NCERT 11.14 Q-25

EE23BTECH11207 -KAILASH.C

I. *QUESTION:*

A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0, and v_0 . [Hint : Start with the equation $x = a\cos(\omega t + \theta)$ and note that the initial velocity is negative.]

II. *SOLUTION:*

Symbols	Definition
x	Displacement
t	Time
v	Velocity
ω	Angular velocity
Θ	Phase constant

TABLE 0
SYMBOLS AND THEIR DEFINITIONS

$$x = A\cos(\omega t + \theta) \tag{1}$$

By differentiating equation-(1) with respect to time t, we get:

$$v = \frac{dx}{dt} \tag{2}$$

$$v = -A\omega\sin(\omega t + \theta) \tag{3}$$

Let at time t=0s, x=x_0
Substituting t=0s in equation (1):

$$x = A\cos(\omega(0) + \theta) \tag{4}$$

$$x_0 = A\cos(\theta) \tag{5}$$

$$v_0 = \frac{dx_0}{dt} \tag{6}$$

$$v_0 = -A\omega\sin(\omega(0) + \theta) \tag{7}$$

$$v_0 = -A\omega\sin(\theta) \tag{8}$$

$$-A\sin(\theta) = \frac{v_0}{\omega} \tag{9}$$

Squaring and Adding eq-(5) and eq-(9):

$$(A\cos(\theta))^2 + (A\sin(\theta)^2) = (x_0)^2 + (\frac{v_0}{\omega})^2 \tag{10}$$

$$A^2(\cos^2(\theta) + \sin^2(\theta)) = (x_0)^2 + (\frac{v_o}{\omega})^2 \tag{11}$$

$$A^2 = (x_0)^2 + (\frac{v_o}{\omega})^2 \tag{12}$$

Therefore by solving the above equation we get:

$$\text{Amplitude=A=}\sqrt{(x_0)^2 + (\tfrac{v_0}{\omega})^2}$$