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```
% ID - 202183504, Name - Choudhary Kailash Project - 2
```

Code Link: [GitHub_Code](#)

```
clc
close all
clear
```

```
data_path = 'I:/Kailash/Course_Project/Turbulence/Project2/Data_set';
addpath(data_path);
```

Reading Grid data

```
h = 1;
grid_x = read_pos([data_path, '/Position/Grid_x.dat']);
grid_x.Properties.VariableNames = ["i", "x"];
grid_x = h*grid_x.x;

grid_y = read_pos([data_path, '/Position/Grid_y.dat']);
grid_y.Properties.VariableNames = ["j", "y"];
grid_y = h*grid_y.y;

grid_z = read_pos([data_path, '/Position/Grid_z.dat']);
grid_z.Properties.VariableNames = ["k", "z"];
grid_z = h*grid_z.z;
```

Reading position with velocity data set

```
% consider the fluid is air
mu = 1.84e-5;
ro = 1.224;
nu = mu/ro;
%nu = 2.33e-04;
h = 1;

% It is given that center line reynolds no is 4200, so
Re_c = 4200;
Uc = Re_c*nu/h;

n_files = dir(fullfile(data_path, '/Velocity_field', '*.dat'));

vel1 = [];

%reading one file to get indices
filename = strcat(data_path, '/Velocity_field/', sprintf('INS3D_Vel_%06d.dat', 1));
data = read_vel(filename);
indexs = [double(data.i), double(data.j), double(data.k)];
parfor i=1:numel(n_files)
    filename = strcat(data_path, '/Velocity_field/', sprintf('INS3D_Vel_%06d.dat', i));
    data = read_vel(filename);
    vel_t = [data.U, data.V, data.W];

    Velocity_Data(:, :, i) = Uc*vel_t;
    %i
```

```
end
vel = mean(Velocity_Data,3);
```

Averaging over t, x, z

```
ii = unique(indexs(:,1)) ; nx = length(ii) ;
jj = unique(indexs(:,2)) ; ny = length(jj) ;
kk = unique(indexs(:,3)) ; nz = length(kk) ;

xi = grid_x(ii);
yj = grid_y(jj);
zk = grid_z(kk);

[XX,YY, ZZ] = meshgrid(xi,yj, zk);
UU = reshape(vel(:,1), nx, ny, nz);
UU(end, :, :) = mean(UU(end-20:end-10, :, :), 1);
VV = reshape(vel(:,2), nx, ny, nz);
WW = reshape(vel(:,3), nx, ny, nz);

u_bar = mean(mean(UU,3),1); % we can take direct mean because x and z grids are uniform
v_bar = mean(mean(VV,3),1);
w_bar = mean(mean(WW,3),1);

vel_bar = [u_bar', v_bar', w_bar'];
```

1. Plot the streamwise, wall-normal, spanwise velocity contours in the x–z plane and the y–z plane. Choose any position and time.

Q1 - Part 1 on x-z plane

```
ind = (indexs(:,2) == 96);

pos = indexs(ind,:);
val = vel(ind,:);

ii = unique(pos(:,1)) ; nx = length(ii) ;
kk = unique(pos(:,3)) ; nz = length(kk) ;

xi = grid_x(ii);
zk = grid_z(kk);

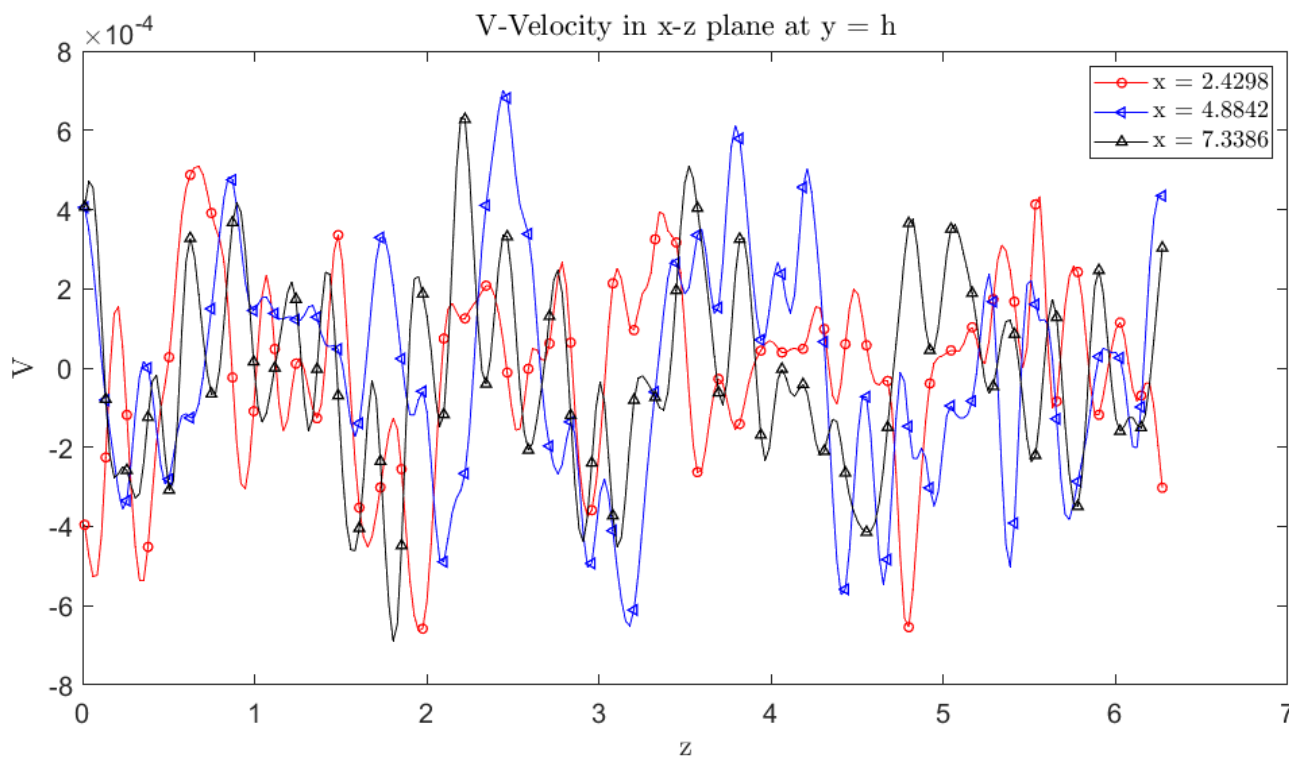
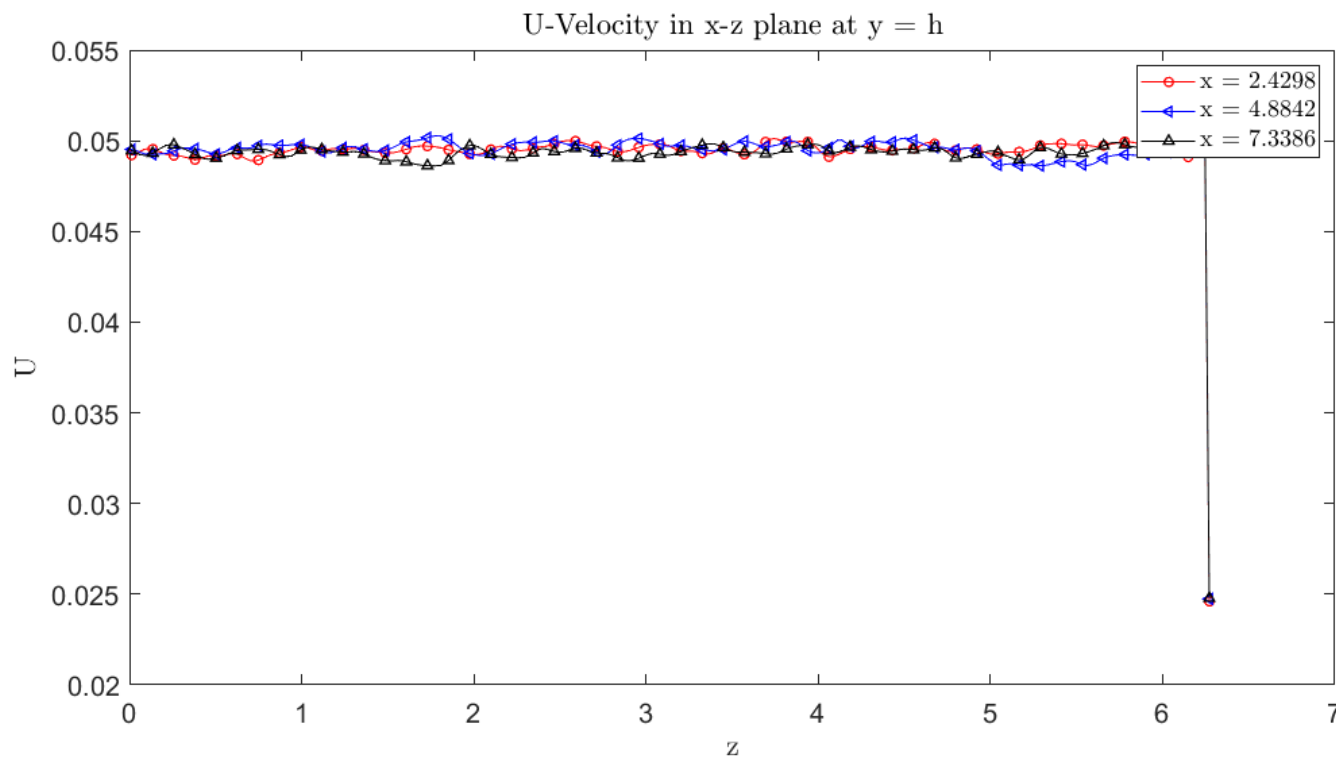
[XX,ZZ] = meshgrid(xi,zk);
UU = reshape(val(:,1), nz, nx);
VV = reshape(val(:,2), nz, nx);
WW = reshape(val(:,3), nz, nx);

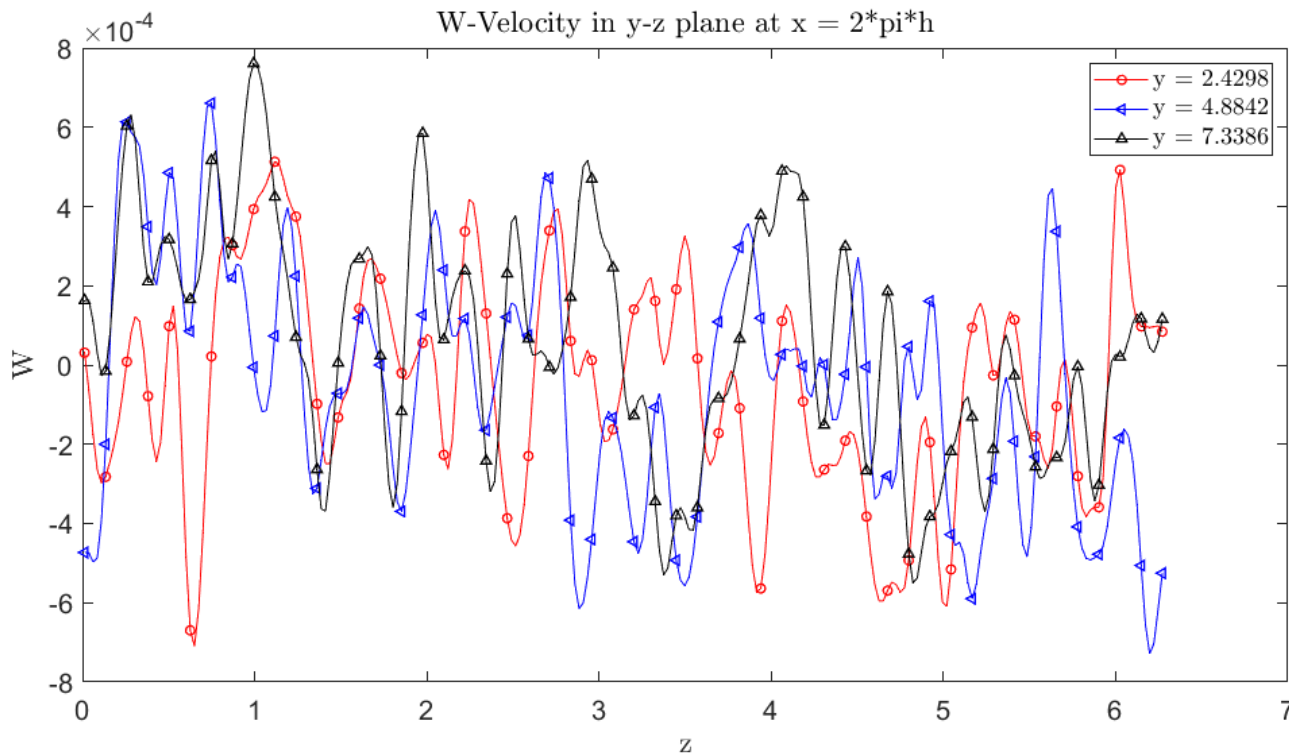
figure
CM = jet(6);
plot(zk(1:end), UU(1:end,50), '-or', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
hold on
plot(zk(1:end), UU(1:end,100), '-<b', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
plot(zk(1:end), UU(1:end,150), '-^k', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
title('U-Velocity in x-z plane at y = h','FontWeight','Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
lgd = {[ 'x = ', num2str(grid_x(50))], [ 'x = ', num2str(grid_x(100))], [ 'x = ', num2str(grid_x(150))]};
legend(lgd,'Interpreter','Latex');
xlabel('z','Interpreter','Latex');
ylabel('U','Interpreter','Latex');

figure
CM = jet(6);
plot(zk(1:end), VV(1:end,50), '-or', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
hold on
plot(zk(1:end), VV(1:end,100), '-<b', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5)
plot(zk(1:end), VV(1:end,150), '-^k', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5)
title('V-Velocity in x-z plane at y = h','FontWeight','Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
lgd = {[ 'x = ', num2str(grid_x(50))], [ 'x = ', num2str(grid_x(100))], [ 'x = ', num2str(grid_x(150))]};
legend(lgd,'Interpreter','Latex');
xlabel('z','Interpreter','Latex')
ylabel('V','Interpreter','Latex');

figure
CM = jet(6);
plot(zk(1:end), WW(1:end,50), '-or', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
hold on
plot(zk(1:end), WW(1:end,100), '-<b', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
plot(zk(1:end), WW(1:end,150), '-^k', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
title('W-Velocity in y-z plane at x = 2*pi*h','FontWeight','Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
lgd = {[ 'y = ', num2str(grid_x(50))], [ 'y = ', num2str(grid_x(100))], [ 'y = ', num2str(grid_x(150))]};
legend(lgd,'Interpreter','Latex');
xlabel('z','Interpreter','Latex');
ylabel('W','Interpreter','Latex');
```

