DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

QUESTION:
Find the sum of n terms of the series: $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

SOLUTION:
In the above series,we have:$a_n = (n + 1)(n + 2)$.
Let sum of n terms be $y(n)$.

$$y(n) = \sum_{n=0}^{n-1} (n + 1) \cdot (n + 2) \tag{1}$$

$$= \sum_{n=0}^{n-1} (n^2 + 3n + 2) \tag{2}$$

$$= \sum_{n=0}^{n-1} n^2 + 3 \sum_{n=0}^{n-1} n + 2 \sum_{n=0}^{n-1} 1 \tag{3}$$

By the formula for sum of series,we have:

$$\sum_{n=0}^{n-1} n^2 = \frac{(n - 1)(n)(2n + 1)}{6} \tag{4}$$

$$\sum_{n=0}^{n-1} n = \frac{(n - 1)(n)}{2} \tag{5}$$

$$\sum_{n=0}^{n-1} 1 = n \tag{6}$$

Using eq-(4),eq-(5) and eq-(6) in eq-(3):

$$y(n) = \frac{(n - 1)n(2n - 1)}{6} + \frac{3(n - 1)n}{2} + 2n \tag{7}$$

$$= \frac{n(n - 1)}{6} [2n - 1 + 9] + 2n \tag{8}$$

$$= \frac{n^3 + 3n^2 + 2n}{3} \tag{9}$$

Z-Transformation of y(n):

$$y(z) = \frac{1}{3} \sum_{n=0}^{\infty} (n^3 + 3n^2 + 2n) \cdot z^{-n} \tag{10}$$

By applying z-transformation to each term:

$$y(z) = \frac{1}{3} \left(\sum_{n=0}^{\infty} n^3 z^{-n} + \sum_{n=0}^{\infty} 3n^2 z^{-n} + \sum_{n=0}^{\infty} 2nz^{-n} \right) \tag{11}$$

Let $X_1(z) = \sum_{n=0}^{\infty} n^3 z^{-n}$,$X_2(z) = \sum_{n=0}^{\infty} n^2 z^{-n}$ and $X_3(z) = \sum_{n=0}^{\infty} nz^{-n}$
By Z-transformation property:

$$Z[nf(n)] = -z \frac{d}{dz} F(z) \tag{12}$$

Using eq-(15) to get $X_1(z), X_2(z), X_3(z)$,we get:

$$y(z) = \frac{1}{3} \left(1 \sum_{n=0}^{\infty} n^3 \cdot z^{-n} + 3 \sum_{n=0}^{\infty} n^2 \cdot z^{-n} + 2 \sum_{n=0}^{\infty} n \cdot z^{-n} \right) \tag{13}$$

$$= \frac{1}{3} \left(\frac{z(z^2 + 4z + 1)}{(z - 1)^4} + \frac{3z(z + 1)}{(z - 1)^3} + \frac{z}{(z - 1)^2} \right) \tag{14}$$

The Z-transformation of eq-(12) is eq-(18) with r.o.c as:$|z| > 1$

Graph of y(n) vs n: