DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:

Find the sum of n terms of the series: $1\times 2+2\times 3+3\times 4+4\times 5+....$

SOLUTION:

In the above series,we have:$a_n = n(n+1)$ where n is n_{th} and $n+1$ is $n+1_{th}$element of the natural number correspondingly and it starts from zero.

$$S = \sum_{n=0}^{n} n\cdot(n+1) \tag{1}$$

$$= \sum_{n=0}^{n}(n^2+n) \tag{2}$$

$$= \sum_{n=0}^{n} n^2 + \sum_{n=0}^{n} \tag{3}$$

By the formula for sum of series,we have:

$$\sum_{n=0}^{n} n^2 = \frac{n(n+1)(2n+1)}{6} \tag{4}$$

$$\sum_{n=0}^{n} n = \frac{n(n+1)}{2} \tag{5}$$

Using eq-(4) and eq-(5) in eq-(3):

$$S = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \tag{6}$$

$$= \frac{n(n+1)}{6}(2n+1+3) \tag{7}$$

$$= \frac{n(n+1)(2n+4)}{6} \tag{8}$$

$$= \frac{n(n+1)(n+2)}{3} \tag{9}$$

Z-Transformation of S(n):

$$S(z) = \sum_{n=0}^{\infty} \frac{n(n+1)(n+2)}{3}\cdot z^{-n} \tag{10}$$

$$= \frac{1}{3}\sum_{n=0}^{\infty} n(n+1)(n+2)\cdot z^{-n} \tag{11}$$

$$= \frac{1}{6}\sum_{n=0}^{\infty}(n^3+3n^2+2n)\cdot z^{-n} \tag{12}$$

By applying z-transformation to each term:

$$S(z) = \frac{1}{3}\left(\sum_{n=0}^{\infty} n^3 z^{-n} + \sum_{n=0}^{\infty} 3n^2 z^{-n} + \sum_{n=0}^{\infty} 2nz^{-n}\right) \tag{13}$$

Let:

$$X_1(z) = \sum_{n=0}^{\infty} n^3 z^{-n} \tag{14}$$

$$X_2(z) = \sum_{n=0}^{\infty} 3n^2 z^{-n} \tag{15}$$

$$X_3(z) = \sum_{n=0}^{\infty} 2nz^{-n} \tag{16}$$

By the z-transformation formulas,we have:

$$Z[n^k] = -Z\frac{d}{dZ}Z[n^{k-1}] \tag{17}$$

By using eq-(17) in eq-(14),(15) and (16) and solving,we get:

$$X_1(z) = z^2\left(\frac{1}{2}+\frac{1}{2}(z-1)^{-1}+\frac{1}{4}(z-1)^{-2}\right) \tag{18}$$

$$X_2(z) = -3\left(\frac{1}{(z-1)^2}+\frac{1}{2}(z-1)^{-3}\right) \tag{19}$$

$$X_3(z) = -2\left(\frac{1}{(z-1)^2}+(z-1)^{-3}\right) \tag{20}$$

By combining eq-(18),(19) and (20) we get S(z):

$$S(z) = \frac{1}{3}\left(\frac{z^2}{2}+\frac{z^2}{2}(z-1)^{-1}+\frac{z^2}{4}(z-1)^{-2} - 9\left(\frac{1}{(z-1)^2}+\frac{1}{2}(z-1)^{-3}\right) + 2z\left(-2\left(\frac{1}{(z-1)^2}+(z-1)^{-3}\right)\right)\right) \tag{21}$$

The Z-transformation of eq-(8) is eq-(21) with r.o.c as:$|z|>1$

Graph of S(n) vs n: