Modulation

https://www.desmos.com/calculator/juog6aelo6

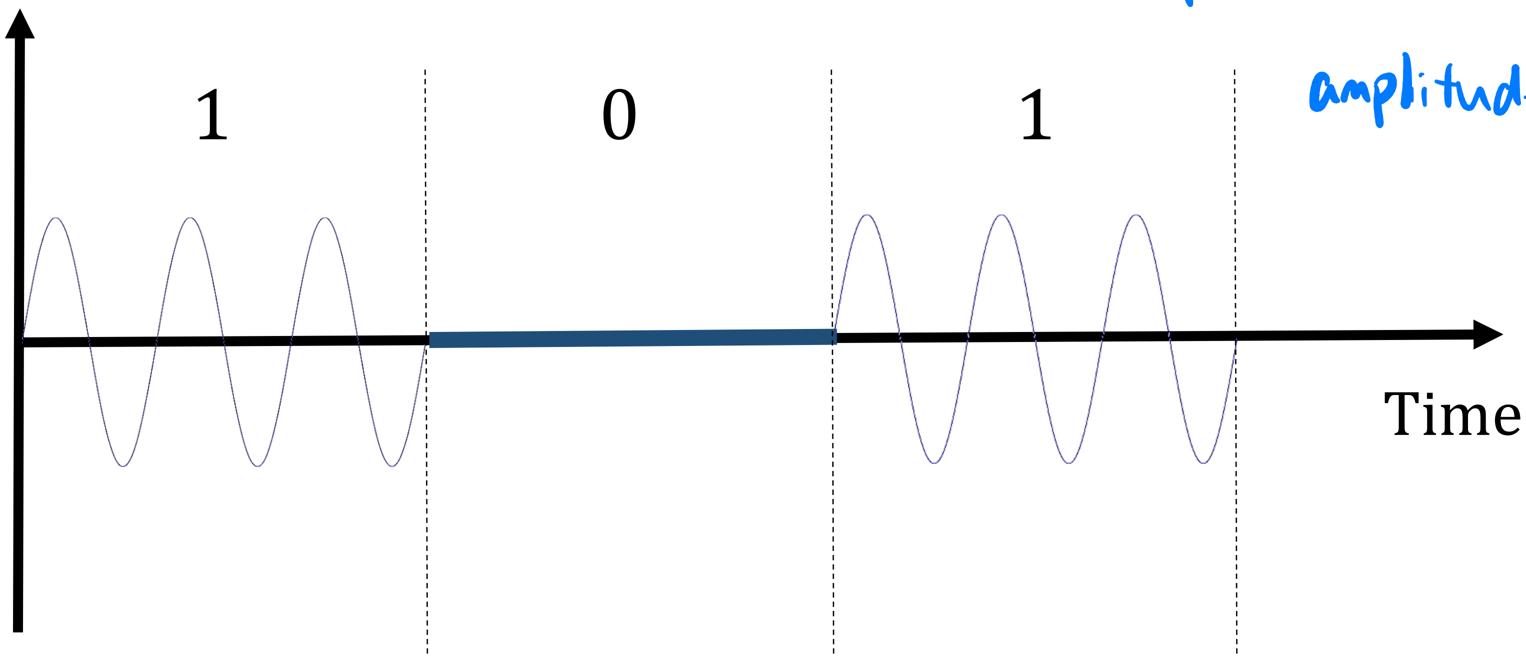
Overview

- Send a message from one node to another
- In particular, we send digital data using analog signal
- We will send digital data by changing the peak amplitude or the phase of a sine wave

Changing Amplitude

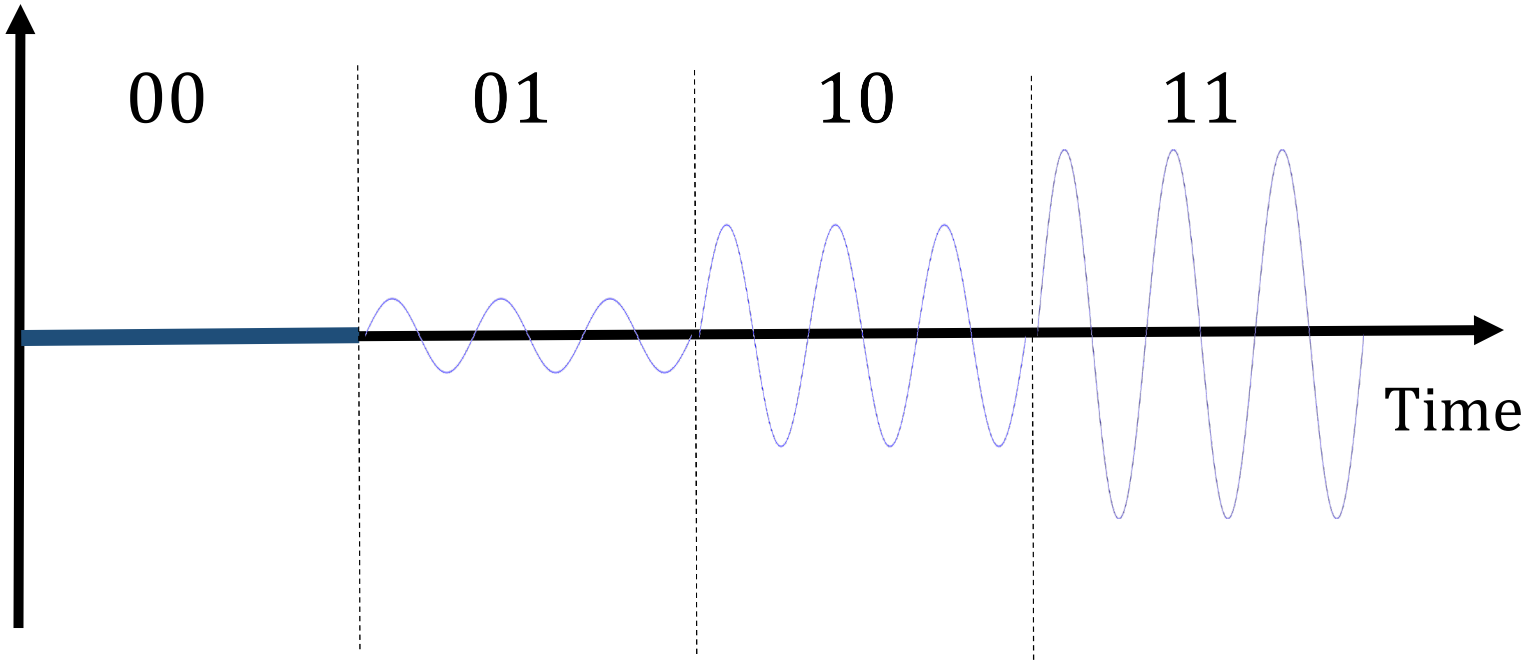
- Amplitude Shift Keying (ASK)
  - Binary Amplitude Shift Keying (BASK)

"Shift Keying" is a modulation method that involves changing or "shifting" a parameter of a carrier wave, such as amplitude, phase or frequency



