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Sensitivity Analysis and Its Application on DICE Model

A Review on Sensitivity Analysis' Role in DICE Model and Climate Policy

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1 Introduction

Sensitivity analysis aims to find how the change in model input affects model output. Input can include the coefficients, assumptions or more (Saltelli, 2004). Intuitively, if the output O is sensitive to input i_a, then input i_a plays an important role in determining the output O. Therefore, the model development could be improved by deepening the understanding and reducing the uncertainty of input i_a.

Sensitivity analysis can measure the intrinsic uncertainty in the model beyond the uncertainty of the input. It plays a crucial role in the model refinement process, particularly when the model is large and complected as more uncertainty might be introduced in the modeling process. Sensitivity analysis would provide a detailed analysis of how the uncertainty in the outcome is driven by the uncertainties in the model itself (Miftakhova, 2021).

Dynamic Integrated Climate-Economy (DICE) model is developed by Nordhaus in 1992 (Nordhaus, 1992). It is an integrated model that includes the mutual effect between socioeconomic dynamics and climate dynamics. Despite it being the most widely used model in climate policy-making, it was criticised for its limitation on the assumption of utility discounting, technology innovation and population growth (Ackerman et al., 2009). Therefore, the question was raised: what if the parameters or assumptions are different? How different the outcome of the model could be? A sensitivity analysis of those parameters could answer it and, therefore support a better understanding of the integration between economics and climate dynamics.

In this review, the author discusses the Sobol' method in sensitivity analysis itself and how it was used on the DICE model to improve the model and climate policy.

2 Sensitivity Analysis

Generally, there are two types of sensitivity analysis: local sensitivity analysis and global sensitivity analysis (Pianosi et al., 2016). Local sensitivity analysis focuses on the sensitivity analysis of parameters around some input. Global sensitivity analysis tries to measure the sensitivity of the output concerning all the potential inputs.

Sensitivity analysis should be performed on the model while taking the model as a black box (Miftakhova, 2018). The purpose of sensitivity analysis is to understand the parameters' behaviour when it is hard to understand each parameter's detailed behaviour. If each input is easy to measure and studied individually, sensitivity would potentially miss some information and therefore would be not preferred. Therefore, the performance of sensitivity analysis should not assume the model is well studied or has known analytical properties with any loss of generality.

For the DICE model specifically, local sensitivity could misrepresent the sensitivity due to it only representing a small part of feasible parameter space (Butler et al., 2014). Most research has applied the global sensitivity analysis method to DICE, like Sobol' method.

Sensitivity analysis can be done analytically if the model is simplified enough. This is usually not the case for a complicated integrated model. Most of the sensitivity analysis is done numerically. The result of numerical sensitivity analysis is related to the sample used in the analysis (Pianosi et al., 2016). The impact of the sampling method will be discussed in section 2.2.

2.1 Theory of Sobol' Indices

The sensitivity analysis applied to the DICE model mainly took the Sobol' method. This is a variance-based method, that aims at finding how the variance in the output is contributed by the input variable. Here the author provides a summary of such a method.

Consider the model is $y = f(x)$, where y is the output vector in the *output space*, and x is the vector into the *input space*. Sobol method made the following assumptions according to the review of Pianosi et al. (2016):

1. The input vector is probabilistic, therefore this model induces a probabilistic output.
2. Variance of the output is a good approximation of the uncertainty.
3. Under 1 and 2, the contribution of each input's variance towards the output measures the sensitivity.

In summary, the assumption requires both input output and output to be stochastic (a random variable), therefore possible to find variance and expected value. We require those random variables to have a certain degree of well-behave. Therefore the variance of such random variables indeed measures the uncertainty.

Sobol' (2001) formally define such indies using analytical method. Since all model inputs are random variables, we can simplify the model into the following:

The input is a vector in an n-dimensional box K^n and standardized between 0 and 1. That is the input $\mathbf{X} = \{x_1, x_2, \dots x_n\}$ is mapped to $\mathbf{Y}$ by a black box mechanism (a function) f from $K^n = \{\mathbf{X} : 0 \leq x_i \leq 1, i = 1, \dots, n\}$ to the output space. There exist a set of functions $f_{\mathbf{A}}$ where $\mathbf{A} \subset \{1, 2, \dots, n\}$ that maps the input entries $\{x_i\}$, $i \in \mathbf{A}$ to the output space, such that

$$f(\mathbf{X}) = f_0 + \textstyle\sum_{i=1}^{n} f_i(x_i) + \sum_{1\leq i \leq j \leq n} f_{ij}(x_i, x_j) + \dots f_{12\dots n}(x_1, x_2 \dots, x_n)$$

while the following condition holds Sobol' (2001):

$$\textstyle\int_0^1 f_{i_1 \dots i_s}(x_{i_1}, \dots, x_{i_s}) dx_{i_k} = 0$$

The above statement argues a n-dimension function can be decomposed into the summation of a sequence of orthogonal functions that maps all the elements of the input power set to the output space.

If such a function is integrable, the following condition holds:

$$f_i(x_i) = \textstyle\int_0^1 \cdots \int_0^1 f(\mathbf{X}) d\mathbf{X}_{-i} - f_0$$
$$f_{ij}(x_i, x_j) = \textstyle\int_0^1 \cdots \int_0^1 f(\mathbf{X}) d\mathbf{X}_{-ij} - f_i(x_i) - f_j(x_j) - f_0$$

While $d\mathbf{X}_{-ij\dots l}$ implies the integral over all input except for $i, j, \dots, l$ (Archer et al., 1997).

The expected value μ of this continuous random variable is $\int f(\mathbf{X}) = \int_0^1 f_0 + \sum \int_0^1 f_{i_1 \dots i_s}(x_{i_1}, \dots, x_{i_s}) dx_{i_k} = f_0$ as $\int_0^1 f_{i_1 \dots i_s}(x_{i_1}, \dots, x_{i_s}) dx_{i_k} = 0$ mentioned. Therefore, the variance of this random variable is:

$$D = \textstyle\int_{K^n} f(\mathbf{X}) d\mathbf{X} - \mu^2 = \int_{K^n} f(\mathbf{X}) d\mathbf{X} - f_0^2$$

For the variance caused by some input variable's function term, we apply the following calculation:

$$\begin{aligned}
D_{i_1 \dots i_s} &= var(f_{i_1 \dots i_s}(x_{i_1}, x_{i_2}, \dots x_{i_s})) \\
&= \int f^2_{i_1 \dots i_s}(x_{i_1}, x_{i_2}, \dots x_{i_s}) dx_{i_1} \dots dx_{i_s} - \int f_{i_1 \dots i_s}(x_{i_1}, x_{i_2}, \dots x_{i_s}) dx_{i_1} \dots dx_{i_s} \\
&= \int_0^1 \cdots \int_0^1 f^2_{i_1, \dots, i_s}(x_{i_1}, \dots x_{i_s}) dx_{i_1} \dots dx_{i_s} \\
&\text{Since} \int f_{i_1 \dots i_s}(x_{i_1}, x_{i_2}, \dots x_{i_s}) dx_{i_1} \dots dx_{i_s} = 0 \text{ due to the orthogonality.} \\
&\text{The } D_{i_1 \dots i_s} \text{ has the following property:} \\
D &= \sum D_{i_1 \dots i_s} \text{ if } i_1 \leq i_2 \leq \dots \leq i_s \text{ and } i_j \in [1, 2, \dots n].
\end{aligned}$$

Intuitively, this statement suggests that each function in the decomposition form of $\mathbf{Y} = f(\mathbf{X})$ has a contribution towards the total variance. For example, for the function $f_{i_1, \dots, i_s}(x_{i_1}, \dots, x_{i_s})$, its corresponding variance $D_{i_1, \dots, i_s}$ measures the contribution of such function towards the total variance in D.

