Welcome and Introduction

Philip Schulz and Wilker Aziz
https://github.com/philschulz/VITutorial

About us . . .

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- VI, sampling methods, NLP

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- Applied Scientist at Amazon
- VI, machine translation, Bayesian models

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Goal

- you should be able to navigate through fresh literature
- and start combining probabilistic models and NNs

Supervised problems

We have data $x^{(1)}, \ldots, x^{(N)}$ e.g. sentences, images generated by some **unknown** procedure which we assume can be captured by a probabilistic model

- with **known** probability (mass/density) function e.g.

$$X \sim \mathrm{Cat}(\theta_1, \ldots, \theta_K) \quad \text{or} \quad X \sim \mathcal{N}(\theta_\mu, \theta_\sigma^2)$$

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and estimate parameters θ that assign maximum likelihood $p(x^{(1)}, \ldots, x^{(N)}|\theta)$ to observations

Supervised NN models

Let y be all side information available
e.g. deterministic *inputs/features/predictors*

Have neural networks predict parameters of our probabilistic model

$$X|y \sim \mathrm{Cat}(\pi_\theta(y)) \quad \text{or} \quad X|y \sim \mathcal{N}(\mu_\theta(y), \sigma_\theta(y)^2)$$

and proceed to estimate parameters θ of the NNs

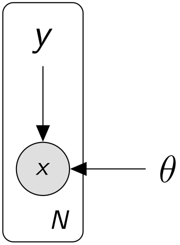
Graphical model

Random variables

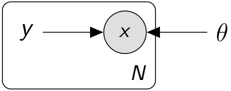
- observed data $x^{(1)}, \ldots, x^{(N)}$

Deterministic variables

- inputs or predictors $y^{(1)}, \ldots, y^{(N)}$
- model parameters θ



Multiple problems, same language



(Conditional) Density estimation

	Side information (y)	Observation (x)
Parsing	a sentence	its syntactic/semantic parse tree/graph
Translation	a sentence	its translation
Captioning	an image	caption in English
Entailment	a text and hypothesis	entailment relation

Task-driven feature extraction

Often our side information is itself some high dimensional data

- y is a sentence and x a tree
- y is the source sentence and x is the target
- y is an image and x is a caption

and part of the job of the NNs that parametrise our models is to also deterministically encode that input in a low-dimensional space

NN as efficient parametrisation

From a statistical point of view, NNs do not generate data

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- compact and efficient way to map from complex side information to parameter space

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Prediction is done by a decision rule outside the statistical model

- e.g. argmax, beam search

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MLE via gradient-based optimisation

If the log-likelihood is **differentiable** and **tractable** then backpropagation gives us the gradient

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$$\nabla_\theta \mathcal{L}(\theta | x^{(1:N)}) = \nabla_\theta \sum_{s=1}^{N} \log p(x^{(s)} | \theta) \quad = \sum_{s=1}^{N} \nabla_\theta \log p(x^{(s)} | \theta)$$

and we can update θ in the direction

$$\gamma \nabla_\theta \mathcal{L}(\theta | x^{(1:N)})$$

to attain a local maximum of the likelihood function

Big Data

For large N, computing the gradient is inconvenient

$$\nabla_\theta \mathcal{L}(\theta|x^{(1:N)}) = \underbrace{\sum_{s=1}^{N} \nabla_\theta \log p(x^{(s)}|\theta)}_{\text{too many terms}}$$

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&= \sum_{s=1}^{N} \mathcal{U}(s | {}^{1}\!/\!{}_{N}) N \nabla_\theta \log p(x^{(s)} | \theta) \\
&= \mathbb{E}_{S \sim \mathcal{U}({}^{1}\!/\!{}_{N})} \left[N \nabla_\theta \log p(x^{(S)} | \theta) \right]
\end{aligned}
$$

S selects data points uniformly at random

Stochastic optimisation

For large N, we can use a gradient estimate

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and take a step in the direction

$$\gamma \frac{N}{M} \underbrace{\nabla_\theta \mathcal{L}(\theta | x^{(s_1 : s_M)})}_{\text{stochastic gradient}}$$

where $x^{(s_1 : s_M)}$ is a random mini-batch of size M

DL in NLP recipe

Maximum likelihood estimation

- tells you which loss to optimise
  (i.e. negative log-likelihood)

Automatic differentiation (*backprop*)

- "give me a tractable forward pass and I will give you gradients"

Stochastic optimisation powered by backprop

- general purpose gradient-based optimisers

Constraints

Differentiability

- intermediate representations must be continuous
- activations must be differentiable

Tractability

- the likelihood function must be evaluated exactly, thus it's required to be tractable

When do we have intractable likelihood?

Unsupervised problems contain unobserved random variables

$$p(x, z|\theta) = \overbrace{p(z)}^{\text{prior}} \quad \underbrace{p(x|z, \theta)}_{\text{observation model}}$$

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$$p(x, z|\theta) = \overbrace{p(z)}^{\text{prior}} \quad \underbrace{p(x|z, \theta)}_{\text{observation model}}$$

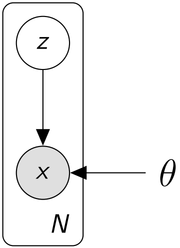
thus assessing the marginal likelihood requires marginalisation of latent variables

$$p(x|\theta) = \int p(x, z|\theta)\,\mathrm{d}z = \int p(z)p(x|z, \theta)\,\mathrm{d}z$$

Latent variable model

Latent random variables

- unobserved
- or unobservable



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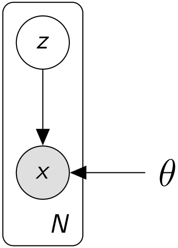

A joint distribution over data and unknowns

$$p(x, z|\theta) = p(z)p(x|z, \theta)$$

Examples of latent variable models

Discrete latent variable, continuous observation

$$p(x|\theta) = \underbrace{\sum_{c=1}^{K} \mathrm{Cat}(c|\pi_1, \ldots, \pi_K) \underbrace{\mathcal{N}(x|\mu_\theta(c), \sigma_\theta(c)^2)}_{\text{forward pass}}}_{\textbf{too many forward passes}}$$

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Continuous latent variable, discrete observation

$$p(x|\theta) = \underbrace{\int \mathcal{N}(z|0, I) \underbrace{\text{Cat}(x|\pi_\theta(z))}_{\text{forward pass}} \mathrm{d}z}_{\textbf{infinitely many forward passes}}$$

Intractable gradient

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  e.g. LDA
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- induce discrete representations
  e.g. parse trees, dependency graphs, alignments
- uncertainty quantification
  e.g. Bayesian NNs

Examples: Lexical alignment

Generate a word x_i in L1 from a word y_{a_i} in L2

$$P(x|y,\theta) \stackrel{\text{ind}}{=} \prod_{i=1}^{|x|} \sum_{a_i=1}^{|y|} P(a_i|y)P(x_i|y_{a_i})$$

a mixture model whose mixture components are labelled by words marginalisation $O(|x||y|)$

Examples: Rationale extraction

Sentiment analysis based on a subset of the input

$$P(x|y,\theta) = \sum_{f_1=0}^{1} \cdots \sum_{f_{|y|}=0}^{1} P(x|f,y) \prod_{i=1}^{|y|} \text{Bernoulli}(f_i|\theta_{y_i})$$

where $P(x|f,y)$ conditions on y_i iff $f_i = 1$.

A factor model whose factors are labelled by words

marginalisation $O(2^{|y|})$

Examples: Language modelling

A (deterministic) RNNLM aways produces the same conditional for a given prefix. Isn't it reasonable to expect the conditional to depend on what we are talking about?: e.g. *Rio de Janeiro* . . .

- history: *was once the Brazilian capital*
- tourism: *offers some of Brazil's most iconic landscapes*
- news: *recently hosted the world cup final*

$$P(x|\theta) = \int \mathcal{N}(z|0, I) \prod_{i=1}^{|x|} P(x_i|z, x_{<i}, \theta)$$

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