Changing Amplitude

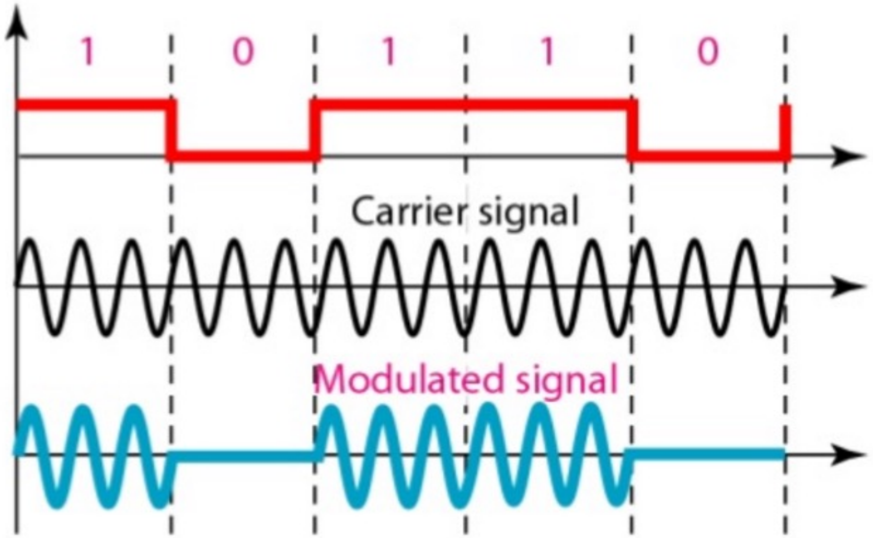
- Amplitude Shift Keying (ASK)

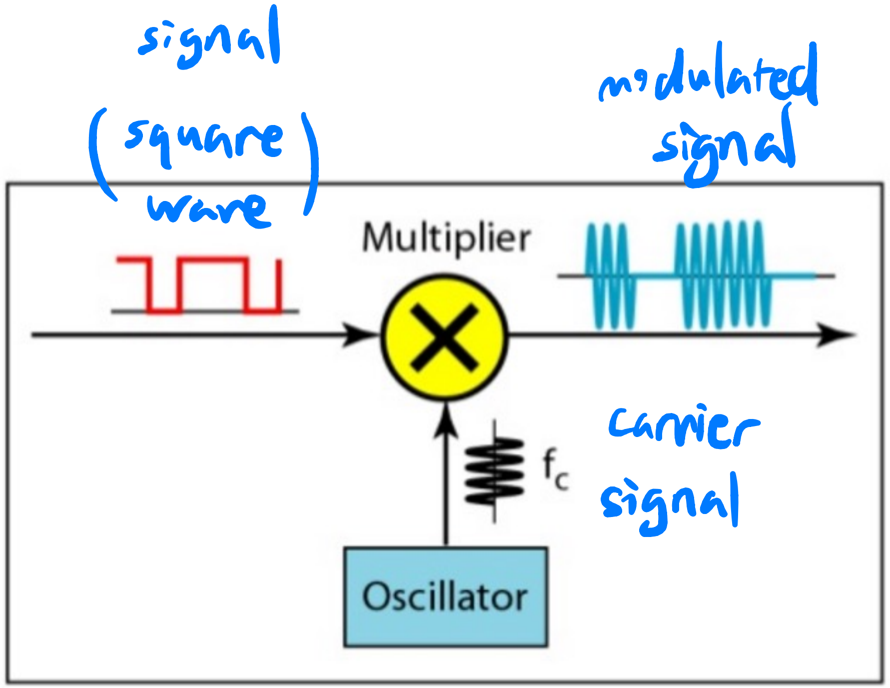


Amplitude $\uparrow \Rightarrow$ power $\uparrow$

binary

Implementation of BASK



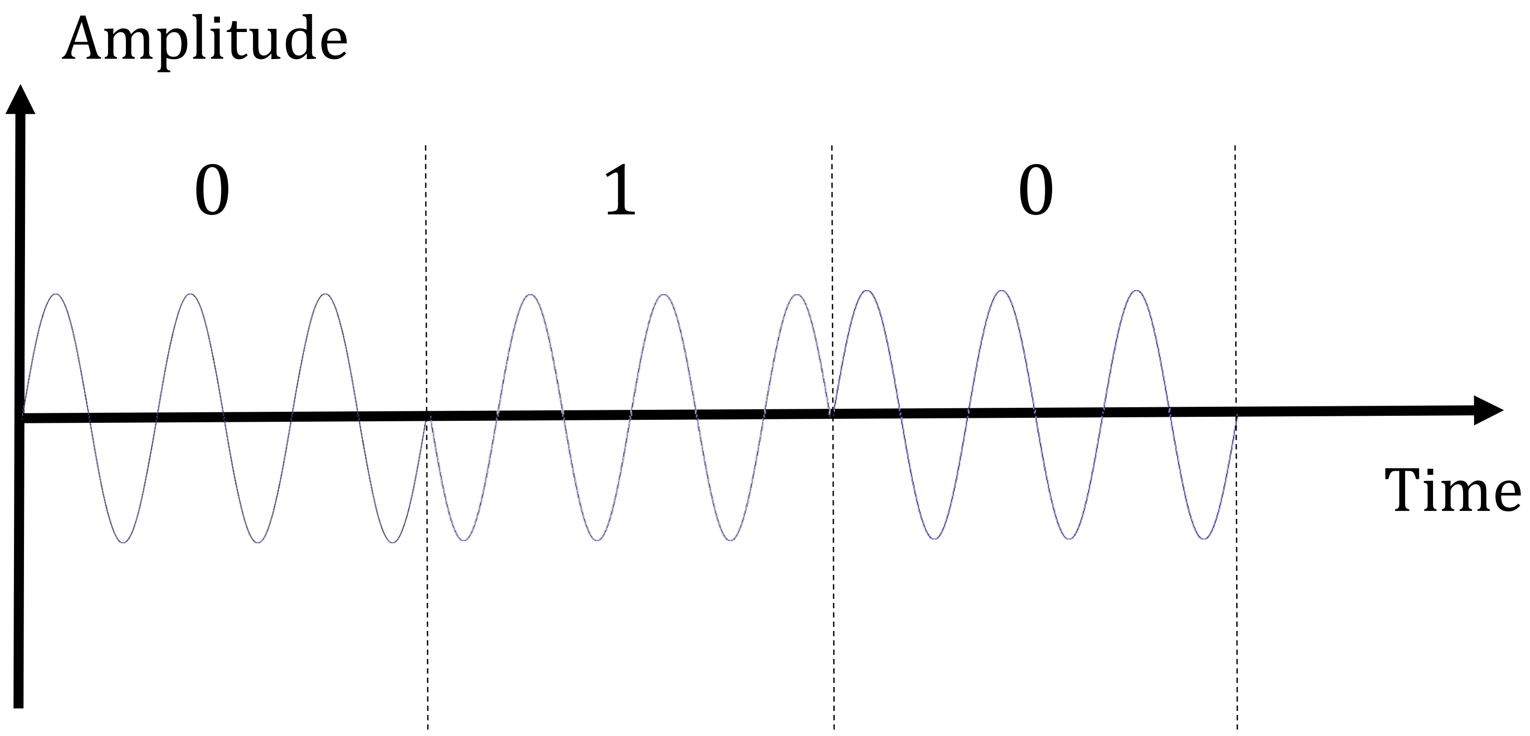


Properties of BASK

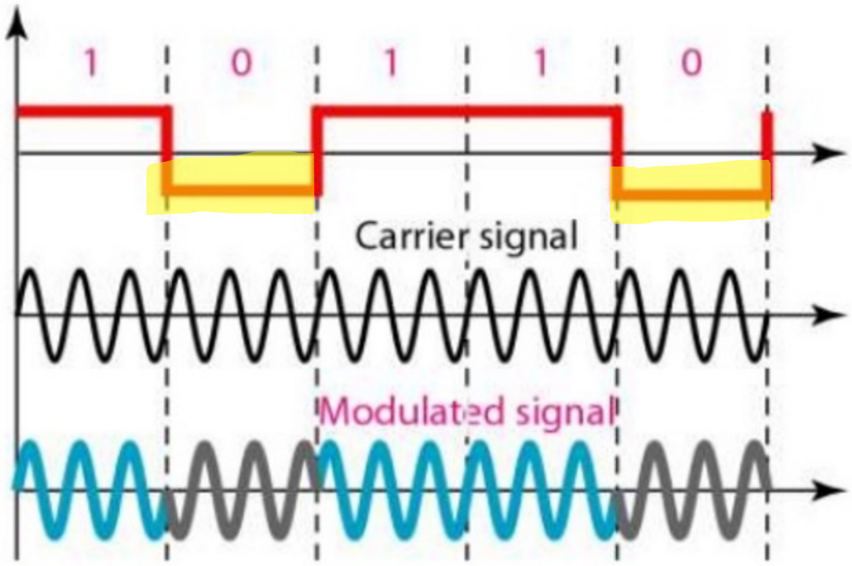
- Formula:
  - $s(t) = \begin{cases} A cos(2\pi f_c t) & \text{bit } 1 \\ 0 & \text{bit } 0 \end{cases}$
  - f_c: carrier frequency
- Bandwidth is proportional to the bit rate
- We can change the spectrum by changing the carrier frequency
  - Different channels use different carrier frequency to avoid interference

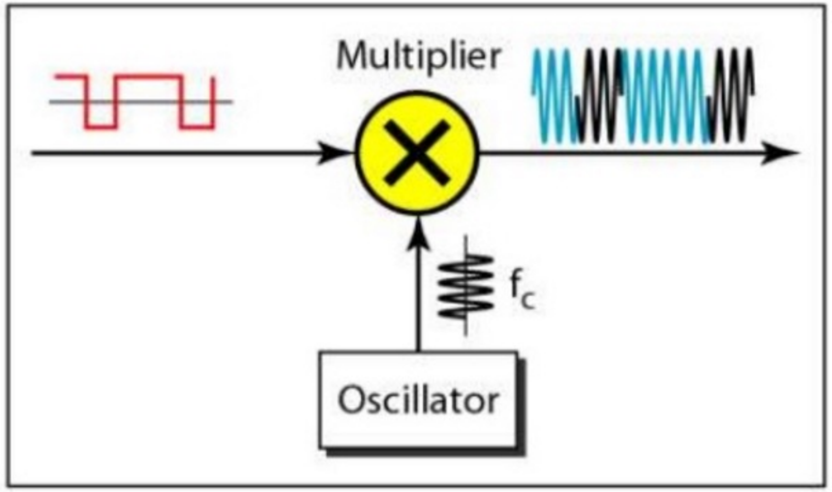
Changing Phase

- Phase Shift Keying (PSK) https://www.desmos.com/calculator/1pyhiaticd
  - Binary Shift Keying (BPSK)



Implementation of BPSK





QPSK (Quadrature Phase Shift Keying)

- In BPSK
  - 1 → phase shift 0°
  - 0 → phase shift 180°
- Can we do the following?
  - 11 → phase shift 45°
  - 01 → phase shift 135°
  - 00 → phase shift 225°
  - 10 → phase shift 315°
- Are the four phases of $\pm\sin(x) \pm \cos(x)$ the same?
  - https://bit.ly/2EcPyNC

quadrature carrier

1st bit controls this

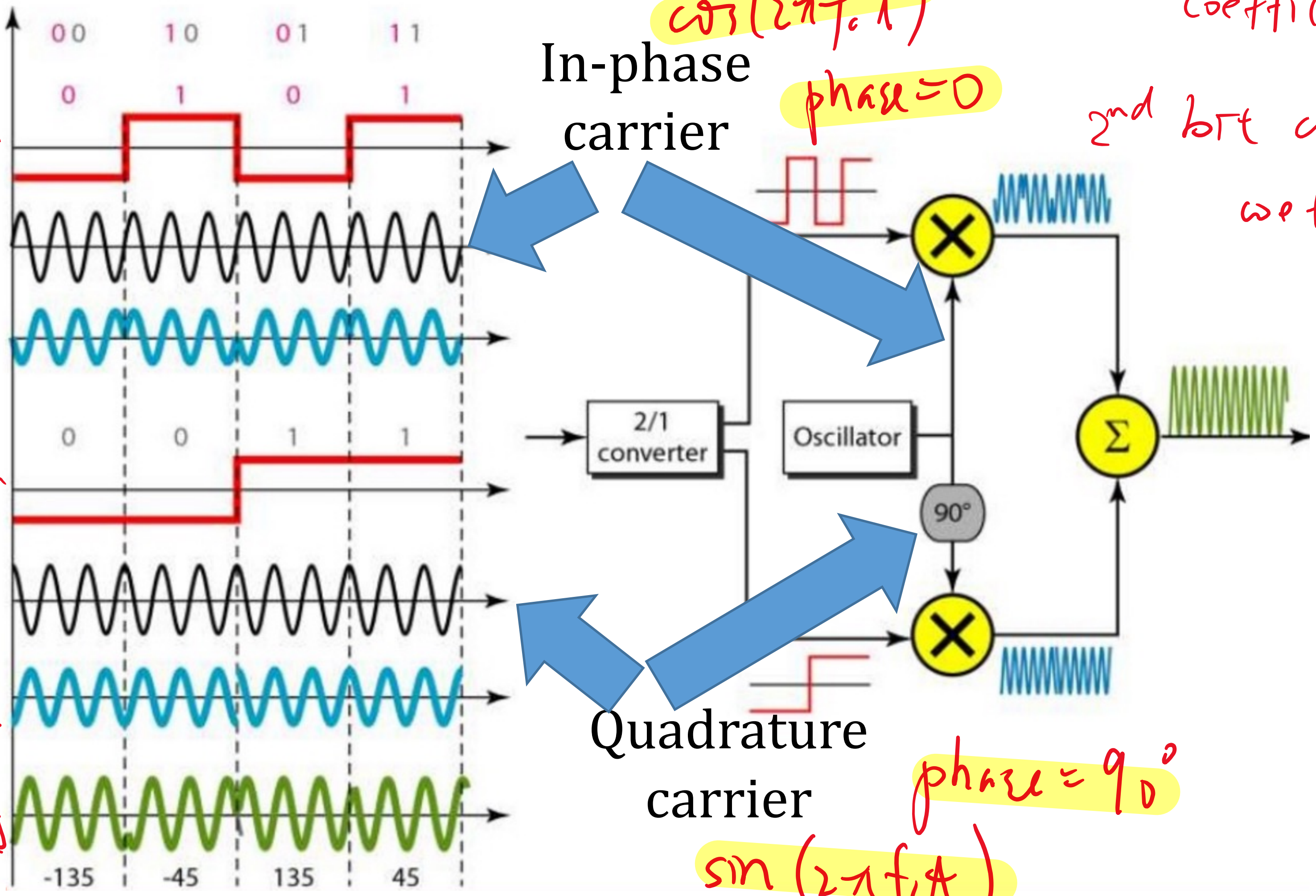
2nd bit controls this

inphase carrier

Implementation of QPSK

BPSK
in-phase carrier

BPSK
quadrature carrier



$\cos(2\pi f_c t)$

phase = 0

phase = 90°

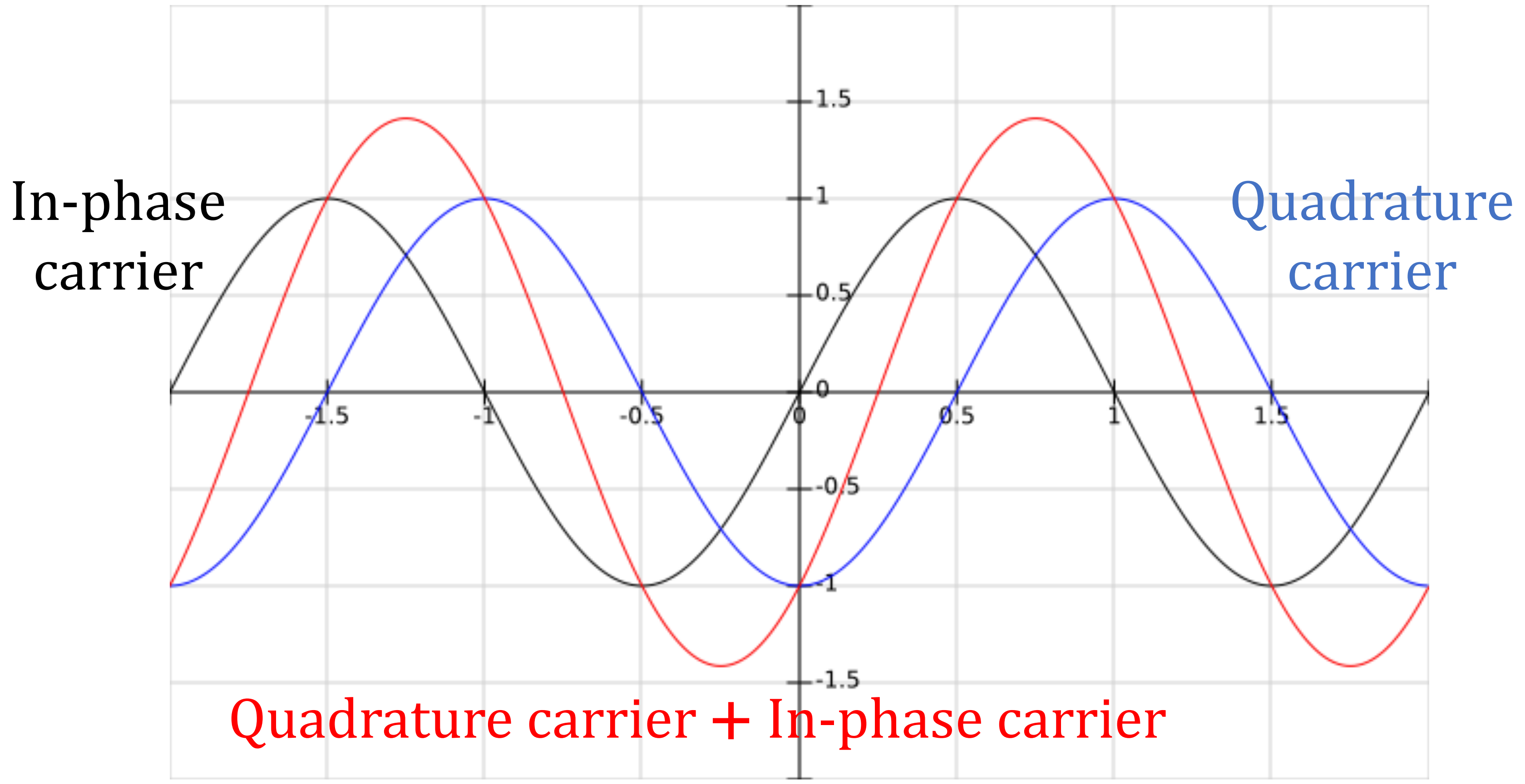
$\sin(2\pi f_c t)$

for each symbol:

1st bit controls the coefficient of cos (in-phase carrier)

2nd bit controls the coefficient of sin (quadrature carrier)

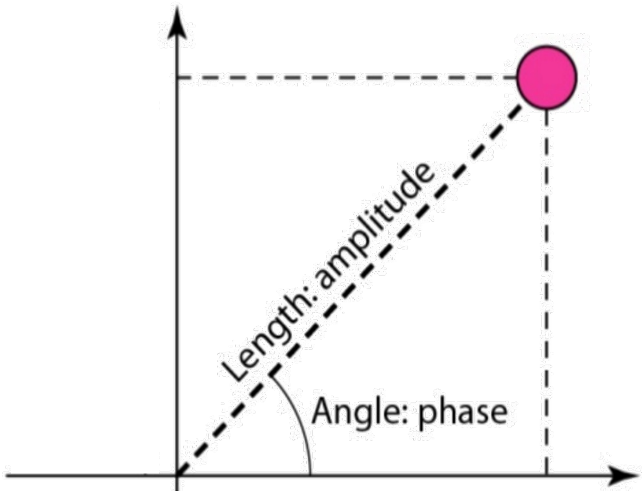
In-Phase and Quadrature Carriers

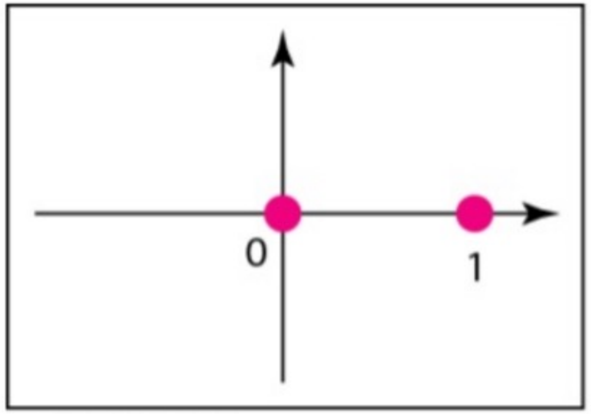


Combination of ASK and PSK

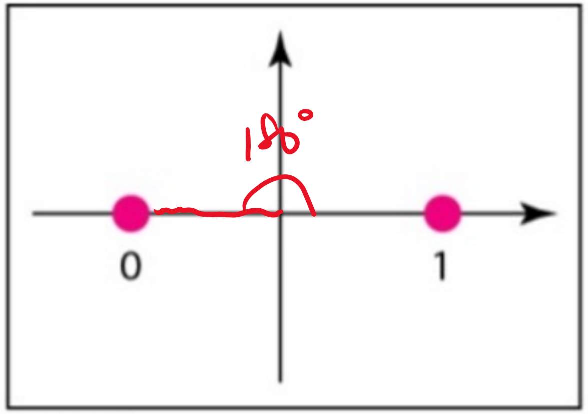
- So far, we have been only altering one of the three characteristics of a sine wave at a time
- Next, we try to alter the amplitude and phase at the same time
- We need a way to represent the encoding: the constellation diagram

