DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

Find the sum of n terms of the series: $1\times2+2\times3+3\times4+4\times5+....$

Solution:

Symbols	Definition
$x(n)$	General term
$y(n)$	Sum of terms till n_{th}term
$Y(z)$	Z-Transformation Of $y(n)$

TABLE 0
PARAMETER TABLE

$$x(n) = (n+1)(n+2)u(n) \tag{1}$$

By Z-transformation property:

$$Z[nf(n)] = -z\frac{d}{dz}F(z) \tag{2}$$

By (2),We have the formulas for:

$$nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1-z^{-1})^2} \tag{3}$$

$$n^2u(n) \xleftrightarrow{Z} \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} \tag{4}$$

Using (3),(4) for z-transformation of $x(n)$

$$X(z) = \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}, \quad |z| > |1| \tag{5}$$

$$Y(z) = X(z) * U(z) \tag{6}$$

$$= \left(\frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}\right)\frac{1}{1-z^{-1}} \tag{7}$$

$$= \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^4} + \frac{3z^{-1}}{(1-z^{-1})^3} + \frac{2}{(1-z^{-1})^2} \tag{8}$$

By using inverse Z transformation,we have:

$$\frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^4} \xleftrightarrow{Z^{-1}} \frac{2n^3+3n^2+n}{6} \tag{9}$$

$$\frac{3z^{-1}}{(1-z^{-1})^3} \xleftrightarrow{Z^{-1}} \frac{3n^2+3n}{2} \tag{10}$$

$$\frac{2}{(1-z^{-1})^2} \xleftrightarrow{Z^{-1}} 2n+2 \tag{11}$$

By adding (9),(10) and (11),we get:

$$y(n) = \frac{2n^3-3n^2+n}{6} + \frac{3n^2+3n}{2} + 2n+2 \tag{12}$$

$$= \frac{2n^3+12n^2+22n+12}{6} \tag{13}$$

$$= \frac{(n+1)(n+2)(n+3)}{3} \tag{14}$$

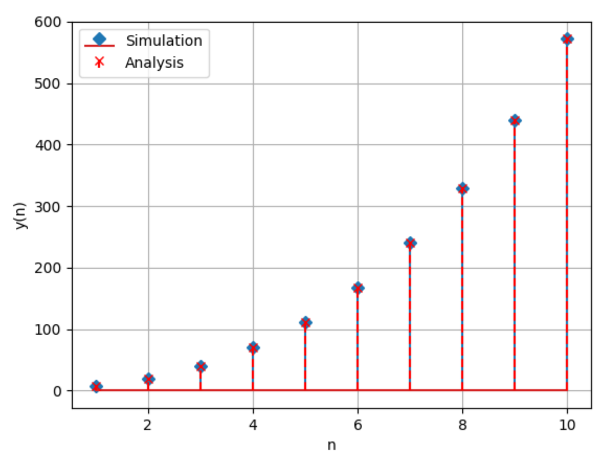

Fig. 0. Simulation v/s Analysis