

# DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C\*

## QUESTION:

Find the sum of n terms of the series:  
 $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$

## TABLE

Symbols	Definition
$x(n)$	General term
$y(n)$	Sum of n terms
$y(z)$	Z-Transformation Of $y(n)$

TABLE 0  
INPUT PARAMETERS

## SOLUTION:

$$x(n) = (n+1)(n+2) \quad (1)$$

$$y(n) = \sum_{n=0}^{n-1} (n+1) \cdot (n+2) \quad (2)$$

$$= \sum_{n=0}^{n-1} n^2 + 3 \sum_{n=0}^{n-1} n + 2 \sum_{n=0}^{n-1} 1 \quad (3)$$

By the formula for sum of series, we have:

$$\sum_{n=0}^{n-1} n^2 = \frac{(n-1)(n)(2n+1)}{6} \quad (4)$$

$$\sum_{n=0}^{n-1} n = \frac{(n-1)(n)}{2} \quad (5)$$

$$\sum_{n=0}^{n-1} 1 = n \quad (6)$$

Using (4), (5) and (6) in (3):

$$y(n) = \frac{n^3 + 3n^2 + 2n}{3} \quad (7)$$

**Z-Transformation of  $y(n)$ :**

$$Y(z) = \frac{1}{3} \sum_{n=0}^{\infty} (n^3 + 3n^2 + 2n) \cdot z^{-n} \quad (8)$$

By applying z-transformation to each term:

$$Y(z) = \frac{1}{3} \left( \sum_{n=0}^{\infty} n^3 z^{-n} + \sum_{n=0}^{\infty} 3n^2 z^{-n} + \sum_{n=0}^{\infty} 2n z^{-n} \right) \quad (9)$$

By Z-transformation property:

$$Z[nf(n)] = -z \frac{d}{dz} F(z) \quad (10)$$

Using (10) to the individual terms in (9):

$$\sum_{n=0}^{\infty} n^3 z^{-n} \xrightarrow{\text{Z-Transformation}} \frac{z(z^2 + 4z + 1)}{(z-1)^4} \quad (11)$$

$$\sum_{n=0}^{\infty} n^2 z^{-n} \xrightarrow{\text{Z-Transformation}} \frac{3z(z+1)}{(z-1)^3} \quad (12)$$

$$\sum_{n=0}^{\infty} n z^{-n} \xrightarrow{\text{Z-Transformation}} \frac{z}{(z-1)^2} \quad (13)$$

Using (11), (12) and (13) in (9):

$$Y(z) = \frac{1}{3} \left( \frac{z(z^2 + 4z + 1)}{(z-1)^4} + \frac{3z(z+1)}{(z-1)^3} + \frac{z}{(z-1)^2} \right), \quad |z| > 1 \quad (14)$$

**Graph:**

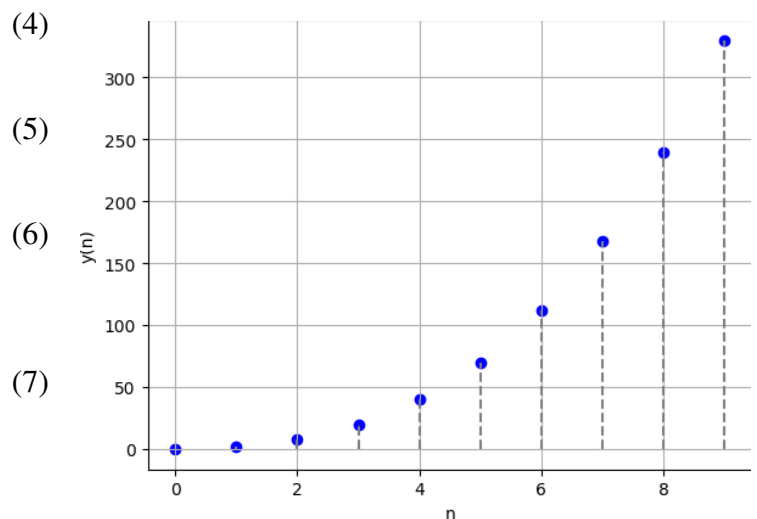


Fig. 0. Graph of  $y(n)$  vs  $n$