DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:
Find the sum of n terms of the series: $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

SOLUTION:
From observing the series above,We can say that is a sum of the series:$\sum_{n=0}^{\infty} n \cdot (n+1)$

$$S = \sum_{n=0}^{\infty} n \cdot (n+1) \tag{1}$$

$$S = \sum_{n=0}^{\infty} (n^2 + n) \tag{2}$$

$$S = \sum_{n=0}^{\infty} n^2 + \sum_{n=0}^{\infty} n \tag{3}$$

$$\tag{4}$$

By the formula for sum of series,we have:

$$\sum_{n=0}^{\infty} n^2 = \frac{n(n+1)(2n+1)}{6} \tag{5}$$

$$\sum_{n=0}^{\infty} n = \frac{n(n+1)}{2} \tag{6}$$

Using eq-(5) and eq-(6) in eq-(3):

$$S = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \tag{7}$$

$$S = \frac{n(n+1)}{6}(2n+1+3) \tag{8}$$

$$S = \frac{n(n+1)(2n+4)}{6} \tag{9}$$

ANSWER:
The sum of b terms of series is:$S = \frac{n(n+1)(2n+4)}{6}$

Z-Transformation of S(n):

$$S(z) = \sum_{n=0}^{\infty} \frac{n(n+1)(2n+4)}{6} \cdot z^{-n} \tag{10}$$

$$S(z) = \frac{1}{6} \sum_{n=0}^{\infty} n(n+1)(2n+4) \cdot z^{-n} \tag{11}$$

$$S(z) = \frac{1}{6} \sum_{n=0}^{\infty} (2n^3 + 6n^2 + 4n) \cdot z^{-n} \tag{12}$$

By applying z-transformation to each term:

$$S(z) = \frac{1}{6} \left(\sum_{n=0}^{\infty} 2n^{3-n} + \sum_{n=0}^{\infty} 6n^{2-n} + \sum_{n=0}^{\infty} 4n^{-n} \right) \tag{13}$$

By the z-transformation formulas,we have:

$$Z\{n^k\} = \frac{1}{(1-z^{-1})^{k+1}} \tag{14}$$

$$Z\{n\} = \frac{z^{-1}}{(1-z^{-1})^2} \tag{15}$$

Using eq-(14) and eq-(15) in eq-(13):

$$S(z) = \frac{1}{6} \left(2 \sum_{n=0}^{\infty} n^3 \cdot z^{-n} + 6 \sum_{n=0}^{\infty} n^2 \cdot z^{-n} + 4 \sum_{n=0}^{\infty} n \cdot z^{-n} \right) \tag{16}$$

$$S(z) = \frac{1}{6} \left(2 \cdot \frac{z^{-1}}{(1-z^{-1})^4} + 6 \cdot \frac{z^{-1}}{(1-z^{-1})^3} + 4 \cdot \frac{z^{-1}}{(1-z^{-1})^2} \right) \tag{17}$$

$$S(z) = \frac{1}{6} \left(\frac{2z^{-1}}{(1-z^{-1})^4} + \frac{6z^{-1}}{(1-z^{-1})^3} + \frac{4z^{-1}}{(1-z^{-1})^2} \right) \tag{18}$$

The Z-transformation of eq-(9) as eq-(18) with r.o.c as:$|z| > 1$

Graph of S(n) vs n: