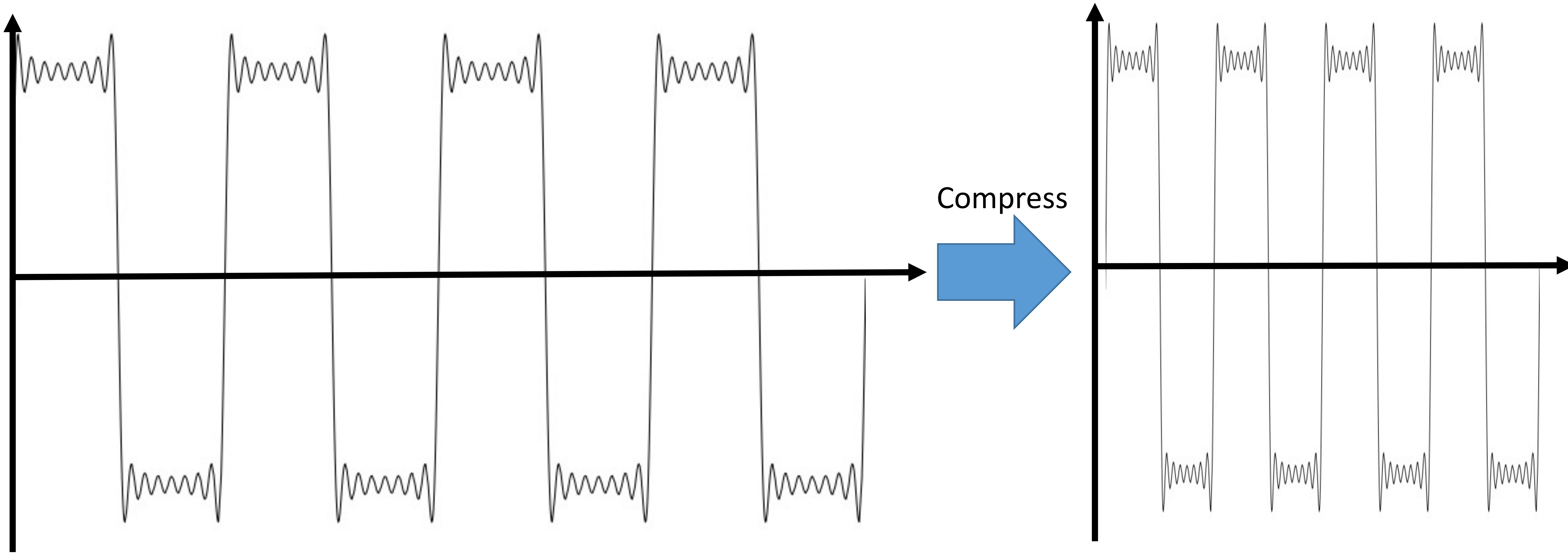
highest freq. = 5 Hz

lowest freq. = 1 Hz

$\Rightarrow$ Bandwidth = 5 - 1 = 4 (Hz)

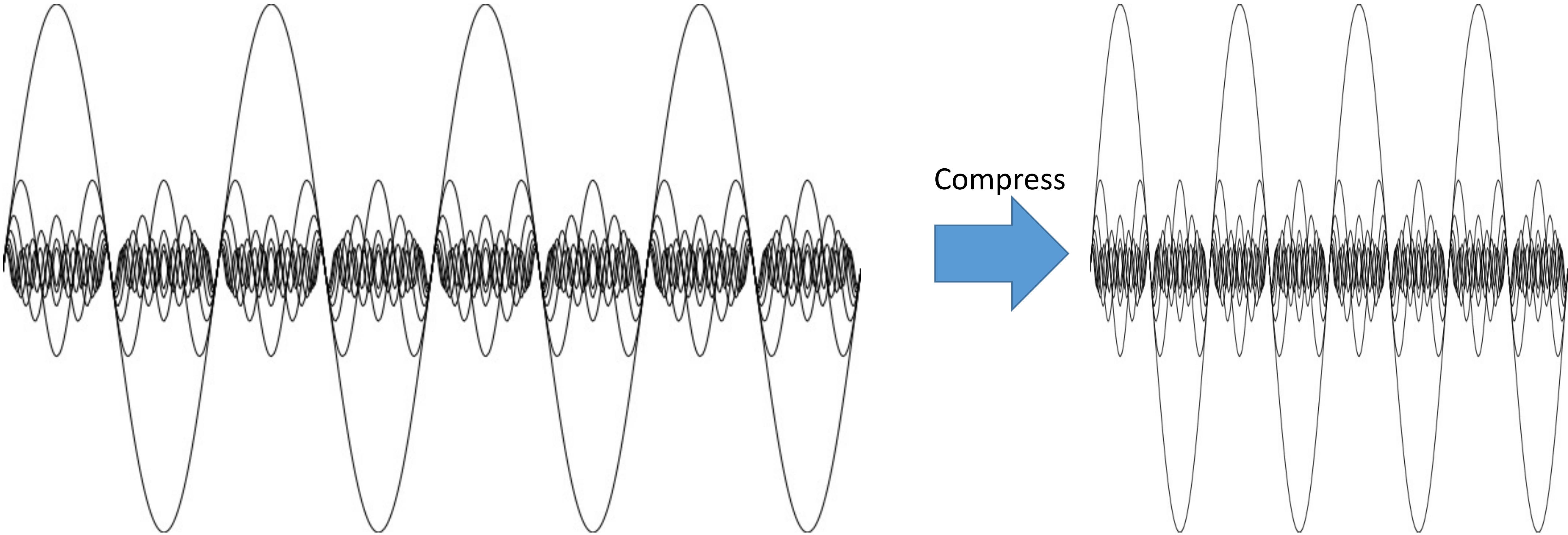
Bandwidth

- Which signal has a larger bandwidth?

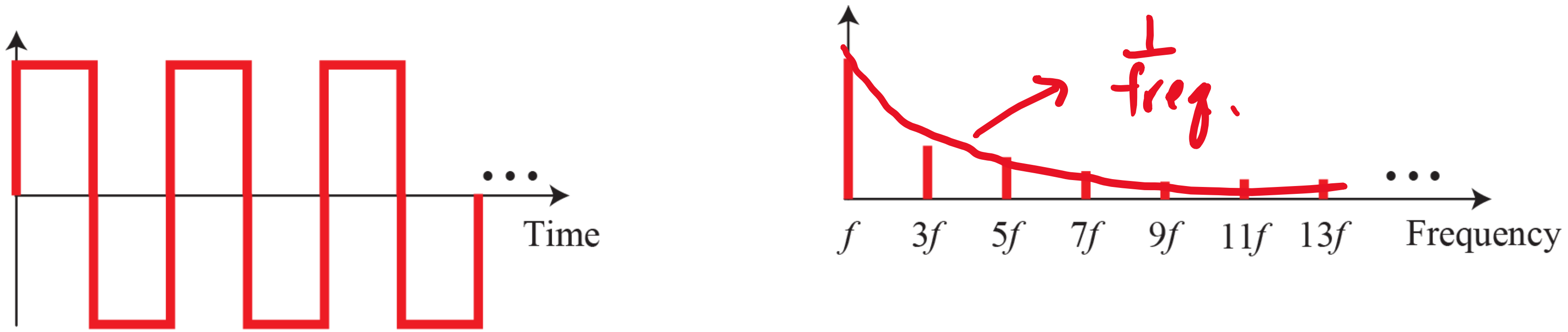


Bandwidth

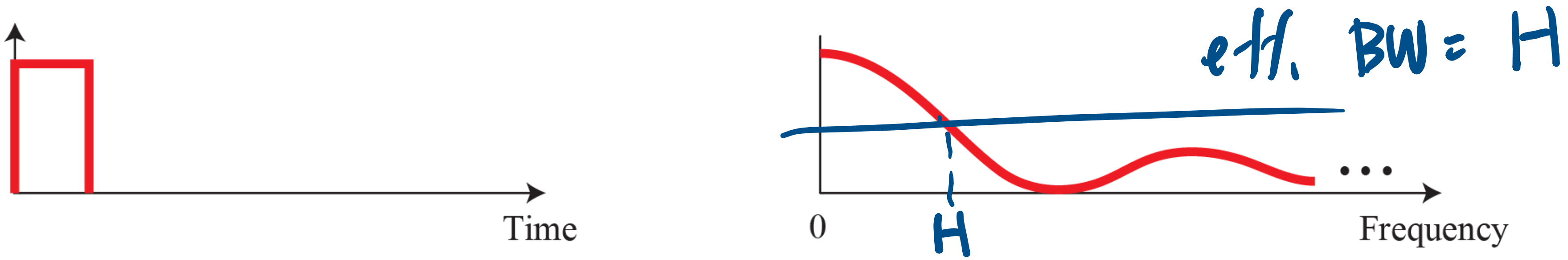
- Which signal has a larger bandwidth?



Fourier Transform of Digital Signals

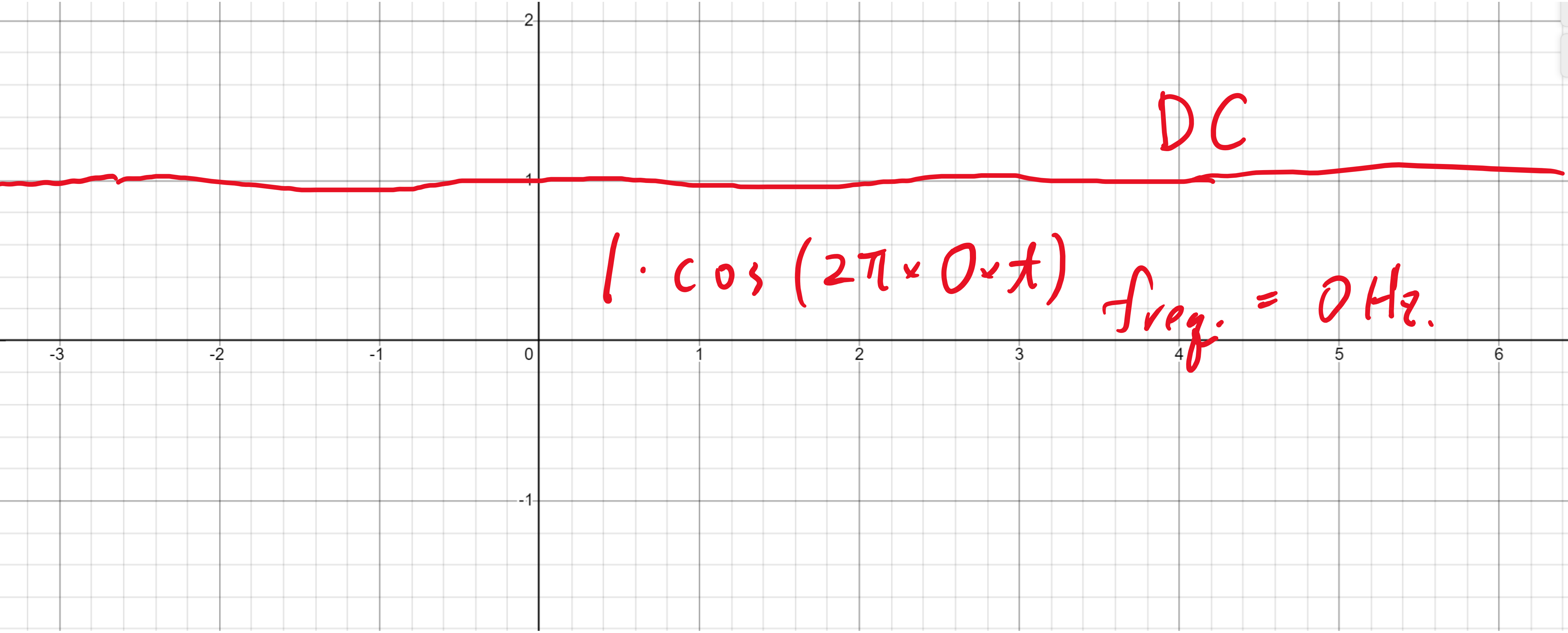


a. Time and frequency domains of periodic digital signal



b. Time and frequency domains of nonperiodic digital signal

$A\cos(2\pi f t)$



Effective Bandwidth

- Ignore weak frequency components
- The effective bandwidth of digital signal is finite
- Can we send digital signal wirelessly?
  - DC component

Simple Properties of Fourier Transform

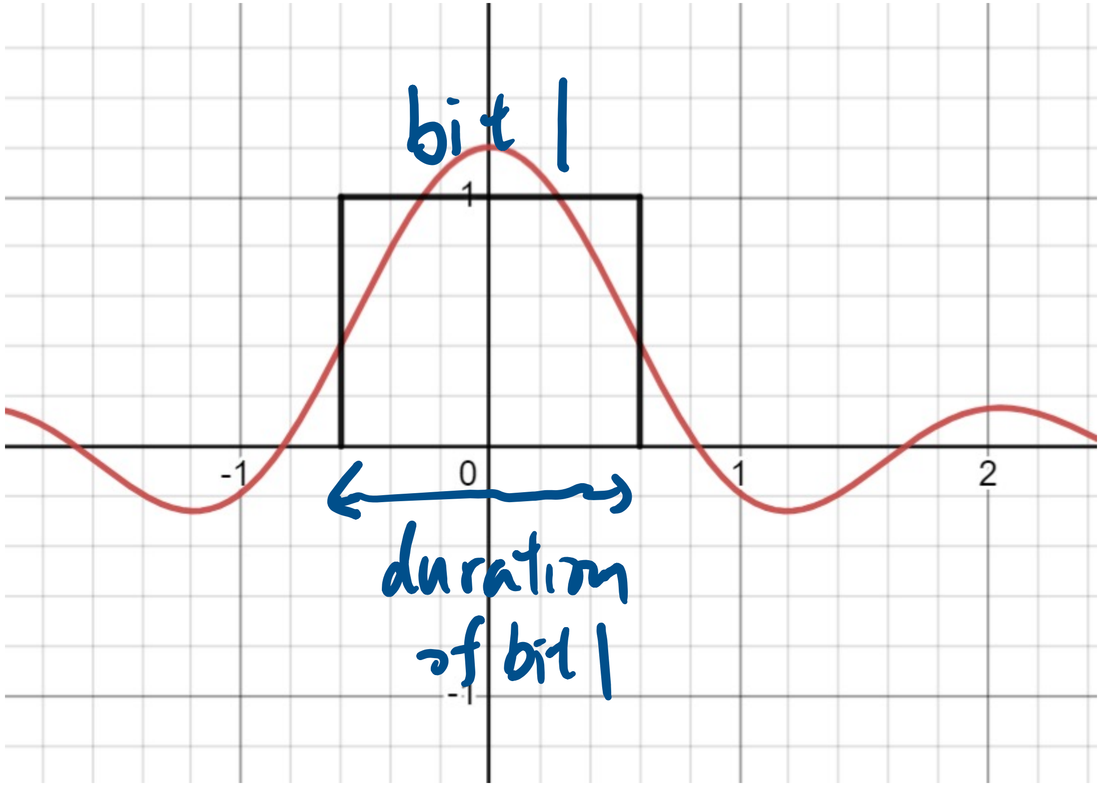
1. Fourier transform of a time domain rectangular function
2. Fourier transform of a time domain function with time shift
3. Fourier transform of a time domain function with time scaling
4. Fourier transform of a sum of time domain functions
5. Fourier transform of a time domain function multiplied by a cosine function

Fourier Transform of Rectangular Function

- Width $\downarrow \Rightarrow$ Effective Bandwidth $\uparrow$
- https://www.desmos.com/calculator/36fmlhlkb0

red → frequency domain

black → time domain



bit duration ↓

⇓

bit rate ↑

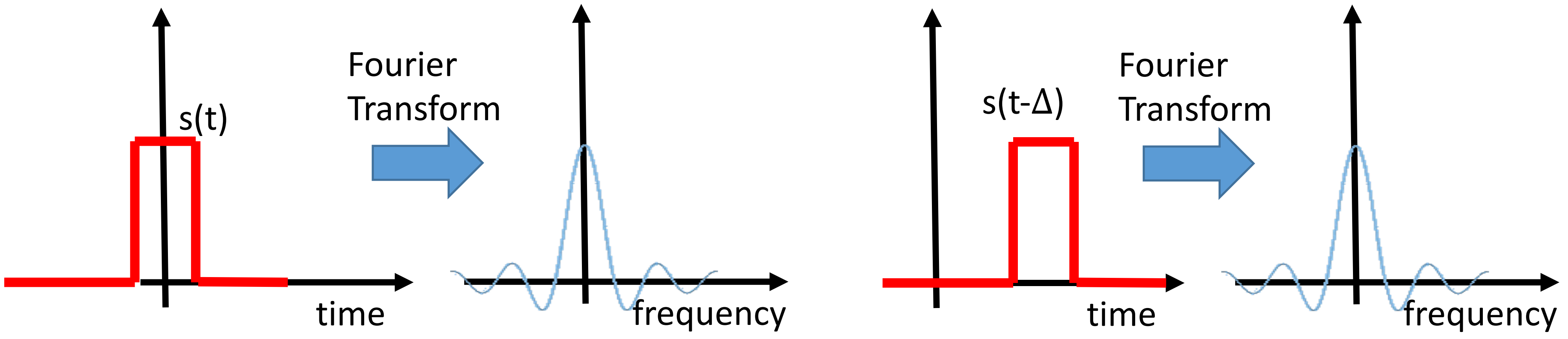
⇓

BW ↑

Fourier Transform of a Time Domain Function with Time Shift

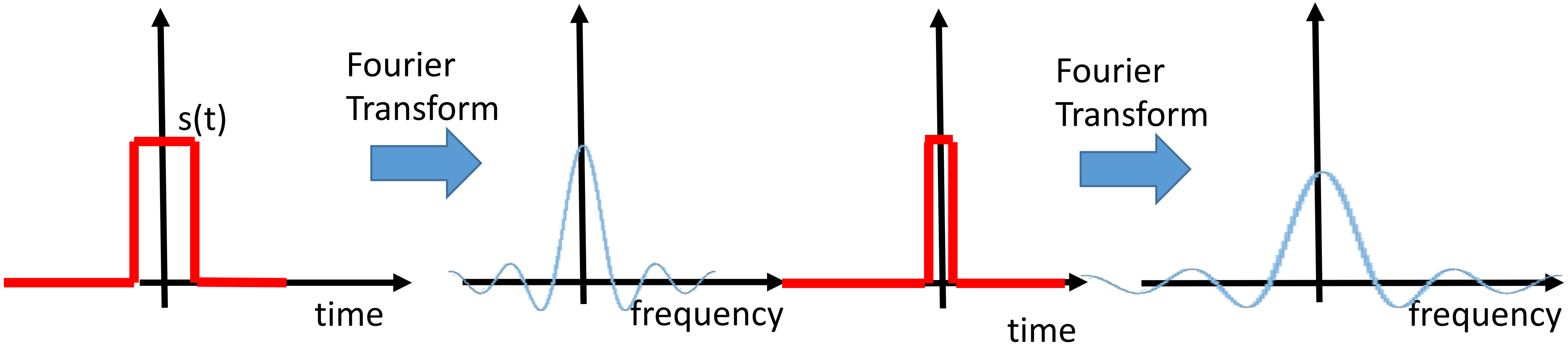
Time Shift Property

- In short, s(t) and s(t-Δ) have the same spectrum
- Intuition: Sending the same signal at different time does not affect the spectrum



Fourier Transform of a Time Domain Function with Time Scaling

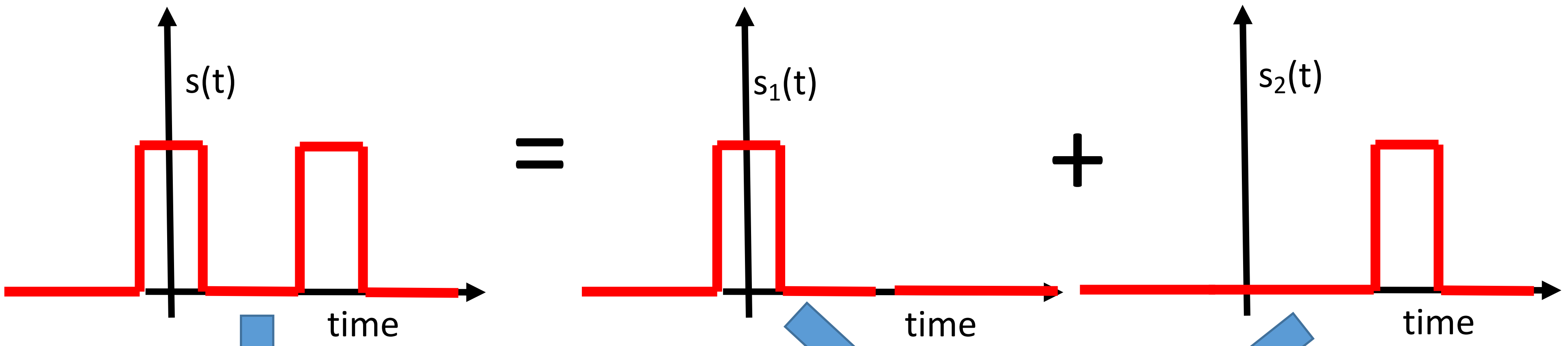
- Time scaling: compress or stretch a signal
- Intuition: compress->bit rate is increased-> bandwidth is increased
- Intuition: stretched->bit rate is decreased->bandwidth is decreased



Fourier Transform of A Sum of Time Domain Functions

- Say $s(t) = s_1(t) + s_2(t)$
- Assume that s_1 and s_2 do not interfere with each other destructively
- Frequency components of $s_1(t)$ are also frequency components of $s(t)$
- Frequency components of $s_2(t)$ are also frequency components of $s(t)$

Fourier Transform of A Sum of Time Domain Functions



These two signals have the same decomposition (by time shift property)

By the sum property, the bandwidth of s(t) = the bandwidth of $s_1(t)$ = the bandwidth of $s_2(t)$

$$\cos(2\pi f_c t) \times \cos(2\pi f_2 t)$$

$$= \frac{1}{2}\Big(\cos\big(2\pi(f_c+f_2)t\big) + \cos\big(2\pi(f_c-f_2)t\big)\Big)$$

2 freq. components: f_c+f_2 , f_c-f_2

$$\cos(2\pi \times 10 \times t) \times \cos(2\pi \times 1000 \times t)$$

freq. comp. : 1010, 990

BW : 20

$$s(t) = 2\cos(2\pi\, 0\, t) + 1\cos(2\pi\, 10\, t) + 0.5\cos(2\pi \cdot 20 \cdot t)$$

$$\times \cos(2\pi \cdot 10000 \cdot t)$$

$$= 2\cos(2\pi\, 0\, t)\cos(2\pi\, 10k\, t) + \cos(2\pi\, 10\, t)\cos(2\pi\, 10k\, t)$$

freq: 10k, 10k

amp: 1 1

freq: 10k − 10, 10k + 10

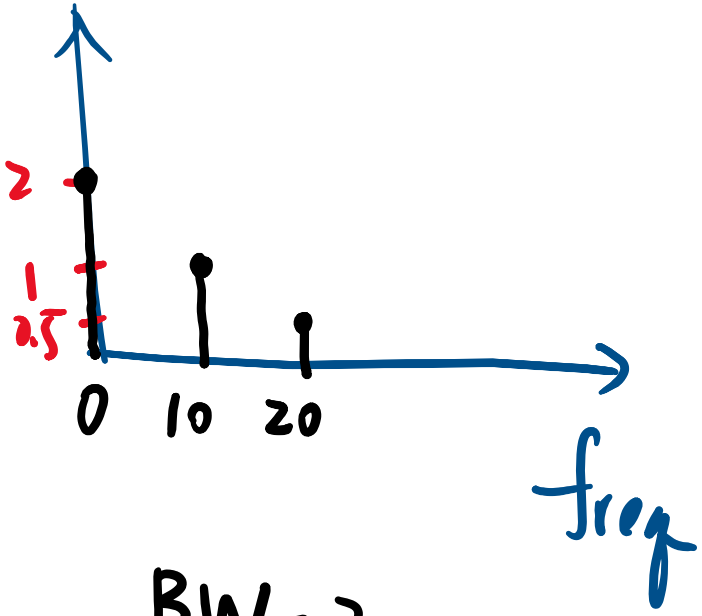
amp: 0.5 0.5

$$+ \frac{1}{2}\cos(2\pi\, 20\, t)\cos(2\pi\, 10k\, t)$$

freq: 10k + 20, 10k − 20

amp: $\frac{1}{4}$ $\frac{1}{4}$

$$s(t) = 2\cos(2\pi\, 0\, t) + 1\cos(2\pi\, 10\, t) + 0.5\cos(2\pi \cdot 20 \cdot t)$$



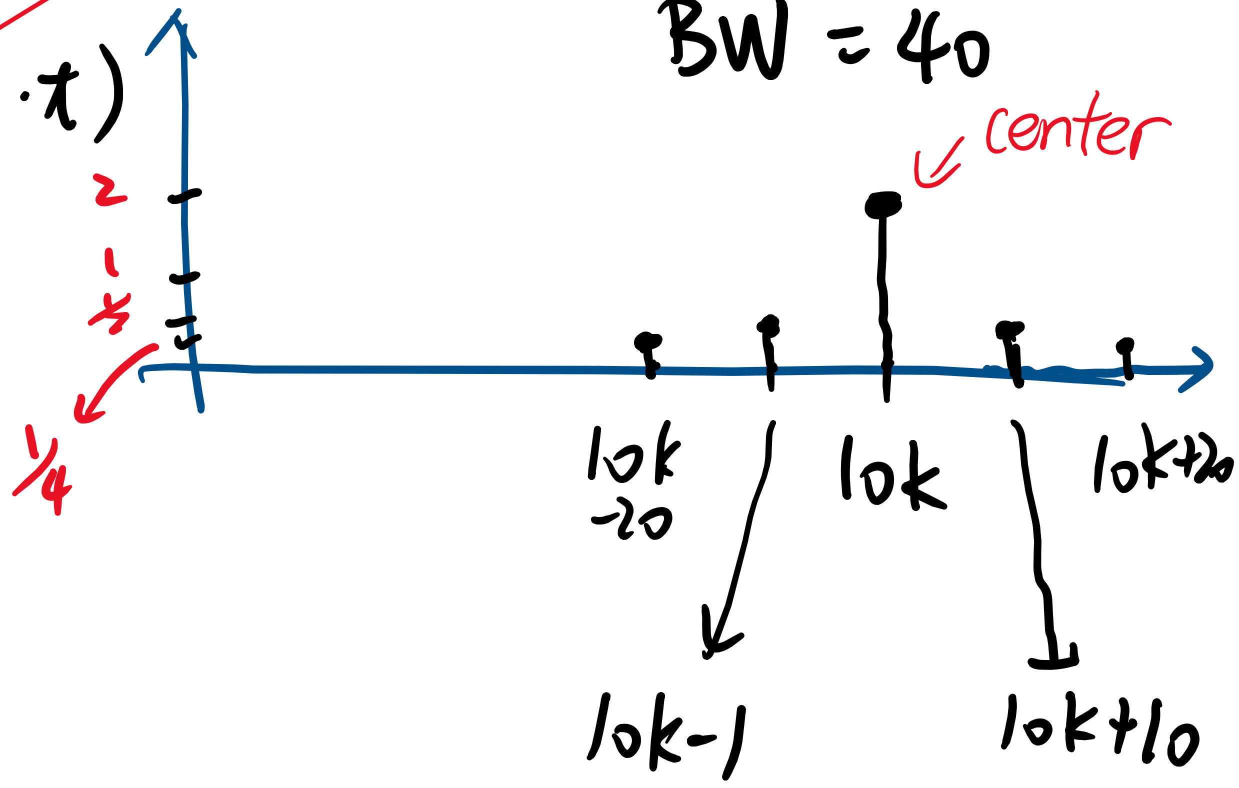
BW = 20

carrier signal

$$\times \cos(2\pi \cdot 10000 \cdot t)$$

carrier frequency

BW = 40



① frequency ↑

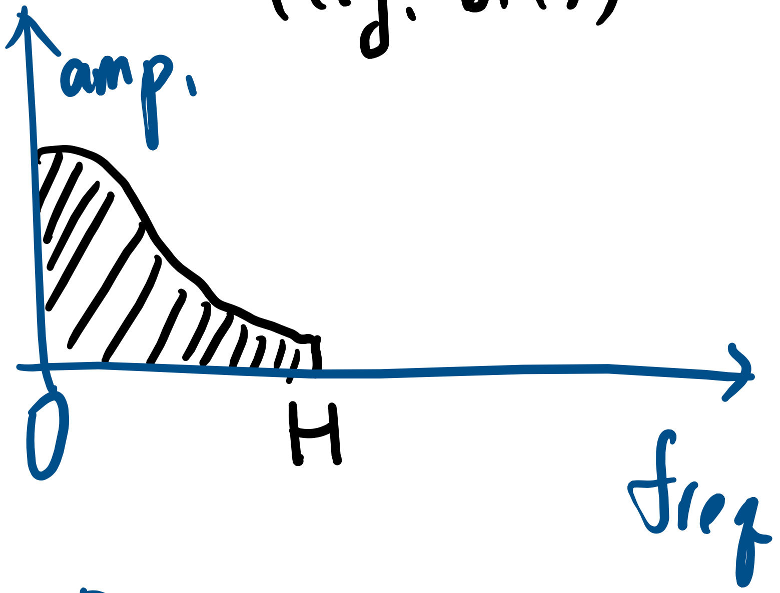
② BW is doubled

③ center freq = central freq.

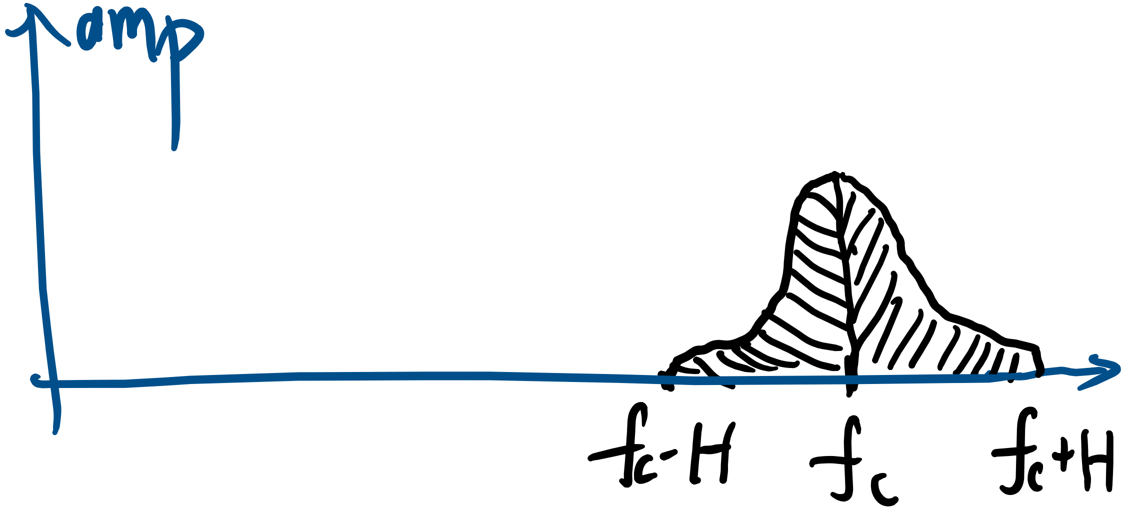
f_c : carrier frequency (e.g., 10^9)

$f_c \gg s(x)$'s freq. comp. (i.e., H)

$s(t)$: orig. signal (e.g. bits)



$\times \cos(2\pi f_c t)$



BW: H

BW: 2H

channel : 100 MHz ~ 120 MHz

We need to decide :

① f_c : $\dfrac{100\text{ MHz} + 120\text{ MHz}}{2}$

$\dfrac{120-100}{2}$

② H (orig. BW) : 10 MHz

↓

freq. comp. of (orig.) signal : 0 ~ H Hz

BW ↑ ⇒ bit rate ↑

BW ↓ ⇒ bit rate ↓

BW ∝ bit rate

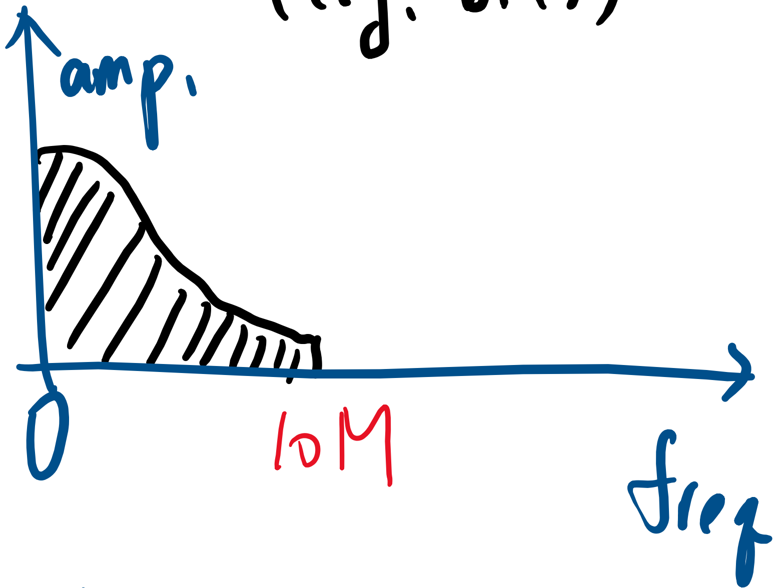
f_c : carrier frequency (e.g., 10^9)

$f_c \gg s(x)$'s freq. comp.

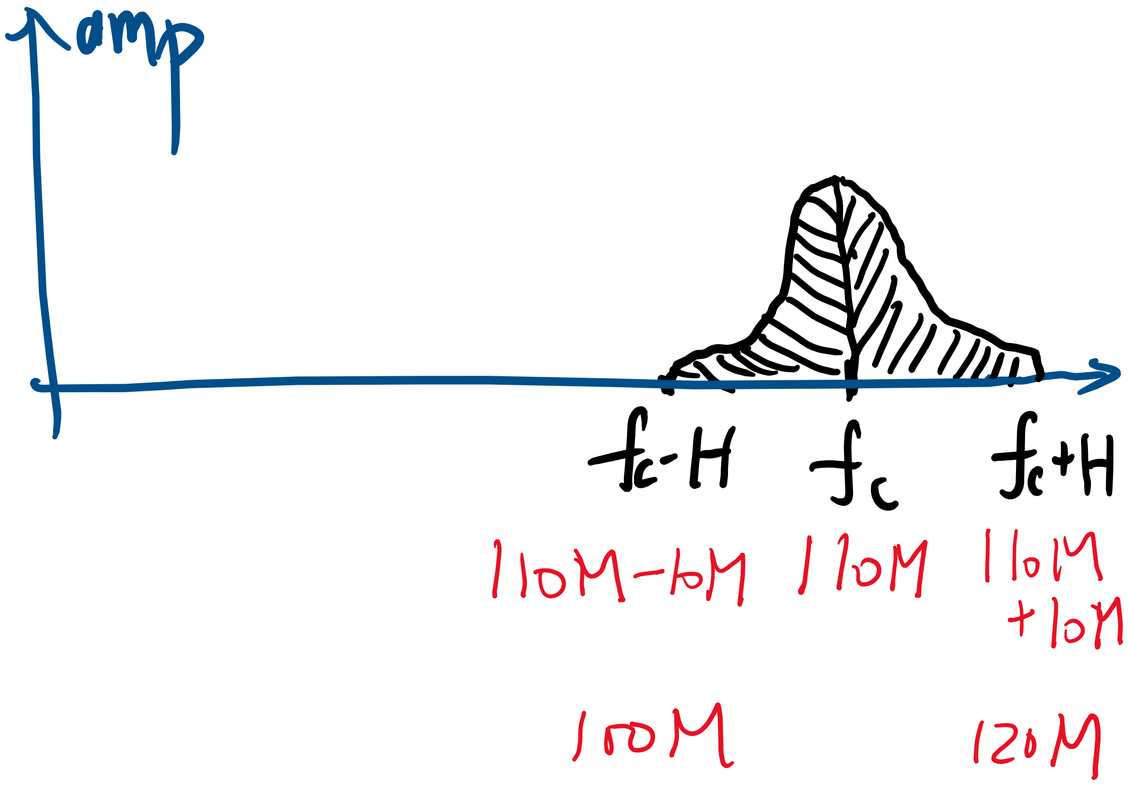
(i.e., H)

$s(t)$: orig. signal

(e.g. bits)



$\times \cos(2\pi f_c t)$



BW: H

Fourier Transform of A Time Domain Function Multiplied by a Cosine Function

- If the frequency components of $s(t)$ are between 0 and f_s, then the frequency components of $s(t)\cos(2\pi f_c t)$ are between $f_c - f_s$ and $f_c + f_s$
  - https://www.cliffsnotes.com/study-guides/trigonometry/trigonometric-identities/product-sum-and-sum-product-identities

- Usually, f_c >> f_s. As a result, after s(t) is multiplied by $\cos(2\pi f_c t)$, its frequency components are shifted to higher frequencies

$s(t) \times \cos(2\pi f_c t) \times \cos(2\pi f_c t)$

↳ demodulatation (at receiver side)

$= s(t) \times \frac{1}{2}\left(\cos(2\pi(2f_c)t) + \cos(2\pi 0 t)\right)$

$= \frac{1}{2} s(t) \cos(2\pi 2f_c t) + s(t) \times \frac{1}{2}$

① filter out frequency $> s(t)$'s highest freq.

(recall $f_c \gg s(t)$'s highest freq.)

