

DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:

Find the sum of n terms of the series:
 $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$

Let $X_1(z) = \sum_{n=0}^{\infty} n^3 z^{-n}$, $X_2(z) = \sum_{n=0}^{\infty} n^2 z^{-n}$ and $X_3(z) = \sum_{n=0}^{\infty} n z^{-n}$

By Z-transformation property:

$$Z[nf(n)] = -z \frac{d}{dz} F(z) \quad (12)$$

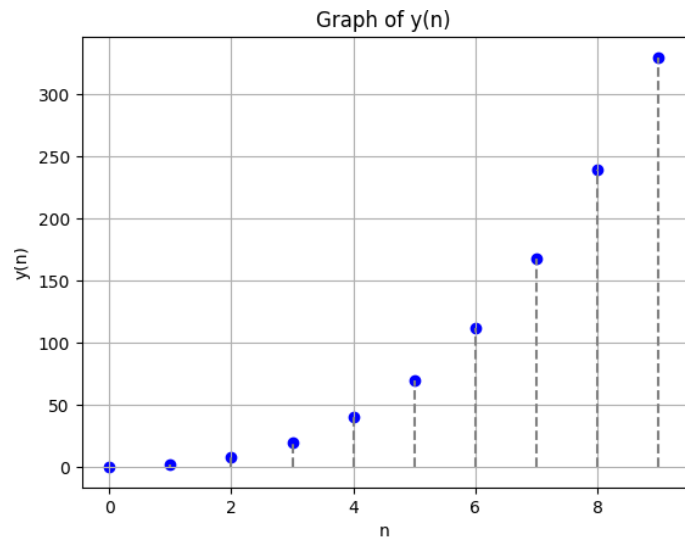
Using (12) to get $X_1(z)$, $X_2(z)$, $X_3(z)$, we get:

$$(1) \quad y(z) = \frac{1}{3} \left(1 \sum_{n=0}^{\infty} n^3 \cdot z^{-n} + 3 \sum_{n=0}^{\infty} n^2 \cdot z^{-n} + 2 \sum_{n=0}^{\infty} n \cdot z^{-n} \right) \quad (13)$$

$$(2) \quad = \frac{1}{3} \left(\frac{z(z^2 + 4z + 1)}{(z-1)^4} + \frac{3z(z+1)}{(z-1)^3} + \frac{z}{(z-1)^2} \right) \quad (14)$$

The Z-transformation of (9) is (14) with r.o.c as: $|z| > 1$

Graph of y(n) vs n:



SOLUTION:

In the above series, we have: $a_n = (n+1)(n+2)$.
 Let sum of n terms be $y(n)$.

$$\begin{aligned} y(n) &= \sum_{n=0}^{n-1} (n+1) \cdot (n+2) \\ &= \sum_{n=0}^{n-1} (n^2 + 3n + 2) \\ &= \sum_{n=0}^{n-1} n^2 + 3 \sum_{n=0}^{n-1} n + 2 \sum_{n=0}^{n-1} 1 \end{aligned}$$

By the formula for sum of series, we have:

$$\begin{aligned} \sum_{n=0}^{n-1} n^2 &= \frac{(n-1)(n)(2n+1)}{6} \\ \sum_{n=0}^{n-1} n &= \frac{(n-1)(n)}{2} \\ \sum_{n=0}^{n-1} 1 &= n \end{aligned}$$

Using (4), (5) and (6) in (3):

$$\begin{aligned} y(n) &= \frac{(n-1)n(2n+1)}{6} + \frac{3(n-1)n}{2} + 2n \\ &= \frac{n(n-1)}{6} [2n+1+9] + 2n \\ &= \frac{n^3 + 3n^2 + 2n}{3} \end{aligned}$$

Z-Transformation of y(n):

$$y(z) = \frac{1}{3} \sum_{n=0}^{\infty} (n^3 + 3n^2 + 2n) \cdot z^{-n} \quad (10)$$

By applying z-transformation to each term:

$$y(z) = \frac{1}{3} \left(\sum_{n=0}^{\infty} n^3 z^{-n} + \sum_{n=0}^{\infty} 3n^2 z^{-n} + \sum_{n=0}^{\infty} 2n z^{-n} \right) \quad (11)$$