DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:
Find the sum of n terms of the series: $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

SOLUTION:
In the above series,we have:$a_n = (n+1)(n+2)$

$$S = \sum_{n=0}^{n-1}(n+1)\cdot(n+2) \tag{1}$$

$$= \sum_{n=0}^{n-1}(n^2+3n+2) \tag{2}$$

$$= \sum_{n=0}^{n-1}n^2 + 3\sum_{n=0}^{n-1}n + 2\sum_{n=0}^{n-1}1 \tag{3}$$

By the formula for sum of series,we have:

$$\sum_{n=0}^{n-1}n^2 = \frac{(n-1)(n)(2n+1)}{6} \tag{4}$$

$$\sum_{n=0}^{n-1}n = \frac{(n-1)(n)}{2} \tag{5}$$

$$\sum_{n=0}^{n-1}1 = n \tag{6}$$

Using eq-(4),eq-(5) and eq-(6) in eq-(3):

$$S(n) = \frac{(n-1)n(2n-1)}{6} + \frac{3(n-1)n}{2} + 2n \tag{7}$$

$$= \frac{n(n-1)}{6}[2n-1+9] + 2n \tag{8}$$

$$= \frac{n}{6}[(n-1)(2n+8)+12] \tag{9}$$

$$= \frac{n}{6}[2n^2+6n+4] \tag{10}$$

$$= \frac{2n^3+6n^2+4n}{6} \tag{11}$$

$$= \frac{n^3+3n^2+2n}{3} \tag{12}$$

Z-Transformation of S(n):

$$S(z) = \frac{1}{3}\sum_{n=0}^{\infty}(n^3+3n^2+2n)\cdot z^{-n} \tag{13}$$

By applying z-transformation to each term:

$$S(z) = \frac{1}{3}\left(\sum_{n=0}^{\infty}n^3z^{-n} + \sum_{n=0}^{\infty}3n^2z^{-n} + \sum_{n=0}^{\infty}2nz^{-n}\right) \tag{14}$$

By the z-transformation formulas,we have:

$$Z[n^k] = -Z\frac{d}{dZ}Z[n^{k-1}] \tag{15}$$

Using eq-(15) in eq-(14):

$$S(z) = \frac{1}{3}\left(1\sum_{n=0}^{\infty}n^3\cdot z^{-n} + 3\sum_{n=0}^{\infty}n^2\cdot z^{-n} + 2\sum_{n=0}^{\infty}n\cdot z^{-n}\right) \tag{16}$$

$$= \frac{1}{3}\left(1\cdot\frac{z(z^2+4z+1)}{(z-1)^4} + 3\cdot\frac{z(z+1)}{(z-1)^3} + 2\cdot\frac{z}{(z-1)^2}\right) \tag{17}$$

$$= \frac{1}{3}\left(\frac{z(z^2+4z+1)}{(z-1)^4} + \frac{3z(z+1)}{(z-1)^3} + \frac{z}{(z-1)^2}\right) \tag{18}$$

The Z-transformation of eq-(12) is eq-(18) with r.o.c as:$|z| > 1$

Graph of S(n) vs n: