Phase Retrieval Under a Generative Prior

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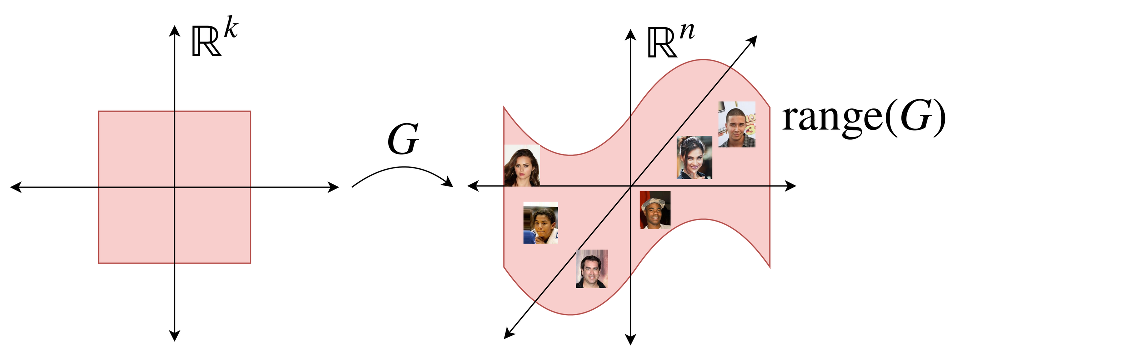
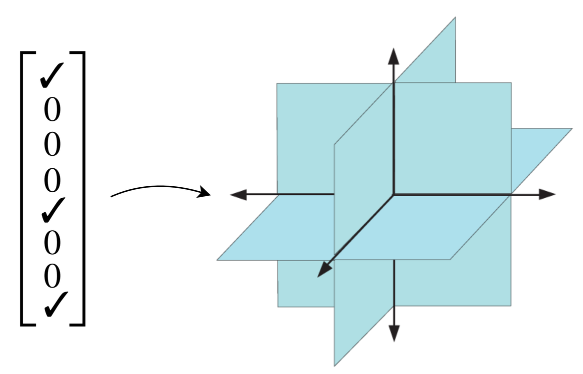
Generative models can outperform sparsity models in signal recovery



Generative models provide explicit low dimensional representation of natural signal manifold



Sparsity-based models view signals as living in union of combinatorially many subspaces

Why generative models in signal recovery?

- Signal dimensionality under generative priors can be smaller than for sparsity priors
- Representation can be directly and efficiently exploited in problem formulation

Case in point: Phase retrieval

No sample efficient sparsity-based algorithm exists BUT we show there does using generative models

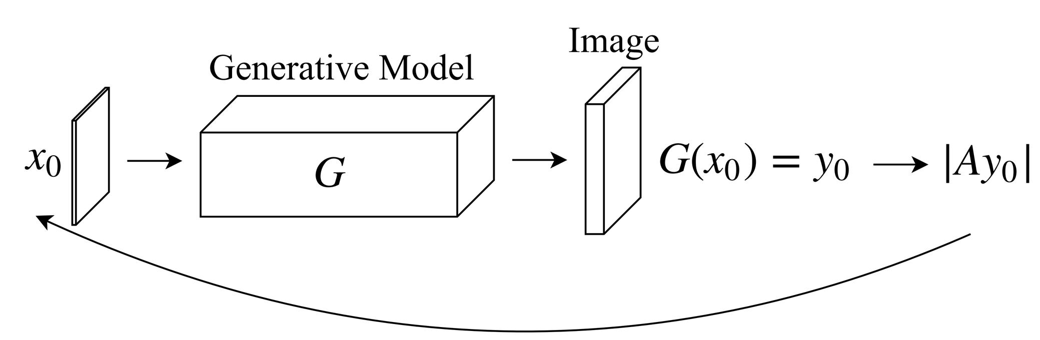
Compressive Phase Retrieval: A problem in which sparsity has not succeeded

$\boxed{\text{Given } m \ll n \text{ meas. } |Ay_0| \text{ find } y_0 \in \mathbb{C}^n}$

Open problem: no algorithm to recover s-sparse y_0 with less than $O(s^2)$ generic measurements exists