Constellation Diagram

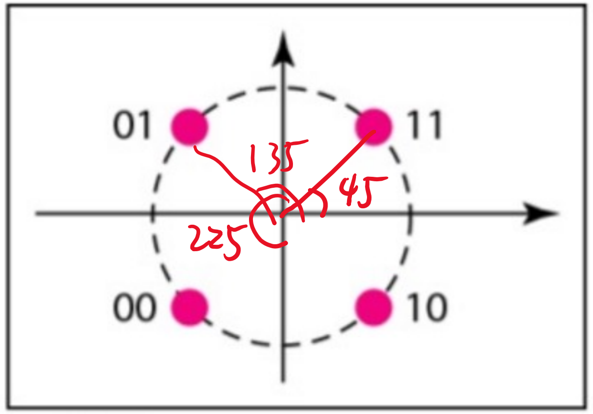




BASK

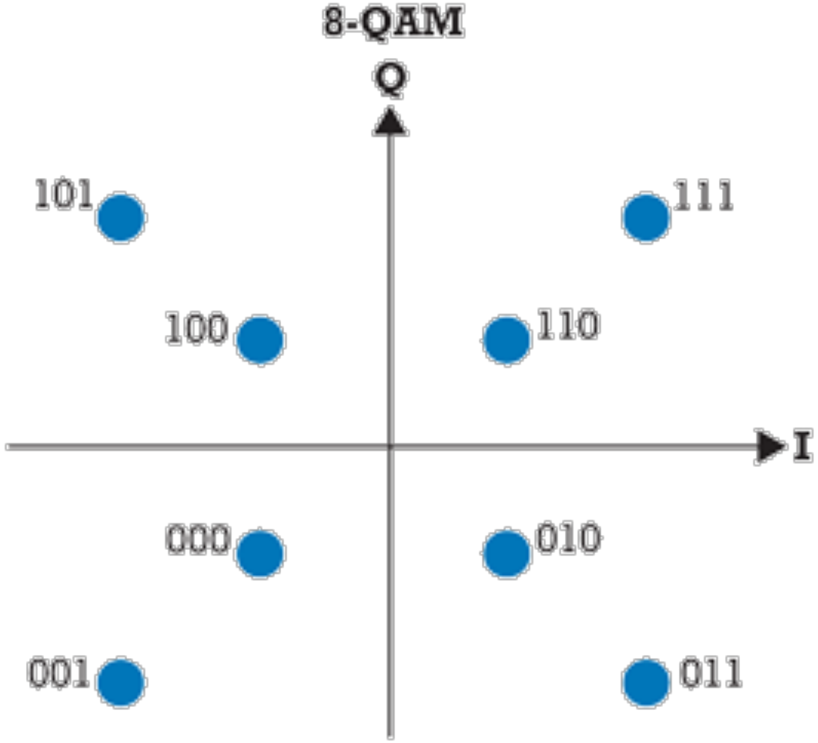


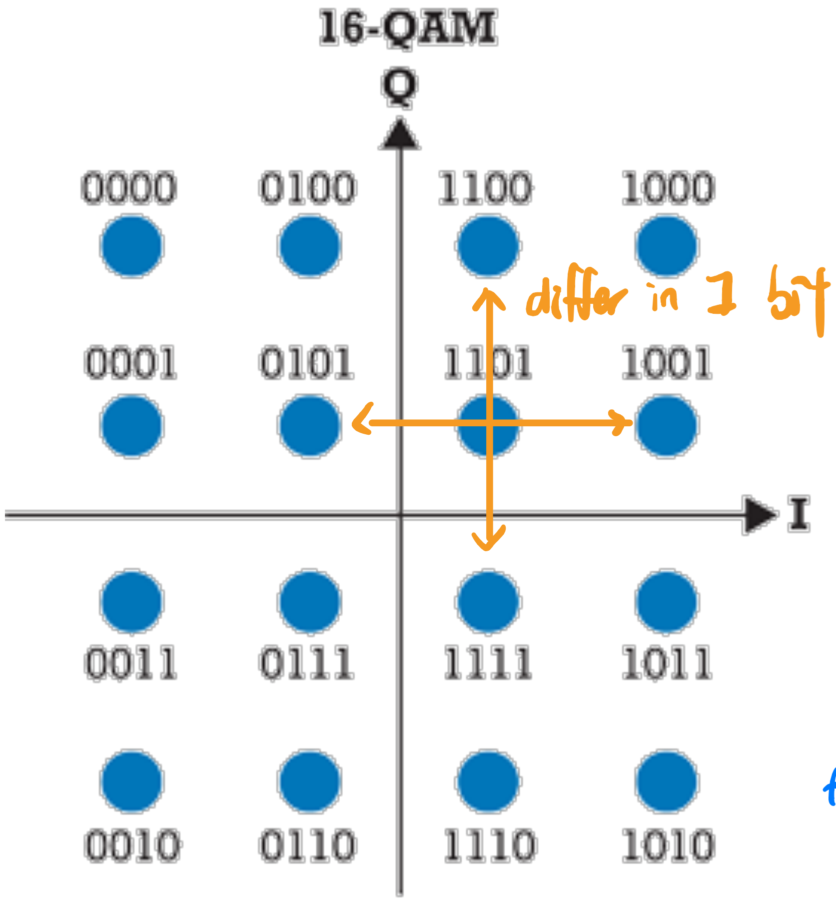
BPSK



QPSK

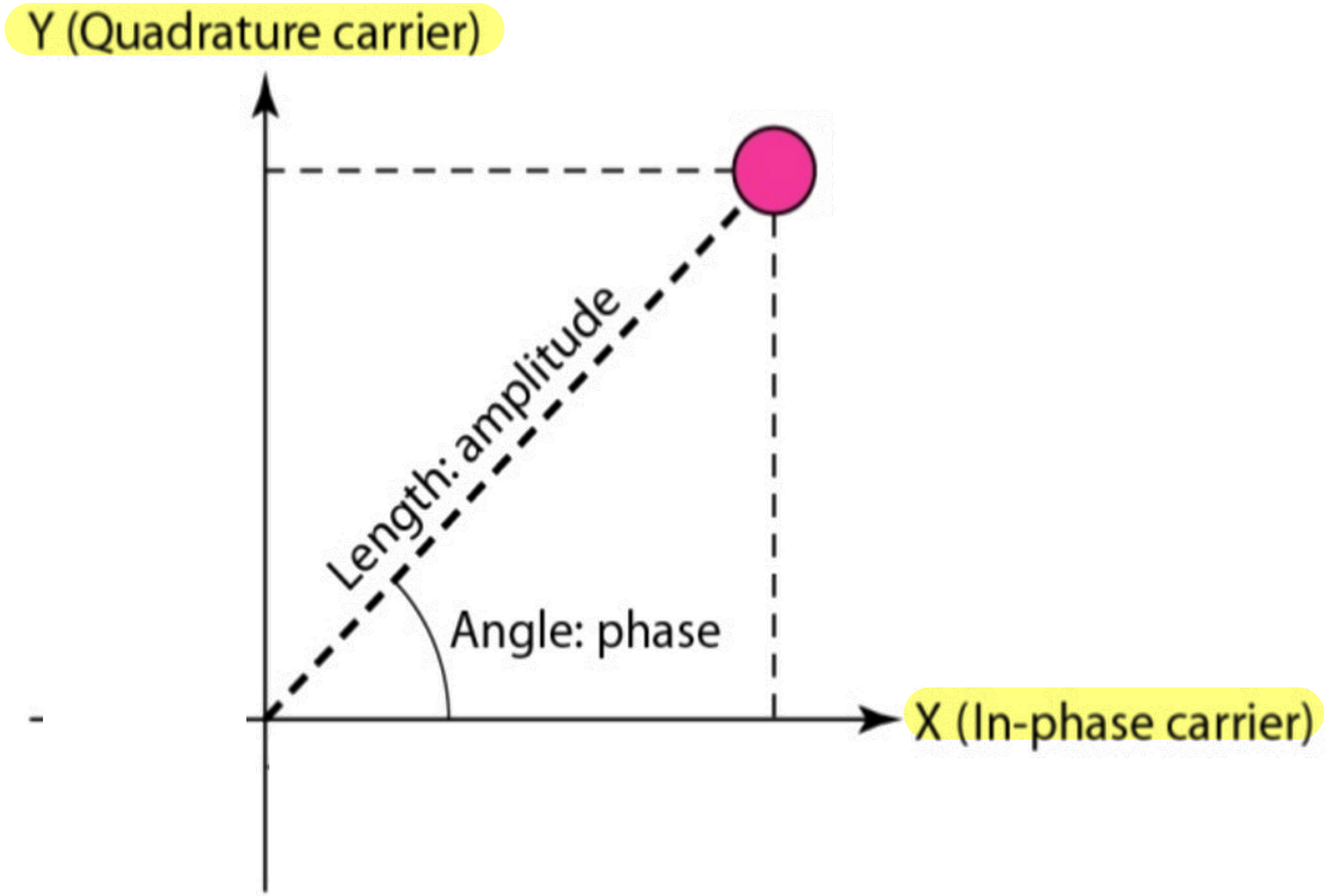
Quadrature Amplitude Modulation (QAM)



