DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:
Find the sum of n terms of the series: $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

TABLE

Symbols	Definition
$x(n)$	General term
$y(n)$	Sum of n terms
$y(z)$	Z-Transformation Of $y(n)$

TABLE 0
INPUT PARAMETERS

SOLUTION:

$$x(n) = (n+1)(n+2)u(n) \tag{1}$$

By Z-transformation property:

$$Z[nf(n)] = -z\frac{d}{dz}F(z) \tag{2}$$

By (2),We have the formulas for:

$$\sum_{n=0}^{\infty} nz^{-n} \xleftrightarrow{Z-Transformation} \frac{z^{-1}}{(1-z^{-1})^2} \tag{3}$$

$$\sum_{n=0}^{\infty} n^2z^{-n} \xleftrightarrow{Z-Transformation} \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3}, \tag{4}$$

$$\sum_{n=0}^{\infty} n^3z^{-n} \xleftrightarrow{Z-Transformation} \frac{z^{-1}\left(z^{-2}+4z^{-1}+1\right)}{(1-z^{-1})^4} \tag{5}$$

Using (3),(4) for z-transformation of (1)

$$X(z) = \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}, \quad |z| > |1| \tag{6}$$

$$Y(z) = X(z) * U(z) \tag{7}$$

$$Y(z) = \left(\frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}\right)\frac{1}{1-z^{-1}} \tag{8}$$

$$= \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^4} + \frac{3z^{-1}}{(1-z^{-1})^3} + \frac{2}{(1-z^{-1})^2} \tag{9}$$

Using contour integration to find inverse Z transformation in (9):

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)z^{n-1}dz \tag{10}$$

$$= \frac{1}{2\pi j}\oint_C \frac{z^{n-2}\left(z^{-1}+1\right)}{(1-z^{-1})^4} + \frac{3z^{n-2}}{(1-z^{-1})^3} + \frac{2z^{n-1}}{(1-z^{-1})^2}\, dz \tag{11}$$

$$= \frac{1}{2\pi j}\oint_C \frac{2z^{n-2}}{(1-z^{-1})^4}\, dz \tag{12}$$

$$= \frac{1}{2\pi j}\oint_C \frac{2z^{n+2}}{(z-1)^4} \tag{13}$$

We can observe that pole is repeated 4 times,thus m=4.

$$R = \frac{1}{(m-1)!}\lim_{z\to z_0}\frac{d^{m-1}}{dz^{m-1}}(f(z)) \tag{14}$$

$$= \frac{1}{(3)!}\lim_{z\to 1}\frac{d^3}{dz^3}\left(z^{n+2}\right) \tag{15}$$

$$y(n) = \frac{n^3+3n^2+2n}{3} \tag{16}$$

Z-Transformation of $y(n)$:

$$Y(z) = \frac{1}{3}\sum_{n=0}^{\infty}\left(n^3+3n^2+2n\right)\cdot z^{-n} \tag{17}$$

By applying z-transformation to each term:

$$Y(z) = \frac{1}{3}\left(\sum_{n=0}^{\infty} n^3z^{-n} + \sum_{n=0}^{\infty} 3n^2z^{-n} + \sum_{n=0}^{\infty} 2nz^{-n}\right) \tag{18}$$

Using (3),(4) and (5) to the individual terms in (**??**):

$$Y(z) = \frac{1}{3}\left(\frac{z\left(z^2+4z+1\right)}{(z-1)^4} + \frac{3z(z+1)}{(z-1)^3} + \frac{z}{(z-1)^2}\right), \quad |z| > |1| \tag{19}$$

Graph:

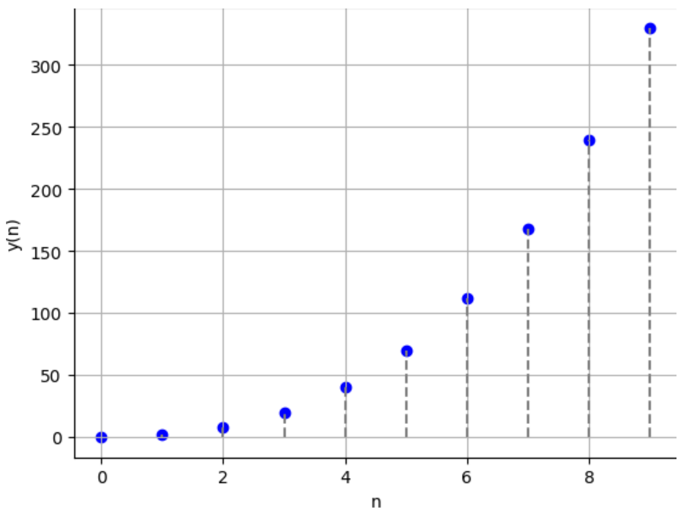

Fig. 0. Graph of $y(n)$ vs n