Proof Workshop #1

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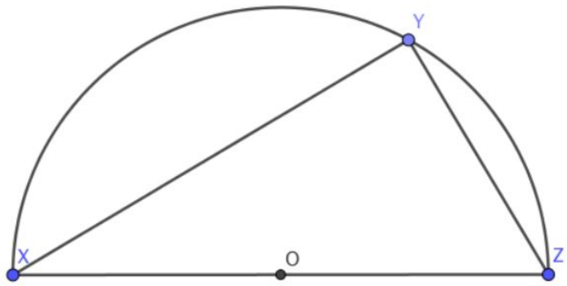
Claim: If a triangle *XYZ* is inscribed in a semicircle (see figure), then it is a right triangle.



Proof: If triangle ΔXYZ is inscribed in a semicircle, where $\overline{XZ}$ is the diameter of the triangle, then $\overline{XO}$, $\overline{OZ}$ are both the radius of the semicircle. If *Y* is a point on the semicircle, then a line drawn between $\overline{OY}$ would be the length of the radius of the semicircle.

Therefore, $\overline{OY}$, $\overline{OZ}$ are equal because they are both the radius of the semicircle. If $\overline{OY}$, $\overline{OZ}$ are equal, then ΔOYZ is an isosceles triangle where angles *OYZ* and *YZO* are equal.

Likewise, $\overline{OX}$, $\overline{OY}$ are equal because they are both the radius of the semicircle. If $\overline{OX}$, $\overline{OY}$ are equal, then ΔOYX is an isosceles triangle where angles *OYX* and *OXY* are equal.

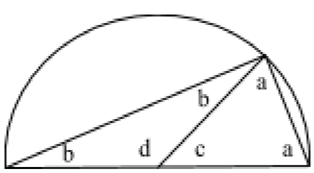


Figure 1

So, we end up with a triangle where *a* represents the equivalent angles *OYZ* and *YZO*, *b* represents the equivalent angles *OYX* and *OXY*, and where *c* and *d* represent the other unknown angles. See Figure 1 for illustration. Now, we know that the sum of all angles in a triangle is always equal to 180 degrees. Because of this, we can look at the main triangle and see that $a + b + (b + a) = 180$. This equation can be reduced to $2a + 2b = 180$, which can then be reduced to $a + b = 90$. In other words, no matter how a triangle is inscribed in the semicircle, *a*+*b* will always equal 90. Thus, a triangle inscribed in a semicircle, so that the diameter is one side of the triangle, will always have a right angle *XYZ*.