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Using Line and Ellipse Features for Rectification of Broadcast Hockey Video

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Abstract—To use hockey broadcast videos for automatic game analysis, we need to compensate for camera viewpoint and motion. This can be done by using features on the rink to estimate the homography between the observed rink and a geometric model of the rink, as specified in the appropriate rule book (top down view of the rink). However, player occlusion, wide range of camera motion, and frames with few reliable key-points all pose significant challenges for robustness and accuracy of the solution. In this work, we describe a new method to use line and ellipse features along with key-point based matches to estimate the homography. We combine domain knowledge (i.e., rink geometry) with an appearance model of the rink to detect these features accurately. This overdetermines the homography estimation to make the system more robust. We show this approach is applicable to real world data and demonstrate the ability to track long sequences on the order of 1,000 frames.

Keywords-Homography; Rectification; Sports; Videos; Geometric error

I. Introduction

Automated sports video analysis is an active and challenging research area in computer vision. One of the important problems in this domain is to automatically estimate player locations and velocities relative to the ground. This information can be used to analyze [1] or even predict [2] game play. The problem is simpler in the case of videos obtained from a stationary camera. In the case of a moving camera, to obtain the trajectories of players on the field or rink (henceforth referred to as the rink), we need to estimate the transformation between the geometric model and each video frame (see Figure 1).

All the images of a plane are related to each other by homographies [3]. Assuming the rink is a planar surface in the world, the geometric model of a rink is also related to its image with a homography. There are various features (lines, markings, logos, etc.) on the rink which can be used to estimate this transformation. Homography estimation given point matches between two images is a well studied problem, but there are no direct point matches available between the geometric model and a video frame (some point matches can be obtained by using curve intersections). However, there are other geometric shapes like lines and circles on
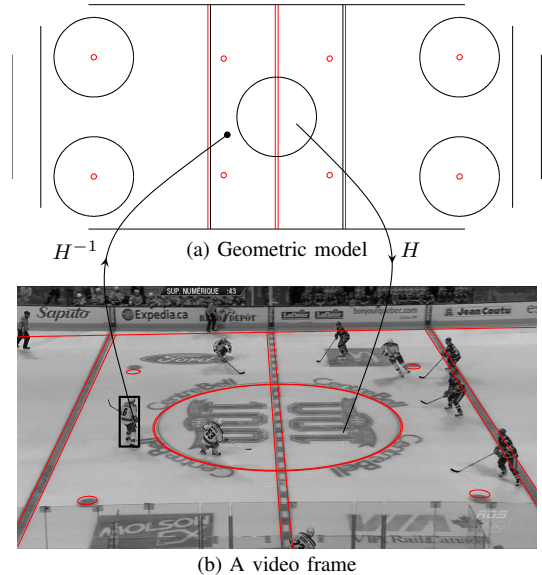

(a) Geometric model H^{-1} H

(b) A video frame

Figure 1. The problem definition: to estimate a best fitting transformation matrix H between (a) the geometric model of the rink and every frame in the sequence. (b) An example frame from the video is also shown with the transformed geometric model superimposed (shown in red). The inverse transformation H^{-1} can be used to map events in the frame coordinates to the world coordinates. This process is known as rectification.

the rink surface which can be utilized to overcome this limitation. Lines transform to lines and circles transform to conics under perspective projection [3]. Please note the transformed conic is an ellipse in all the cases we encounter in this particular problem. These features can be detected and tracked in the sports video.

In this work, we present a novel method to combine point, line and ellipse matches to get a homography estimate by extending the linear method for point matches (the DLT algorithm). We also propose an area-based geometric error measure, which can be minimized to fine-tune our linear estimate. We combine an appearance model (key-frames) with the geometric model of the rink to estimate the homography robustly over time. We test this system

on a hockey video sequence. However, it can be easily generalized to other sports where there are similar features on the playing surface.

This paper is organized as follows. In the next section, we discuss related work. Section III outlines mathematical preliminaries for homography estimation from point and line correspondences. Section IV describes our new approach to combine ellipses in the same framework. We discuss a new area based geometric error measure for homography estimation in Section V. In Section VI, we combine all these methods together to complete our system implementation. Experiments are described in Section VII, followed by discussion in Section VIII.

II. RELATED WORK

We are looking at the problem of sports video rectification. There are similar systems developed for hockey [4], soccer [5], tennis [6], and American football [7]. However, these systems differ in goals and scope. They often comprise multiple modules each dealing with different functionality e.g., feature detection, tracking and homography estimation. We look at the related work in each of these subproblems in the context of sports video rectification.

A. Homography estimation

A homography transformation can be estimated given a set of feature matches between two images. Four or more point correspondences provide enough constraints to obtain the homography using the DLT algorithm [3]. Lines being the dual of points can be similarly used for homography estimation [8]. Dubrofsky and Woodham [9] show how to combine line and point matches in the same image to estimate the homography using the DLT.

Conic correspondences have also been used to estimate homographies as described in [10]–[13]. However, these methods deal only with conics, they do not combine these constraints with other features. Conomis [13] suggests that a new set of invariant points can be obtained using conic correspondences. These point correspondences are then used to estimate the homography using the DLT. It can be shown that two conic correspondences are enough to solve for a homography [11]. Based on these methods ellipse features on the rink can be used to estimate the homography. However, there may not be two ellipses visible in the field of view of the camera in every frame.

The DLT based algorithm for point (and line) matches is fast and easy to implement. However, one major limitation is that the DLT minimizes algebraic error which does not correspond to any geometrically meaningful quantity (see Section III for details). The homography estimate obtained using point matches with DLT is often refined by minimization of geometric error. Transfer error [3] is a commonly used error measure (see Figure 3(a)). However, there is no clear way to deal with combined minimization of geometric error in the case of line and ellipse features.

B. Feature detection and tracking

Detecting and tracking lines is one of the popular methods for estimating homographies over a sequence of frames [14], [15]. On a textureless field like a soccer pitch, lines prove to be useful features. However, usually there are not enough lines visible in each frame to uniquely determine the homography. The idea of using line features (boundary lines) to avoid drift while tracking planar surfaces is explored by Xu et al. [16]. They show that line features make tracking more accurate. However, when they do correction based on lines the point feature information is discarded. Farin et al. [6] use lines to calculate real and virtual points of intersection. These points are used to establish the homography between image and the model. They also define a geometric error measure which they minimize for estimating the homography based on lines. They project the white pixels (court lines in case of tennis) onto the model. The error measure is defined as the sum of the geometric distance between model lines and these projected points.

Okuma et al. [4] also tackle the problem of rink rectification for hockey videos. Their approach is based on tracking point correspondences (using KLT [17]) to estimate the homography between consecutive frames (using RANSAC [18] for robustness). However, this leads to significant drift in homography estimate over time. They correct their estimate based on a geometric model of the rink by generating additional point correspondences. They achieve this by searching for points on the edges in the image along the normals at sampled points on lines and circles in the transformed model (using an approximate homography estimate). These additional point correspondences are then used to estimate the homography using the DLT. The two major limitations of this approach are: first, the nearest point chosen along the normal may not correspond to the actual ellipse or line feature on the frame. Second, final drift correction is based on the DLT; there is no geometric error minimization used to refine the estimate.

Hess and Fern [7] demonstrate that using local features (e.g., SIFT [19]) can also be an alternative way to rectify sports video frames. They use a set of frames as reference images (or key-frames) with a known homography transform (obtained by manually establishing point correspondences). These reference images are then used to assemble a set of local features registered to the rink model. This model with registered key-frames is used to rectify frames based on point matches with each new frame. This approach is robust. However, its effectiveness is subject to the availability of sufficient point features well distributed across the rink. Also, this does not exploit any other information available apart from point matches.