The notion of sensitivity for the terms $x_{i_1}, x_{i_2}, \dots x_{i_s}$ is $S_{i_1 \dots, i_s}$, and $S_{i_1 \dots, i_s} = \frac{D_{i_1 \dots i_s}}{D}$ and is defined as global sensitivity indices (Sobol' indices) by Sobol' (2001). Integer s is known as the dimension of this index. Sobol' indices are now known as first-order indices or main effect usually notated as:

$$S^F_{i_1\ldots,i_s} = \frac{D_{i_1\ldots i_s}}{D}$$

Homma and Saltelli (1996) induce another set of indices induced from Sobol's work, known as total-order indices or total effect, defined as following:

Say the research intends to study the sensitivity around input variable x_i, following the notion presented in the previous part, we can separate the functions by whether or not they include x_i. That is (Homma and Saltelli, 1996):

$$f(\mathbf{X}) = f_0 + f_{x_i}(x_i) + f_{-x_i}(\mathbf{X}_{-x_i}) + f_{-x_i,x_i}(\mathbf{X}_{-x_i}, x_i)$$

Where the first term address the constant, the second term address the functions that only contain x_i, the third term address the functions that do not contain x_i and the fourth one address functions that contain x_i but not only x_i. The functions $f_{x_i}(x_i)$, $f_{-x_i}(\mathbf{X}_{-x_i})$ and $f_{-x_i,x_i}(\mathbf{X}_{-x_i}, x_i)$ are all orthogonal as they could be represented by the summation of a sequence of orthogonal functions. Homma and Saltelli (1996) defined the total-order indices as all the variance that x_i has a contribution in. The following is a formalization:

$$\begin{aligned}
\text{Since: } f(\mathbf{X}) &= f_0 + f_{x_i}(x_i) + f_{-x_i}(\mathbf{X}_{-x_i}) + f_{-x_i,x_i}(\mathbf{X}_{-x_i}, x_i) \\
\text{Then we have: } D &= D_i + D_{-x_i} + D_{-x_i,x_i} \\
S^T_{x_i} &= \frac{D_i + D_{-x_i,x_i}}{D} = 1 - \frac{D_{-x_i}}{D} \\
&= S^F_i + S^F_{-x_i,x_i} = 1 - S^F_{-x_i}
\end{aligned}$$

In a general sense, without the notion of integral, Pianosi et al. (2016) offered the following definition of first-order and total-order global sensitivity indices:

$$\begin{aligned}
S^F_{x_i} &= \frac{V_{x_i}[E_{x_{-i}}(y|x_i)]}{V(\mathbf{y})} = 1 - \frac{E_{x_i}[V_{x_{-i}}(y|x_i)]}{V(\mathbf{y})} \\
S^T_{x_i} &= \frac{E_{x_{-i}}[V_{x_i}(y|x_{-i})]}{V(\mathbf{y})} = 1 - \frac{V_{x_{-i}}[E_{x_i}(y|x_{-i})]}{V(\mathbf{y})}
\end{aligned}$$

Where V implies the variance, and $E(X)$ is the expected value of the random variable X.

For $S^F_{x_i}$, the contribution of $f_i(x_i)$'s variance towards the total variance is equivalent to the expected reduction of total variance by constraining x_i (Pianosi et al., 2016). While $S^T_{x_i}$ intend to measure the contribution of all terms including x_i. It could be interpreted as an expected variance increment if everything else but changes x_i. The equivalent in the equations could be interpreted by the total variance theorem:

$$V_{x_i}[E_{x_{-i}}(y|x_i)] + E_{x_i}[V_{x_{-i}}(y|x_i)] = V(\mathbf{y})$$

2.2 Numerical Estimation and Sampling Methods

First-order and total-order indices' value is hard to compute analytically, most popular numerical estimation method is Monte-Carlo estimation (Nossent et al., 2011). Such estimation requires some specific sampling strategies.