- Sparsity creates an $O(s^2)$ computational bottleneck
- But a k-dim. latent code can be recovered from $O(k)$ measurements

Our formulation: Deep Phase Retrieval



Given: matrix $A \in \mathbb{R}^{m \times n}$,
generative model $G : \mathbb{R}^k \to \mathbb{R}^n$,
$m \ll n$ measurements $|AG(x_0)|$,
Find: $x_0 \in \mathbb{R}^k$

Non-convex optimization

Solve

$$\min_{x \in \mathbb{R}^k} f(x) := \frac{1}{2}\Big\| |AG(x)| - |AG(x_0)| \Big\|^2$$

where

$$G(x) := \text{relu}(W_d \ldots \text{relu}(W_2 \text{relu}(W_1 x)) \ldots)$$

Phase Retrieval Under a Generative Prior

Under a generative prior, the empirical risk minimization problem exhibits favorable geometry for gradient descent with optimal sample complexity. In particular, the objective function f exhibits a strict descent direction if $m = O(k)$, the network has sufficiently expansive layers, and the weights are Gaussian with high probability.

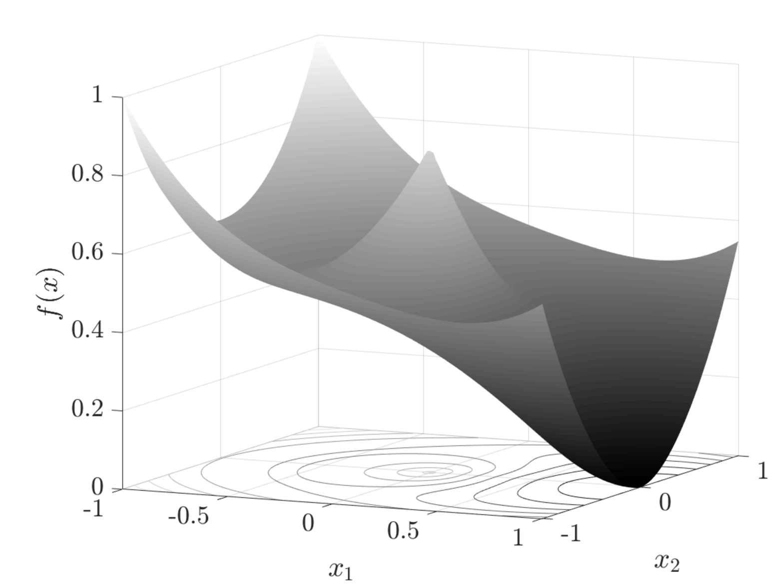
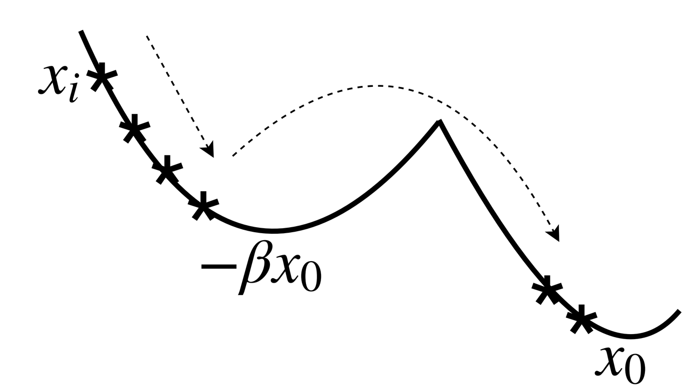


Figure 1: Expectation of f with $x_0 = [1, 0] \in \mathbb{R}^2$

Variation of gradient descent

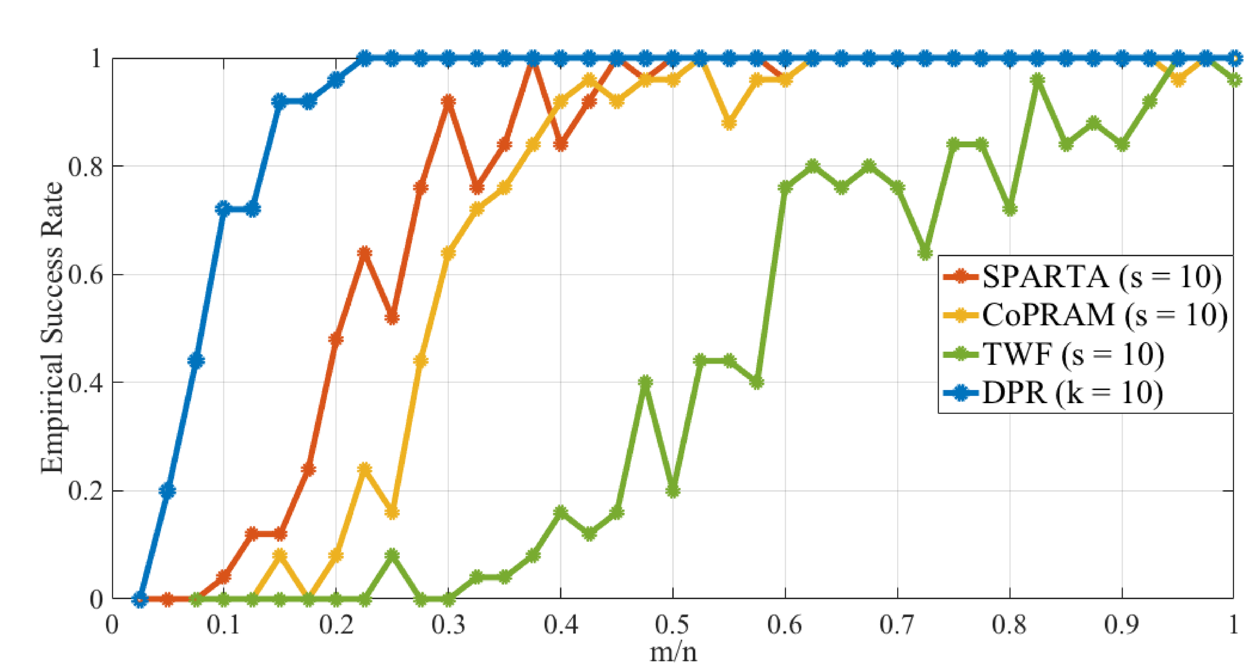
Algorithm 1 DPR Gradient method

Require: W_i, A, $|AG(x_0)|$, & step size $\alpha > 0$
Choose an arbitrary initial point $x_1 \in \mathbb{R}^k \setminus \{0\}$
for $i = 1, 2, \ldots$ **do**
  if $f(-x_i) < f(x_i)$ **then**
    $x_i \leftarrow -x_i$;
  end if
  $x_{i+1} = x_i - \alpha \nabla f(x_i)$
end for



Synthetic experiments

Gaussian signals:



