Faraday's Law

4. **T** A long solenoid has $n = 400$ turns per meter and carries a current given by $I = 30.0(1 - e^{-1.60t})$, where I is in amperes and t is in seconds. Inside the solenoid and coaxial with it is a coil that has a radius of $R = 6.00$ cm and consists of a total of $N = 250$ turns of fine wire (Fig. P30.4). What emf is induced in the coil by the changing current?

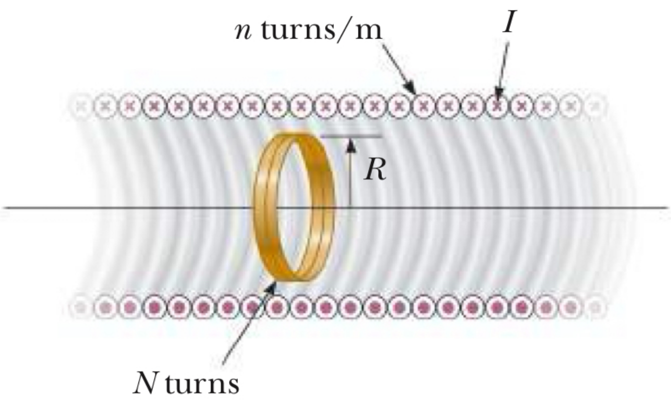

Figure P30.4

13. **S** A metal rod of mass m slides without friction along two parallel horizontal rails, separated by a distance ℓ and connected by a resistor R, as shown in Figure P30.13. A uniform vertical magnetic field of magnitude B is applied perpendicular to the plane of the paper. The applied force shown in the figure acts only for a moment, to give the rod a speed v. In terms of m, ℓ, R, B, and v, find the distance the rod will then slide as it coasts to a stop.

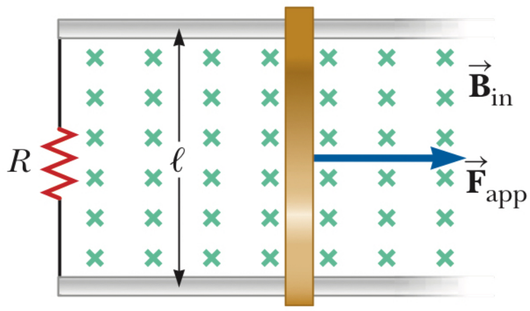

Figure P30.13

4.

$$1^\circ \quad B = \mu_0 n i = (4\pi \times 10^{-7})\, 400 \times 30\, (1-e^{-1.6t})$$
$$= 1.5 \times 10^{-2}\, (1-e^{-1.6t})$$

$$2^\circ \quad A = \pi\, (6\times 10^{-2})^2 = 0.011$$

$$3^\circ \quad \phi_B = N\, B A = 4.1 \times 10^{-2}\, (1-e^{-1.6t})$$

$$4^\circ \quad \varepsilon = -\frac{d\phi_B}{dt} = 6.56 \times 10^{-2}\, e^{-1.6t}$$

13.

$$F_B = F_{app} = i l B \qquad i = \frac{|\varepsilon|}{R}$$
$$= \frac{B^2 l^2 v}{R} \qquad = \frac{B l v}{R}$$

$$-\frac{B^2 l^2 v}{R} = ma = m\frac{dv}{dt}$$

$$\Rightarrow \int_V^{v(t)} \frac{dv}{v} = \int_0^t \frac{-B^2 l^2}{mR}\, dt$$

$$\Rightarrow \ln\left(\frac{v(t)}{V}\right) = \frac{-B^2 l^2 t}{mR}$$

$$\Rightarrow v(t) = V e^{-\frac{B^2 l^2}{mR} t}$$

$$x = \int_0^\infty v(t)\, dt$$

$$= \int_0^\infty v e^{-\frac{B^2 l^2}{mR} t}\, dt$$

Let $x = -\frac{B^2 l^2 t}{mR}$

$\Rightarrow dx = -\frac{B^2 l^2}{mR} dt$

$$= \int_0^\infty v \frac{-mR}{B^2 l^2} e^{x}\, dx$$

$$= \frac{-mRv}{B^2 l^2} \int_{t=0}^{t=\infty} e^{-\frac{B^2 l^2}{mR} t}\, dx$$

$$= \frac{mRv}{B^2 l^2}$$

#

39. Figure P30.39 shows a stationary conductor whose shape is similar to the letter e. The radius of its circular portion is $a = 50.0$ cm. It is placed in a constant magnetic field of 0.500 T directed out of the page. A straight conducting rod, 50.0 cm long, is pivoted about point O and rotates with a constant angular speed of 2.00 rad/s. (a) Determine the induced emf in the loop POQ. Note that the area of the loop is $\theta a^2/2$. (b) If all the conducting material has a resistance per length of 5.00 Ω/m, what is the induced current in the loop POQ at the instant 0.250 s after point P passes point Q?

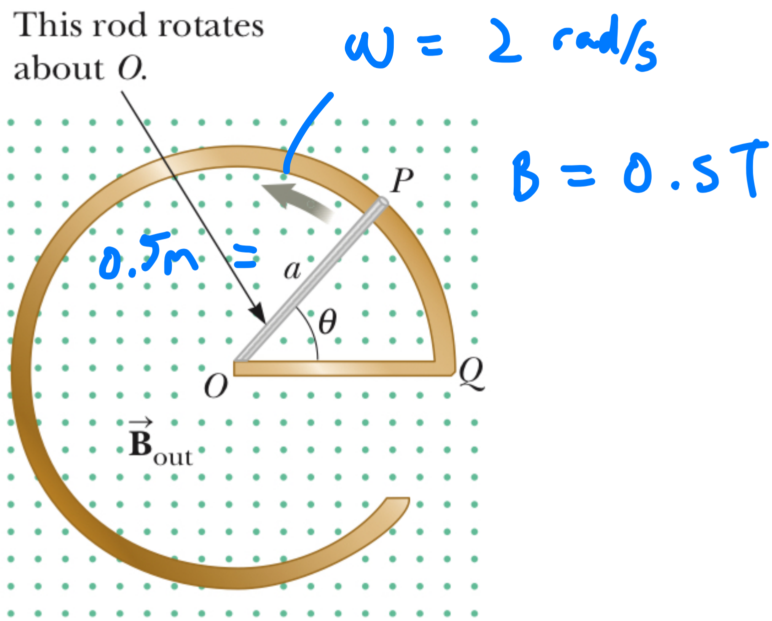

Figure P30.39

Generator

25. The rotating loop in an AC generator is a square 10.0 cm on each side. It is rotated at 60.0 Hz in a uniform magnetic field of 0.800 T. Calculate (a) the flux through the loop as a function of time, (b) the emf induced in the loop, (c) the current induced in the loop for a loop resistance of 1.00 Ω, (d) the power delivered to the loop, and (e) the torque that must be exerted to rotate the loop.

39.

(a) $$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d\left(0.5\cdot\theta\cdot(0.5)^2/2\right)}{dt}$$

$$= (0.5)^4\omega = (0.5)^4\, 2 = 0.125\ (V)$$

(b) 1° $$\ell = 0.5\times 2 + 0.5\,\theta$$

$$\theta = \omega t$$

$$= 1 + 0.5\,\omega t$$

$$= 1 + 0.5(2)(0.25) = 1.25$$

2° $$I = \frac{\varepsilon}{R} = \frac{0.125}{1.25\times 5} = 0.02\ (A)$$

Q

25.

(a) $$\phi_B = BA\cos\omega t$$

$$= (0.8)(0.01)\cos(2\pi\times 60\, t)$$

$$= 8\times 10^{-3}\cos(120\pi t)$$ #

(b) $$\varepsilon = -\frac{d\phi_B}{dt} = -0.96\pi\cos(120\pi t)$$

(c) $I = \frac{|\varepsilon|}{R} = 0.96\pi \cos(120\pi t)$

(d) $P_{avg} = \frac{\varepsilon_{max}^2}{2R} = \frac{(0.96\pi)^2}{2} = 4.6\ (W)$

or

$P = \frac{[-0.96\pi \cos(120\pi t)]^2}{1} = 9.1 \cos^2(120\pi t)$

(e) $\tau_{avg} = \frac{P_{avg}}{\omega} = \frac{4.6}{2\times 60\times\pi} = 1.2\times10^{-2}\ (N\cdot m)$ #

Inductance

3. An emf of 24.0 mV is induced in a 500-turn coil when the current is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil at an instant when the current is 4.00 A?

5. S A self-induced emf in a solenoid of inductance L changes in time as $\mathcal{E} = \mathcal{E}_0 e^{-kt}$. Assuming the charge is finite, find the total charge that passes a point in the wire of the solenoid.

15 The switch in Figure P31.15 is open for $t < 0$ and is then thrown closed at time $t = 0$. Assume $R = 4.00\ \Omega$, $L = 1.00$ H, and $\mathcal{E} = 10.0$ V. Find (a) the current in the inductor and (b) the current in the switch as functions of time thereafter.

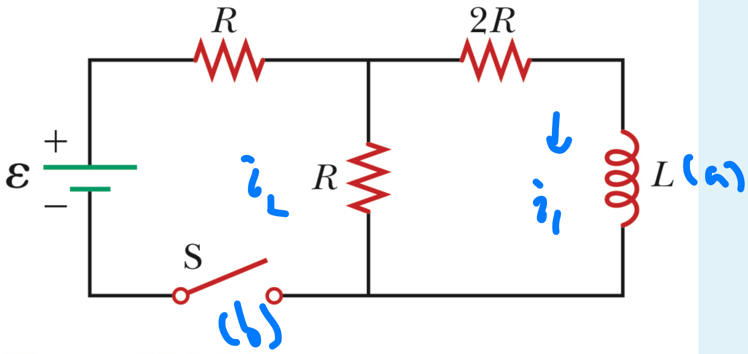

Figure P31.15 Problems 15, 16, and 38.

3.

1° $\mathcal{E} = L\dfrac{di}{dt} \Rightarrow 24\times10^{-3} = L\times10$

$\Rightarrow L = 2.4\times10^{-3}$

2° $2.4\times10^{-3} = \dfrac{500\,\phi_B}{4} \Rightarrow \phi_B = 1.9\times10^{-5}$ (Wb)

5. $\mathcal{E} = \mathcal{E}_0 e^{-kt} = -L\dfrac{di}{dt}$

$\Rightarrow di = -\dfrac{\mathcal{E}_0}{L} e^{-kt} dt$

$\Rightarrow i = -\dfrac{\mathcal{E}_0}{L}\int_0^t e^{-kt} dt$

let $-kt = x$

$\Rightarrow -k\,dt = dx$

$\Rightarrow dt = -\dfrac{dx}{k}$

$= \dfrac{\mathcal{E}_0}{kL}\left[e^{-kt}\right]_0^t$

$= \dfrac{\mathcal{E}_0}{kL}\left(1 - e^{-kt}\right)$

$dq = \dfrac{\mathcal{E}_0}{kL}\left(1 - e^{-kt}\right) dt$

$\Rightarrow q =$

15. (a) $i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

$= \frac{10}{4}\left(1 - e^{-\frac{4}{1}t}\right)$

$= 2.5\left(1 - e^{-4t}\right)$

(b)

RLC Circuit

29. In the circuit of Figure P31.29, the battery emf is 50.0 V, the resistance is 250 Ω, and the capacitance is 0.500 μF. The switch S is closed for a long time interval, and zero potential difference is measured across the capacitor. After the switch is opened, the potential difference across the capacitor reaches a maximum value of 150 V. What is the value of the inductance?