Say there is a set of input $\{\mathbf{X}_m\}$ with sample size p is a sample with proper sampling method. An estimation of $D = \int_{K^n} f(\mathbf{X})d\mathbf{X} - f_0^2$ is $\hat{D} = \frac{1}{n-1}\sum_{m=1}^{p} f^2(\mathbf{X}_m) - f_0^2$ (Nossent et al., 2011). Where $\hat{D}$ is the estimation of total variance.

Saltelli et al. (2010) Suggested then use 2 $p \times n$ matrix called sampling matrix M_1 and re-sampling matrix M_2 to compute the variance to decrease the sampling bias when applying Monte-Carlo integration. Here p is the sample size and n is the number of input variables. Each column vector in M_1 or M_2 is one sample in the input.

Homma and Saltelli (1996) suggested the following estimation:

$$\hat{f}_0^2 = \frac{1}{p}\sum_{m=1}^{p} f(x_m^{M_1}) * f(x_m^{M_2})$$

$$\hat{D} = \frac{1}{2(n-1)}\sum_{m=1}^{2p} f^2(x_m^{M_1,M_2}) - \hat{f}_0^2$$

$$\hat{D}_i = \frac{1}{n-1}\sum_{m=1}^{p} f(x_{-im}^{M_1}, x_{im}^{M_1}) * f(x_{-im}^{M_2}, x_{im}^{M_2}) - \hat{f}_0^2$$

$$\hat{D_{ij}} = \frac{1}{n-1}\sum_{m=1}^{p} f(x_{-im}^{M_1}, x_{im}^{M_1}) * f(x_{-jm}^{M_2}, x_{jm}^{M_2}) - \hat{f}_0^2 - \hat{D}_i - \hat{D}_j$$

Where p is the sample size, $x_m^{M_a}$ is the m^{th} sample in the matrix M_a, $x_m^{M_1,M_2}$ is the m^{th} sample in the combined $m \times 2p$ matrix by M_1 and M_2. $(x_{-jm}^{M_a}, x_{jm}^{M_b})$ is the m^{th} sample in matrix M_a expect it's j^{th} column is replaced by the j^{th} column of M_b.

Such calculation allows a numerical estimation of the analytical result based on some basic sampling matrix. M_1 and M_2 played an important role in this method and should satisfy some quality to make this sample is representative enough.

Sobol' quasi-random sampling technique (Sobol, 1976), also known as quasi-random Monte Carlo sampling was used to find the appropriate sample of M_1 and M_2 (Nossent et al., 2011). The intuition is to use the Monte-Carlo random sampling to achieve random sampling in each input sample dimension. That is, for each sample, each entry is uniformly distributed. This method is only representative if there are plenty of samples, therefore to achieve a representative, it takes a lot of samples. A quasi-random Monte Carlo sampling is a lot more efficient.

According to Nossent et al. (2011)'s summarizing, such a quasi-random Monte-Carlo method picks an initial starting point and puts all the points in a convergent sequence. Such sequences perform a more organized sampling converging at a higher ratio, therefore the samples spread out in the input space more uniformly, presented almost as a grid, but with a higher degree of randomness and more likely to be representative.

Overall, such numerical estimation method proposed by Homma and Saltelli (1996); Saltelli et al. (2010) and the quasi-random Monte-Carlo sampling (Sobol, 1976) provides an achievable way to calculate the sensitivity of some input without knowing the detailed decomposition of an analytical model. This is consistent with the black-box mechanism of sensitivity analysis in general by allowing the heuristic in the model process.

It is possible to estimate the confidence interval of the Sobol' indices using the bootstrap method (Nossent et al., 2011). Here the author does not intend to discuss it further as the confidence interval was only included in one piece of work during the research.

2.3 Benefits and Restrictions

Before the maturity of Sobol' variance-based method, the sensitivity analysis relies on the one-at-a-time (OAT) method. Analytically, such method requires a "solution" $\mathbf{Y}^*$ of the objective function $f(\mathbf{X})$ and say such solution is $\mathbf{X}^* = (x_1^*, x_2^* \dots x_n^*)$. OAT method changes some x_i^* locally and detects its effect on the output $\mathbf{Y}$ and defines the change as the sensitivity.