III. PRELIMINARIES

Let $\mathbf{p}_i = \begin{bmatrix} x_i & y_i & w_i \end{bmatrix}^T$ and $\mathbf{p}'_i = \begin{bmatrix} x'_i & y'_i & w'_i \end{bmatrix}^T$ be corresponding points related by a homography, written in homogeneous coordinates. The homography matrix, H, by definition relates these points as

$$\mathbf{p}'_i = H\mathbf{p}_i \quad \forall i \in \{1...n_p\} \tag{1}$$

where n_p is the number of point correspondences and H is a 3x3 matrix given by

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \tag{2}$$

Equation 1 can be rewritten in the form

$$A_i \mathbf{h} = 0 \tag{3}$$

where $\mathbf{h} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 & h_9 \end{bmatrix}^T$ and A_i is a 2×9 matrix given by

$$A_i = \begin{bmatrix} \mathbf{0}^T & -w'_i \mathbf{p}_i^T & y'_i \mathbf{p}_i^T \\ w'_i \mathbf{p}_i^T & \mathbf{0}^T & -x'_i \mathbf{p}_i^T \end{bmatrix} \tag{4}$$

The matrix A_i for each point correspondence can be stacked to form a matrix $A = \begin{bmatrix} A_1 & A_2 & \cdots & A_{n_p} \end{bmatrix}^T$ which satisfies the relation

$$A\mathbf{h} = 0 \tag{5}$$

In case of an over-constrained system, a solution can be obtained by minimizing the cost function (algebraic distance): $\|A\mathbf{h}\|$. This is the DLT algorithm for point correspondences (see Hartley and Zisserman [3] for details).

A. Normalization for points

The DLT algorithm is sensitive to the choice of the coordinate frame (origin and scale). Hartley and Zisserman [3] suggest a normalization step to make the data well conditioned. A similarity transformation, S, is applied to transform points such that their centroid is at the origin and the average distance from the origin is $\sqrt{2}$

$$\tilde{\mathbf{p}}_i = S\mathbf{p}_i \quad \forall i \in \{1...n_p\} \tag{6}$$

where S is defined as

$$S = \begin{bmatrix} s & 0 & t_x \\ 0 & s & t_y \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Corresponding points are also normalized by a similar transform S'. The homography matrix $\tilde{H}$ is computed using the DLT on these normalized correspondences. It is denormalized to get the homography estimate for original correspondences.

$$H = S'^{-1} \tilde{H} S \tag{8}$$

B. Adding lines

A line $ax + by + c = 0$ can be represented as a vector of coefficients $\begin{bmatrix} a & b & c \end{bmatrix}^T$. Using this representation, the transformation of a line $\mathbf{l}_i = \begin{bmatrix} p_i & q_i & r_i \end{bmatrix}^T$ under the homography H is given by

$$\mathbf{l}'_i = H^{-T}\mathbf{l}_i \quad \text{or} \quad \mathbf{l}_i = H^T \mathbf{l}'_i \tag{9}$$

This is analogous to the point case described above and a similar relation as Equation 4 can be obtained. Additional rows corresponding to the line correspondences are appended to the matrix A in Equation 5. However, including lines in the same framework as points requires lines to be normalized with the same similarity transform S. Dubrofsky and Woodham [9] extend point normalization to lines as

$$\tilde{\mathbf{l}}_i = s \begin{bmatrix} p_i \\ q_i \\ sr_i - t_x p_i - t_y q_i \end{bmatrix} \tag{10}$$

Now, these lines can be treated uniformly along with the normalized points to estimate the homography.

IV. ADDING ELLIPSES

The coefficients of a conic cannot be treated in a similar way to lines and points. However, the constraints obtained from ellipses using existing points and lines in the scene can be transformed into additional line and point correspondences.

A. Pole-polar relationship

Let C be a matrix of coefficients of a conic. Any point $\mathbf{x}$ lying on the conic satisfies the relationship $\mathbf{x}^T C \mathbf{x} = 0$. The transformed conic under a homography H is given by

$$C' = H^{-T}CH^{-1} \tag{11}$$

A polar line corresponding to a point $\mathbf{x}$ in the plane is defined as $\mathbf{l} = C\mathbf{x}$. It is straightforward to prove that if two points correspond in two images (transformed by a homography), their polar lines with respect to the corresponding conics in the images also transform under the same homography [13]. Let $\mathbf{x}$ and $\mathbf{x}'$ be two matching points, C and C' be matching conics in the images and $\mathbf{l} = C\mathbf{x}$ be the polar corresponding to pole $\mathbf{x}$ with respect to conic C. The polar in the corresponding image is given by

$$\begin{aligned} \mathbf{l}' &= C'\mathbf{x}' \\ &= (H^{-T}CH^{-1})(H\mathbf{x}) \\ &= H^{-T}C\mathbf{x} \\ &= H^{-T}\mathbf{l} \end{aligned} \tag{12}$$

We can similarly prove that if two lines $\mathbf{l}$, $\mathbf{l}'$ are transformed under a homography, H, then their poles $\mathbf{x}$, $\mathbf{x}'$ with respect to ellipses C, C' also satisfy the relation $\mathbf{x}' = H\mathbf{x}$.
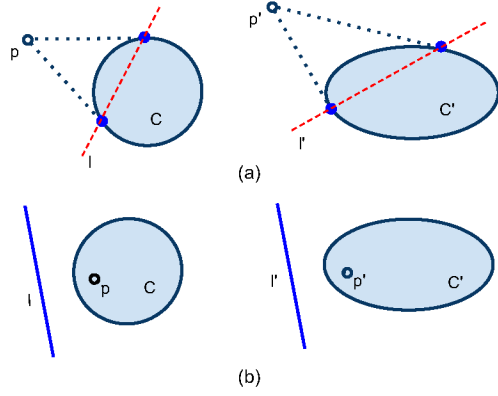
Figure 2. New correspondences generated using points and ellipses. There are two images related by a homography transformation with point correspondences p, p' and ellipse correspondences C, C'. (a) When points lie outside the ellipse, we get two new point correspondences (shown with solid blue dots). (b) When points lie inside or on the ellipse we calculate one new line correspondence (shown as blue solid line).

B. Generating new correspondences

For each corresponding point and line, we generate additional point or line correspondences using an ellipse as follows (also summarized in Figure 2). For each corresponding:

1) *point lying outside the ellipse*: calculate the two points of intersection of its polar with the ellipse (two new points correspondences, see Figure 2(a)).
2) *point lying inside or on the ellipse*: generate a new line correspondence by calculating its polar with respect to the ellipse (one new line correspondence, see Figure 2(b)).
3) *line intersecting the ellipse*: use the corresponding intersection points (two new point correspondences). Please note that instead of these two points we can alternatively use the pole of the intersecting line with respect to the ellipse as the additional point. However, we have not experimentally verified if this would be a better choice.
4) *line not intersecting the ellipse or is a tangent*: calculate the corresponding pole (one new point correspondence).

We repeat the same procedure for each ellipse in the scene. Once all new correspondences are calculated, we append them to the existing matches. These augmented point and line correspondences, now also include constraints due to ellipses, are used to estimate the homography (as described in Section III).

