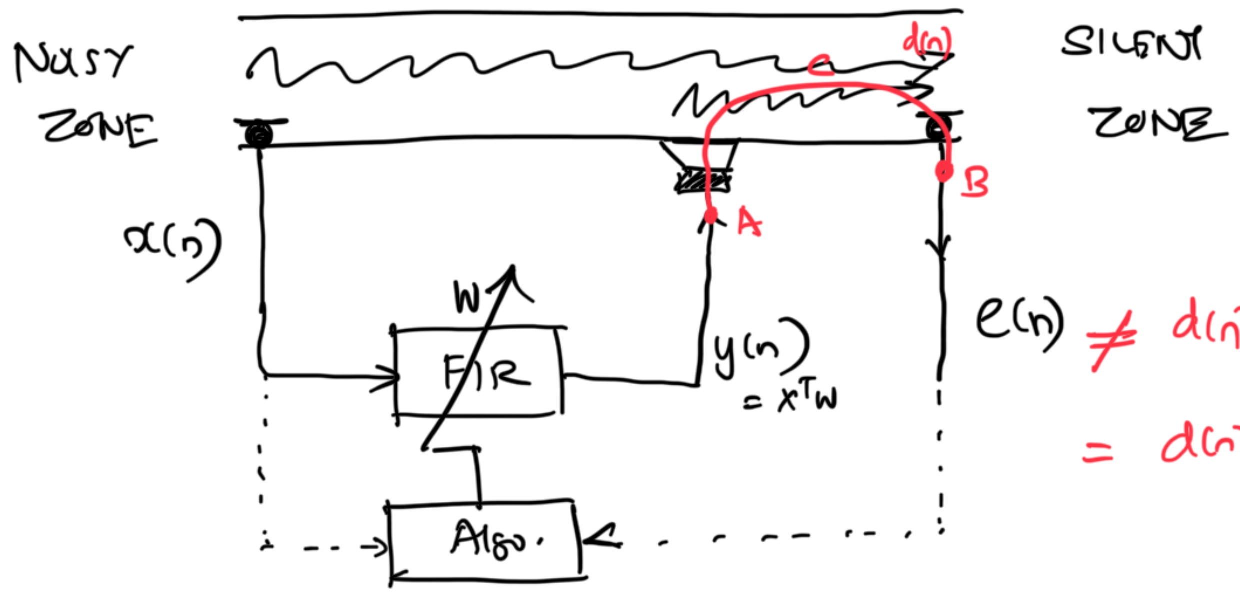
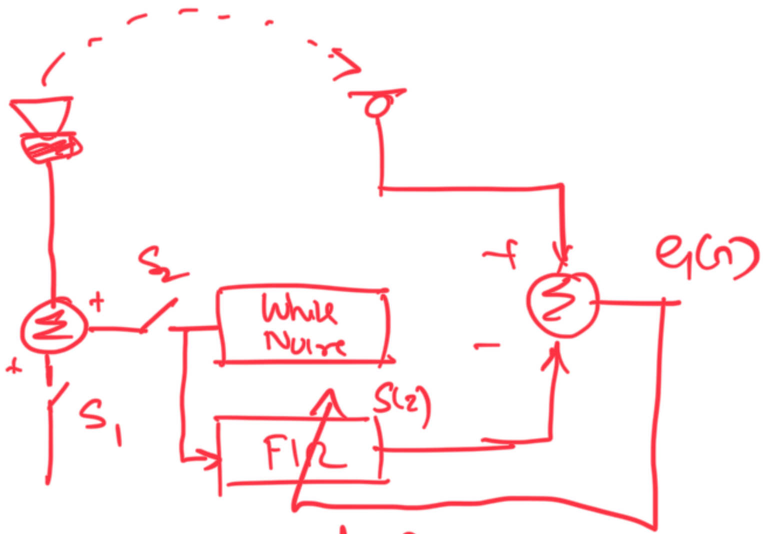
06 FEB 2025



$e(n) \neq d(n) - y(n)$

$= d(n) - y(n) * s(n)$

$e^2(n) \Downarrow$ e(n)



$e(n) = d(n) - y_s(n)$ — ①

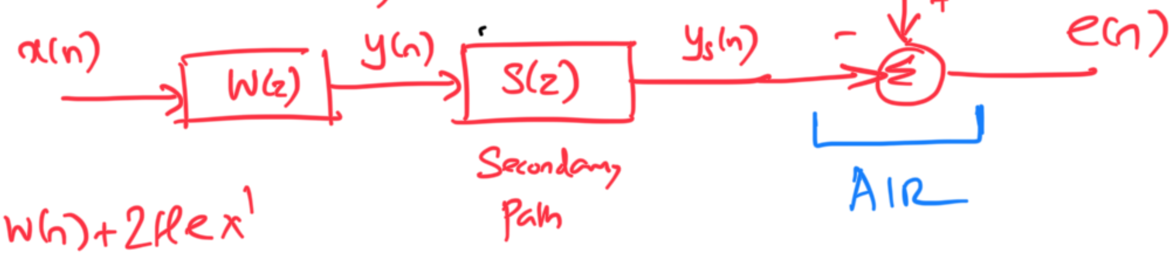
$W(n+1) = W(n) - \mu \nabla W$

$\nabla W = \frac{\partial e^2}{\partial W} = 2 \cdot e \cdot \frac{\partial e}{\partial W}$ — ②

$= -2 \cdot e \cdot \frac{\partial y_s}{\partial W}$

$y_s = s(n) * y(n) = s(n) * [x^T W]$

$= x'(n)$



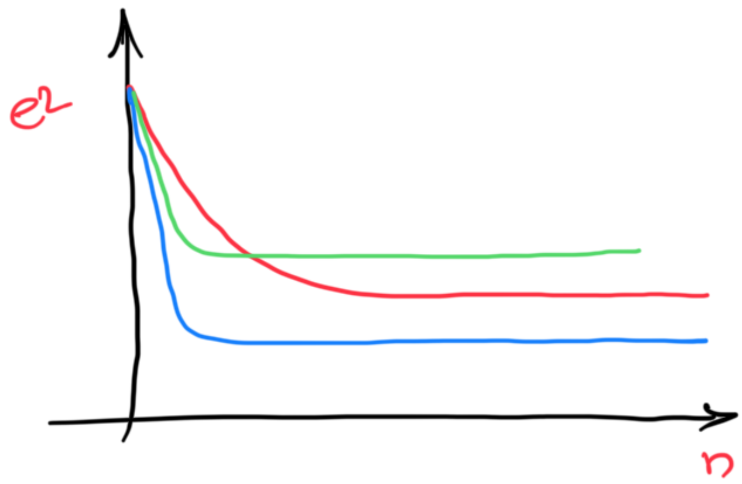
$W(n+1) = W(n) + 2\mu e x'$

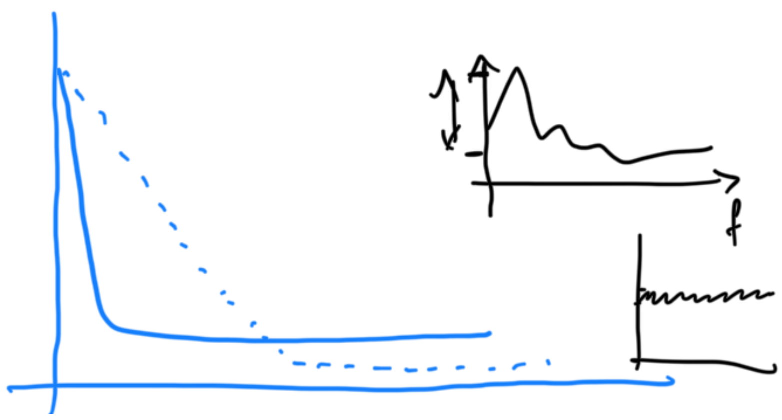
FxLMS Algo.

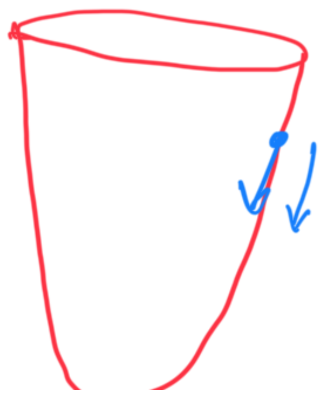
LMS

$\Rightarrow W(n+1) = W(n) + 2\mu e X$

$\mu \Uparrow$ $\mu \Downarrow$







Eigen value spread

$\mu(n)$ $\quad e^2(k)$ $\quad e(k) = d(k) - w^T(k)x(k)$ — (1a)

$$e^2(k) = d^2(k) + w^T(k)x(k)x^T(k)w(k) - 2d(k)w^T(k)x(k) \quad -(1)$$

$$\tilde{w}(k) = w(k) + \Delta\tilde{w}(k) \quad -(2)$$

$$\tilde{e}^2(k) = e^2(k) + 2\Delta\tilde{w}(k)x(k)x^T(k)w(k) + \Delta\tilde{w}^T(k)x(k)x^T(k)\Delta\tilde{w}(k) - 2d(k)\Delta\tilde{w}^T(k)x(k) \quad -(3)$$

$$\Delta e^2(k) = \tilde{e}^2(k) - e^2(k) \quad -(4)$$

$$= -2\Delta\tilde{w}^T(k)x(k)e(k) + \Delta\tilde{w}^T(k)x(k)x^T(k)\Delta\tilde{w}(k) \quad -(5)$$

$$\Delta\tilde{w}(k) = 2\mu_k e(k)x(k) \quad -(6)$$

μ_k $\quad$ (5)

$$\Delta e^2(k) = -4\mu_k e^2(k)x^T(k)x(k) + 4\mu_k^2 e^2(k)[x^T(k)x(k)]^2 \quad -(7)$$

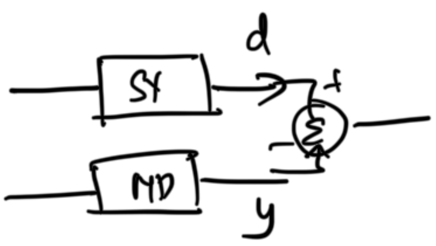
$$\frac{\partial \Delta e^2(k)}{\partial \mu_k} = 0 \quad ; \ \mu_k\ ?$$

$$\mu_k = \frac{1}{2x^T(k)x(k)} \quad -(8)$$

$$w(k+1) = w(k) + \frac{e(k)x(k)}{x^T(k)x(k)} \quad -(9)$$

$$\boxed{w(k+1) = w(k) + \mu\frac{e(k)x(k)}{x^T(k)x(k)+\gamma}} \quad -(10)$$

Normalized LMS (NLMS)



$$A\,\xi(k) = \sum_{i=0}^{k}\lambda^{k-i}\epsilon^2(i) \quad -(11)$$

$$= e^2(k) + \underline{\quad}$$

$$\epsilon(i) = d(i) - x^T(i)w(k) \quad -(12)$$

$$e(i) = d(i) - x^T(i)w(i) \quad -(13)$$

100 $\quad e_1^2 + e_2^2 + \ldots\ \ldots$

$\epsilon(i)$: a posteriori error

$\boxed{k = 500}$

$e(500) \Rightarrow e(500)$

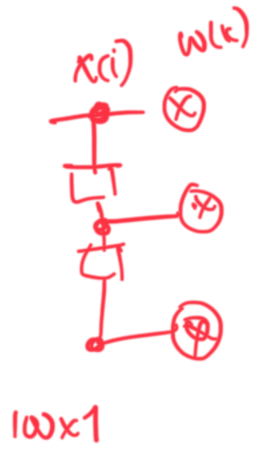
$e(499) \neq e(499)$

$= d(499) - x^T(499)\, w(500)$

$e(99) = d(99) - x^T(99)\, w(500)$

$\vdots$

$e(0) = d(0) - x^T(0)\, w(500)$



$$A(k) = \sum_{i=0}^{k} \lambda^{k-i} \left[d(i) - x^T(i)\, w(k) \right]^2 \quad \text{—(14)}$$

$$\frac{\partial A(k)}{\partial w(k)} = 0 = -2 \sum_{i=0}^{k} \lambda^{k-i} x(i) \left[d(i) - x^T(i)\, w(k) \right]$$

$$\boxed{\underset{100\times1}{w(k)} = \underset{(100\times100)}{R_D^{-1}(k)}\; \underset{(100\times1)}{P_D(k)}} \quad \text{—(15)}$$

$$\underset{100\times100}{R_D(k)} = \sum_{i=0}^{k} \lambda^{k-i} \underset{(100\times1)(1\times100)}{x(i)\, x^T(i)} \quad \text{—(16)} \qquad \underset{100\times1}{P_D(k)} = \sum_{i=0}^{k} \lambda^{k-i} \underset{(100\times1)(1\times1)}{x(i)\, d(i)} \quad \text{—(17)}$$

$$R_D^{-1}(k) = R_D^{-1}(k-1) + \dots\dots$$

$$R_D(k-1) = \sum_{i=0}^{k-1} \lambda^{k-i-1} x(i)\, x^T(i) \quad \text{—(18)}$$

$x(1)$	$x(2)$			$x(1000)$
1	2		999	1000
$d(1)$	$d(2)$			$d(1000)$
$e(1)$	$e(2)$			

$$\underset{A+BCD}{R_D(k)} = \underset{A}{\lambda R_D(k-1)} + \underset{BCD}{x(k)\, x^T(k)} \quad \text{—(19)}$$

$R_D^{-1}(1000)$ $\qquad$ $R_D^{-1}(999)$

$O(N^3)$

$\frac{1}{4}400^3$

$R_D^{-1}(k)$ from $R_D^{-1}(k-1)$?? 10^6 $\qquad \frac{100\times100}{10^2}$

Matrix Inversion Lemma

$$[A + BCD]^{-1} = A^{-1} - A^{-1}B[DA^{-1}B + C^{-1}]^{-1}DA^{-1} \quad -(20)$$

$$R_D^{-1}(k) = R_D^{-1}(k-1) - \quad \text{—} \quad (21)$$

$$R_D^{-1}(1) = R_D^{-1}(0) + \dots$$

$$R_D^{-1}(2) = R_D^{-1}(1) - \dots$$

100×100

(100×100)(100×1)(1×100)(100×100)

$$\underset{100\times100}{S_D(k)} = \underset{100\times100}{R_D^{-1}(k)} = \frac{1}{\lambda}\left[\underset{100\times100}{S_D(k-1)} - \frac{S_D(k-1)\,x(k)\,x^T(k)\,S_D(k-1)}{\lambda + x^T(k)\,S_D(k-1)\,x(k)}\right] - (22)$$

(1×100)(100×100)(100×1)

Algorithm 5.1 Conventional RLS algorithm

Initialization
$\mathbf{S}_D(-1) = \delta\mathbf{I}$
where δ can be the inverse of the input signal power estimate times $1 - \lambda$
$\mathbf{p}_D(-1) = \mathbf{x}(-1) = [0\,0 \ldots 0]^T$
Do for $k \geq 0$:
$\mathbf{S}_D(k) = \frac{1}{\lambda}[\mathbf{S}_D(k-1) - \frac{\mathbf{S}_D(k-1)\mathbf{x}(k)\mathbf{x}^T(k)\mathbf{S}_D(k-1)}{\lambda+\mathbf{x}^T(k)\mathbf{S}_D(k-1)\mathbf{x}(k)}]$
$\mathbf{p}_D(k) = \lambda\mathbf{p}_D(k-1) + d(k)\mathbf{x}(k)$
$\mathbf{w}(k) = \mathbf{S}_D(k)\mathbf{p}_D(k)$
If necessary compute
$y(k) = \mathbf{w}^T(k)\mathbf{x}(k)$
$\varepsilon(k) = d(k) - y(k)$
