DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

Find the sum of n terms of the series: $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

Solution:

Symbols	Definition
$x(n)$	General term
$y(n)$	Sum of terms till n_{th}term
$Y(z)$	Z-Transformation Of $y(n)$

TABLE 0
INPUT PARAMETERS

$$x(n) = (n+1)(n+2)u(n) \tag{1}$$

By Z-transformation property:

$$Z[nf(n)] = -z\frac{d}{dz}F(z) \tag{2}$$

By (2),We have the formulas for:

$$nu(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \tag{3}$$

$$n^2u(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3}, \tag{4}$$

Using (3),(4) for z-transformation of (1)

$$X(z) = \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}, \quad |z| > |1| \tag{5}$$

$$Y(z) = X(z) * U(z) \tag{6}$$

$$= \left(\frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}\right)\frac{1}{1-z^{-1}} \tag{7}$$

$$= \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^4} + \frac{3z^{-1}}{(1-z^{-1})^3} + \frac{2}{(1-z^{-1})^2} \tag{8}$$

Using contour integration to find inverse Z transformation in (8):

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)z^{n-1}dz \tag{9}$$

$$= \frac{1}{2\pi j}\oint_C \frac{2z^{n+3}}{(z-1)^4} \tag{10}$$

We can observe that pole is repeated 4 times,thus m=4.

$$R = \frac{1}{(m-1)!}\lim_{z\to z_0}\frac{d^{m-1}}{dz^{m-1}}(f(z)) \tag{11}$$

$$= \frac{1}{(3)!}\lim_{z\to 1}\frac{d^3}{dz^3}\left(2z^{n+3}\right) \tag{12}$$

$$y(n) = \frac{(n+1)(n+2)(n+3)}{3} \tag{13}$$

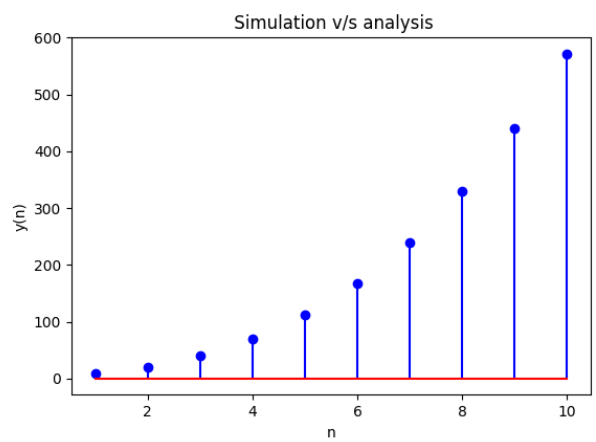

Fig. 0. Graph of $y(n)$ vs n