ANALOG 11.14 Q-25

EE23BTECH11207 -KAILASH.C*

QUESTION:

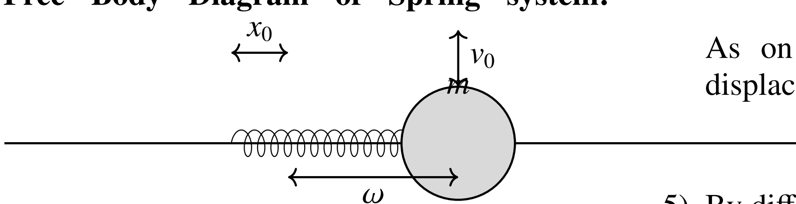
A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0, and v_0 . [Hint : Start with the equation $x = a\cos(\omega t + \theta)$ and note that the initial velocity is negative.]

SOLUTION:

1) **Input Parameters:**

Symbols	Definition
x	Displacement
t	Time
v	Velocity
ω	Angular velocity
θ	Phase constant
a	Acceleration

2) **Free Body Diagram of Spring system:**



3) Let $x(t)$ be a twice differentiable function of time t

$$\text{Let } x(t) = x \tag{1}$$
$$\frac{dx}{dt} = v \tag{2}$$
$$\frac{dv}{dt} = a \tag{3}$$

By applying the chain rule:

$$\frac{dx}{dt} = \frac{dx}{dt} \tag{4}$$
$$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} \tag{5}$$

We know that:

$$\frac{dv}{dt} = -\omega^2 x \tag{6}$$

Substituting eq-(6) in eq-(5):

$$\frac{d^2x}{dt^2} = -\omega^2 x \tag{7}$$
$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{8}$$

4) Let us assume a solution of:

$$x(t) = A\cos(\omega t + \theta) \tag{9}$$
$$\frac{dx}{dt} = -A\omega\sin(\omega t + \theta) \tag{10}$$
$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \theta) \tag{11}$$

Now substituting eq-(9) and eq-(11) in eq(8):

$$-A\omega^2\cos(\omega t + \theta) + \omega^2 A\cos(\omega t + \theta) = 0 \tag{12}$$
$$\omega^2 A\cos(\omega t + \theta)(-1 + 1) = 0 \tag{13}$$
$$0 = 0 \tag{14}$$

As on both sides we get same value, the displacement equaion is:

$$x(t) = A\cos(\omega t + \theta) \tag{15}$$

5) By differentiating equation-(15) with respect to time t, we get:

$$v = -A\omega\sin(\omega t + \theta) \tag{16}$$

Let at time t=0s, x=x_0
Substituting t=0s in equation (15):

$$x = A\cos(\omega(0) + \theta) \tag{17}$$
$$x_0 = A\cos(\theta) \tag{18}$$
$$v_0 = \frac{dx_0}{dt} \tag{19}$$
$$v_0 = -A\omega\sin(\omega(0) + \theta) \tag{20}$$
$$v_0 = -A\omega\sin(\theta) \tag{21}$$
$$-A\sin(\theta) = \frac{v_0}{\omega} \tag{22}$$

6) Squaring and Adding eq-(18) and eq-(22):

$$(A\cos(\theta))^2 + (A\sin(\theta)^2) = (x_0)^2 + \left(\frac{v_0}{\omega}\right)^2 \quad (23)$$

$$A^2(\cos^2(\theta) + \sin^2(\theta)) = (x_0)^2 + \left(\frac{v_o}{\omega}\right)^2 \quad (24)$$

$$A^2 = (x_0)^2 + (\frac{v_o}{\omega})^2 \quad (25)$$

Answer:
Amplitude of the resulting oscillation is: $\sqrt{(x_0)^2 + (\frac{v_0}{\omega})^2}$