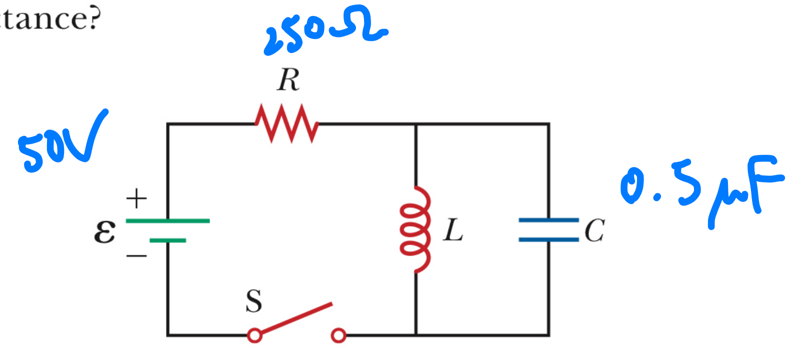

Figure P31.29

37. **S** A capacitor in a series *LC* circuit has an initial charge Q and is being discharged. When the charge on the capacitor is $Q/2$, find the flux through each of the N turns in the coil of the inductor in terms of Q, N, L, and C.

Mutual Inductance

53. **S** Two inductors having inductances L_1 and L_2 are connected in parallel as shown in Figure P31.53a. The mutual inductance between the two inductors is M. Determine the equivalent inductance L_{eq} for the system (Fig. P31.53b).

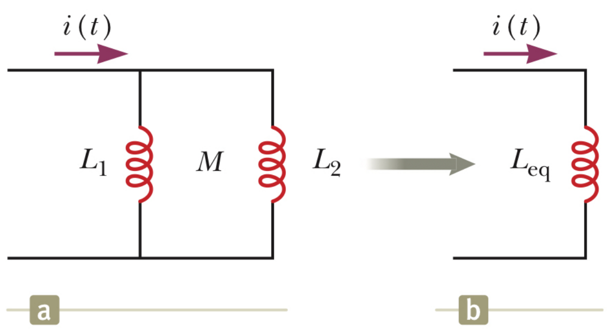

Figure P31.53

29. $$\frac{1}{2} L \left(\frac{50}{250}\right)^2 = \frac{1}{2}(0.5\times 10^{-6})(150)^2$$

$$\Rightarrow L \approx 0.281 \ (H) \ \#$$

37. $$\frac{Q^2}{2C} - \frac{(Q/2)^2}{2C} = \frac{Li^2}{2} \Rightarrow i = \frac{Q}{2}\sqrt{\frac{3}{LC}}$$

$$L = \frac{N\Phi_B}{i} \Rightarrow N\Phi_B = Li = \frac{Q}{2N}\sqrt{\frac{3L}{C}} \ \#$$

53. $$\begin{cases} i = i_1 + i_2 \\ V = V_1 = V_2 \end{cases} \qquad \begin{cases} V_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \\ V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \end{cases}$$

$$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$\Rightarrow (L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$$

$$\Rightarrow \frac{di_1}{dt} = \frac{L_2 - M}{L_1 - M}\frac{di_2}{dt}$$

$$\frac{di}{dt} = \left(\frac{L_2 - M}{L_1 - M} + 1\right)\frac{di_2}{dt}$$

Goal:

$$\boxed{V = L\frac{di}{dt}}$$

$$V = V_2 = L_2 \frac{di_2}{dt} + M \frac{L_2 - M}{L_1 - M} \frac{di_2}{dt}$$

$$= \frac{L_1 L_2 - M^2}{L_1 - M} \frac{L_1 - M}{L_1 + L_2 - 2M} \frac{di}{dt}$$

$$= \underbrace{\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}_{L_{eq}} \frac{di}{dt}$$