DISCRETE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

Find the sum of n terms of the series: $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

Solution:

Symbols	Definition
$x(n)$	General term
$y(n)$	Sum of terms till n_{th} term
$Y(z)$	Z-Transformation Of $y(n)$

TABLE 0
PARAMETER TABLE

$$x(n) = (n+1)(n+2)u(n) \tag{1}$$

By Z-transformation property:

$$Z[nf(n)] = -z\frac{d}{dz}F(z) \tag{2}$$

By (2),We have the formulas for:

$$nu(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \tag{3}$$

$$n^2u(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} \tag{4}$$

Using (3),(4) for z-transformation of $x(n)$

$$X(z) = \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}, \quad |z| > |1| \tag{5}$$

$$Y(z) = X(z) * U(z) \tag{6}$$

$$= \left(\frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{3z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}}\right)\frac{1}{1-z^{-1}} \tag{7}$$

$$= \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{3z^{-1}}{(1-z^{-1})^3} + \frac{2}{(1-z^{-1})^2} \tag{8}$$

By using inverse Z transformation,we have:

$$\frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} \overset{Z^{-1}}{\longleftrightarrow} \frac{2n^3 - 3n^2 + n}{6} \tag{9}$$

$$\frac{3z^{-1}}{(1-z^{-1})^3} \overset{Z^{-1}}{\longleftrightarrow} \frac{3n^2 + 3n}{2} \tag{10}$$

$$\frac{2}{(1-z^{-1})^2} \overset{Z^{-1}}{\longleftrightarrow} n^2 + 2n + 2 \tag{11}$$

By adding (??),(??) and (??),we get:

$$y(n) = \frac{2n^3 + 12n^2 + 22n + 12}{6} \tag{12}$$

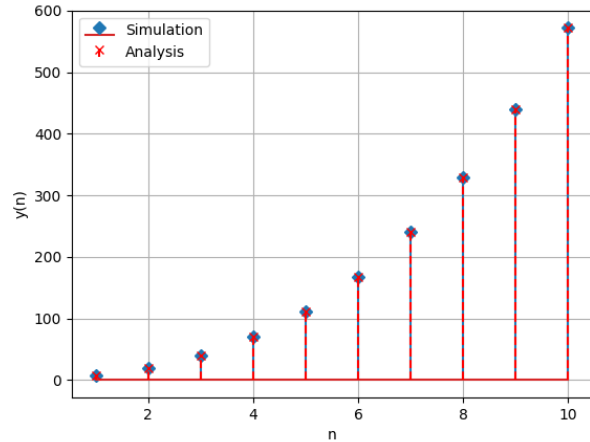$$= \frac{n^3 + 6n^2 + 11n + 6}{3} \tag{13}$$

$$= \frac{(n+1)(n+2)(n+3)}{3} \tag{14}$$

Fig. 0. Simulation v/s Analysis