DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

Find the sum of n terms of the series: $1\times2+2\times3+3\times4+4\times5+....$

Solution:

Symbols	Definition
$x(n)$	General term
$y(n)$	Sum of terms till n_{th}term
$Y(z)$	Z-Transformation Of $y(n)$

TABLE 0

PARAMETER TABLE

$$x(n) = (n+1)(n+2)u(n) \tag{1}$$

By Z-transformation property:

$$Z[nf(n)] = -z\frac{d}{dz}F(z) \tag{2}$$

By (2),We have the formulas for:

$$nu(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \tag{3}$$

$$n^2u(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} \tag{4}$$

Using (3),(4) for z-transformation of $x(n)$

$$X(z) = \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}, \quad |z| > |1| \tag{5}$$

$$Y(z) = X(z) * U(z) \tag{6}$$

$$= \left(\frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}\right)\frac{1}{1-z^{-1}} \tag{7}$$

By using inverse z transformation property:

$$\frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} \overset{Z^{-1}}{\longleftrightarrow} n^2 \tag{8}$$

$$\frac{z^{-1}}{(1-z^{-1})^2} \overset{Z^{-1}}{\longleftrightarrow} n \tag{9}$$

$$\frac{1}{1-z^{-1}} \overset{Z^{-1}}{\longleftrightarrow} 1 \tag{10}$$

We get:

$$y(n) = \left(n^2+3n+2\right) * 1^n \tag{11}$$

By convolution property,we have:

$$f(n) * g(n) = \sum_{r=0}^{n} f(r)\,g(n-r) \tag{12}$$

Using the convolution property:

$$y(n) = 1^n * n^2 + 1^n * 3n + 1^n * 2 \tag{13}$$

By solving (13),we get:

$$y(n) = \frac{n^3+6n^2+11n+6}{3} \tag{14}$$

$$= \frac{(n+1)(n+2)(n+3)}{3} \tag{15}$$

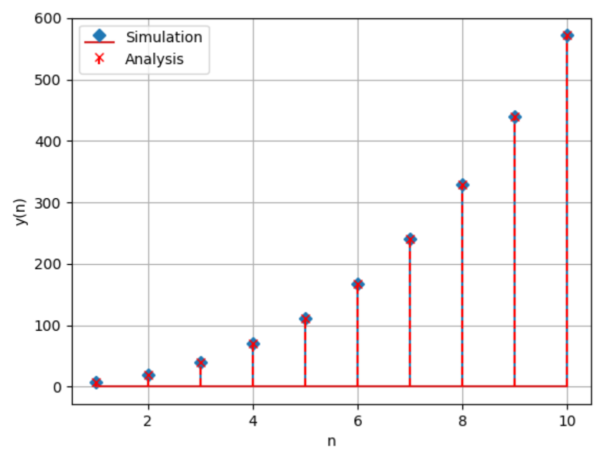

Fig. 0. Simulation v/s Analysis