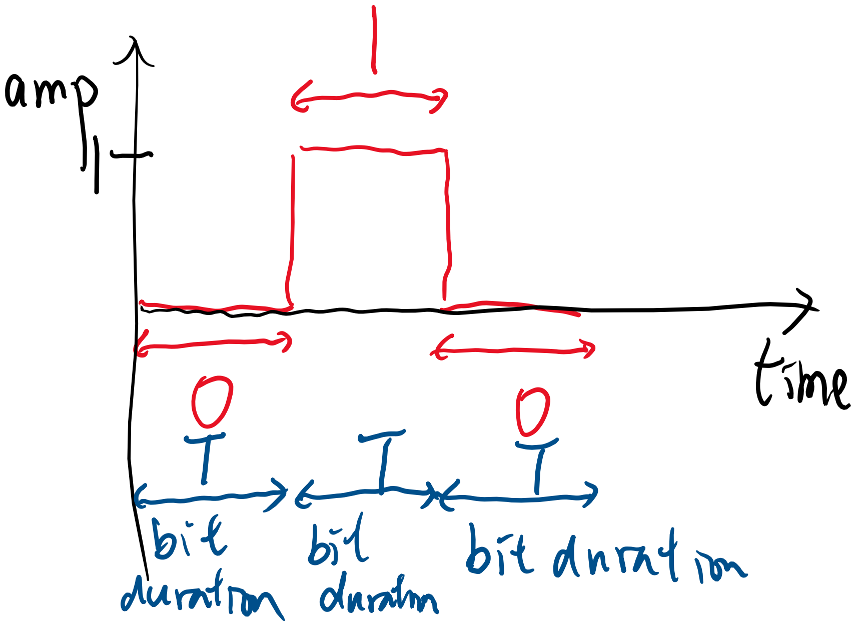
② amplify

$\Rightarrow s(t)$

Modulation

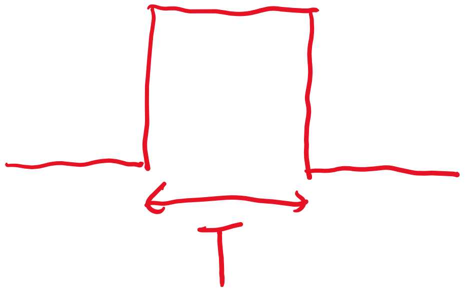
- To send $s(t)$, we can send $s(t)\cos(2\pi f_c t)$ (modulation)
- The receiver receives $r(t) = s(t)\cos(2\pi f_c t)$
  - Not realistic due to attenuation and noise
- How does the receiver obtain $s(t)$?
- $r(t)\cos(2\pi f_c t) = s(t)\cos(2\pi f_c t)\cos(2\pi f_c t)$
  $= s(t)[0.5(\cos(2\pi 2 f_c t) + 1)] = 0.5s(t) + 0.5\cos(2\pi 2 f_c t)$
- Filter out $0.5\cos(2\pi 2 f_c t)$ to obtain $0.5s(t)$
- In short:1. Modulate. 2. Filter 3. Amplify

Bits to Waves



Bit 1: amp = 1

Bit 0: amp = 0



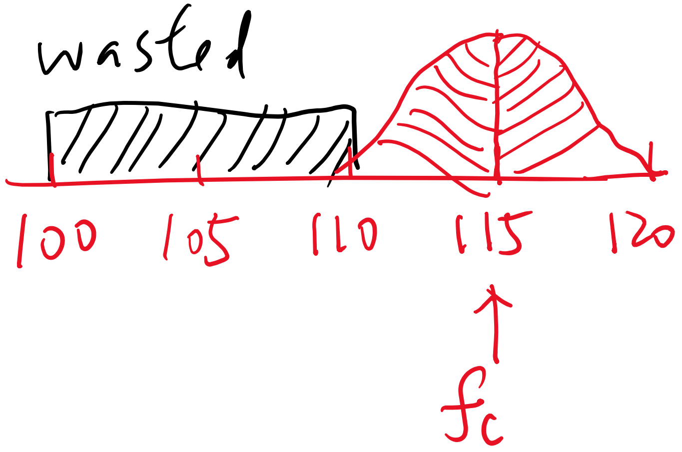
effective BW: $\frac{1}{T}$

Example: $T = 10^{-6}$ sec

eff. BW = 10^{6} Hz

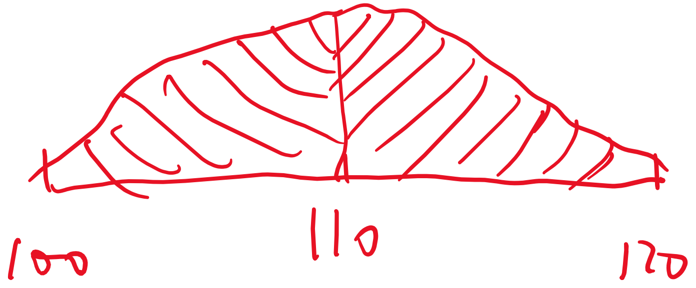
Goal: Given a channel between 100 MHz and 120 MHz, determine the highest bit rate.

$f_c = 115\,MHz$ X



$f_c = 110\,MHz$

BW of the (original) digital signal $= 10\,MHz$

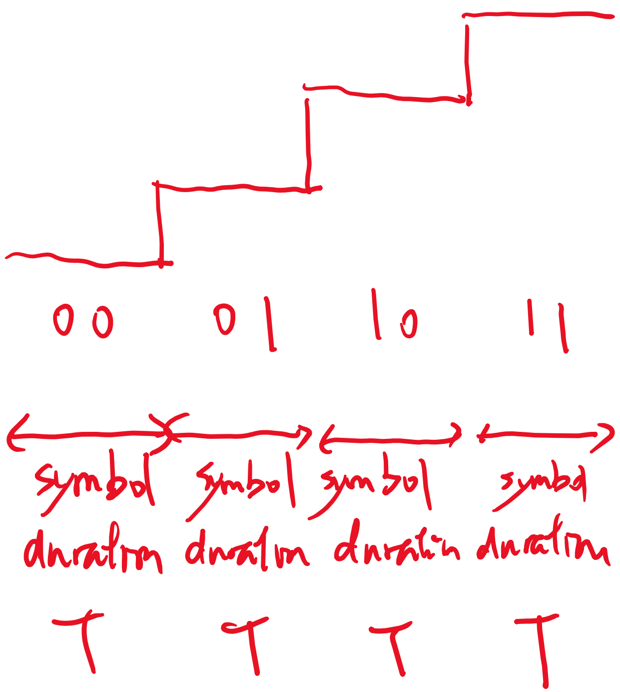


$$\text{eff. } BW = \frac{1}{T} = 10\ MHz = 10^7\ Hz$$

$$\frac{1}{T} = 10^7\ Hz \Rightarrow T = 10^{-7}\ sec$$

$$\text{Bit rate} = 10^7\ \text{bits/sec}$$

(number of bits per second)



00 → amp : 0

01 → amp : $1/3$

10 → amp : $2/3$

11 → amp : 1

Drawbacks: ① # symbol ↑ ⇒ BER (bit error rate) ↑

different

(due to noise & interference)

② # diff symbol ↑ ⇒ power ↑ (Amplitude ↑)