V. Residual Area Minimization

For fine tuning the homography estimate we minimize a geometric error measure. This is based on the residual area between the detected shapes and the projected model. This is analogous to transfer error [3] for point correspondences

(a) Transfer error
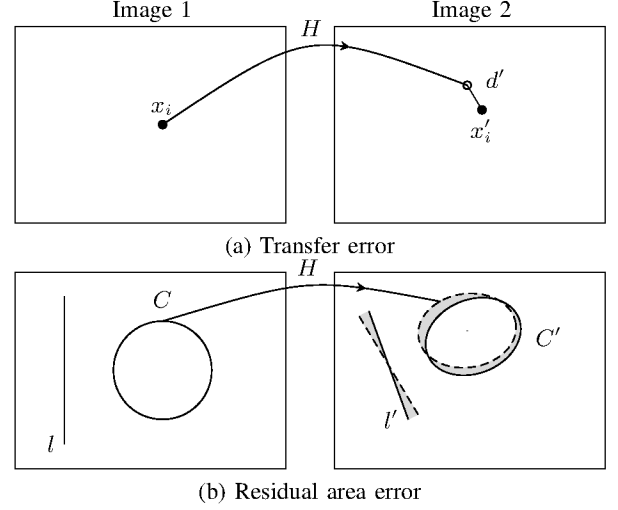

(b) Residual area error

Figure 3. Geometric error measures for homography estimation. (a) Transfer error is defined for point correspondences. We extend this to lines and ellipses by using (b) residual area (area of the shaded region) as our error measure. Dotted lines show features from Image 1 transformed under the homography H.

(see Figure 3(a)). This is closely related to parametric correspondence for chamfer matching [20] which is also effectively minimizing the integral area between the curves.

To define residual area error, let the correspondences between features (lines, ellipses and points) in the model and a video frame be $\{l_i \leftrightarrow l'_i\}$, $\{C_i \leftrightarrow C''_i\}$ and $\{x_i \leftrightarrow x'_i\}$ respectively. For a homography, H, the projections of these model features on the frame are given by

$$\hat{l}'_i = H^{-T} l_i \tag{13}$$
$$\hat{C}'_i = H^{-T} C_i H^{-1} \tag{14}$$
$$\hat{x}'_i = H x_i \tag{15}$$

Using these projections, we can compute the residual area (see Figure 3(b)) for all the matching point, line and ellipse features detected in the frame. We propose the weighted sum of the residual area as our error measure

$$A_{res}(H) = A_l(H) + w_c A_c(H) + w_p A_p(H) \tag{16}$$

where A_l and A_c are the sum of residual area due to all the lines and ellipses features visible in the image. A_p is the sum of geometric errors corresponding to the point correspondences. w_c and w_p are weights to adjust the relative importance of each feature type in the optimization.

To calculate the area between projected model line segments and lines detected in the image, we draw perpendiculars from the two extremes of a projected model line segment onto the detected line. We then calculate the area of resulting triangles or trapezoid. The residual area between two ellipses is calculated by approximating ellipses with small circular

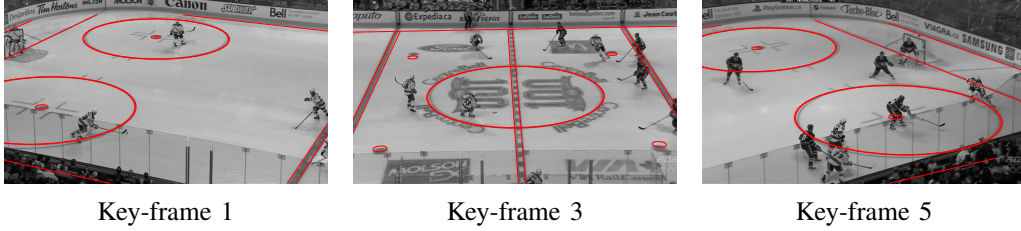|		
Key-frame 1	Key-frame 3	Key-frame 5

Figure 4. The key-frames used in appearance model of the rink. Figure shows three key-frames with the transformed geometric model superimposed. The homography between these frames and the geometric model is obtained by manually selecting point correspondences.

arcs (see Section 5.2 in [21] for details on area calculation). The error term for point matches is defined as

$$A_p(H) = \sum_i d(\hat{x}_i', x_i')^2 \qquad (17)$$

Once we have the area calculation framework in place, the homography estimation problem can be formulated as

$$H_{est} = \underset{H}{\mathrm{argmin}}(A_{res}(H)) \qquad (18)$$

VI. System Implementation

We initialize the system by choosing a set of key-frames. Key-frames are images with overlapping features to cover the whole range of camera motion. In the current implementation, we manually select five frames from the sequence (see Figure 4). We also manually choose point correspondences between key-frames and the geometric model to estimate the homography for all the key-frames.