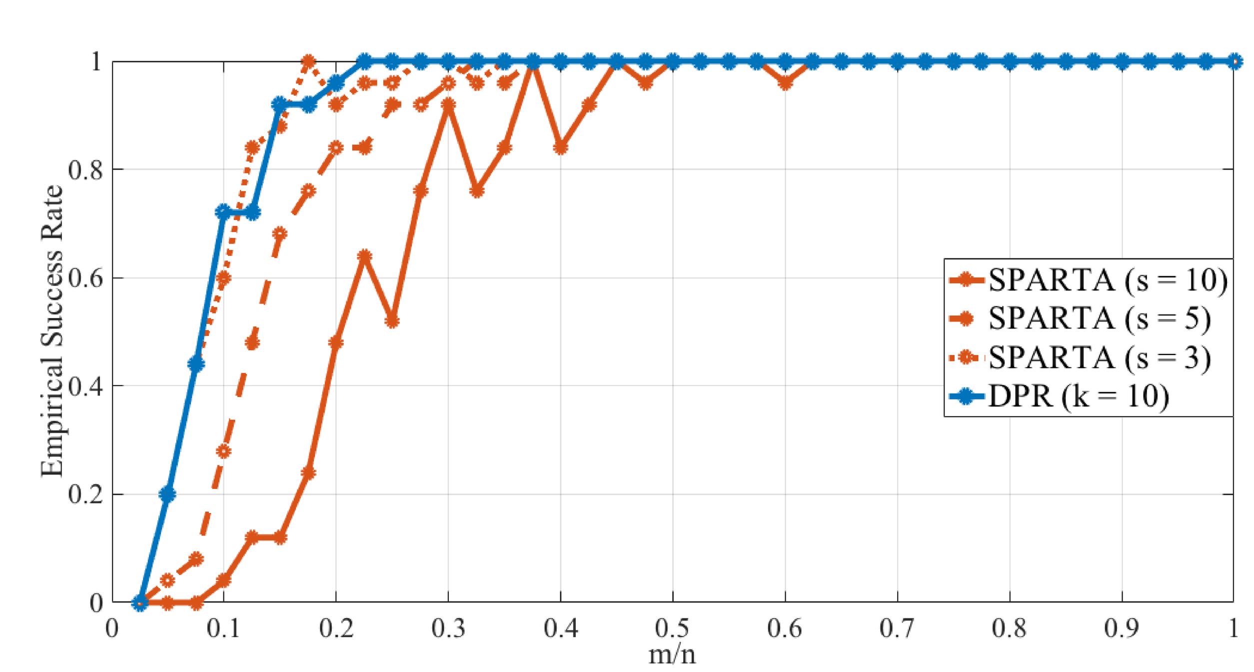
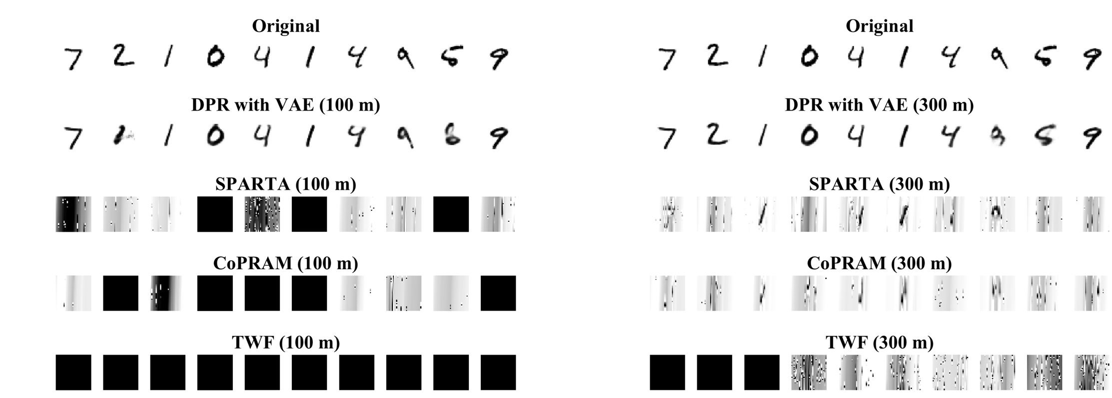
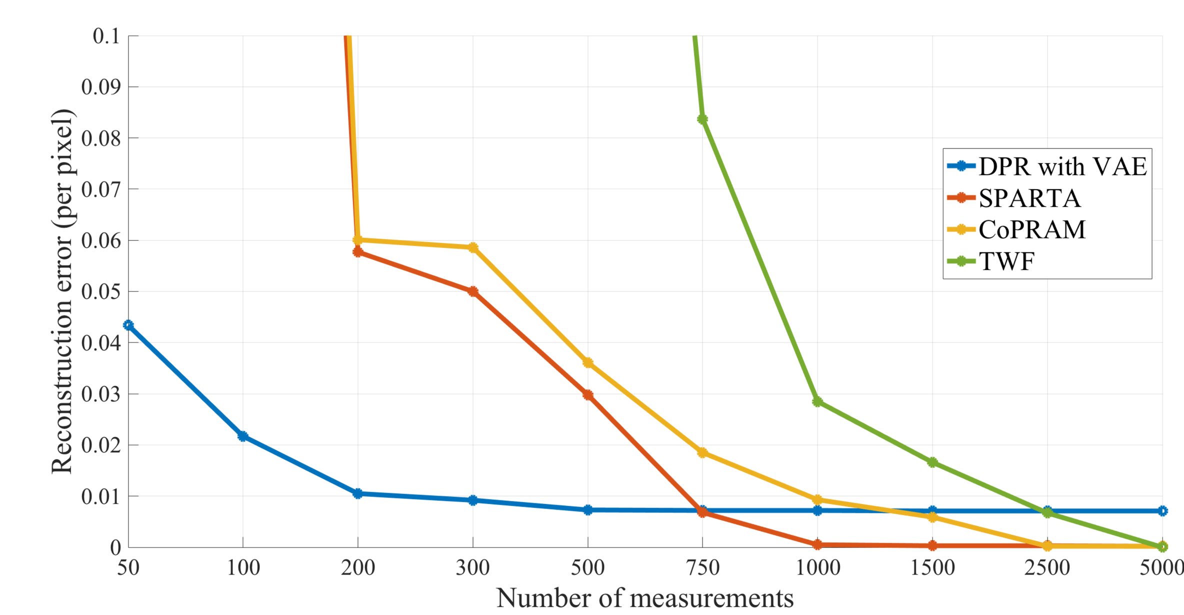
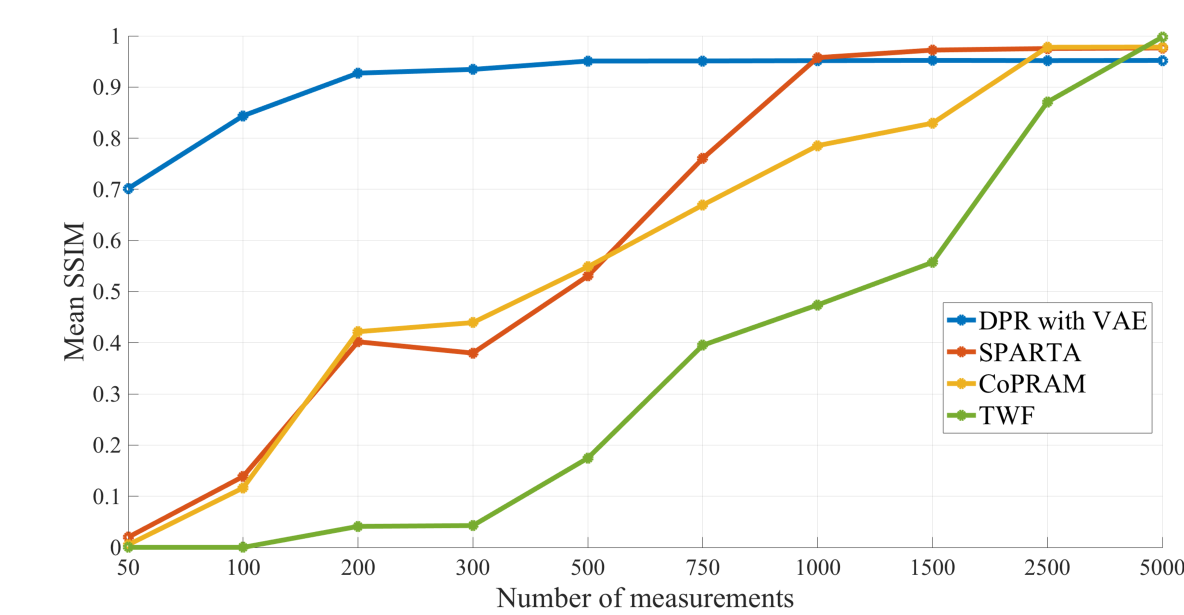


Image experiments

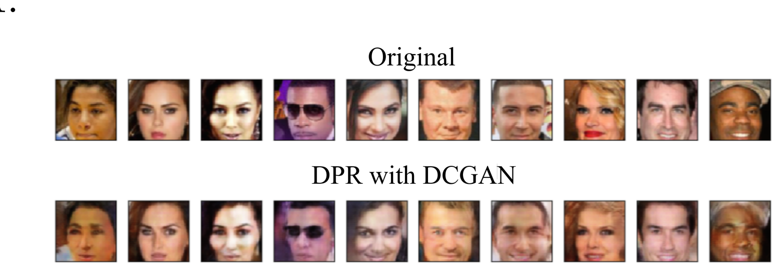
MNIST:







CelebA:



500 measurements

Conclusions

Generative models can solve signal recovery problems with lower sample complexity by:

- providing a lower dimensional representation of the data
- exploiting the low dimensionality directly & efficiently through empirical risk minimization

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