

DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:

Find the sum of n terms of the series:
 $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$

Let $X_1(z) = \sum_{n=0}^{\infty} n^3 z^{-n}$, $X_2(z) = \sum_{n=0}^{\infty} n^2 z^{-n}$ and $X_3(z) = \sum_{n=0}^{\infty} n z^{-n}$

By Z-transformation property:

$$Z[nf(n)] = -z \frac{d}{dz} F(z) \quad (12)$$

SOLUTION:

In the above series, we have: $a_n = (n+1)(n+2)$.

Let sum of n terms be $y(n)$.

Using eq-(15) to get $X_1(z)$, $X_2(z)$, $X_3(z)$, we get:

$$y(n) = \sum_{n=0}^{n-1} (n+1) \cdot (n+2) \quad (1) \quad y(z) = \frac{1}{3} \left(1 \sum_{n=0}^{\infty} n^3 \cdot z^{-n} + 3 \sum_{n=0}^{\infty} n^2 \cdot z^{-n} + 2 \sum_{n=0}^{\infty} n \cdot z^{-n} \right) \quad (13)$$

$$= \sum_{n=0}^{n-1} (n^2 + 3n + 2) \quad (2) \quad = \frac{1}{3} \left(\frac{z(z^2 + 4z + 1)}{(z-1)^4} + \frac{3z(z+1)}{(z-1)^3} + \frac{z}{(z-1)^2} \right) \quad (14)$$

$$= \sum_{n=0}^{n-1} n^2 + 3 \sum_{n=0}^{n-1} n + 2 \sum_{n=0}^{n-1} 1 \quad (3)$$

The Z-transformation of eq-(12) is eq-(18) with r.o.c as: $|z| > 1$

By the formula for sum of series, we have:

$$\sum_{n=0}^{n-1} n^2 = \frac{(n-1)(n)(2n+1)}{6} \quad (4)$$

$$\sum_{n=0}^{n-1} n = \frac{(n-1)(n)}{2}$$

$$\sum_{n=0}^{n-1} 1 = n$$

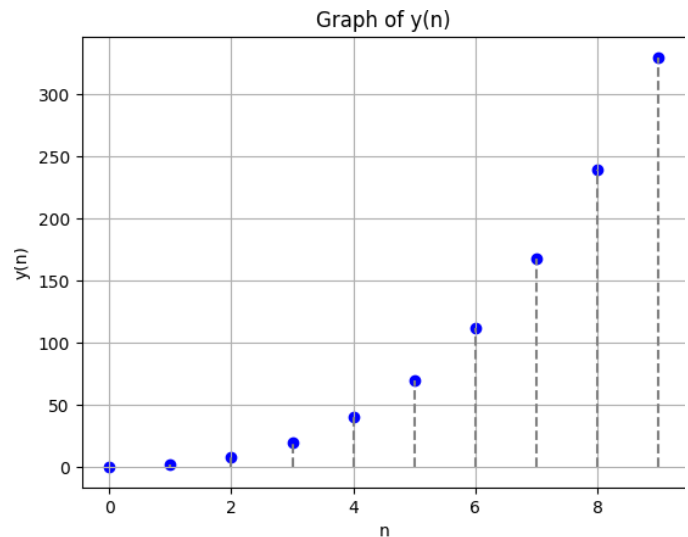
Graph of y(n) vs n:

Using eq-(4), eq-(5) and eq-(6) in eq-(3):

$$y(n) = \frac{(n-1)n(2n+1)}{6} + \frac{3(n-1)n}{2} + 2n \quad (5)$$

$$= \frac{n(n-1)}{6} [2n+1+9] + 2n \quad (6)$$

$$= \frac{n^3 + 3n^2 + 2n}{3} \quad (7)$$



Z-Transformation of y(n):

$$y(z) = \frac{1}{3} \sum_{n=0}^{\infty} (n^3 + 3n^2 + 2n) \cdot z^{-n} \quad (8)$$

By applying z-transformation to each term:

$$y(z) = \frac{1}{3} \left(\sum_{n=0}^{\infty} n^3 z^{-n} + \sum_{n=0}^{\infty} 3n^2 z^{-n} + \sum_{n=0}^{\infty} 2n z^{-n} \right) \quad (9)$$