

DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:

Find the sum of n terms of the series:
 $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots$

TABLE

Symbols	Definition
$x(n)$	General term
$y(n)$	Sum of n terms
$Y(z)$	Z-Transformation Of $y(n)$

TABLE 0
INPUT PARAMETERS

SOLUTION:

$$x(n) = (n+1)(n+2)u(n) \quad (1)$$

By Z-transformation property:

$$Z[nf(n)] = -z \frac{d}{dz} F(z) \quad (2)$$

By (2), We have the formulas for:

$$n \xleftrightarrow{\text{Z-Transformation}} \frac{z^{-1}}{(1-z^{-1})^2} \quad (3)$$

$$n^2 \xleftrightarrow{\text{Z-Transformation}} \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3}, \quad (4)$$

$$n^3 \xleftrightarrow{\text{Z-Transformation}} \frac{z^{-1}(z^{-2}+4z^{-1}+1)}{(1-z^{-1})^4} \quad (5)$$

Using (3),(4) for z-transformation of (1)

$$X(z) = \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}, \quad |z| > 1 \quad (6)$$

$$Y(z) = X(z) * U(z) \quad (7)$$

$$= \left(\frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}} \right) \frac{1}{1-z^{-1}} \quad (8)$$

$$= \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{3z^{-1}}{(1-z^{-1})^3} + \frac{2}{(1-z^{-1})^2} \quad (9)$$

Using contour integration to find inverse Z transformation in (9):

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \quad (10)$$

$$= \frac{1}{2\pi j} \oint_C \frac{2z^{n+2}}{(z-1)^4} dz \quad (11)$$

We can observe that pole is repeated 4 times, thus $m=4$.

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow z_0} \frac{d^{m-1}}{dz^{m-1}} (f(z)) \quad (12)$$

$$= \frac{1}{(3)!} \lim_{z \rightarrow 1} \frac{d^3}{dz^3} (z^{n+2}) \quad (13)$$

$$y(n) = \frac{n^3 + 3n^2 + 2n}{3} \quad (14)$$

Graph:

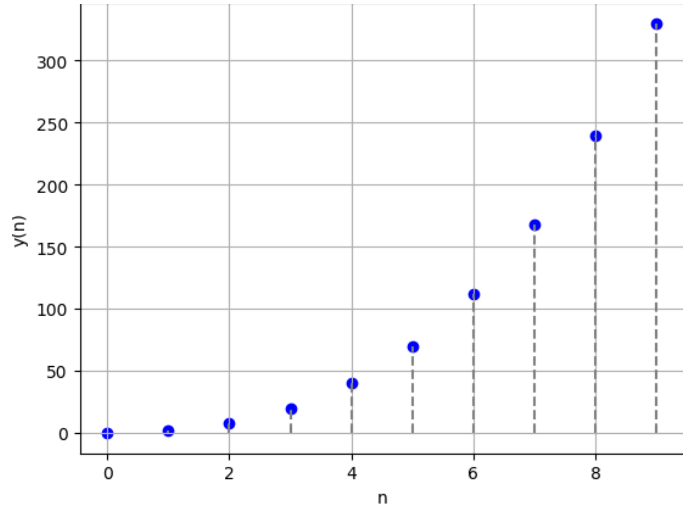


Fig. 0. Graph of $y(n)$ vs n