OAT method is a lot more simple to calculate compared to Sobol' method. It is more intuitive and the effect of one parameter is directly measured. However, its requirement on solution sometimes has no meaning in the model if the solution is not optimal or equilibrium due to the structure of the model. Therefore the corresponding $\mathbf{X}^*$ might not be representative of the entire model. This is the major criticism towards the OAT method or local sensitivity analysis generally.

The Sobol' method aims to reduce the limitation of local sensitivity analysis by not requiring a solution in the analysis process. Indeed it achieved such a goal. Sobol' translated the impact of some input x_i into the contribution of x_i towards the total variance by the function's Fourier decomposition is well supported by theories despite the computation complexity. A necessary development in the numerical estimation method should remove the requirement of such complex decomposition as we assume the model is a black-box and impossible to calculate the precise notion of decomposed function. The Monte-Carlo integration using sampling re-sampling matrices and quasi-random Monte-Carlo sampling method provides such well-performed estimation and, at the same time greatly reduces the calculation complexity.

However since the method assume the variance represent the uncertainty and treat all the input variable as a stochastic random variable, the uncertainty aspect of analysis cannot be separated from sensitivity analysis. Pianosi et al. (2016) suggested sensitivity analysis does not necessarily include the uncertainty aspect. Therefore, the sensitivity that does not include uncertainty is impossible to calculate using Sobol' method.

Other major criticisms towards Sobol' method are its computational complexity and the implicit assumption that variance captures all the uncertainty (Pianosi et al., 2016). In the following performance of performing sensitivity analysis on the DICE model, Ackerman et al. (2009) has shown that the computation of Sobol' indices is affordable and doable with proper performance. Regarding the implicit assumption, it should be assessed case by case, depending on the data used to fit the model, the object of the item being measured, etc.

3 DICE Model

One of the applications of the DICE model is the integrated assessment model (IAM) (Miftakhova, 2018). IAM is the model that integrates the main aspect of several related models and discovers their relationships with each other. Due to its complexity and huge uncertainty in each step of the modelling, sensitivity analysis would be a good approach to assess the model.

Dynamic Integrated Climate-Economy (DICE) model is developed by Nordhaus in 1992 (Nordhaus, 1992). It is an integrated model that includes the mutual effect between socioeconomic dynamics and climate dynamics. Despite it being the most widely used model in climate policy-making, it was criticised for its limitation on the assumption of utility discounting, technology innovation and population growth (Ackerman et al., 2009). Therefore, the question was raised: what if the parameters or assumptions are different? How different the outcome of the model could be? A sensitivity analysis of those parameters could answer it and, therefore support a better understanding of the integration between economics and climate dynamics.

The DICE model as the most popular IAM in the field of climate policy has demonstrated the possibility to model the social economics dynamics, welfare, carbon emission and the climate together. This complected model has many uncertainties in measuring the impact of production towards carbon, climate change's damage toward welfare etc, making it hard (and almost unnecessary) to solve the model itself directly but still requires a high level of understanding in each input to provide a general direction (e.g. increase or decrease) and magnitude (e.g. aggressive and harsh, or reserving and gentle) of adjustment towards the current policy regulation.

3.1 Overview

The original DICE model developed by Nordhaus (1992) contains 2 parts: an economics module and a climate module. The economics part contains the factors that generate the social production, for example, population, technology, capital stock, labour participation, initial national production output (GDP at period 0), etc. It also contains the damage it causes and the objective function the model intends to optimize.

The climate part of the model consists of 2 parts: carbon cycle and climate dynamics. The carbon cycle is directly affected by the production behaviour. The climate dynamics are affected by the carbon cycle and would affect the damage directly. Further study suggested it also impacts population growth (Repetto and Easton, 2014). The climate sector also includes a Greenhouse gas reduction cost function, which measures the potential investment that could be used to reduce carbon emissions.

The DICE model has some intrinsic assumptions, that challenged its constructional validity to a degree. For the economics component, the model assumes a neoclassical model with hyperbolic discounting utility over generation. This assumption implies the following (Butler et al., 2014);

- Every participant in this economy has a well-behaved utility function and always behaves rationally according to their utility function.
- Every participant cares about the future generation to some degree. As the generation goes, they care about them less and less proportional to the previous generation, but will never not care about them all at all.
- The production function is a Cobb-Douglas function, meaning it will be a constant return to scale, concave at all intervals.

- There is only 1 producer in this economy, which transforms the world into a big factory. This producer only produces 1 commodity (in this case, dollars).

These are all extremely strong assumptions in economics modelling and has been challenged by experts from economics background multiple times. The second assumption on discounted generational utility is essential towards the optimization as it makes sure the objective function on individual welfare is a convex function with the property of local non-satiation. However, it is extremely lack of empirical evidence and has been challenged by behavioural economists for decades.

The model assumes the carbon cycles in 3 parts: atmospheric, upper stratum and lower stratum. This is known as the 3-reservoir model. It also assumes It also assumed the heat in the climate model raise the temperature through a heat cycle in 2 component: deep ocean and atmospheric (Butler et al., 2014).

The author does have further comments towards the modelling from the climate side. The optimization problem is mainly on the economics aspect and the climate model itself is almost treated as exogenous factors in the optimization question. However, it is crucial to keep in mind which model is used here for potential improvement.

3.2 Outcome

Multiple input variables would contribute to the variation of the optimal policy subjected to the welfare optimization. The outcome concluded from the DICE model is the optimal climate policy, that is a chosen investment level into production technology and capital stock, and the optimal level of carbon tax to maximize the social welfare (Nordhaus, 1993). In some literature, the optimal policy is simplified to the social carbon cost. Due to the assumption of externality and a well-behaved market, social carbon cost is the externality and should be equal to the carbon tax.

Due to the complexity of input space, it is hard to capture the effect of each input variable. To understand the model better, the researchers usually treat the model as a black box and study the relationship between the model output (optimal climate policy) and some interesting input variables.

3.3 Limitation

Other than the assumptions discussed before. The major limitation of the DICE model is its high underlying uncertainties in the parameters.

Nordhaus (1993) suggested that he does not use the sensitivity analysis in this piece of work as if the policy aims to maximize the welfare at this stage only requires the knowledge (the inputs) known in the current stage, which has very little uncertainty.

If we are faced with an inter-temporal optimizing problem, that is we are optimizing the social welfare from now to the infinite future, high uncertainty will be introduced. Besides, the uncertainty of the input does not only lie in the value of the parameters itself, it also contains the uncertainty in the mechanism. For example, the uncertainty in how the damage function is produced. It is impossible to measure this value directly, therefore the function exhibits large uncertainty even if only the optimal polity aims to optimize the current state of the world at a time now is targeted.

4 The Role of Sensitivity Analysis in DICE model

One of the most influential works published on the DICE model's sensitivity analysis is made by Miftakhova (2021), where a systematic sensitivity analysis on the DICE model was performed and provided some intuition in its translation in policy.

4.1 Overview

Other than the uncertainty mentioned in the previous part, Sobol' method as an assessment can compare the performance of a model under 2 different non-probabilistic assumptions (Miftakhova, 2021). If the results of both sensitivity analyses are extremely similar, it is not necessary to discuss the sensitivity of the assumption. Some literature uses the OAT method to perform a local sensitivity analysis before performing Sobol' global sensitivity analysis. Local sensitivity analysis is achievable as there exists an optimal solution in the DICE model.

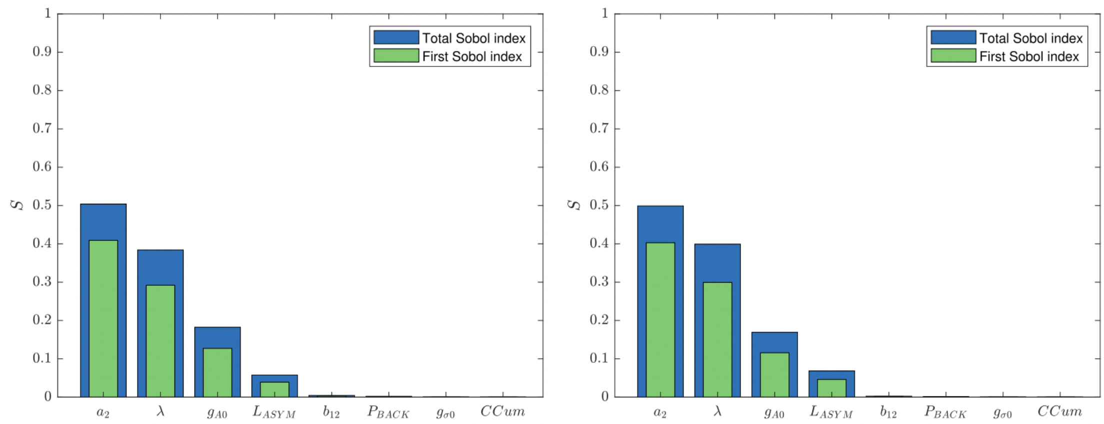

Figure 1: Sobol' indices under different distribution assumptions: left: Uniformed distribution, right: Normal distribution

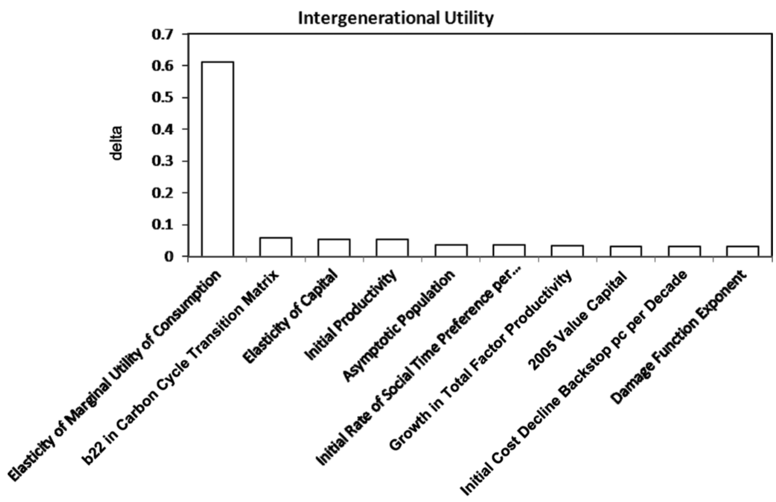

Figure 2: 10 most important exogenous variables for inter-temporal utility

4.2 Selected Result Summary

4.2.1 Parameters' Uncertainty in Distribution

Miftakhova (2021) compare 2 sets of Sobol' indices with different assumptions on whether parameters are distributed uniformly or normally. The Sobol' indices are identical between the two assumptions. Figure 1 indicates the result.

This indicates a low impact of the distribution assumption towards the model performance, which could potentially imply a low sensitivity of the assumption. A detailed description of each parameter's notation can be found in table B.2 of Miftakhova (2021)'s work.

4.2.2 Ranking of the Sensitivity

Anderson et al. (2014) provided an overall basic Sobol' indices of most of the parameters. Due to the limitation of the page, the author will not present it here. The most sensitive exogenous input is the elasticity of the marginal rate of the marginal utility of consumption, which is directly affected by the inter-temporal hyperbolic discounted rate. Figure 2 shows the dominance of sensitivity of such input compared to other exogenous inputs. Where delta is a monotonic transformation of Sobol' indices.

4.2.3 Dependency Assumption

Instead of only running a scanning style Sobol' sensitivity analysis, Miftakhova (2021) provided an alternative argument. They suggested the inter-temporal discount rate should not be treated as an exogenous variable.

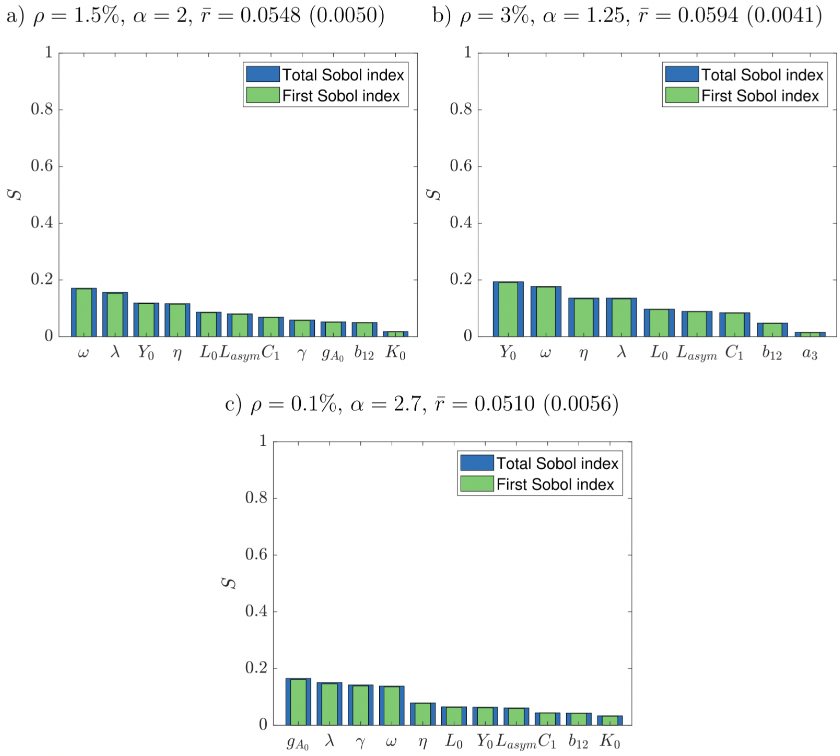

Figure 3: Sobol' indices with fixed ρ (discount rate), α (elasticity of marginal utility) and $\hat{r}$ (interest rate)

They proposed such a discount rate is related to the saving rate and interest rate.

Indeed, in economics analysis, one has a consistent or at least correlated hyperbolic discount rate in considering their utility and considering the trade-off between spending and saving. Therefore, the discount rate should "match" the given saving rate and interest rate.

Miftakhova (2021) also uses the scanning method to reduce the number of parameters. Achieve a simplified form of the social cost of carbon. By setting the hyperbolic discount rate at some different level, Miftakhova (2021)'s work generates the visualization in Figure 3.

5 Discussion and Potential Improvement

The theoretic framework of Sobol' method and related sampling strategies discussed in this review are widely applied in DICE model assessment. It helps eliminate non-influential inputs and achieve a more simple form of optimal taxation policy. Sobol' method detects the main contributor to the model. Making it possible for researchers to notice the important input and reconstruct the model if necessary. However, some potential improvements in sensitivity analysis could be made.

In Figure 3, the sensitivity of Y_0 is higher as the hyperbolic discount rate is higher. This might be due to the model structure itself. However, this initial condition setting is generally more sensitive to the current state (in this case, the social cost of carbon's value in the current year). By extending the time to longer, the sensitivity of the initial condition could be uncertain and can be discussed.

Confidence interval generated by bootstrap is only used once in Anderson et al. (2014)'s work and the author failed to find more of it presented in the literature. Presenting the top 10 most sensitive parameters is indeed a solution for not having a confidence level, but it would be more rigorous to include such intervals in the ranking.

Overall, there is great potential in the DICE model and global sensitivity analysis is a piratical methodology for its improvement. The improvement in the model could be easily translated into a refined climate policy like the carbon tax. More effective policy responses in our rapidly changing world are worth expecting.

(word count: 4286)

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