For each new frame from the video first we identify the closest key-frame. We choose it on the basis of total number of local feature matches between a key-frame and the current frame, combined with the area covered by these matches (see Section 3.2.2 in [21] for details). We use SFOP [22] based key-point detection along with SIFT [19] descriptors to generate point correspondences. We also use these point matches to obtain a rough estimate of the homography between the selected key-frame and the current frame. As we already have the homography for each of the key-frames, we can also calculate an initial homography estimate between the geometric model and the current frame by chaining these two estimates together.

We use this approximate homography estimate to project the geometric model onto the current frame and use the location of transformed lines and circles as the basis to search for line and ellipse features in the frame. This model guided approach simplifies the line and ellipse detection problem (for details see Section 3.3 in [21]). We detect all the lines and ellipses corresponding to the features in the geometric model. However, there are no direct point matches available between the model and the current frame. We solve this problem by back-projecting point matches from the closest key-frame onto the model to obtain a set of point matches. We combine these features (line, point and

ellipse) matches between the model and current frame to obtain a linear estimate for the homography (referred to as H_{lin}) using the approach described in Section IV.

Consecutive frames in the video have a lot of overlapping features (assuming smooth camera motion). We again use SFOP-SIFT based local features to establish point correspondences between the last frame and the current frame. We estimate the homography using these point matches. Given the homography estimate for the last frame, we can multiply it with this frame to frame homography estimate to obtain another estimate for the homography between the model and the current frame. We refer to it as H_{tr}.

We can use one of these estimates (H_{lin} or H_{tr}) as an initial value for the geometric minimization step (described in Section V). As observed by Okuma et al. [4], frame to frame estimation is prone to drift due to accumulation of error. On the other hand, H_{lin} is sensitive to errors in detection. We choose between the two based on the residual area error for each of these initial estimates. A complete system diagram is shown in Figure 5 (see Section 5.3 in [21] for details).

VII. Experiments

We test our system on a high-definition (HD) broadcast hockey video sequence with 1000 frames.

A. Ground truth

It is hard to generate ground truth for all the frames in the dataset. Ground truth in this case means the best possible homography fit for each frame. A good fit has to be visually evaluated by a user, as we do not have a clear way to quantitatively measure it. To simplify this problem, we only annotate a subset of frames from the 1000 frame sequence by selecting point correspondences between these frames and the geometric model. An initial estimate of the homography is obtained by these point matches which is used to detect line and ellipse features on these frames. We further refine the estimate by using geometric minimization of the residual area. The error measure does not go to zero even for these ground truth frames as features never align perfectly with the projected model. We refer to this error as the *ground truth residual area*. These annotated frames represent a close approximation to the perfect transformation
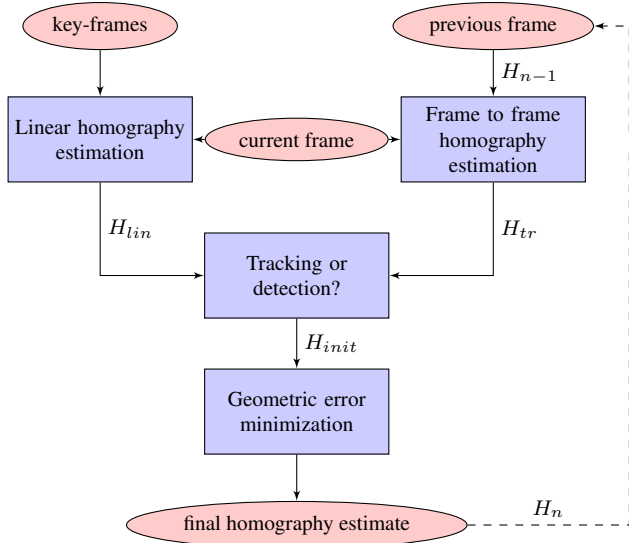
Figure 5. Outline of the system implementation. Ovals represent data and rectangles denote software modules.