```
% It looks like there is a problem in U-velocity data at z=zmax point. Even  
% it caused me some problem in plots related to U in z direction.
```





Q1 - Part 2 - on y-z plane

say $x = 2\pi h$

```
ind = (indexs(:,1) == 256/2);

pos = indexs(ind,:);
val = vel(ind,:);

jj = unique(pos(:,2)) ; ny = length(jj) ;
kk = unique(pos(:,3)) ; nz = length(kk) ;

yi = grid_y(jj);
zk = grid_z(kk);

[YY,ZZ] = meshgrid(yi,zk);
UU = reshape(val(:,1), nz, ny);
VV = reshape(val(:,2), nz, ny);
WW = reshape(val(:,3), nz, ny);

figure
CM = jet(6);
plot(zk(1:end), UU(1:end,50), '-or', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
hold on
plot(zk(1:end), UU(1:end,100), '-<b', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
plot(zk(1:end), UU(1:end,150), '-^k', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
title('U-Velocity in y-z plane at x = 2*pi*h','FontWeight','Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
lgd = {[ 'y = ', num2str(grid_y(50))], [ 'y = ', num2str(grid_y(100))], [ 'y = ', num2str(grid_y(150))]};
legend(lgd,'Interpreter','Latex');
xlabel('z','Interpreter','Latex');
ylabel('U','Interpreter','Latex');

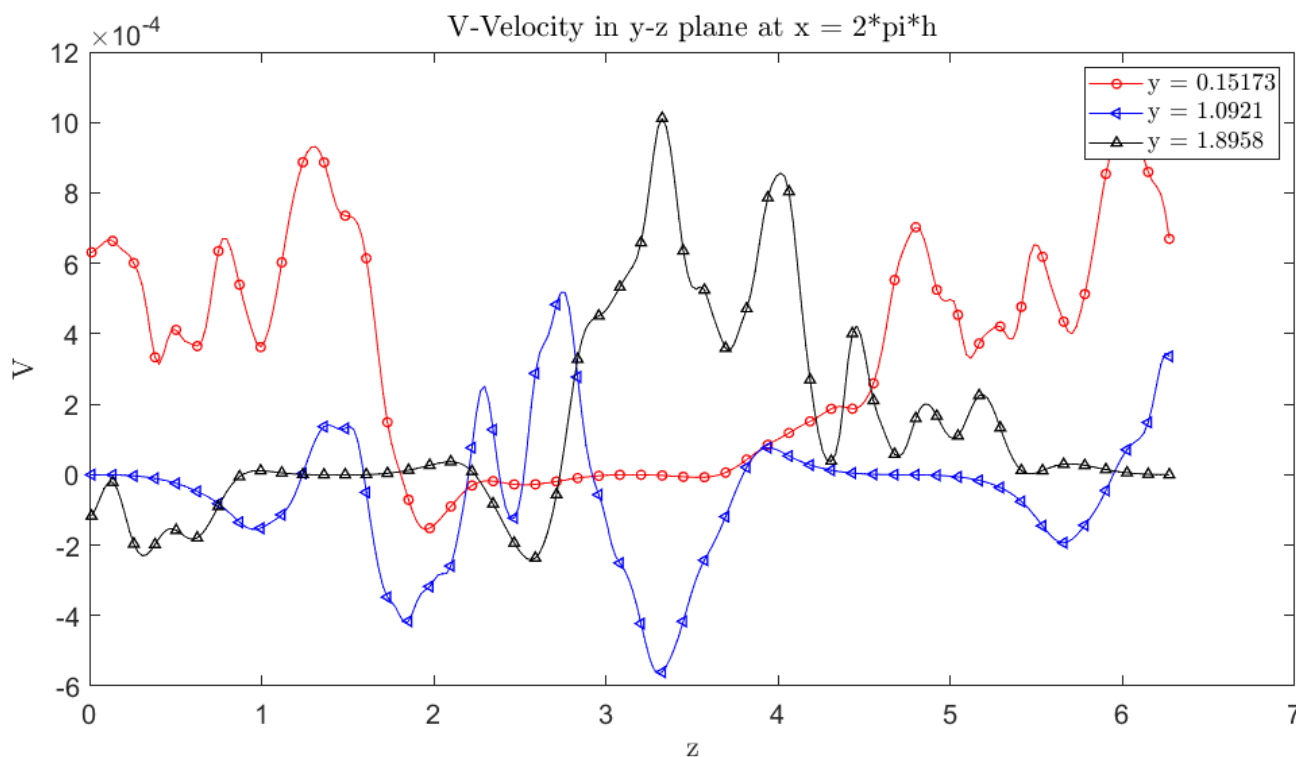
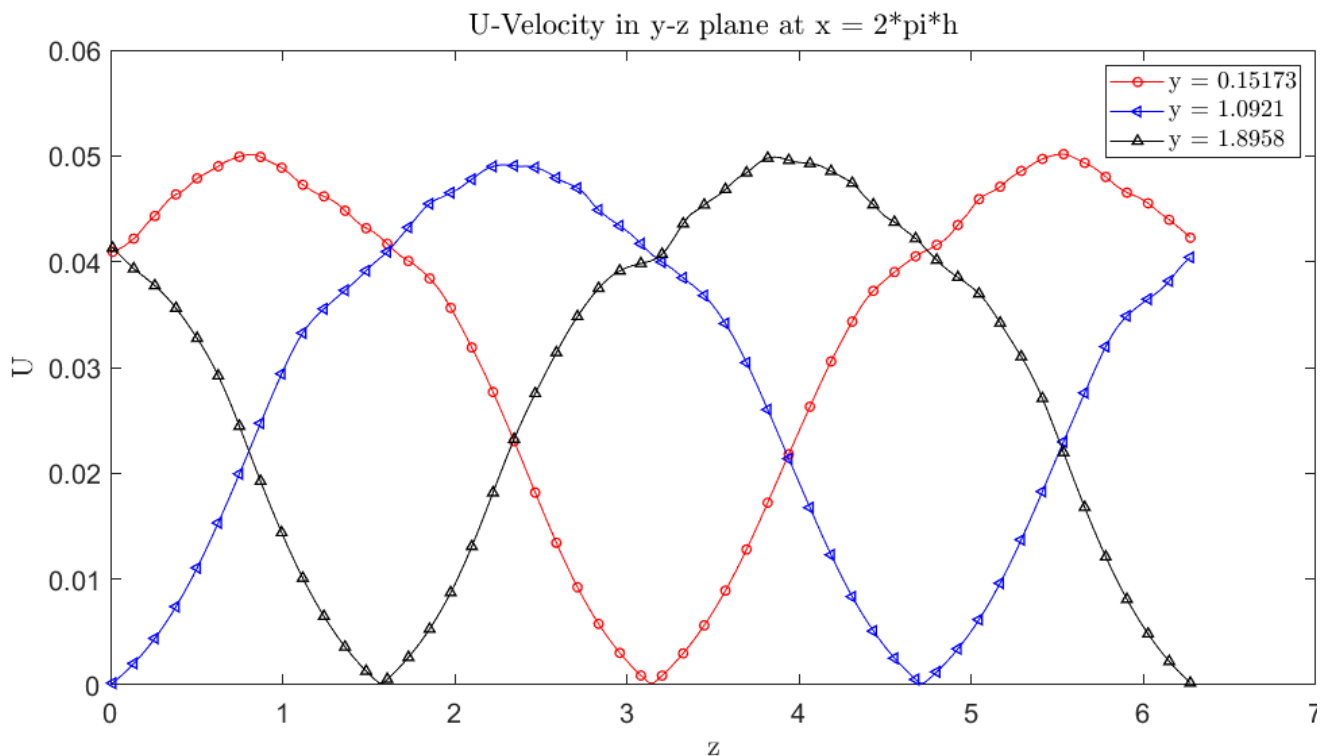
figure
CM = jet(6);
plot(zk(1:end), VV(1:end,50), '-or', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
hold on
plot(zk(1:end), VV(1:end,100), '-<b', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
plot(zk(1:end), VV(1:end,150), '-^k', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
title('V-Velocity in y-z plane at x = 2*pi*h','FontWeight','Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
lgd = {[ 'y = ', num2str(grid_y(50))], [ 'y = ', num2str(grid_y(100))], [ 'y = ', num2str(grid_y(150))]};
legend(lgd,'Interpreter','Latex');
xlabel('z','Interpreter','Latex');
ylabel('V','Interpreter','Latex');

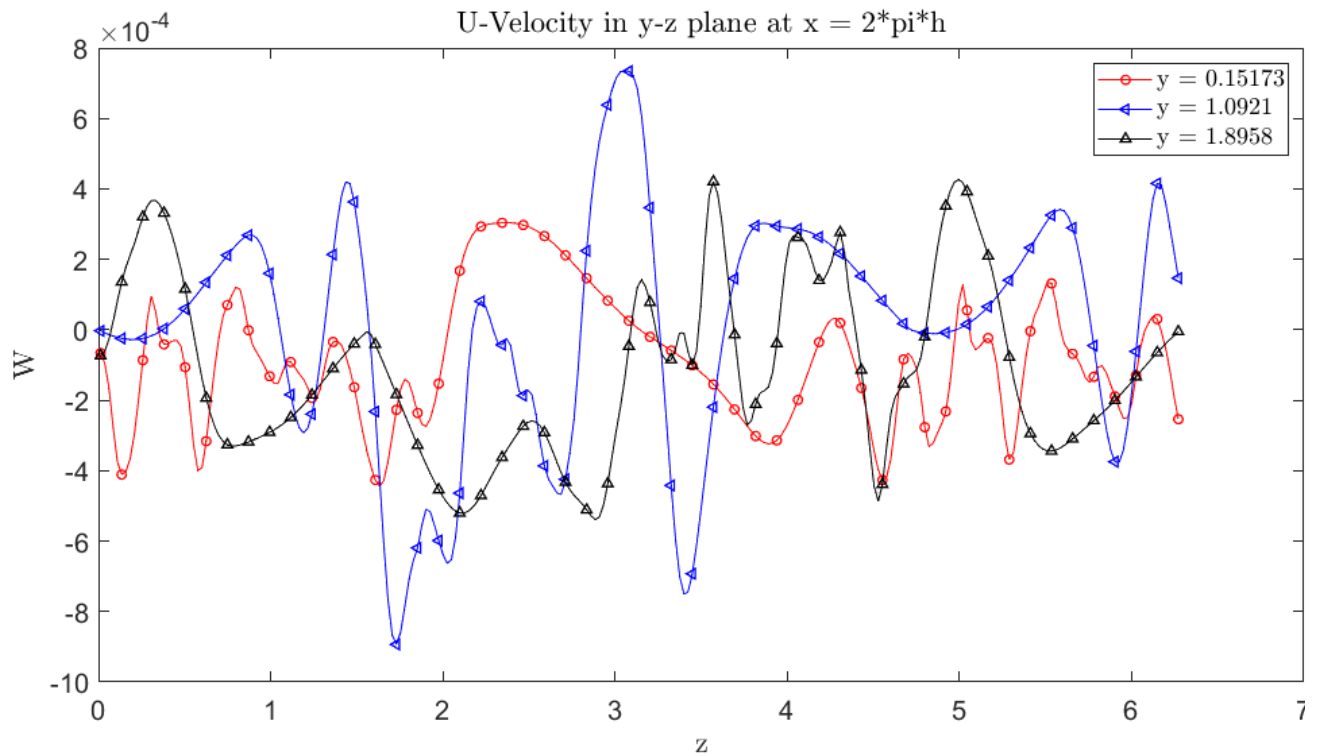
figure
plot(zk(1:end), WW(1:end,50), '-or', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
hold on
plot(zk(1:end), WW(1:end,100), '-<b', 'MarkerIndices', 1:5:length(zk), 'LineWidth', 1, 'markersize', 5);
```

```

plot(zk(1:end), WW(1:end,150), '-^k', 'MarkerIndices', 1:length(zk), 'LineWidth', 1, 'markersize', 5);
title('U-Velocity in y-z plane at x = 2*pi*h', 'FontWeight', 'Bold', 'Interpreter', 'Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
lgd = {'y = ', num2str(grid_y(50))}, {'y = ', num2str(grid_y(100))}, {'y = ', num2str(grid_y(150))};
legend(lgd, 'Interpreter', 'Latex');
xlabel('z', 'Interpreter', 'Latex');
ylabel('W', 'Interpreter', 'Latex');

```





```
clearvars('-except', 'Velocity_Data', 'data_path', 'indexs', 'vel_tavg', 'grid_x', 'grid_y', 'grid_z', 'vel_bar', 'nx',...
'ny', 'nz', 'Uc', 'nu', 'Re_c', 'h');
```

Q2 Compute the friction Reynolds number (Re_τ), displacement thickness (δ^*/h), momentum

thickness (θ/h) and shape factor ($H = \delta^*/\theta$). Compare your results with Table 1 in KMM.

```
% Lets consider t=20sec for the computation of the friction velocity.

