DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:
Find the sum of n terms of the series: $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

INPUT PARAMETERS

Symbols	Definition
$x(n)$	General term
$y(n)$	Sum of n terms
$y(z)$	Z-Transformation Of $y(n)$

SOLUTION:
We have $x(n) = (n+1)(n+2)$

$$y(n) = \sum_{n=0}^{n-1} (n+1) \cdot (n+2) \tag{1}$$

$$= \sum_{n=0}^{n-1} n^2 + 3\sum_{n=0}^{n-1} n + 2\sum_{n=0}^{n-1} 1 \tag{2}$$

By the formula for sum of series,we have:

$$\sum_{n=0}^{n-1} n^2 = \frac{(n-1)(n)(2n+1)}{6} \tag{3}$$

$$\sum_{n=0}^{n-1} n = \frac{(n-1)(n)}{2} \tag{4}$$

$$\sum_{n=0}^{n-1} 1 = n \tag{5}$$

Using (3),(4) and (5) in (2):

$$y(n) = \frac{n^3 + 3n^2 + 2n}{3} \tag{6}$$

Z-Transformation of $y(n)$:

$$y(z) = \frac{1}{3}\sum_{n=0}^{\infty}\left(n^3 + 3n^2 + 2n\right) \cdot z^{-n} \tag{7}$$

By applying z-transformation to each term:

$$y(z) = \frac{1}{3}\left(\sum_{n=0}^{\infty} n^3 z^{-n} + \sum_{n=0}^{\infty} 3n^2 z^{-n} + \sum_{n=0}^{\infty} 2nz^{-n}\right) \tag{8}$$

By Z-transformation property:

$$Z[nf(n)] = -z\frac{d}{dz}F(z) \tag{9}$$

Using (9) to the individual terms in (8):

$$\sum_{n=0}^{\infty} n^3 z^{-n} = \frac{z\left(z^2 + 4z + 1\right)}{(z - 1^4)} \tag{10}$$

$$\sum_{n=0}^{\infty} n^2 z^{-n} = \frac{3z(z+1)}{(z-1)^3} \tag{11}$$

$$\sum_{n=0}^{\infty} nz^{-n} = \frac{z}{(z-1)^2} \tag{12}$$

Using (10),(11) and (12) in (8)

$$y(z) = \frac{1}{3}\left(\frac{z\left(z^2 + 4z + 1\right)}{(z-1)^4} + \frac{3z(z+1)}{(z-1)^3} + \frac{z}{(z-1)^2}\right), \quad |z| > |1| \tag{13}$$

Graph:

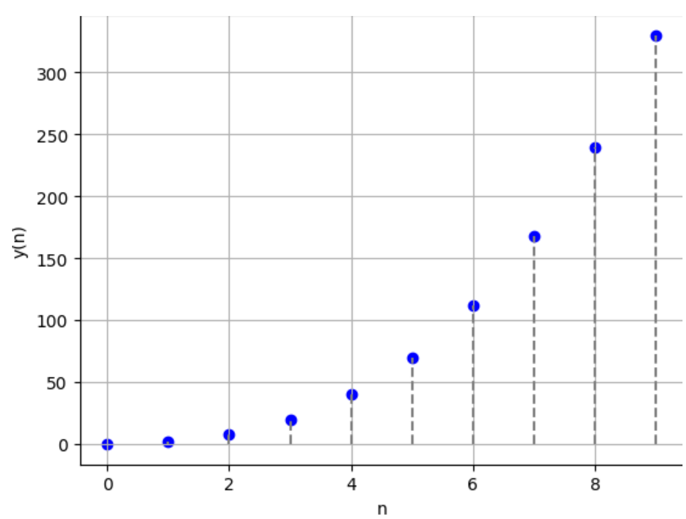

Fig. 0. Graph of $y(n)$ vs n