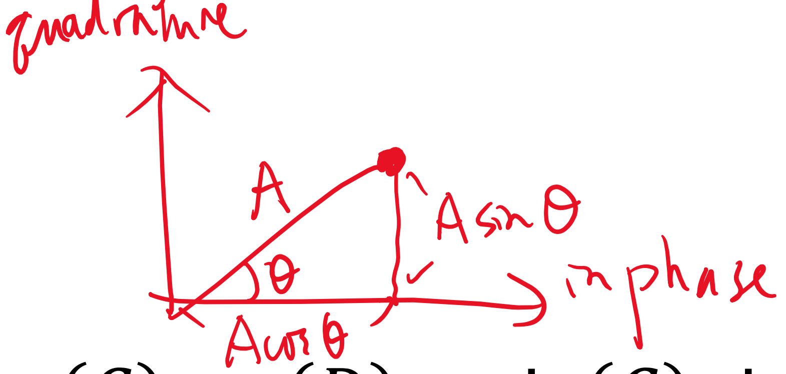


If noise is not big enough, then the # of incorrect bits ↓ ⇒ BER ↓

Implementation of QAM



Proof



- $\cos(C + D) = \cos(C)\cos(D) - \sin(C)\sin(D)$

$$A\cos(\underbrace{2\pi f_c t}_{C} + \underbrace{\theta}_{D}) = A\cos(2\pi f_c t)\cos(\theta) - A\sin(2\pi f_c t)\sin(\theta)$$

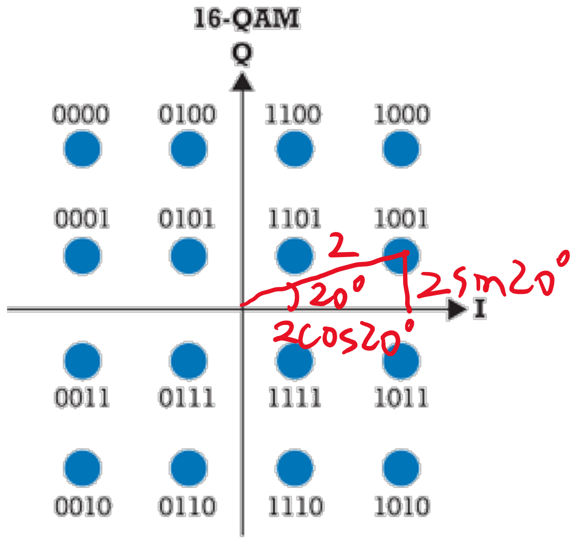
$$= A\cos(\theta)\cos(2\pi f_c t) - A\sin(\theta)\sin(2\pi f_c t)$$

$$= A\cos(\theta)\cos(2\pi f_c t) + A\sin(\theta)(-\sin(2\pi f_c t))$$

$$= A\cos(\theta) \times \underset{\text{carrier}}{\text{inphase}} + A\sin(\theta) \times \underset{\text{carrier}}{\text{quadrature}}$$

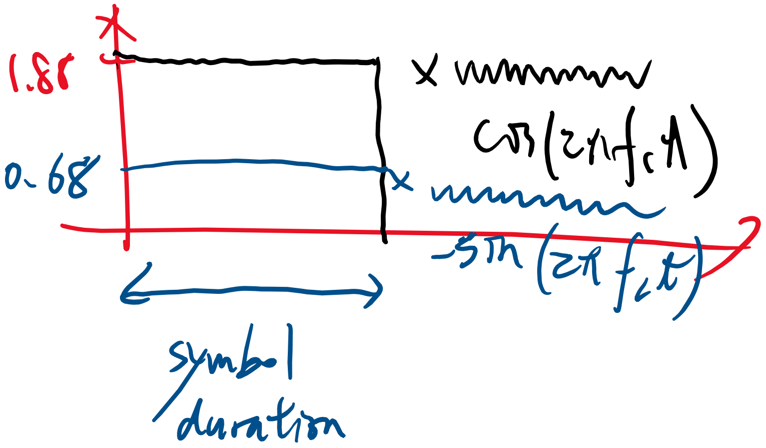
$$= x\text{-coordinate} \times \text{inphase} + y\text{-coordinate} \times \text{quadrature}$$

Example send bits : 1001 using 16 QAM.