% filename = strcat(data_path, '/Velocity_field/', sprintf('INS3D_Vel_%06d.dat', 41));
% data = read_vel(filename);
```

Q2 - Part1 - Friction Reynolds no (Re_τ)

```
% Velocity at y=0 (wall normal direction)
% viscous stress on the wall
% for that we need to consider the stream and span wise direction
% velocities

u1 = sqrt( vel_bar(1,1).^2 + vel_bar(1,3).^2 );
u_tua = sqrt(nu*u1/grid_y(1));

format short
Re_tua = u_tua*h./nu;
disp(['friction Reynolds number (Ret) = ', num2str(Re_tua)]);
```

friction Reynolds number (Re_τ) = 178.4352

Q2 - Part2 - displacement thickness (δ^*/h)

```
%  $\delta^*$  = integration(1-u/Um, dy) with limit 0 to h
% say x = 2*pi*h and z = pi*h(mid points)

u_bar = vel_bar(:,1);
U0 = max(u_bar); % (1/(2*h))*trapz(grid_y, u_bar);

ind = U0 > u_bar(1:round(end/2));
u = u_bar(ind);
y = grid_y(ind);

%plot(u, yi);
del_star = trapz(grid_y((1:floor(end/2))), (1-u_bar(1:floor(end/2))./U0));
del_star_by_h = del_star/h;
disp(['displacement thickness ( $\delta^*/h$ ) = ', num2str(del_star_by_h)]);
```

displacement thickness (δ^*/h) = 0.148

Q2 - Part-3 momentum thickness (θ/h)

```
%plot(u, yi);
tha = trapz(grid_y(1:floor(end/2)), (u_bar(1:floor(end/2))./U0).*(1-u_bar(1:floor(end/2))./U0));
tha_by_h = tha/h;
disp(['momentum thickness( $\theta/h$ ) = ', num2str(tha_by_h)]);
```

momentum thickness(θ/h) = 0.091489

Q2 - Part-4 shape factor

```
H = del_star/tha;
disp(['shape factor (H) = ', num2str(H)]);

heading={'Variable' 'KMM' 'Our Results'};
data = ["Ret", 180, Re_tua; " $\delta^*/h$ ", 0.141, del_star_by_h; " $\theta/h$ ", 0.087, tha_by_h; "H", 1.62, H];
Q2_Results = array2table(data, 'VariableNames', heading)

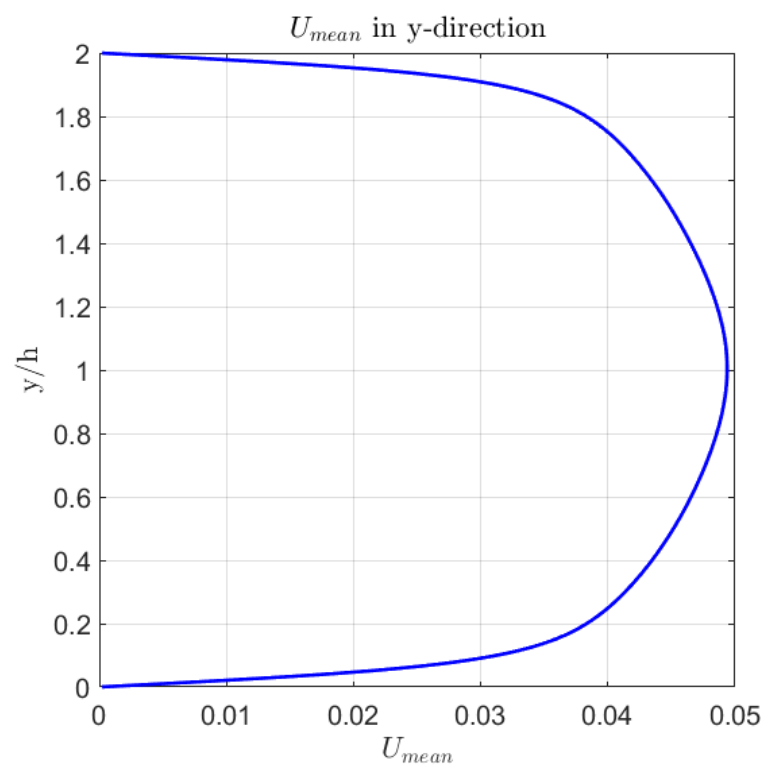
CM = jet(6);
figure
plot(u_bar, grid_y, 'color', CM(1, :), 'LineWidth', 2);
title('$U_{mean}$ in y-direction', 'FontWeight', 'Bold', 'Interpreter', 'Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
axis square
grid on
xlabel('$U_{mean}$', 'Interpreter', 'Latex');
ylabel('y/h', 'Interpreter', 'Latex');
```

shape factor (H) = 1.6176

Q2_Results =

4x3 table

Variable	KMM	Our Results
"Ret"	"180"	"178.4352"
" δ^*/h "	"0.141"	"0.148"
" θ/h "	"0.087"	"0.091489"
"H"	"1.62"	"1.6176"



```
clearvars('-except', 'Velocity_Data', 'data_path', 'indexs', 'vel_tavg', 'grid_x', 'grid_y', 'grid_z', 'vel_bar', 'h...',
, 'Re_tua', 'del_star', 'tha', 'H', 'u_tua', 'CM', 'nx', 'ny', 'nz', 'Uc', 'h', 'nu');
```

Q3 - Plot the mean velocity profile and root-mean-square velocity fluctuations in wall units.

Compare your results with figure 5 and 6 in KMM.

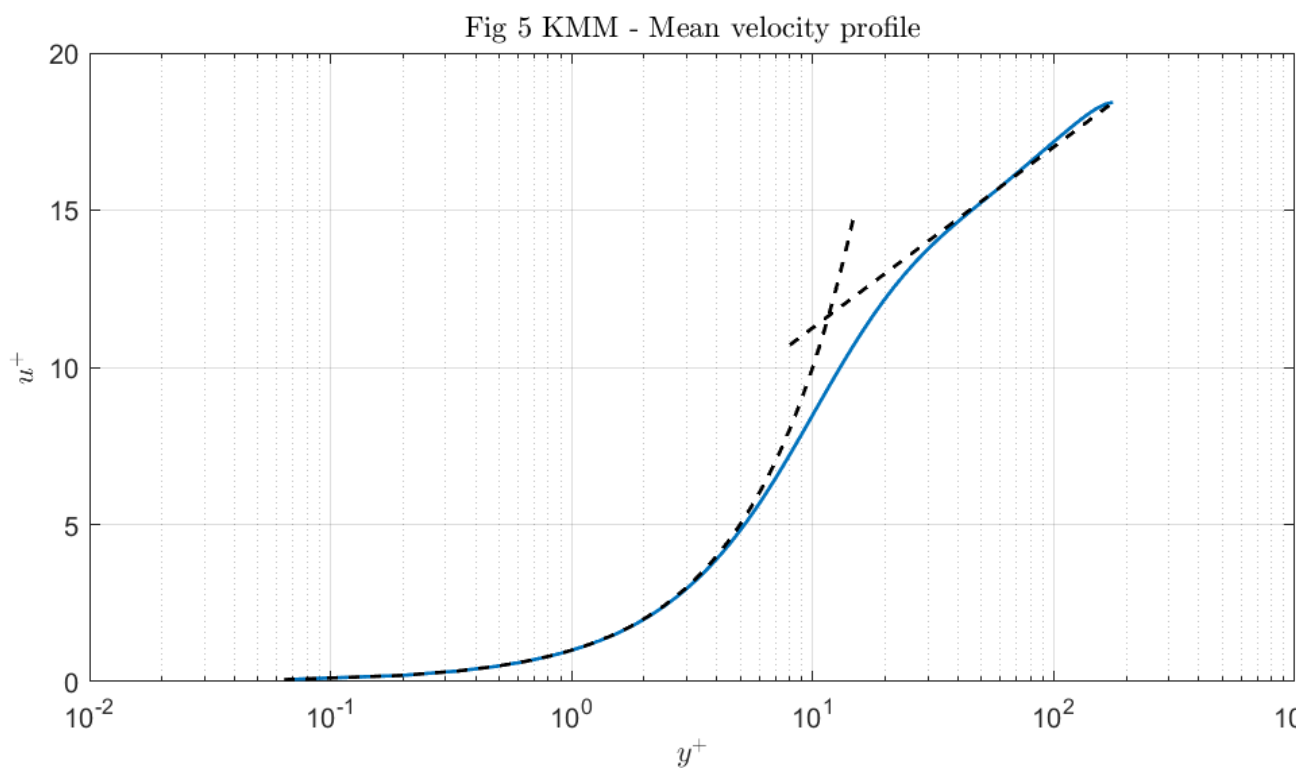
```
% y-plus

y_plus = grid_y(1:round(end/2)) * u_tua / nu;

u_plus = vel_bar(:,1)./(u_tua);
u_plus1 = y_plus;
u_plus2 = 2.5*log(y_plus) + 5.5;

figure
plot(y_plus, u_plus(1:round(end/2)), 'LineWidth', 2);
hold on
plot(y_plus(y_plus<15), u_plus1(y_plus<15), '--k', 'LineWidth', 2);
plot(y_plus(y_plus>8), u_plus2(y_plus>8), '--k', 'LineWidth', 2);

set(gca, 'XScale', 'log')
title('Fig 5 KMM - Mean velocity profile', 'FontWeight', 'Bold', 'Interpreter', 'Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
%xlim([0, 300])
grid on
xlabel('$y^+$', 'Interpreter', 'Latex');
ylabel('$u^+$', 'Interpreter', 'Latex');
```



Q3 - Part 2

In this paper an overbar indicates an average over x , z and t , and a prime indicates perturbation from this average. We have the time and x, z averaged data. We need the one data file to get the perturbation from.

```
n_files = dir(fullfile(data_path, 'Velocity_field', '*.dat'));
N = numel(n_files);
Mean_U_all = zeros(size(indexs));
for i=1:size(indexs,1)
    Mean_U_all(i,:) = vel_bar(indexs(i,2),:);
end

uvw = Velocity_Data - Mean_U_all;

uvw_rms = sqrt(sum(uvw.^2, 3)/N);

u_rms = reshape(uvw_rms(:,1), nx, ny, nz);
v_rms = reshape(uvw_rms(:,2), nx, ny, nz);
w_rms = reshape(uvw_rms(:,3), nx, ny, nz);

u_rms_mean = mean(mean(u_rms, 3), 1)';
v_rms_mean = mean(mean(v_rms, 3), 1)';
w_rms_mean = mean(mean(w_rms, 3), 1)';
```

```
clear uvw uvw_rms u_rms v_rms w_rms
```



```

%
% uu_sqr_sum = 0;
% vv_sqr_sum = 0;
% ww_sqr_sum = 0;
%
% for i=1: numel(n_files)
%     filename = strcat(data_path, '/Velocity_field/', sprintf('INS3D_Vel_%06d.dat', i));
%     data = read_vel(filename);
%     vel_t = Uc.*(data.U, data.V, data.W);
%
%
%
%     uu = UU-vel_bar(:,1)';
%     vv = VV-vel_bar(:,2)';
%     ww = WW-vel_bar(:,3)';
%
%
%     vel_t = [data.U, data.V, data.W];
%     vel1(:, :, i) = vel_t;
%     i
%
%     uu_sqr_sum = uu_sqr_sum + uu.^2;
%     vv_sqr_sum = vv_sqr_sum + vv.^2;
%     ww_sqr_sum = ww_sqr_sum + ww.^2;
%     i
% end
%
%
% N = numel(n_files);
% u_rms = (uu_sqr_sum./N).^0.5;
% v_rms = (vv_sqr_sum./N).^0.5;
% w_rms = (ww_sqr_sum./N).^0.5;
%
% u_rms_mean = mean(mean(u_rms, 3),1)';
% v_rms_mean = mean(mean(v_rms, 3),1)';
% w_rms_mean = mean(mean(w_rms, 3),1)';

```

```

% vel_pertub_t = velocity_mid_line - vel_bar;
% N = 41*size(vel_pertub_t,3);
% vel_rms = sqrt((1/N) * sum(vel_pertub_t.^2, 3));
figure
plot(grid_y, u_rms_mean/u_tua, '-or', 'MarkerIndices', 1:5:length(grid_y), 'LineWidth', 1, 'markersize', 5);
hold on
plot(grid_y, v_rms_mean/u_tua, '-<b', 'MarkerIndices', 1:5:length(grid_y), 'LineWidth', 1, 'markersize', 5);
plot(grid_y, w_rms_mean/u_tua, '-^k', 'MarkerIndices', 1:5:length(grid_y), 'LineWidth', 1, 'markersize', 5);

title('Fig. 6(a) KMM - RMS velocity fluctuations normalized by the wall shear velocity','FontWeight',...
'Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
xlabel('y/h','Interpreter','Latex');
ylabel('$u_{rms}$, $v_{rms}$, $w_{rms}$','Interpreter','Latex');
legend('$u_{rms}$', '$v_{rms}$', '$w_{rms}$', 'Location', 'EastOutside','Interpreter','Latex');

figure

plot(y_plus(y_plus<80), u_rms_mean(y_plus<80)./u_tua, '-or', 'MarkerIndices', 1:5:length(y_plus(y_plus<80)), 'LineWidth', 1, 'markersize', 5);
hold on
plot(y_plus(y_plus<80), v_rms_mean(y_plus<80)./u_tua, '-<b', 'MarkerIndices', 1:5:length(y_plus(y_plus<80)), 'LineWidth', 1, 'markersize', 5);
plot(y_plus(y_plus<80), w_rms_mean(y_plus<80)./u_tua, '-^k', 'MarkerIndices', 1:5:length(y_plus(y_plus<80)), 'LineWidth', 1, 'markersize', 5);

title('Fig. 6(b) KMM - RMS velocity fluctuations normalized by the wall shear velocity','FontWeight', ...
'Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
xlabel('y+');
ylabel('$u_{rms}$, $v_{rms}$, $w_{rms}$','Interpreter','Latex');
legend('$u_{rms}$', '$v_{rms}$', '$w_{rms}$', 'Location', 'EastOutside','Interpreter','Latex');

```

Fig. 6(a) KMM - RMS velocity fluctuations normalized by the wall shear velocity

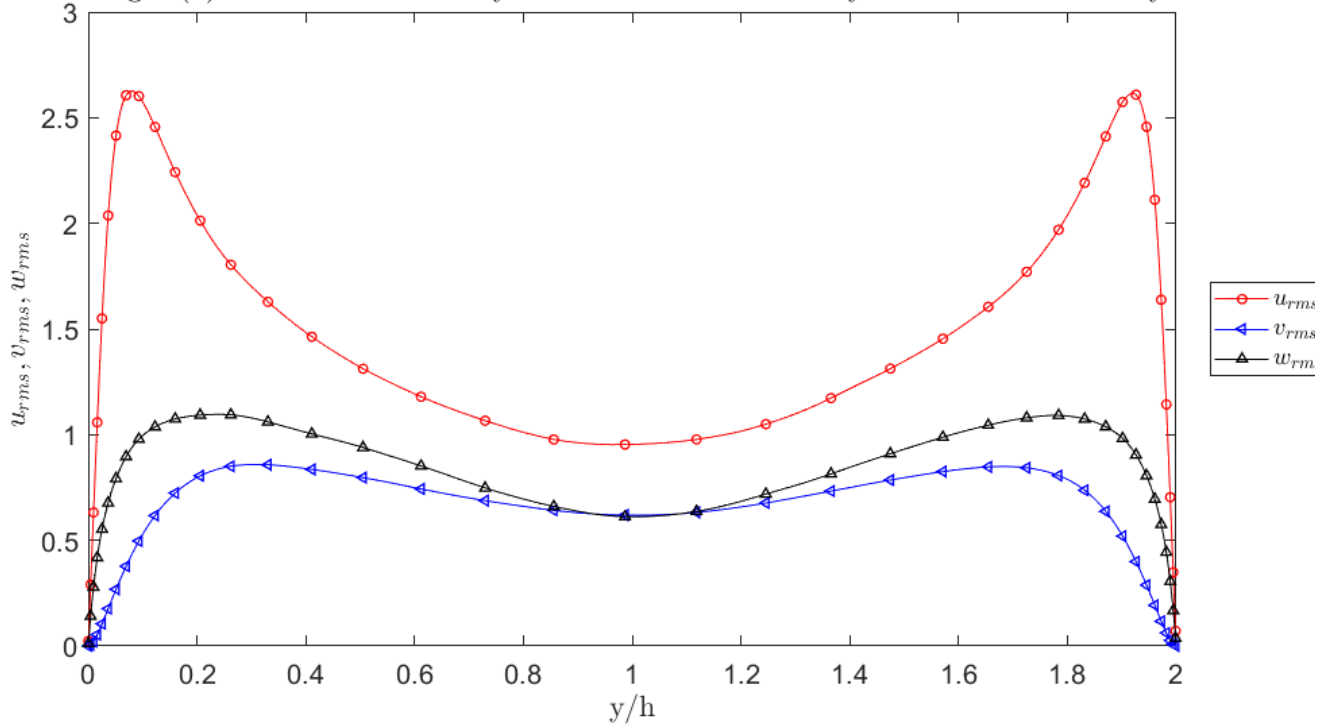
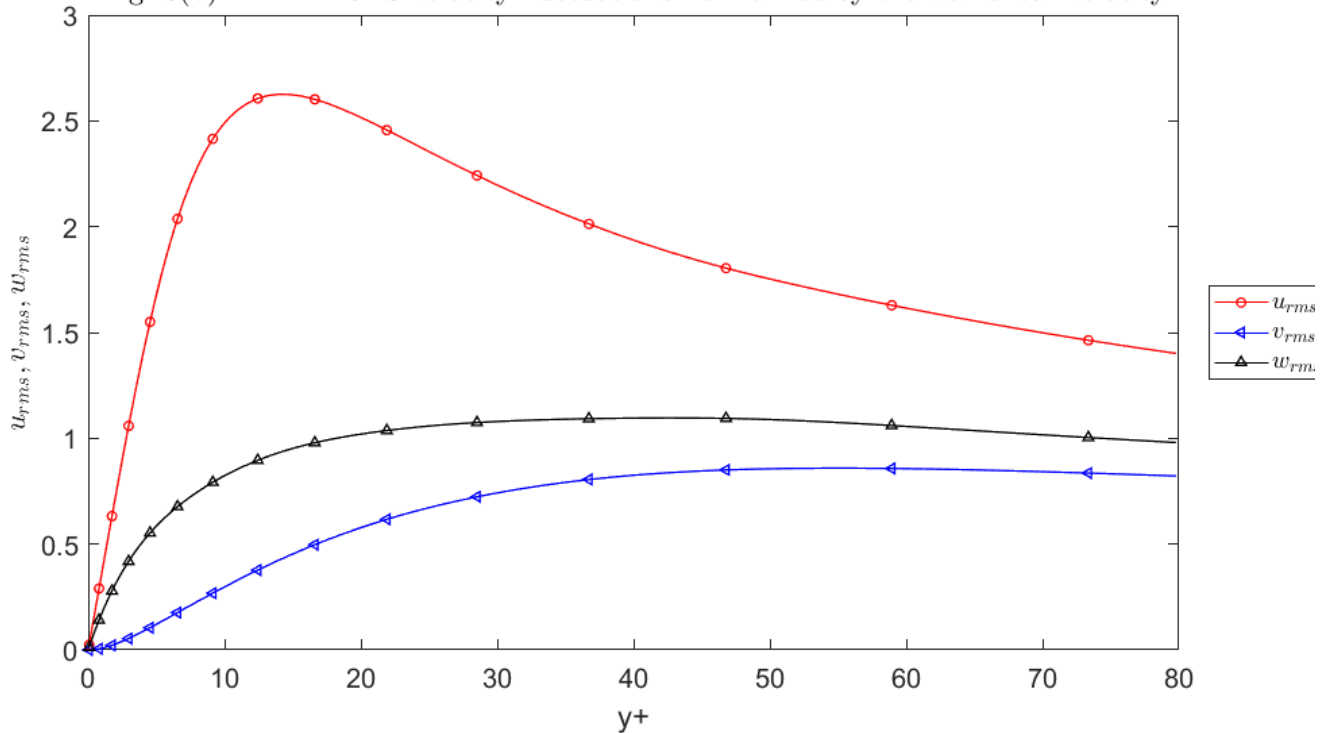


Fig. 6(b) KMM - RMS velocity fluctuations normalized by the wall shear velocity



```
clearvars('-except', 'Velocity_Data', 'data_path', 'indexes', 'vel_tavg', 'grid_x', 'grid_y', 'grid_z', 'vel_bar', 'h'...
    , 'Re_tua', 'del_star', 'tha', 'H', 'u_tua', 'CM', 'N', 'nu', 'u_rms_mean', 'v_rms_mean', 'w_rms_mean',...
    'nx', 'ny', 'nz', 'Uc', 'h', 'nu', 'Mean_U_all');
```

Q-4 -- . Plot the wall-normal profiles of the total shear stress, viscous shear stress and Reynolds shear

stress. Compare your results with figure 10 of KMM.

```
uvw_pert = Velocity_Data - Mean_U_all;
uv_pert = mean(uvw_pert(:,1,:).*uvw_pert(:,2,:), 3);
uv_pert = reshape(uv_pert, nx, ny, nz);

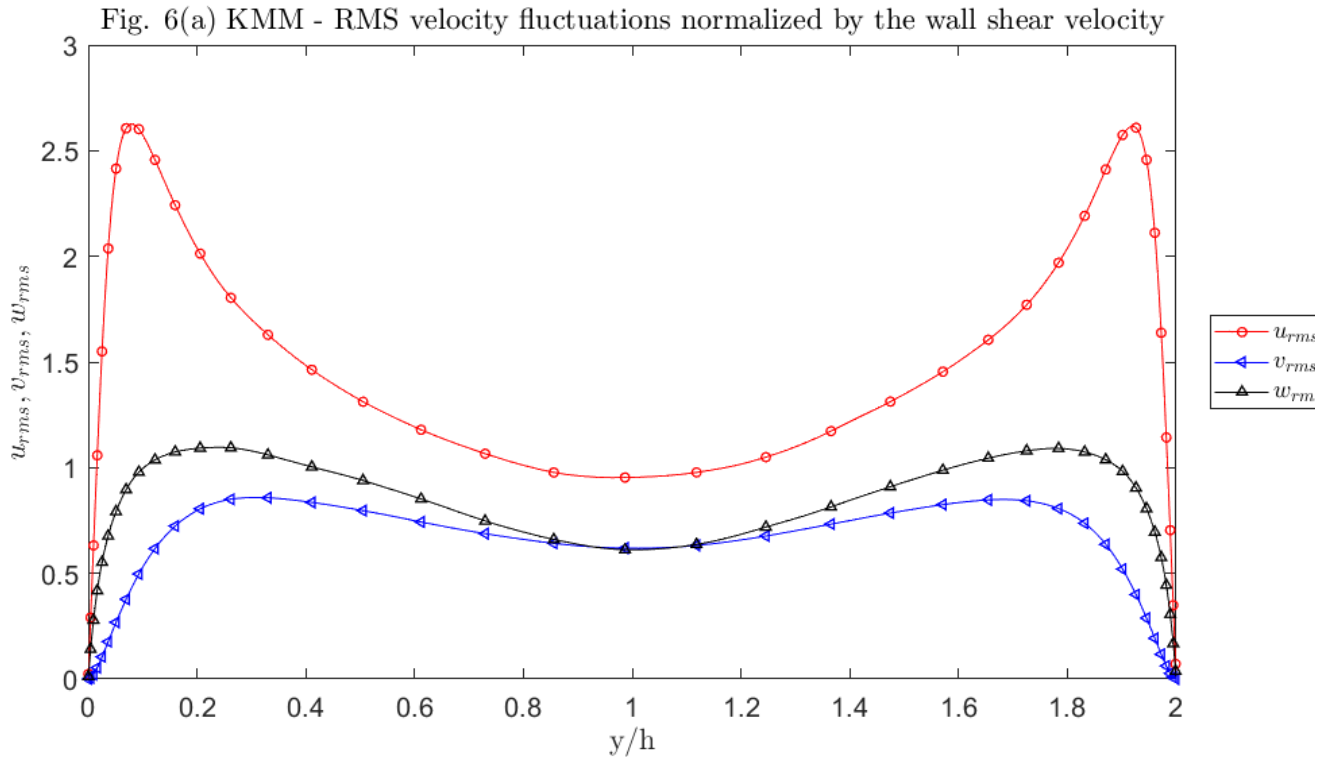
uv_pert_mean = mean(mean(uv_pert, 3),1)';

% n_files = dir(fullfile(data_path, '/Velocity_field', '*.dat'));
% uv_pert = 0;
%
%
% for i=1:numel(n_files)
```

```

% filename = strcat(data_path, '/Velocity_field/', sprintf('INS3D_Vel_%06d.dat', i));
% data = read_vel(filename);
% vel_t = Uc*[data.U, data.V, data.W];
%
% UU = reshape(vel_t(:,1), nx, ny, nz);
% VV = reshape(vel_t(:,2), nx, ny, nz);
% WW = reshape(vel_t(:,3), nx, ny, nz);
%
% u_pert = UU-vel_bar(:,1)';
% v_pert = VV-vel_bar(:,2)';
% w_pert = WW-vel_bar(:,3)';
%
% uv_pert = uv_pert + u_pert.*v_pert;
% i
% end
%
% uv_pert = uv_pert./N;
%
% uv_pert_mean = mean(mean(uv_pert, 3),1)';

```



```

% Normalized Reynolds stress
uv_N = -uv_pert_mean./(u_tua*u_tua);

u_plus = vel_bar(:,1)./u_tua;
y = grid_y-h; % will be in range -h to h
y_plus = grid_y*u_tua/nu;

u_plus1 = [0; u_plus; 0];
y1 = [-h; y; h];

grad_u_plus = (u_plus1(2:end)-u_plus1(1:end-1))./((u_tua/nu)*(y1(2:end) - y1(1:end-1)));

stress_N = uv_N + grad_u_plus(1:end-1);

figure
plot(y, uv_N, '-k','LineWidth', 2);
hold on
plot(y, stress_N, '--r','LineWidth', 2);

title('Fig. 10(a) KMM - Normalized shear stress','FontWeight','Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 6], 'PaperUnits', 'Inches', 'PaperSize', [12,6])
xlabel('y/h','Interpreter','Latex');
ylabel('Normalized stress','Interpreter','Latex');
legend('Normalized Reynolds stress', 'Normalized Total stress', 'Location', 'EastOutside');

figure
plot(y_plus(y_plus<80), uv_N(y_plus<80), '-k','LineWidth', 2);

title('Fig. 10(b) KMM - Normalized shear stress','FontWeight','Bold','Interpreter','Latex');
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 8], 'PaperUnits', 'Inches', 'PaperSize', [12,8])

```

