# 1 Description Lab 1

# 1.1 Task 1

## 1.1.1 Subtask 1a

Insert  $x_w = v_w = \frac{dv_w}{dt} = i_A = 0$  into equation 3 of the mathematical model.

$$\frac{dv_w}{dt} = 0 = \frac{1}{m_w + m_s(1 - \cos(\phi)^2)} (m_s l\omega^2 \sin(\phi) - m_s g \sin(\phi) \cos(\phi))$$
(1)

$$m_s l\omega^2 \sin(\phi) = m_s g \sin(\phi) \cos(\phi)$$
 (2)

$$l\omega^2 = g\cos(\phi) \tag{3}$$

Than change the fourth equation of the mathematical model to get the form  $\dot{\omega}(t) = p_1 f_1(\phi(t), \omega(t), \dot{\omega}(t))$ .

$$\frac{d\omega}{dt} = \frac{1}{l\left(1 - \frac{m_s}{m_w + m_s}\cos(\phi)^2\right)} \left(g\sin(\phi) - \frac{m_s}{m_w + m_s}\underbrace{l\omega^2}_{g\cos(\phi)}\sin(\phi)\cos(\phi)\right)$$
(4)

$$\frac{d\omega}{dt} = \frac{1}{l\left(1 - \frac{m_s}{m_w + m_s}\cos(\phi)^2\right)} \left(g\sin(\phi)\left(1 - \frac{m_s}{m_w + m_s}\cos(\phi)^2\right)\right)$$
(5)

$$\frac{d\omega}{dt} = \frac{g}{l}\sin(\phi) \tag{6}$$

#### 1.1.2 Subtask 1b

#### 1.1.3 **Subtask 1c**

## 1.1.4 Subtask 1d

### 1.2 Task 2

#### 1.2.1 Subtask 1a

This is the 4th equation of the mathematical model:

$$\dot{\omega} = \frac{1}{l\left(1 - \frac{m_s}{m_w + m_s}\cos(\phi)^2\right)} \left[g\sin(\phi) - \frac{\cos(\phi)F}{m_w + m_s} - \frac{m_s}{m_w + m_s}l\omega^2\sin(\phi)\cos(\phi)\right]$$
(7)

By multiplying with the denominator we get:

$$\dot{\omega}l\left(1 - \frac{m_s}{m_w + m_s}\cos(\phi)^2\right) = g\sin(\phi) - \frac{\cos(\phi)F}{m_w + m_s} - \frac{m_s}{m_w + m_s}l\omega^2\sin(\phi)\cos(\phi)$$

$$\dot{\omega}l - g\sin(\phi) = \frac{lm_s}{m_w + m_s}\dot{\omega}\cos(\phi)^2 - \frac{V}{m_w + m_s}\cos(\phi)i_A + \frac{k_1}{m_w + m_s}\cos(\phi)v_w - \frac{lm_s}{m_w + m_s}\omega^2\sin(\phi)\cos(\phi)$$

$$\dot{\omega}l - g\sin(\phi) = \frac{lm_s}{m_w + m_s}\left(\dot{\omega}\cos(\phi)^2 - \omega^2\sin(\phi)\cos(\phi)\right)$$

$$-\frac{V}{m_w + m_s}\cos(\phi)i_A + \frac{k_1}{m_w + m_s}\cos(\phi)v_w$$
(8)

The 3 additive functions with its constant prefactors are:

$$p_1 = \frac{lm_s}{m_w + m_s} ; f_1(\dot{\omega}, \phi, \omega) = \dot{\omega}\cos(\phi)^2 - \omega^2\sin(\phi)\cos(\phi)$$
 (9)

$$p_2 = \frac{V}{m_w + m_s} \; ; \; f_2(\phi, i_A) = -\cos(\phi)i_A$$
 (10)

$$p_3 = \frac{k_1}{m_w + m_s} \; ; \; f_3(\phi, v_w) = \cos(\phi)v_w \tag{11}$$

### 1.2.2 Subtask 1b

#### 1.2.3 Subtask 1c

$$p_{1} = \frac{lm_{s}}{m_{w} + m_{s}}$$

$$p_{2} = \frac{V}{m_{w} + m_{s}}$$

$$p_{3} = \frac{k_{1}}{m_{w} + m_{s}}$$

$$(12)$$

 $\rightarrow$  three equations but 4 unknowns, thus not possible. Therefore, we use  $m_s = 0.5kg$ .

$$m_w = \left(\frac{l}{p_1} - 1\right) m_s$$

$$V = (m_w + m_s) p_2$$

$$k_1 = (m_w + m_s) p_3$$

$$(13)$$

- 1.2.4 Subtask 1d
- 1.2.5 Subtask 1e
- 1.2.6 Subtask 1f
- 1.3 Task 3