Optimization and Control, Laboratory

Model Predictive Control of a “Two Tank System”

Winter term 2021/22

1 Introduction

Consider a system consisting of two tanks and a reservoir, as shown in Figure 1. Two pumps can be used to transfer water into the tanks. The goal of the lab is to design and

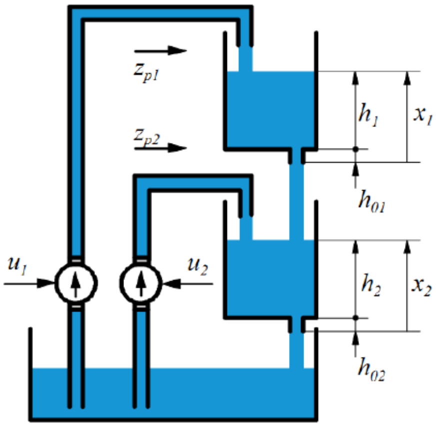

Figure 1: Lab experiment “two tank system”

test a model predictive controller to track a desired water level for the bottom tank.

2 Mathematical Model

The level in the top tank x_1 and in the bottom tank x_2 are used as state variables. They are defined as the sum of the length of the drains h_{01}, h_{02} and the water levels h_1, h_2, i.e. $x_i = h_i + h_{0i}$ $(i = 1, 2)$. A mathematical model for the two tank system is given in form of the following nonlinear differential equations:

$$\begin{aligned}
\frac{dx_1}{dt} &= -k_1\sqrt{x_1} + z_{p,1} \\
\frac{dx_2}{dt} &= -k_2\sqrt{x_2} + k_1\sqrt{x_1} + z_{p,2} \\
h_1 &= x_1 - h_{01} \\
h_2 &= x_2 - h_{02} \\
y &= h_2
\end{aligned}$$

where parameters k_1 and k_2 are constant. The output is defined as the water level h_2 in the bottom tank. The inflow due to pump 1 is symbolized by $z_{p,1}$, the inflow due to pump 2 by $z_{p,2}$. The nonlinear characteristics of the pumps are given as

$$z_{p,i}(u_i) = \begin{cases} \alpha_i + \sqrt{\beta_i + \gamma_i u_i} & \text{for} \quad \beta_i + \gamma_i u_i > 0 \\ 0 & \text{else} \end{cases}$$

Pump voltages u_1 and u_2 represent the inputs of the system. Parameter α_i, β_i and γ_i are constant. All parameters of the lab experiment are listed in Table 1.

Parameter	Tank 1	Tank 2
k_i $[cm^{1/2}s^{-1}]$	0.391	0.386
h_{0i} $[cm]$	10.17	10.53
α_i $[cms^{-1}]$	0.055	0.059
β_i $[cm^2s^{-2}]$	-3.077	-3.246
γ_i $[cm^2s^{-2}V^{-1}]$	3.551	3.610

Table 1: Parameters of the two tank system

3 Tasks

Task 1 Linearize the model in the equilibrium $y_E = 15\,cm$.

Task 2 Build Simulink files for the linearized and the nonlinear model. Check the values of the calculated equilibrium by means of simulation.

Task 3 Discretize the linearized model ($T_S = 0.5\,s$).

Task 4 The first pump is used as an actuator to control the level in the second tank. Pump 2 will be used in the lab to apply disturbances. Implement a MPC for the *unconstrained* case and test the controller in combination with the linearized model and the nonlinear model. Compare the results!

Task 5 Modify your MPC to *take into account the constraints* for the pump voltages and the states that are given by $0 \leq u_i \leq 5\,V$ and $h_{0i} \leq x_i \leq 20\,cm + h_{0i}$ $(i = 1, 2)$. Test the closed loop behavior using the linearized and the nonlinear model. Compare the results!

Task 6 Change the prediction horizon N_P, the control horizon N_C and the matrices Q and R. What about the effect of those parameters?

Task 7 Extend your MPC implementation by introducing *soft constraints* (with only one additional optimizaion variable). What are the effects when choosing different values of ρ?