```
xlabel('y+', 'Interpreter', 'Latex');
ylabel('Normalized stress', 'Interpreter', 'Latex');
```

Fig. 10(a) KMM - Normalized shear stress

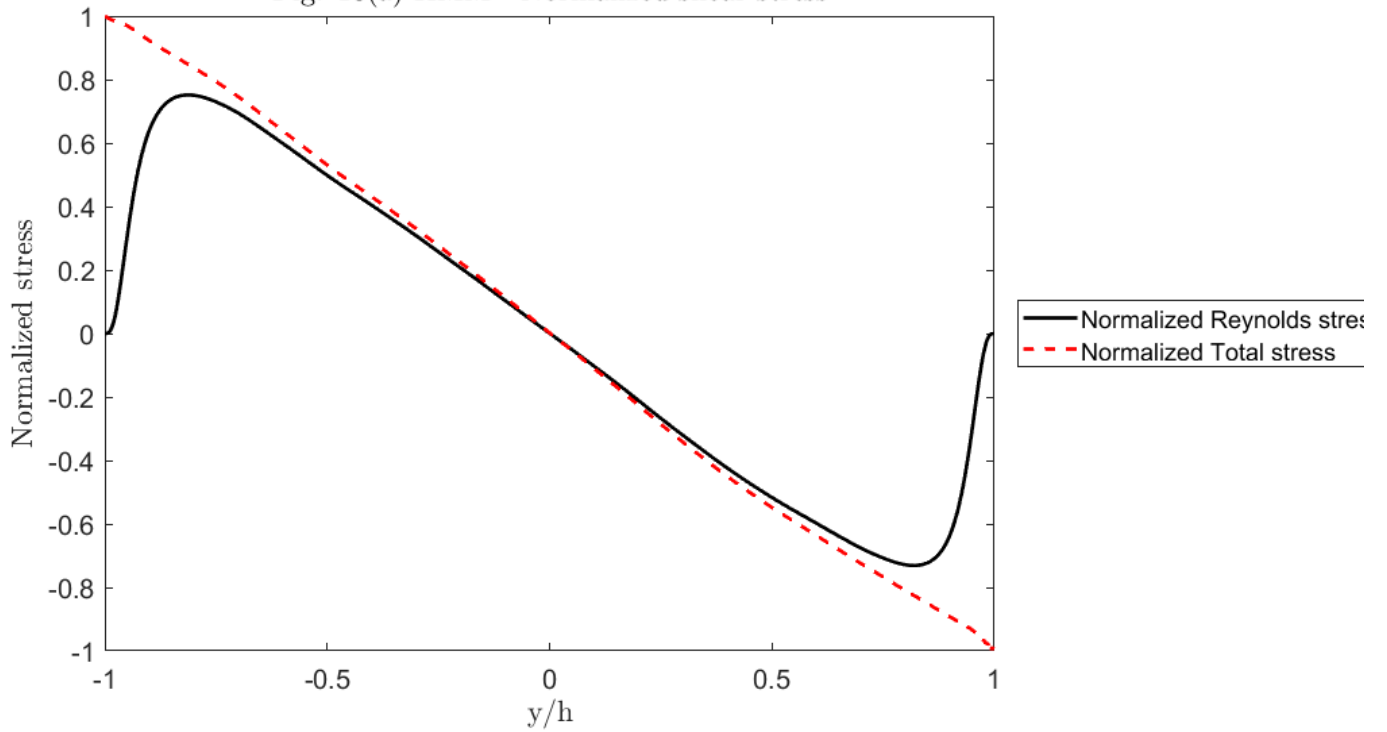
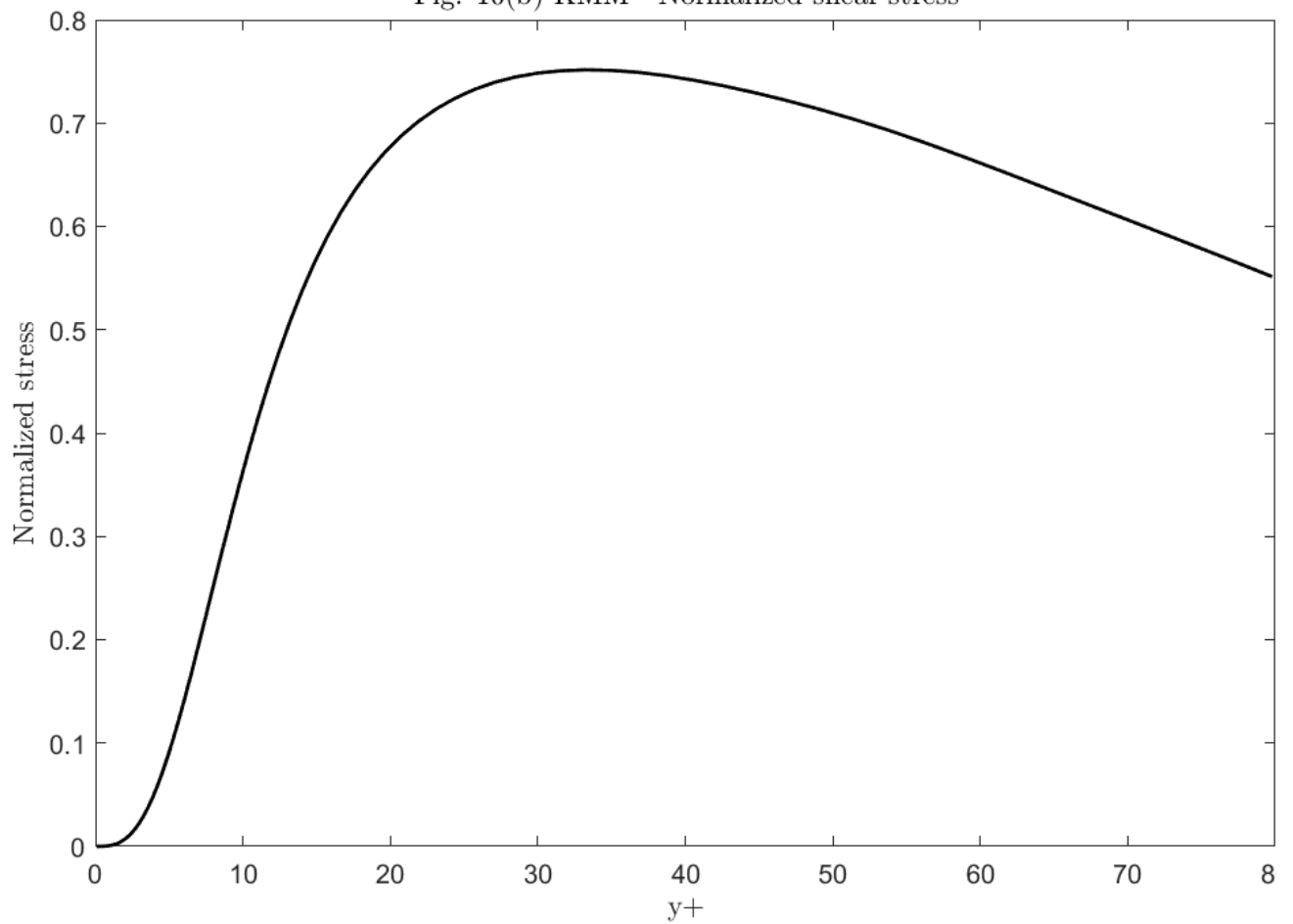


Fig. 10(b) KMM - Normalized shear stress



```
clearvars('-except', 'Velocity_Data', 'data_path', 'indexs', 'vel_tavg', 'grid_x', 'grid_y', 'grid_z', 'vel_bar', 'h'...
, 'Re_tua', 'del_star', 'tha', 'H', 'u_tua', 'CM', 'N', 'nu', 'u_rms_mean', 'v_rms_mean', 'w_rms_mean',...
'nx', 'ny', 'nz', 'uv_pert', 'uv_pert_mean', 'uv_pert_mean', 'stress_N', 'Uc', 'h', 'nu', 'Mean_U_all');
```

Q-5 - . Plot the joint probability density function of u' and v' at

$y^+ = 5, 20, 100$ and 180 . Compare your results with figure 21 of KMM.

```
y_p = [5, 20, 100, 180];
y_i = nu*y_p/u_tua;

y_ind = [];
for i=1:numel(y_i)
    dist = abs(grid_y-y_i(i));
    [val, k] = min(dist);
    y_ind = [y_ind, k];
end
ind = ismember(indexs(:,2), y_ind);
data_ypl = Velocity_Data(ind, :, :);

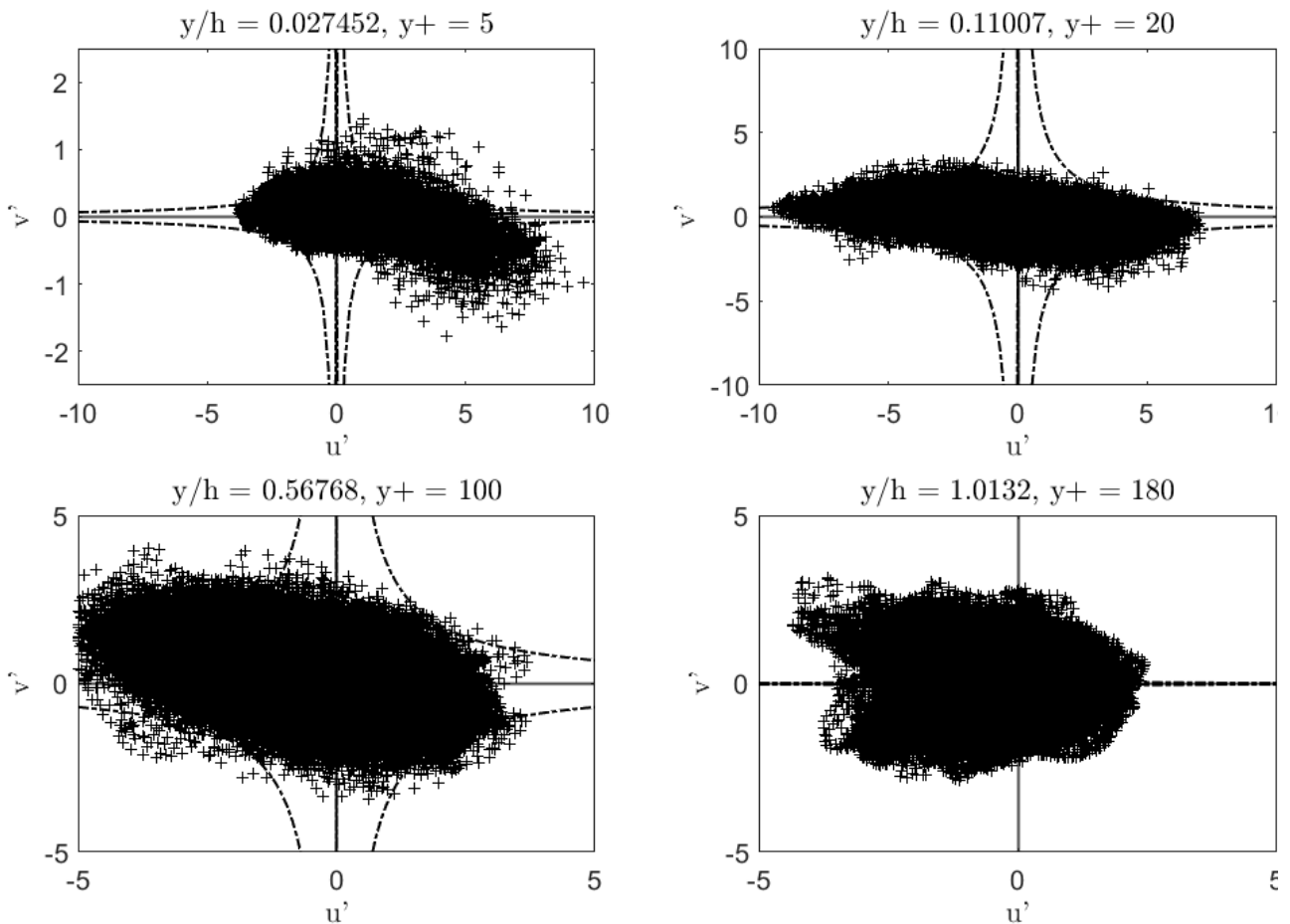
% parfor n=1:numel(n_files)
%     filename = strcat(data_path, '/Velocity_field/', sprintf('INS3D_Vel_%06d.dat', n));
%     data = read_vel(filename);
%     vel_t = Uc*[data.U, data.V, data.W];
%     data_ypl(:, :, n) = vel_t(ind, :);
%     n
% end
```

```
clear data_ypl
data_ypl(:, :, :) = data_ypl(:, :, 35:41);
indexs_ypl = indexs(ind, :);
x_lim = [10, 10, 5, 5];
y_lim = [2.5, 10, 5, 5];
figure
for j=1:numel(y_ind)
    ind_j = (indexs_ypl(:,2) == y_ind(j));
    vel_ypl = (data_ypl(ind_j, :, :)) - vel_bar(y_ind(j), :);
    vel_ypl = vel_ypl./u_tua;
    vel_j = [];
    for n=1:size(data_ypl, 3)
        vel_j = [vel_j; vel_ypl(:, :, n)];
    end
    subplot(2,2,j)
    scatter(vel_j(:,1), vel_j(:,2), '+k');
    xline(0, '-k', 'linewidth', 1.5);
    yline(0, '-k', 'linewidth', 1.5)
    title(['y/h = ', num2str(grid_y(y_ind(j))), ', y+ = ', num2str(y_p(j))], 'Interpreter', 'Latex');
    box('on')
    set(gca, 'FontSize', 15);
    set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 8], 'PaperUnits', 'Inches', 'PaperSize', [12,8])
    xlabel("u'", 'Interpreter', 'Latex');
    ylabel("v'", 'Interpreter', 'Latex');
    xlim([-x_lim(j), x_lim(j)]);
    ylim([-y_lim(j), y_lim(j)]);

    u_desh = linspace(-x_lim(j)*u_tua, x_lim(j)*u_tua, 100);
    v_desh = 8*(-uv_pert_mean(y_ind(j)))./u_desh;
    hold on
    plot(u_desh/u_tua, v_desh/u_tua, '-.k', 'linewidth', 1.5);
    plot(u_desh/u_tua, -v_desh/u_tua, '-.k', 'linewidth', 1.5);

end
sgtitle('Fig. 21 KMM - Distribution of (u', v'), Dashed lines are hyperbolas of  $|u'v'| = 8x - \overline{u'v'}$ ', 'FontWeight', 'Bold', 'Interpreter', 'Latex')
```

Fig. 21 KMM - Distribution of (u', v') , Dashed lines are hyperbolas of $|u'v'| = 8x-\overline{u'v'}$



```
clearvars('-except', 'Velocity_Data', 'data_path', 'indexs', 'vel_tavg', 'grid_x', 'grid_y', 'grid_z', 'vel_bar', 'h'...
, 'Re_tua', 'del_star', 'tha', 'H', 'u_tua', 'CM', 'N', 'nu', 'u_rms_mean', 'v_rms_mean', 'w_rms_mean',...
'nx', 'ny', 'nz', 'uv_pert', 'uv_pert_mean', 'uv_pert_mean', 'stress_N', 'Uc', 'h', 'nu', 'Mean_U_all');
```

Q6 - Using the velocity field data (INS3D_Vel_000001.dat), plot the streamwise velocity

fluctuations signals with respect to x/h at $y+ = 5$ and 150

```
y_p = [5, 150];
y_i = nu*y_p/u_tua;

y_ind = [];
for i=1:numel(y_i)
    dist = abs(grid_y-y_i(i));
    [val, k] = min(dist);
    y_ind = [y_ind, k];
end
ind = ismember(indexs(:,2), y_ind);
%
% n_files = dir(fullfile(data_path, '/Velocity_field', '*.dat'));
%
% filename = strcat(data_path, '/Velocity_field/', sprintf('INS3D_Vel_%06d.dat', 1));
% data = read_vel(filename);
% vel_t = Uc*[data.U, data.V, data.W];
data_yp = Velocity_Data(ind,:,1); % because one one t=1sec file
```

```
indexs_yp = indexs(ind, :);
x_lim = [13, 13];
y_lim = [10, 10];
figure
for j=1:numel(y_ind)
    ind_j = (indexs_yp(:,2) == y_ind(j));
    vel_yp = (data_yp(ind_j, :)) - vel_bar(y_ind(j),:);
    vel_yp = vel_yp./u_tua;
    ind_yp = indexs_yp(ind_j, :);

    ind_z = [50, 100, 150, 200, 250];
    subplot(2,1,j)
    lgd = {};
    mrks = {'-or', '-<b', '-sk', '-*m', '-dg'};
```

```

for k=1:numel(ind_z)

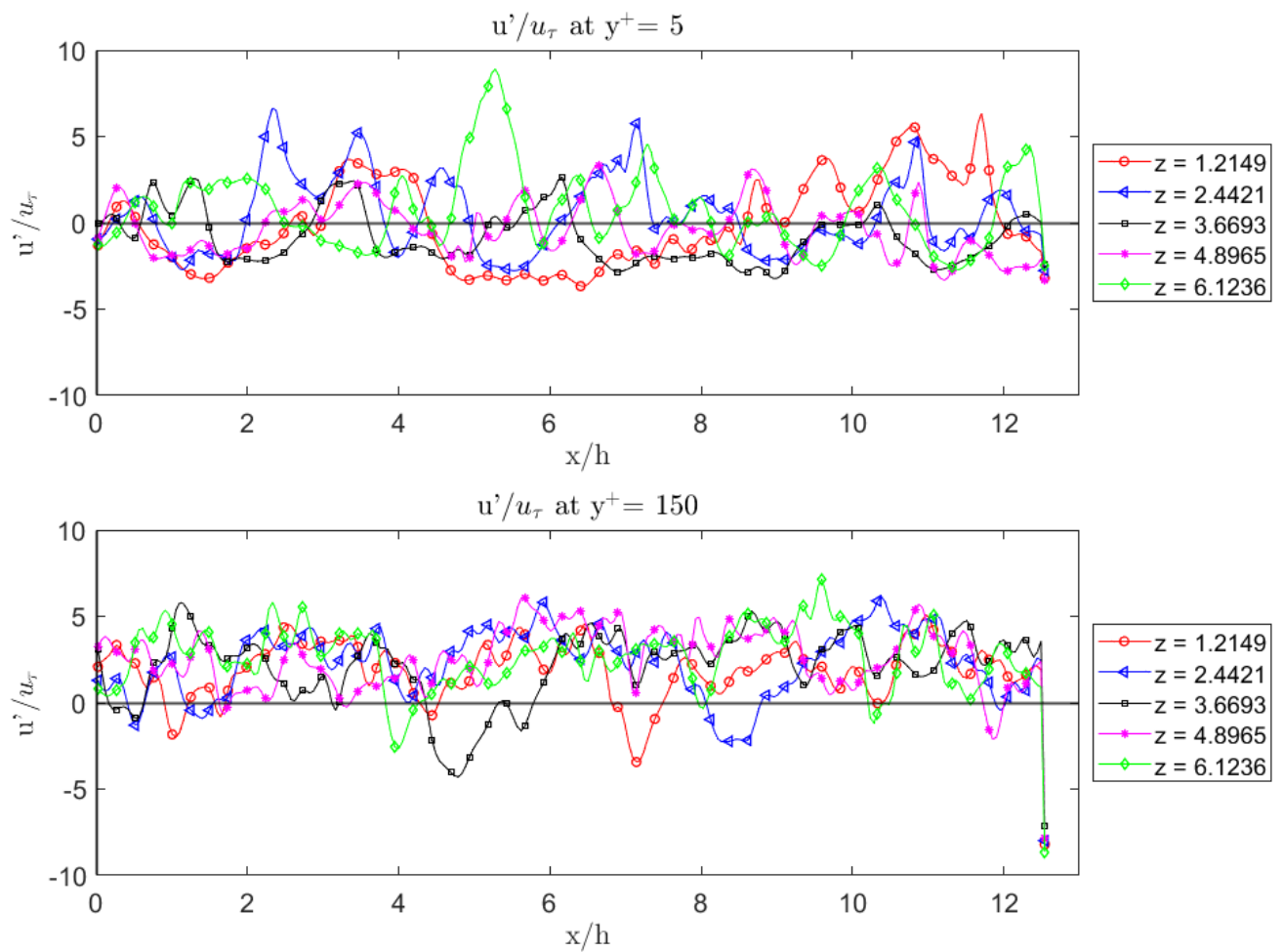
    x_ind = (ind_yp(:, 3) == ind_z(k));
    x_val = grid_x(ind_yp(x_ind, 1));
    y_val = vel_yp(x_ind, 1);
    plot(x_val, y_val, mrks{k}, 'MarkerIndices', 1:5:length(x_val), 'LineWidth', 1, 'markersize', 5);
    hold on
    lgd{k} = ['z = ', num2str(grid_z(ind_z(k)))];

end

legend(lgd,'location', 'eastoutside');
xline(0, '-k', 'linewidth', 1.5, 'HandleVisibility', 'off');
yline(0, '-k', 'linewidth', 1.5, 'HandleVisibility', 'off');
title(strcat("u'/$u_{\tau}$ at $y^+=$ ", num2str(y_p(j))), 'Interpreter', 'Latex');
box('on')
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 8], 'PaperUnits', 'Inches', 'PaperSize', [12,8])
xlabel("x/h", 'Interpreter', 'Latex');
ylabel("u'/$u_{\tau}$", 'Interpreter', 'Latex');
xlim([0, x_lim(j)]);
ylim([-y_lim(j), y_lim(j)]);

end
% As I already pointed out that there is sudden drop in velocity as is
% in perturbation at end point in x and z direction. So in 2nd graph we
% there is sudden drop in perturbation.

```



```

clearvars('except', 'Velocity_Data', 'data_path', 'indexs', 'vel_tavg', 'grid_x', 'grid_y', 'grid_z', 'vel_bar', 'h'...
, 'Re_tua', 'del_star', 'tha', 'H', 'u_tua', 'CM', 'N', 'nu', 'u_rms_mean', 'v_rms_mean', 'w_rms_mean',...
'nx', 'ny', 'nz', 'uv_pert', 'uv_pert_mean', 'uv_pert_mean', 'stress_N', 'Uc', 'h', 'nu');

```

Q7 - Plot the streamwise and spanwise autocorrelation functions $R11(rx/h)$ and $R11(rz/h)$ at a given

wall-normal location ($y_{ref}^+ = 5$ and 150). Compare your results with figure 2 of KMM. Compute the Taylor micro length scale.

```

% Dear Sir, Even though the question is asking for R11, I am here plotting
% the R22, because data of U-velocity have some strange behavior at end
% points. So, Kindly compare the R22(which is Rvv) of KMM paper.
% Regards

```

```

y_p = [5, 150];
y_i = nu*y_p/u_tua;

y_ind = [];
for i=1:numel(y_i)
    dist = abs(grid_y-y_i(i));
    [val, k] = min(dist);
    y_ind = [y_ind, k];
end
ind = ismember(indexs(:,2), y_ind);
data_yp = Velocity_Data(ind, :, :);

```

```

indexs_yp = indexs(ind, :);

for j=1:numel(y_ind)
    ind_j = (indexs_yp(:,2) == y_ind(j));
    vel_yp = (data_yp(ind_j, :, :));%vel_bar(y_ind(j),:);
    ind_pos = indexs_yp(ind_j, :);

    u_yp = vel_yp(:,2, :); % v-velocity (to find Rvv)
    x_yp = grid_x(ind_pos(:,1));
    z_yp = grid_z(ind_pos(:,3));

    u_grid = reshape(u_yp, nx, nz, N);% (in cols, x direction and in rows z direction)
    x_grid = reshape(x_yp, nx, nz);
    z_grid = reshape(z_yp, nx, nz);

    r22_x = 0;
    var_x = 0;
    for n=1:size(u_grid,3)
        for k=1:size(u_grid,2)
            [r11_xt, lags_r11_x] = autocorr(u_grid(:,k,n), size(u_grid,1)-1);
            r22_x = r22_x + r11_xt;
        end
    end

    r22_x = r22_x./(size(u_grid,2)*size(u_grid,3));
    % var_x = var_x./(size(u_grid,2)*size(u_grid,2));
    % r11_x = r11_x./var_x;

    % second derivative of R11_x at x=0 using 2nd order forward difference
    df2 = (2*r22_x(1)-5*r22_x(2)+4*r22_x(3)-r22_x(4))./(grid_x(2)-grid_x(1)).^2;
    lambda_x = abs((-df2/2).^(-0.5));

    r11_z = 0;
    var_z = 0;

    for n=1:size(u_grid,3)
        for k=1:size(u_grid,1)
            [r11_zt, lags_r11_z] = autocorr(u_grid(k,:,n), size(u_grid,1)-1);
            r11_z = r11_z + r11_zt;
        end
    end

    r11_z = r11_z./(size(u_grid,2)*size(u_grid,3));

    % second derivative of R11_x at x=0 using 2nd order forward difference
    df2 = (2*r11_z(1)-5*r11_z(2)+4*r11_z(3)-r11_z(4))./(grid_z(2)-grid_z(1)).^2;
    lambda_z = (-df2/2).^(-0.5);

    disp(['For y= ', num2str(y_p(j)), ', Stream direction  $\lambda_f = ', num2str(lambda_x)]);
    disp(['For y= ', num2str(y_p(j)), ', Span direction  $\lambda_f = ', num2str(lambda_z)]);

    figure
    x11 = linspace(0,5,1000);
    y11 = 1-(x11/lambda_x).^2;

    plot(grid_x(lags_r11_x+1), r22_x, '-k', 'linewidth', 1.5);
    hold on
    plot(x11(y11>-0.5), y11(y11>-0.5), '--k', 'linewidth', 1.5)
    xline(0, '-k', 'linewidth', 1.5);
    yline(0, '-k', 'linewidth', 1.5)
    title(['Fig 2(a) - KMM, Stream wise separations y/h = ', num2str(grid_y(y_ind(j))),...
        ',  $y^{+}$  = ', num2str(y_p(j))], 'Interpreter', 'Latex');
    box('on')
    set(gca, 'FontSize', 15);
    set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 8], 'PaperUnits', 'Inches', 'PaperSize', [12,8])
    xlabel("x/h(index)", 'Interpreter', 'Latex');
    ylabel("$R_{22}(r_x)$", 'Interpreter', 'Latex');
    xlim([0,6]);
    ylim([-0.5, 1.2]);
    legend("$R_{22}(r_x)$", "$p(r_x) = 1-r_x^2/(\lambda_f^2)$", 'Interpreter', 'Latex')

    figure
    x11 = linspace(0,5,100);$$ 
```



```

y11 = 1-(x11/lambda_z).^2;

plot(grid_z(lags_r11_z+1), r11_z, '-k', 'linewidth', 1.5);
hold on
plot(x11(y11>-0.5), y11(y11>-0.5), '--k', 'linewidth', 1.5)

xline(0, '-k', 'linewidth', 1.5);
yline(0, '-k', 'linewidth', 1.5)
title(['Fig 2(b) - KMM, Span wise seperations y/h = ', num2str(grid_y(y_ind(j))),...
      ', $y^{+}$ = ', num2str(y_p(j))], 'Interpreter', 'Latex');
box('on')
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 8], 'PaperUnits', 'Inches', 'PaperSize', [12,8])
xlabel("z/h(index)", 'Interpreter', 'Latex');
ylabel("$R_{22}(r_z)$", 'Interpreter', 'Latex');
xlim([0,3]);
ylim([-0.5 1.2]);
legend("$R_{22}(r_z)$", "$p(r_z) = 1-r_z^2/(\lambda_{f})^2$", 'Interpreter', 'Latex')
end

```

For y+=5, Stream direction $\lambda_f = 0.2199$
 For y+=5, Span direction $\lambda_f = 0.052873$
 For y+=150, Stream direction $\lambda_f = 0.18644$
 For y+=150, Span direction $\lambda_f = 0.16273$

Fig 2(a) - KMM, Stream wise seperations $y/h = 0.027452$, $y^+ = 5$

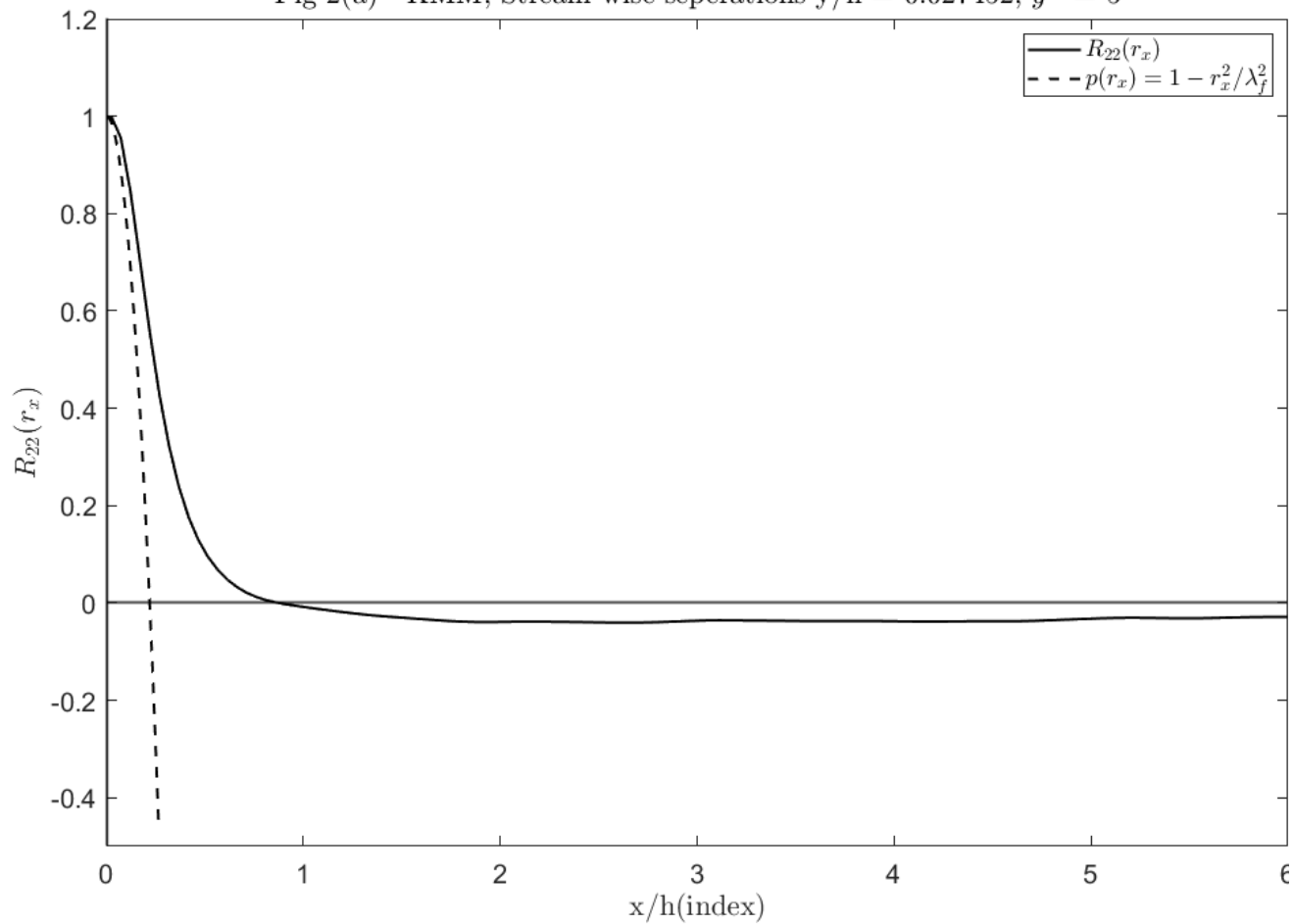


Fig 2(b) - KMM, Span wise seperations $y/h = 0.027452$, $y^+ = 5$

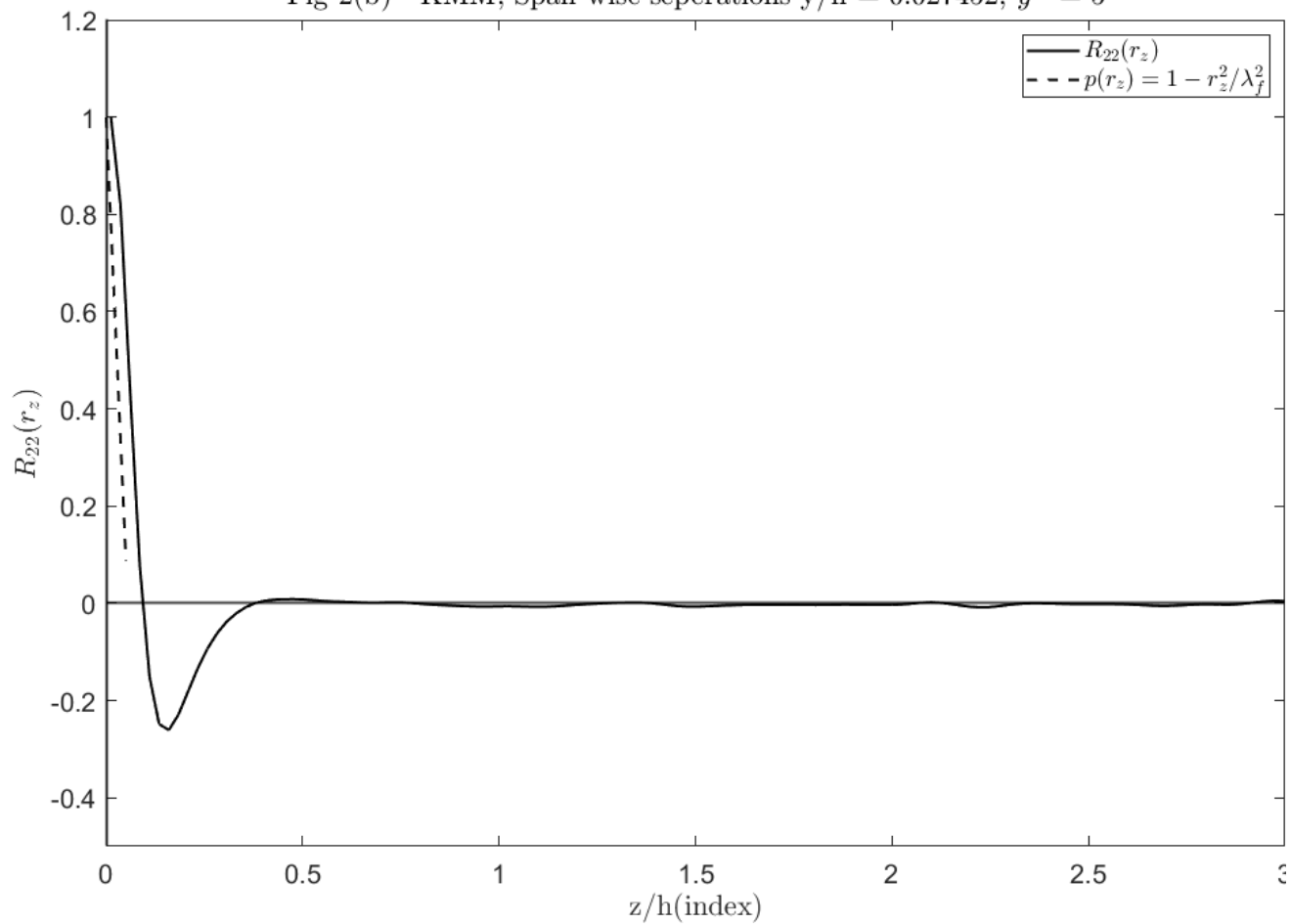


Fig 2(a) - KMM, Stream wise separations $y/h = 0.83008$, $y^+ = 150$

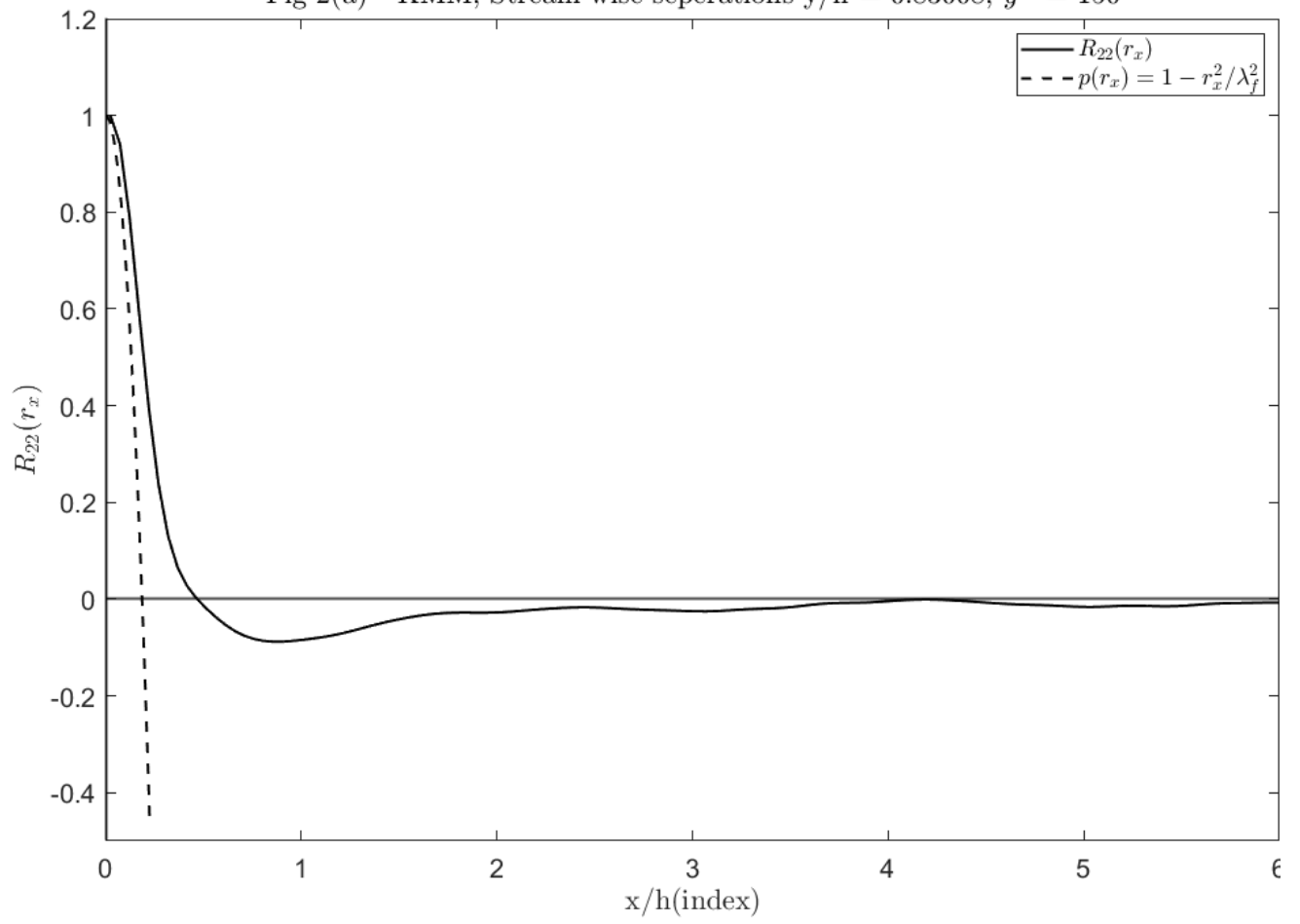
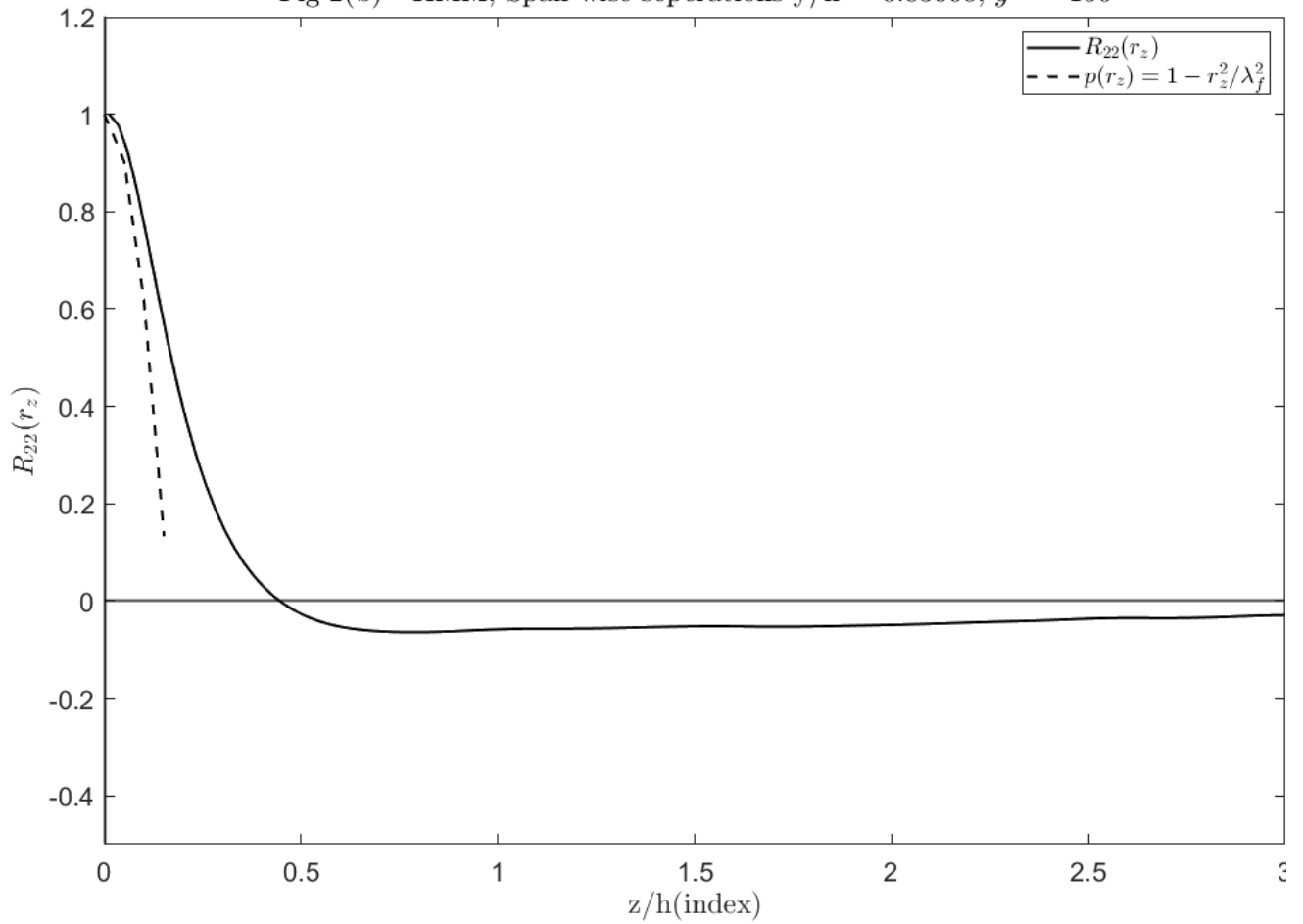


Fig 2(b) - KMM, Span wise separations $y/h = 0.83008$, $y^+ = 150$



8. Compute the one-dimensional energy spectra of streamwise, wall-normal, and spanwise

velocity at $y^+ = 5$ and 150. Compare your results with figure 3 of KMM. Explain your results in terms of Kolmogorov's theory

```

indexs_yp = indexs(ind, :);

for j=1:numel(y_ind)
    ind_j = (indexs_yp(:,2) == y_ind(j));
    vel_yp = (data_yp(ind_j, :, :)) - vel_bar(y_ind(j),:);
    ind_pos = indexs_yp(ind_j, :);

    u_yp = vel_yp(:,1,:);
    v_yp = vel_yp(:,2,:);
    w_yp = vel_yp(:,3,:);

    x_yp = grid_x(ind_pos(:,1));
    y_yp = grid_z(ind_pos(:,2));
    z_yp = grid_z(ind_pos(:,3));

    %u_grid = reshape(u_yp, nx, nz, N);% (in cols, z direction and in rows x direction)
    %v_grid = reshape(v_yp, nx, nz, N);% (in cols, z direction and in rows x direction)
    %w_grid = reshape(w_yp, nx, nz, N);

    %x_grid = reshape(x_yp, nx, nz);
    %y_grid = reshape(y_yp, nx, nz);
    %z_grid = reshape(z_yp, nz, nx)';

    % In stream wise
    % r11_x = 0;
    % r22_x = 0;
    % r33_x = 0;
    % var_11x = 0;
    % var_22x = 0;
    % var_33x = 0;
    v_avg = 0;
    fft_avg = 0;

    r22_x = 0;
    for n=1:size(u_grid,3)

```

```

    for k=1:size(u_grid,2)
        [r22_xt, lags_r22_x] = autocorr(v_grid(:,k,n), size(v_grid,1)-1);
        r22_x = r22_x + r22_xt;
    end
end

r22_x = r22_x./(size(v_grid,2)*size(v_grid,3)^2);

figure
Fs = length(grid_x)/max(grid_x);
L = 256;

f = 2*pi*Fs*(0:L/2)/L;
E_11x = ((1/(pi))*fft(r22_x));

P1 = real(E_11x(1:L/2+1));
plot(f, P1);
xlim([0.3, 50])
set(gca, 'XScale', 'log')
set(gca, 'YScale', 'log')
title('Fig 3(a) - KMM, 1-D Energy Spectra','Interpreter','Latex');
legend(['y/h = ', num2str(grid_y(y_ind(j))),...
        ', $y^{+}$ = ', num2str(y_p(j))], 'Interpreter','Latex');
box('on')
set(gca, 'FontSize', 15);
set(gcf, 'Units', 'Inches', 'Position', [0, 0, 12, 8], 'PaperUnits', 'Inches', 'PaperSize', [12,8])
xlabel("$k_x$", 'Interpreter','Latex');
ylabel("$E_{vv}$", 'Interpreter','Latex');

end

% The energy spectra plot shows that the distance between the grids is good
% enough. The energy density of Evv for higher Kx is very low compared to
% the energy spectrum at low kx. Energy spectra show the contribution in
% total energy by different length scales. In Evv, most of the energy is
% contained by kx<10.

```

Fig 3(a) - KMM, 1-D Energy Spectra

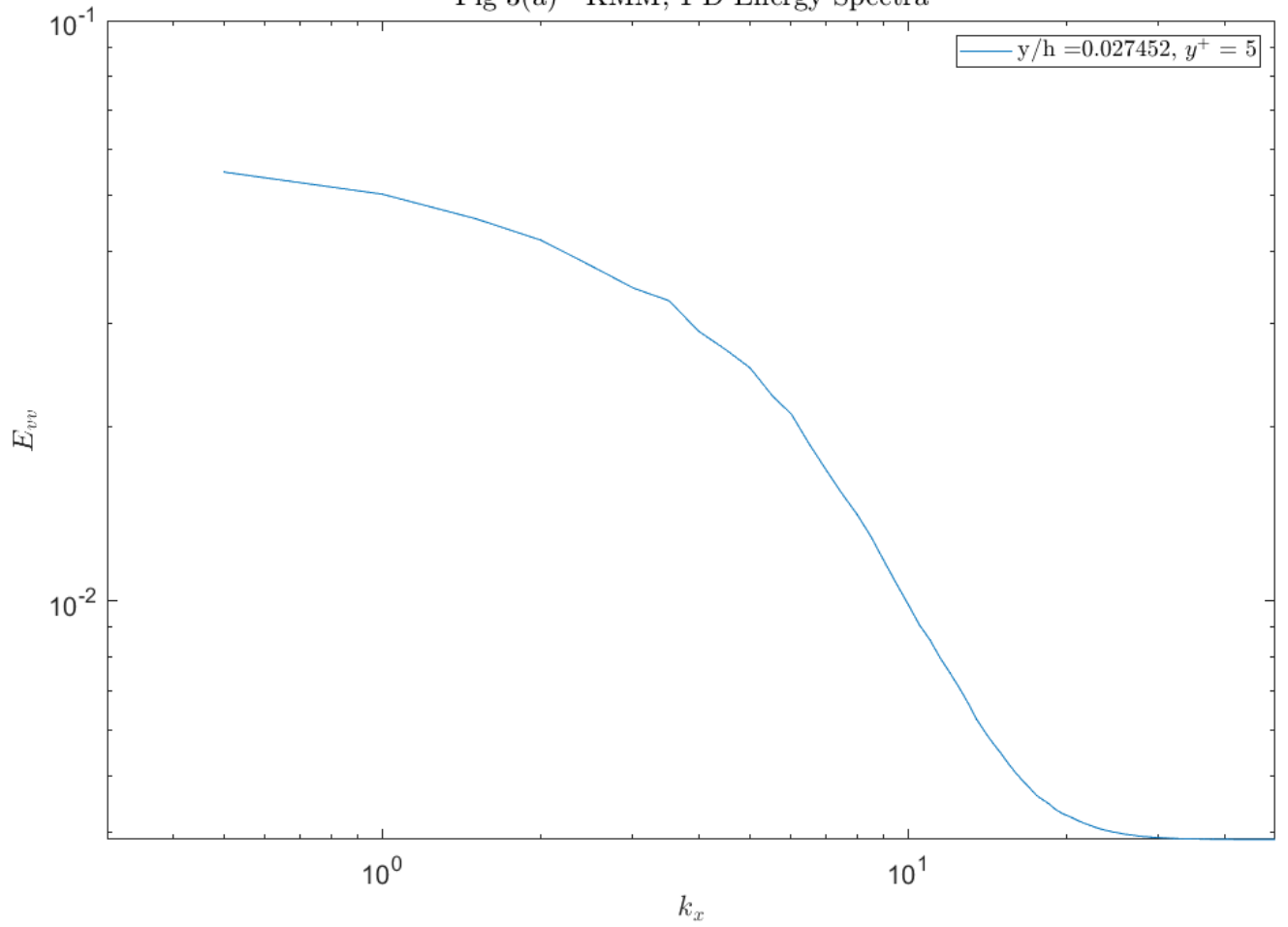
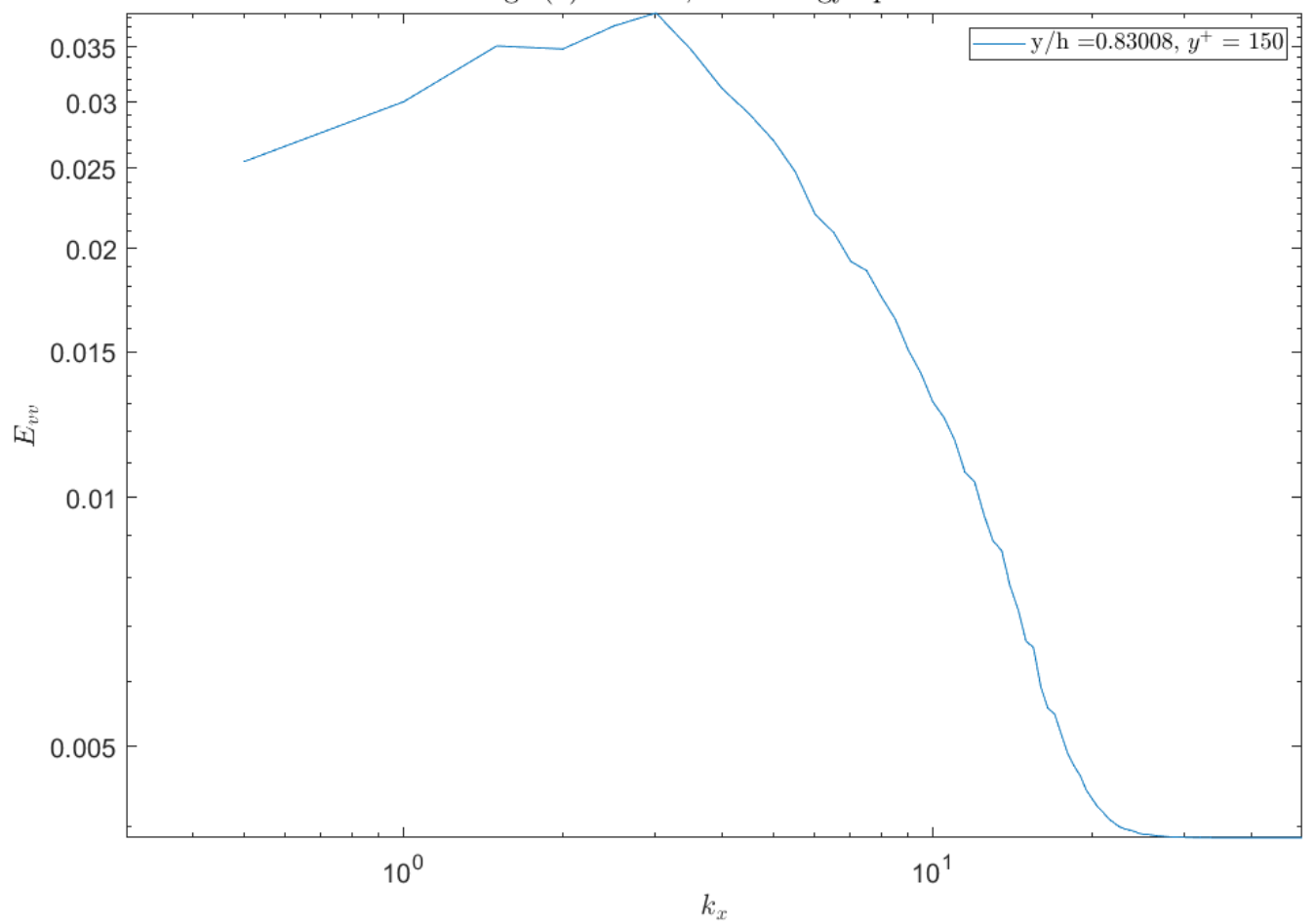


Fig 3(a) - KMM, 1-D Energy Spectra



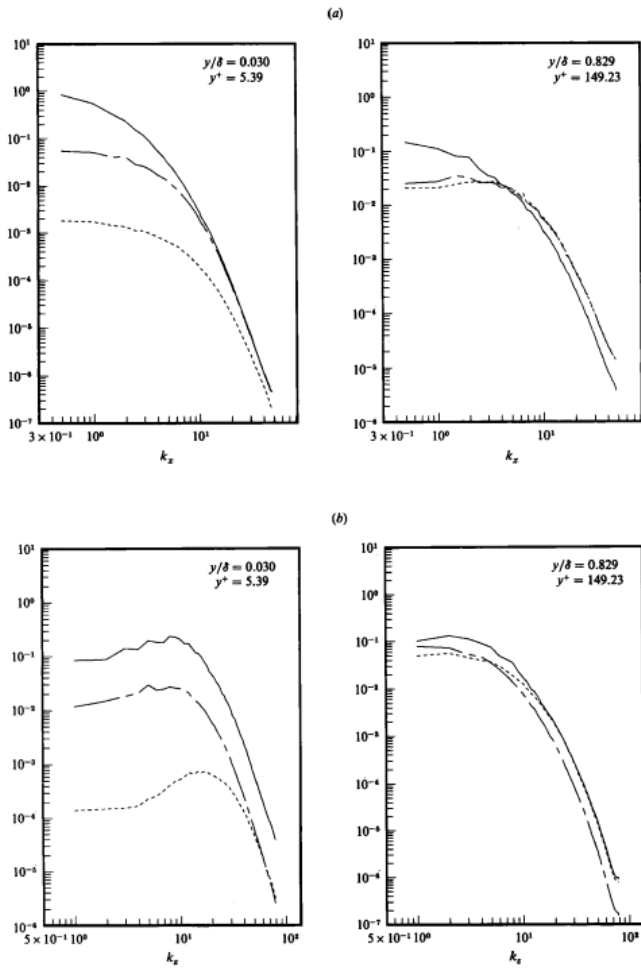


FIGURE 3. One-dimensional energy spectra: —, E_{uu} ; ---, E_{vv} ; — · —, E_{ww} . (a) Streamwise; (b) spanwise.