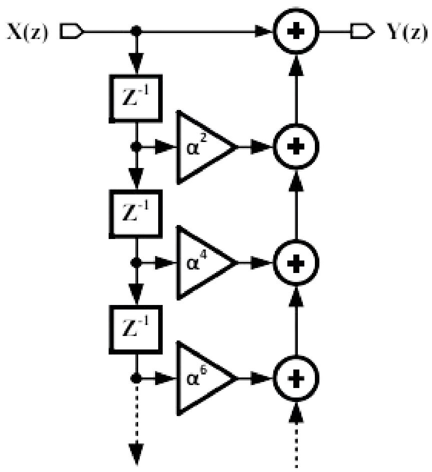
GATE 11.9.4 Q-1

EE23BTECH11207 -KAILASH.C*

In the block diagram shown below, an infinite tap FIR filter with transfer function $H(z) = \frac{Y(z)}{X(z)}$ is realized. If $H(z) = \frac{1}{1-0.5z^{-1}}$. the value of α is



Solution:

Parameter	Definition	Value
$H(z)$	Input Transfer Function	$\frac{1}{1-0.5z^{-1}}$
a	First Term of G.P	
r	Common Ratio of G.P	
S_r	Sum of infinite terms in G.P	

TABLE 0
PARAMETER TABLE

From diagram we have:

$$Y(z) = X(z)\left(\sum_{n=0}^{\infty}\left(z^{-1}\alpha^2\right)^n\right) \tag{1}$$

Dividing by $X(z)$ in both sides:

$$\frac{Y(z)}{X(z)} = \sum_{n=0}^{\infty}\left(z^{-1}\alpha^2\right)^n \tag{2}$$

$$H(z) = \sum_{n=0}^{\infty}\left(z^{-1}\alpha^2\right)^n \tag{3}$$

$$\frac{1}{1-0.5z^{-1}} = \sum_{n=0}^{\infty}\left(z^{-1}\alpha^2\right)^n \tag{4}$$

In the R.H.S we have a geometric series with:

$$a = 1 \tag{5}$$

$$r = z^{-1}\alpha^2 \tag{6}$$

$$S_r = \frac{a}{1-r} \tag{7}$$

Using (**??**),(**??**) and (**??**) in (**??**):

$$\frac{1}{1-0.5z^{-1}} = \frac{1}{1-z^{-1}\alpha^2} \tag{8}$$

$$1-0.5z^{-1} = 1-z^{-1}\alpha^2 \tag{9}$$

$$\alpha^2 = 0.5 \tag{10}$$

$$\alpha = \sqrt{0.5} \tag{11}$$

$$\alpha = \frac{1}{\sqrt{2}} \tag{12}$$