

Optimization and Control, Laboratory

Parameter Identification for the “Inverted Pendulum”

Winter term 2021/22

1 Laboratory Experiment

Consider the inverted pendulum setup as depicted in Figure 1. This setup consists of a cart (with mass m_W) which is actuated by a force F . A pendulum is connected via a rotating joint to this cart. The goal of this laboratory class is to perform parameter identification

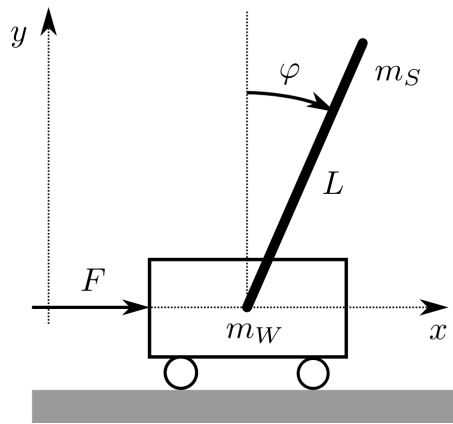


Figure 1: Laboratory experiment "inverted pendulum"

based on measurements from laboratory experiments. Additionally, a controller should be developed which stabilizes the upper (unstable) equilibrium.

2 Mathematical Model

The states consist of the **cart position x_W** , its **velocity v_W** , the **angle of the pendulum φ** and its corresponding **angular velocity ω** . Applying the Newton's laws and considering the connection between cart and pendulum, results in the mathematical model

$$\begin{aligned} \frac{dx_W}{dt} &= v_W \\ \frac{d\varphi}{dt} &= \omega \\ \frac{dv_W}{dt} &= \frac{1}{m_W + m_S(1 - \cos^2 \varphi)} \left[F + m_S l \omega^2 \sin \varphi - m_S g \sin \varphi \cos \varphi \right] \\ \frac{d\omega}{dt} &= \frac{1}{l \left(1 - \frac{m_S}{m_W + m_S} \cos^2 \varphi \right)} \left[g \sin \varphi - \frac{\cos \varphi}{m_W + m_S} F - \frac{m_S}{m_W + m_S} l \omega^2 \sin \varphi \cos \varphi \right] \end{aligned} \quad (1)$$

where g denotes the gravitational acceleration and **l half the length of the pendulum.**

The force F is generated by a DC-motor. Two different models of the inverted pendulum will be used in the laboratory class. The first model (**large pendulum**) takes the current i_A

as input to the system. The force F consists then of a force which is directly proportional to this current and a second force which represents **viscous friction**, i.e.

$$F = V i_A - k_1 v_W. \quad (2)$$

In the second model (**small pendulum**), the input is represented by the motor voltage u . The resulting force is proportional to **this input voltage and also to the velocity v_w** , i.e.

$$F = \frac{k_M k_G}{r R_A} u - \frac{k_M^2 k_G^2}{r^2 R_A} v_W \quad (3)$$

where k_M denotes the engine constant, k_G the transmission ratio, R_A the rotor resistance und r the equivalent radius which is used to translate rotational to transitional movements.

The state vector of the system is chosen as $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4]^T = [x_W \ \varphi \ v_W \ \omega]^T$.

3 Tasks

As already stated above, we want to identify the unknown parameters of the inverted pendulum followed by designing a controller which stabilizes the pendulum in the upper equilibrium. Unless mentioned otherwise, **the tasks below refer to the large pendulum where the unknown parameters are given by l , m_S , m_W , V and k_1** .

Task 1 First, we want to identify the parameter l . In order to do so, an experiment was recorded where the position of the cart was fixed and the current i_A was zero.

- a) A differential equation describing the dynamics of ω when the cart is fixed can be obtained by inserting $x_W = v_W = \frac{dv_W}{dt} = i_A = 0$ in the mathematical model of the inverted pendulum (1). Write down the resulting differential equation in the form

$$\dot{\omega}(t) = p_1 f_1(t) = p_1 f_1(\varphi(t), \omega(t), \dot{\omega}(t)) \quad (4)$$

where p_1 is constant but unknown and the function $f_1(t)$ does not depend on any unknown parameters.

- b) The evolution of φ , ω and $\dot{\omega}$ were recorded during this experiment and are available in the file `measurements_1.mat`. Identify the unknown parameter p_1 in (4) via L_1 , L_2 and L_∞ optimization using Yalmip. Calculate l from the identified parameter p_1 resulting from all three identification methods.
- c) Implement the model of the pendulum with fixed cart

$$\dot{\varphi} = \omega \qquad \dot{\omega} = p_1 f_1(\varphi, \omega)$$

in Matlab/Simulink; determine the initial values of the two state variables $\varphi(t)$ and $\omega(t)$ from the recorded data in `measurements_1.mat`. Run the simulation with the parameter p_1 obtained via L_1 optimization and store the resulting evolution of the angle $\varphi_{L_1}(t)$. Repeat the simulation with p_1 identified via L_2 and L_∞ optimization to obtain $\varphi_{L_2}(t)$ and $\varphi_{L_\infty}(t)$. Plot $\varphi_{L_1}(t)$, $\varphi_{L_2}(t)$, $\varphi_{L_\infty}(t)$ and the evolution of the angle recorded in `measurements_1.mat` $\varphi_{meas.}(t)$ together in one Matlab-plot.

- d) In order to evaluate how well the mathematical model with the identified parameters represents the real system, simulation results and measurements are compared for a different experiment (an experiment that was not used for parameter identification). Repeat the simulations from the previous task for the experiment recorded in `measurements_2.mat` and plot simulation results and measured angle as before.

Task 2 Now that we have identified the parameter l , we want to identify additional parameters from another recorded experiment where the position of the cart is no longer fixed, i.e. the system dynamics are given by (1).

- a) Write down the dynamics of ω from (1) in the form

$$\dot{\omega}(t)l - g \sin(\varphi(t)) = \sum_{i=1}^N p_i f_i(t) = \sum_{i=1}^N p_i f_i(i_A(t), x_W(t), v_W(t), \varphi(t), \omega(t), \dot{\omega}(t))$$

(where $N \in \mathbb{N}$ is some positive constant) similar to Task 1a. Note that l is assumed to be known since it has already been identified.

- b) The evolution of i_A , x_W , φ , v_w and ω was recorded in the file `measurements_3.mat` (the first column of the recorded values is the evolution of i_A ; the remaining columns are x_W , φ , v_w , ω and $\dot{\omega}$). Identify the unknown parameters p_i using L_1 , L_2 and L_∞ optimization.
- c) Can you calculate the values of the unknown parameters m_S , m_W , V and k_1 from the identified parameters p_i ? If you can, do so. Otherwise, use $m_S = 0.5 \text{ kg}$ and calculate the remaining parameters. Compare the values of m_S , m_W , V and k_1 resulting from the three identification methods.
- d) What will change with respect to the parameter identification when the small pendulum is used?
- e) Create a Simulink plan for the nonlinear model. Determine the initial state vector from the experiment recorded in `measurements_3.mat` and use i_A recorded during this experiment as the input of the plant. Run the simulation with the parameters l , m_S , m_W , V and k_1 obtained via L_1 optimization, with the parameters obtained via L_2 optimization and with the parameters obtained via L_∞ optimization. Compare the evolution of φ and x_W from all three simulations to the evolution of φ and x_W recorded in `measurements_3.mat` like in Task 1c.
- f) Repeat the simulations from the previous task for the experiment recorded in `measurements_4.mat` and plot simulation results and measured angle as before.

Task 3 Linearize the nonlinear model around the unstable (upper) equilibrium point. Design a linear state controller $u = -\mathbf{k}^T \mathbf{x}$ for the linearized model using the Matlab command `lqr()`; choose reasonable values for the weighting matrices.