$x : 1.88$

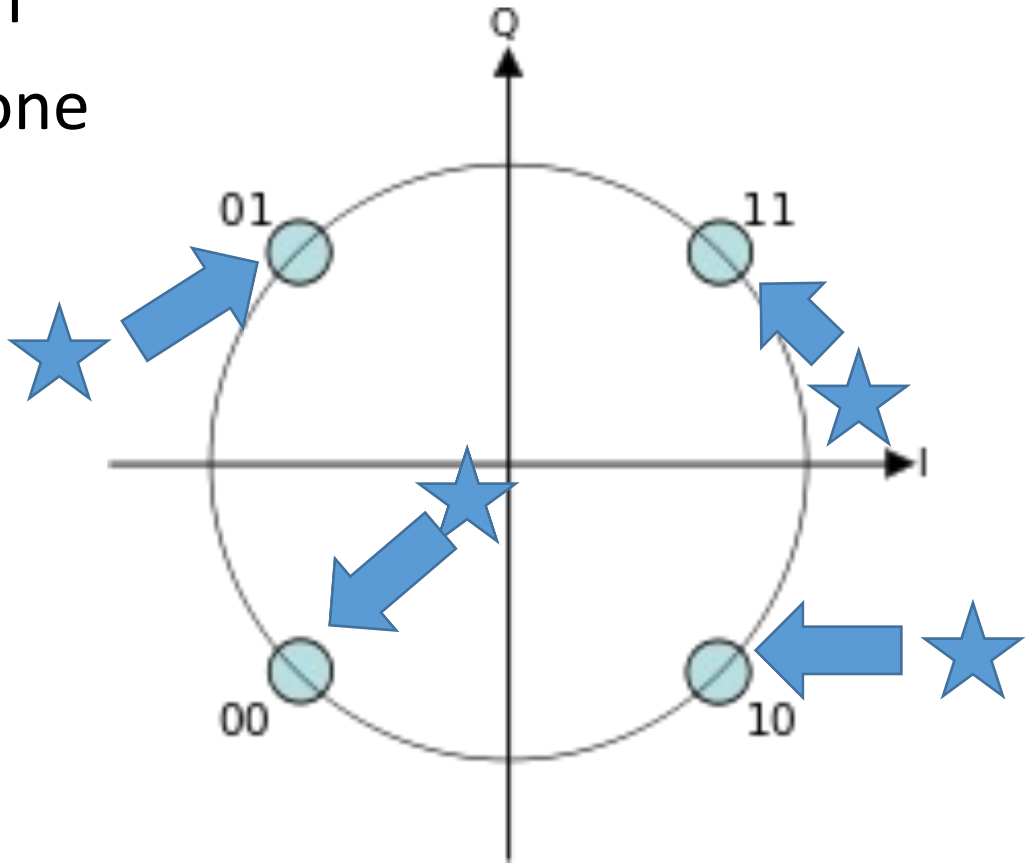
$y : 0.68$



Send : $1.88 \times \cos(2\pi f_c t) + 0.68 \times (-\sin(2\pi f_c t))$

Demodulation

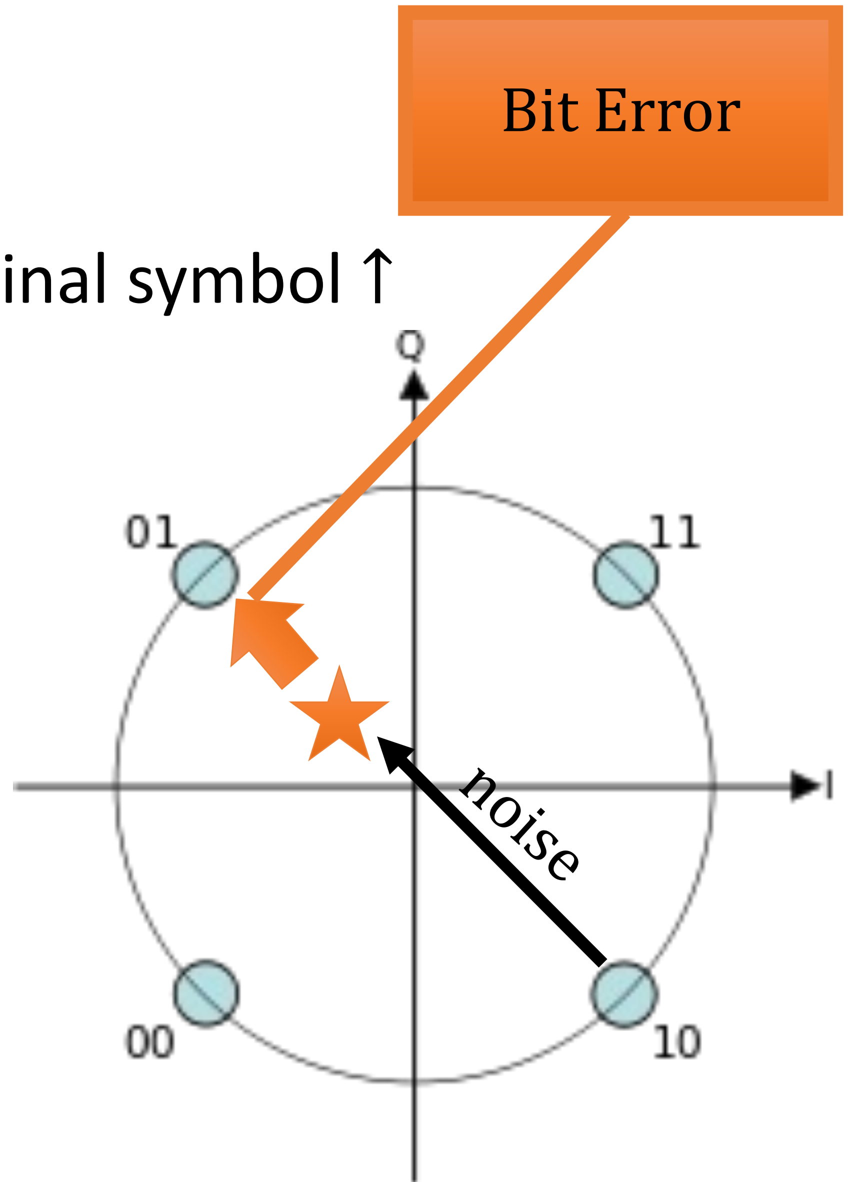
- : the received symbol
- : demodulation
- Choose the closest one



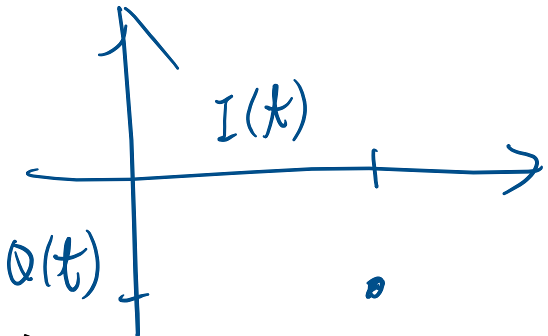
Demodulation

- ★ : bits 10
- $SNR \downarrow \Rightarrow$ distance to the original symbol $\uparrow$

$\Rightarrow$ Bit error rate $\uparrow$



A Closer Look at Demodulation



- The transmit signal $s(t) = I(t)\cos(2\pi f t) + Q(t)\sin(2\pi f t)$
- The receiver gets $s(t)$. How to get $I(t)$ and $Q(t)$?
- $s(t) \times \cos(2\pi f t) =$

$$
\begin{aligned}
I(t)\cos(2\pi f t)\cos(2\pi f t) &+ Q(t)\sin(2\pi f t)\cos(2\pi f t) \\
&= 0.5I(t)(1+\cos(2\pi(2f)t)) + 0.5Q(t)\sin(2\pi(2f)t) \\
&= 0.5I(t) + 0.5I(t)\cos(2\pi(2f)t) + 0.5Q(t)\sin(2\pi(2f)t)
\end{aligned}
$$

- Modulate ($\times \cos(2\pi f t)$) and then use lowpass filter

→ only low-frequency waves pass

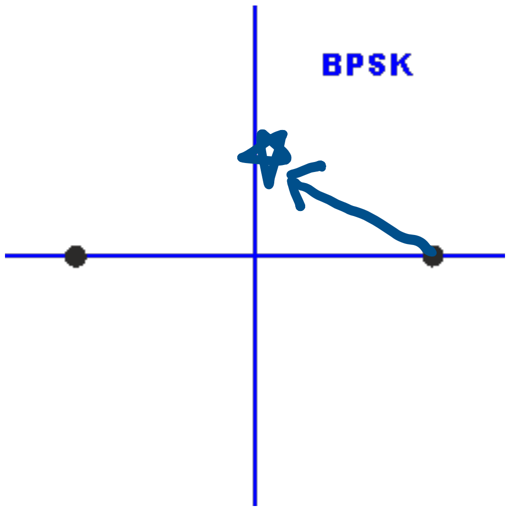
We can send two streams, one by in-phase carrier and another one by quadrature carrier

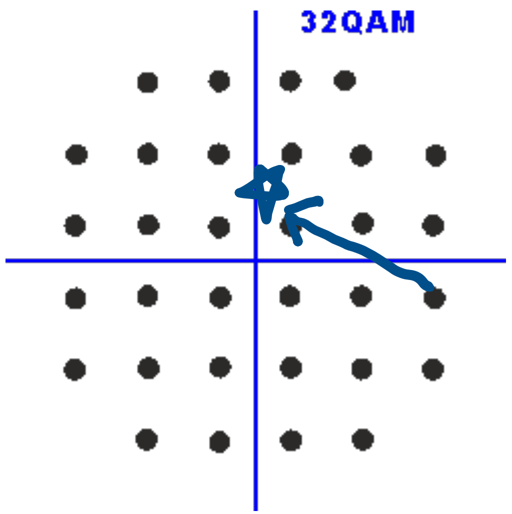
Choose a Proper Modulation

- If SNR is very large, which modulation should be used, BPSK or 32 QAM?
- If SNR is very small, which modulation should be used, BPSK or 32 QAM?

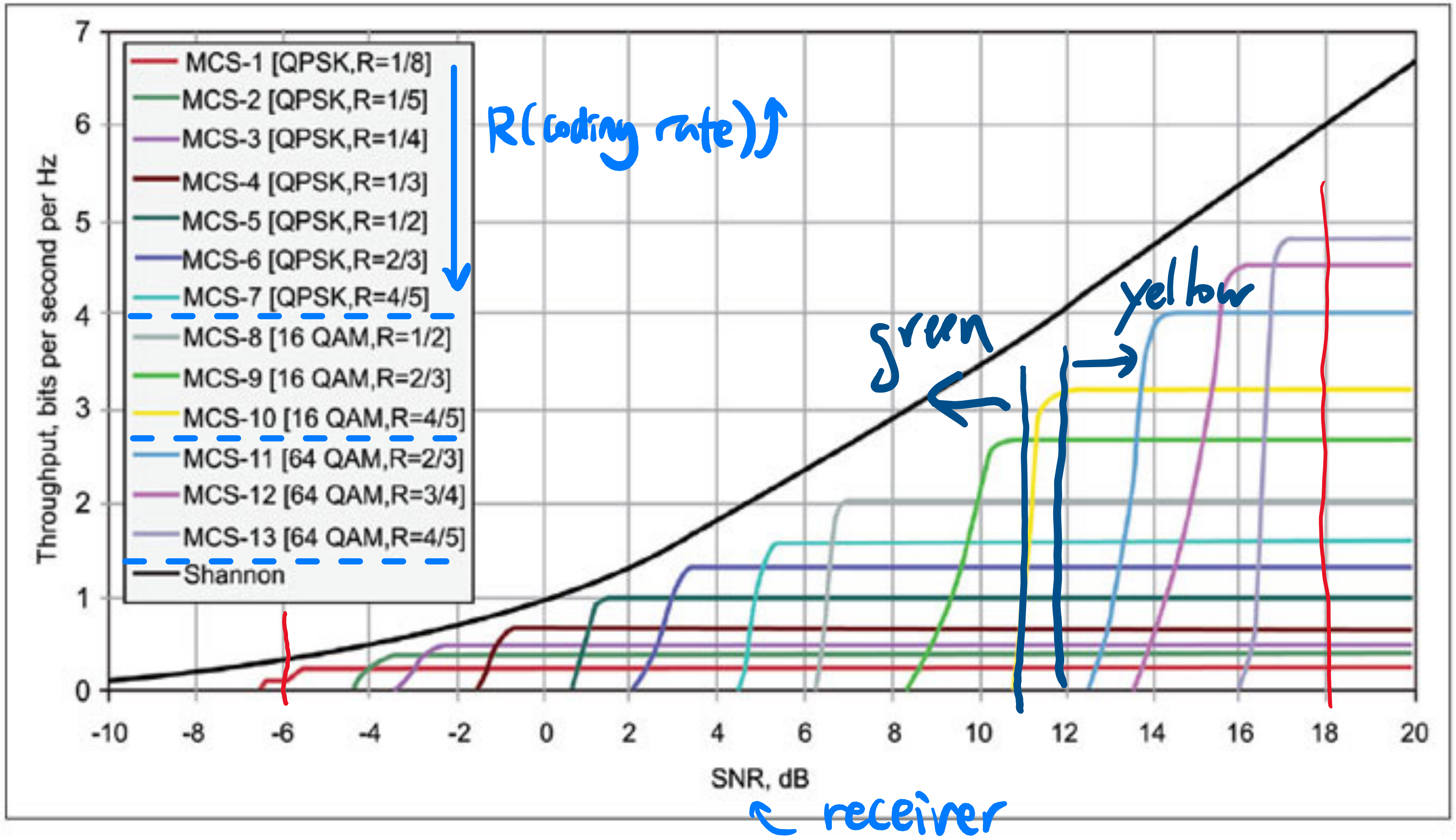
signal to noise ratio

$$= \frac{\text{signal strength}}{\text{noise strength}}$$





Throughput Under Different SNRs



$$SNR = \frac{\text{received signal strength}}{\text{noise strength}}$$

diff. SNR

⇓

diff. modulation.

How to choose a proper modulation?

Coding Rate

$$R: \text{coding rate} = \frac{\#\text{ useful bits}}{\#\text{ bits}}$$

example: parity bit: all bits sum up to 0 (mod 2)

$11011 \longrightarrow 11011\boxed{0}$ parity bit

$10011 \longrightarrow 10011\boxed{1}$ parity bit

$$\text{coding rate} = \frac{5}{6}$$

Parity bit

send $11011 \xrightarrow[\text{encode}]{} 110110$

single-bit error

receive 110010

$1+1+0+0+1+0 \neq 0 \rightarrow$ error

Error Correction

send 1 1 0 1 1 $\xrightarrow{\text{encode}}$ 111 111 000 111 111

single-bit error

coding rate = $\frac{1}{3}$

1 1 0 1 1 $\xleftarrow{\text{decode}}$ 110 111 000 111 111

Error correction for 2-bit error

$1 \rightarrow 11111$

$0 \rightarrow 00000$

coding rate $= \frac{1}{5}$

SNR $\downarrow$ $\Rightarrow$ code rate $\downarrow$ $\Rightarrow$ throughput $\downarrow$


noise
sender
receiver

Adaptive Modulation and Coding

modulation	throughput
BPSK	10
QPSK	3
8QAM	2
16QAM	15
32QAM	1
⋮	⋮

for each frame transmission {

with prob. 0.9 ⇒ use the best modulation M (according to the table)

with prob. 0.1 ⇒ choose modulation M randomly.

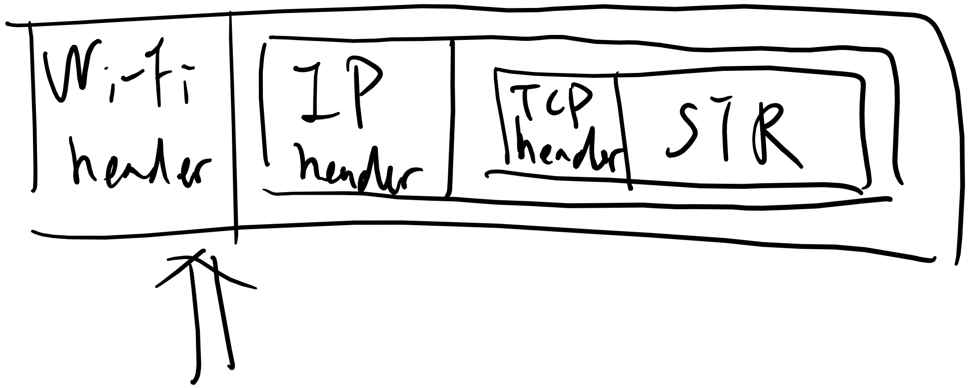
send the frame using M, update M's throughput

}

Example

STR

"application layer packet" "Google map: request "shortest path from A to B""



frame

Update throughput

case 1: frame is received successfully (i.e., sender received an ACK before timeout)

1° compute $\frac{\text{\# useful bits}}{\text{transmission time}}$ for the frame transmission.

↳ tput′

2° M's throughput ← M's original throughput × (1−w)

+ tput′ × w

Update throughput

case 2: transmission failed

1° $tput' = 0$.

2° M's throughput $\leftarrow$ M's original throughput $\times (1-w)$

$+ \; tput' \times w$