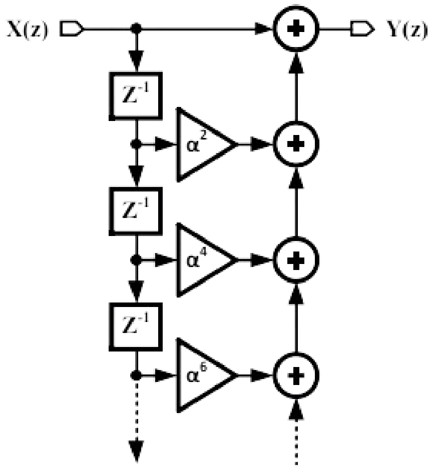
GATE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

In the block diagram shown below, an infinite tap FIR filter with transfer function $H(z) = \frac{Y(z)}{X(z)}$ is realized. If $H(z) = \frac{1}{1-0.5z^{-1}}$.
the value of α is



Solution:

Parameter	Function
$H(z)$	$\frac{1}{1-0.5z^{-1}}$
a	First Term of G.P
r	Common Ratio of G.P
S_r	Sum of infinite terms in G.P

TABLE 0
PARAMETER TABLE

From diagram we have:

$$Y(z) = X(z) + X(z)z^{-1}\alpha^2 + X(z)z^{-2}\alpha^4 + ... \quad (1)$$

Dividing by $X(z)$ in both sides:

$$\frac{Y(z)}{X(z)} = 1 + z^{-1}\alpha^2 + z^{-2}\alpha^4 + ... \quad (2)$$

$$H(z) = 1 + z^{-1}\alpha^2 + z^{-2}\alpha^4 + ... \quad (3)$$

$$\frac{1}{1-0.5z^{-1}} = 1 + z^{-1}\alpha^2 + z^{-2}\alpha^4 + ... \quad (4)$$

In the R.H.S we have a geometric series with:

$$a = 1 \quad (5)$$

$$r = z^{-1}\alpha^2 \quad (6)$$

$$S_r = \frac{a}{1-r} \quad (7)$$

Using (**??**),(**??**) and (**??**) in (**??**):

$$\frac{1}{1-0.5z^{-1}} = \frac{1}{1-z^{-1}\alpha^2} \quad (8)$$

$$1-0.5z^{-1} = 1-z^{-1}\alpha^2 \quad (9)$$

$$\alpha^2 = 0.5 \quad (10)$$

$$\alpha = \sqrt{0.5} \quad (11)$$

$$\alpha = \frac{1}{\sqrt{2}} \quad (12)$$