between the geometric model and the video. We make sure the frames we choose have line and ellipse detections which are closely aligned with the actual features in the image.

B. Error measure

To evaluate a homography estimate we use the following error measure: we project the geometric model using the homography and calculate the residual area between projected features (only lines and ellipses, no points) and the detections in the ground truth frames. In the subsequent discussion, this error is referred as the *residual area error* for a given homography estimate in a particular frame.

C. Results

We evaluate the quantitative reduction in the residual area error due to this non-linear optimization. In Figure 6 we compare the error in homography estimation after the geometric error minimization to the linear homography estimate. We observe that there is a significant reduction in the error after the optimization step. We also find that the tracking is more stable (observe the variation in the error corresponding to the linear estimate in Figure 6 (top)).

We test our system, using all the components and running it over a long image sequence. Figure 7 (left column) shows a few selected frames from the sequence with the model transformed by the estimated homography superimposed (in red). This shows that we are able to robustly estimate the homography for a long sequence accurately. We also observe that there is no error accumulation. The last frame is well aligned with the projected features from the model (see Frame:1299). This shows that the system can possibly continue to track a longer sequence.

Finally, we demonstrate an application based on our video rectification system (see Figure 7). The right column shows the player trajectories for the last 100 frames in the rink coordinates. Using this approach, given the scale of the geometric model, we can estimate player position and velocity with respect to the ground.

VIII. DISCUSSION

We effectively combine the geometry, appearance and motion information to get a homography estimate between a geometric model of the rink and each frame in the sports video sequence. In this work, we focus on using the geometric shapes in the model as features to estimate the homography. To achieve this, we develop a method to incorporate ellipse features in homography estimation along with line and point features (which have been traditionally used to solve similar problems). We show that the minimization of an area based geometric error measure can be used to refine the linear estimate and stabilize tracking. We also combine the geometric model with an appearance model using the key-frame idea to add robustness to the system.

The results we present show that our system is able to robustly track long sequences of the order of 1000 frames. We have tested our system only on hockey videos. However, as the geometric model of the rink is an input to the system, we expect it can be easily generalized to other sports.

The major limitations of our current system are: we rely on line and ellipse features which are more robust to occlusion and motion blur compared to point matches. However, this makes our approach sensitive to errors in detections. RANSAC [18] can be applied in case of points, dealing with outliers in a mixed correspondence case is a topic for future work. We have also ignored the "normalization for lines" issue highlighted by Zeng et al. [8]. We do not deal with lens distortion in the image. Sports footage may have visible radial distortion and hence straight lines in the real world appear curved in the image, making the assumption of a homography inaccurate. Our method also assumes an accurate geometric model. However, not all rinks conform to the standard specifications. Building a model from the data itself can be an interesting direction for future work.

The problem of automatic rectification holds great challenges and possibilities for interesting research. Even with its limitations, our approach is a significant next step towards combining a wider variety of heterogeneous scene information for homography estimation and also building an application that deals with actual broadcast video data.
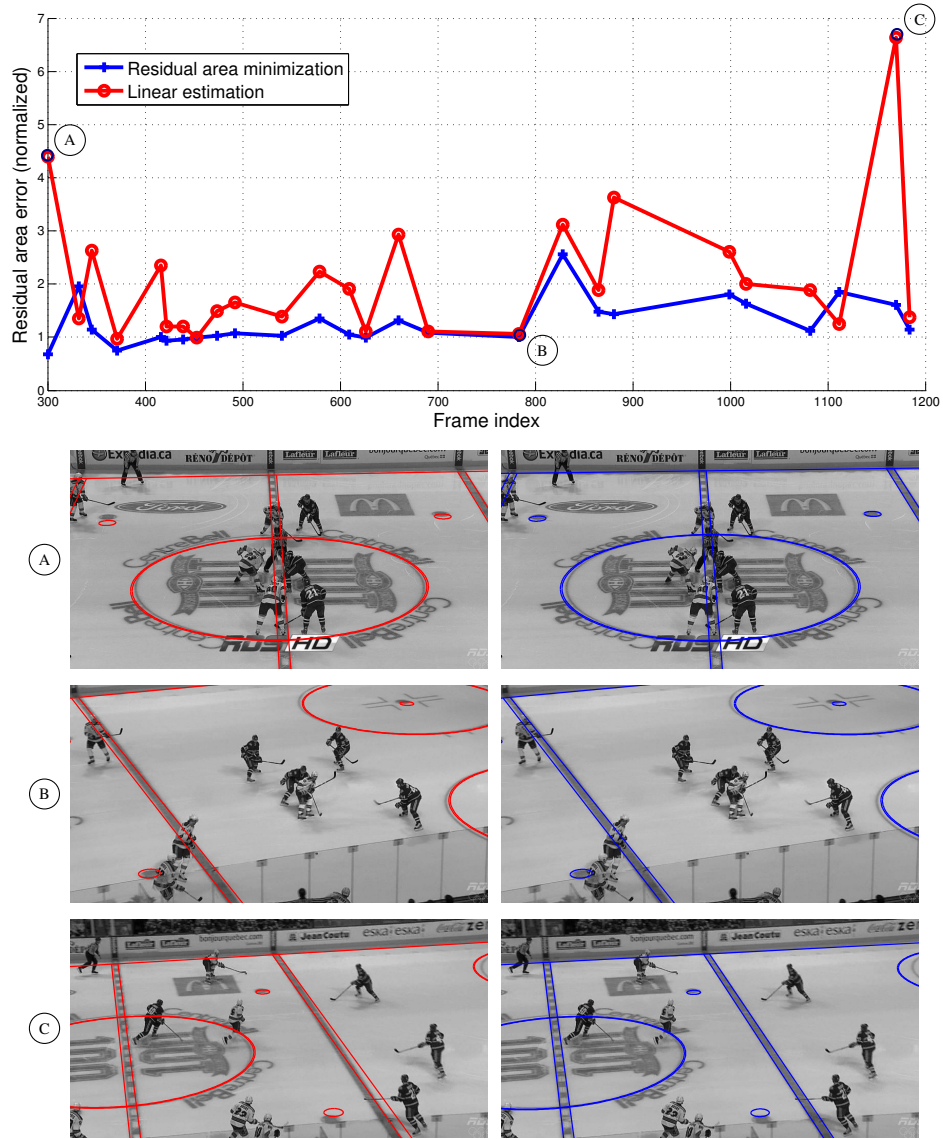
Figure 6. The error in homography estimation after minimization of the geometric error compared with the linear estimate used as the initial value (top). Along the y-axis we have the residual area error, normalized by the *ground truth residual area* (as defined in SectionVII-B). The frame numbers are plotted along the x-axis. We also show homography estimates for three selected frames, denoted by A, B, and C. Left and right column (bottom) show the model superimposed on the frame using linear homography estimate and final output of the system for these three frames.

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Figure 7. Application in automated game analysis: left column shows the system output for selected frames from the 1000 frames long sequence. Player detections obtained using a player tracker are shown in the middle column. Using these player locations in image coordinates combined with the rectification output, we can obtain player trajectories with respect to the ground plane (shown as dots with trails in the right column).

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