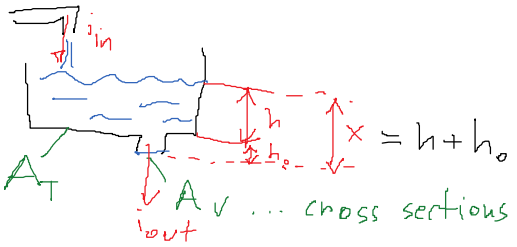
Model (1 Tank):

$(h \geq 0)$



- Volume V: $V = A_T \cdot h$

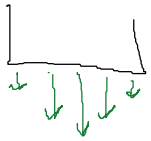
$$\frac{dV}{dt} = A_T \frac{dh}{dt} = A_T \frac{dx}{dt}$$

$$\frac{dV}{dt} = i_{in} - i_{out}$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{A_T}\left(i_{in} - i_{out}\right)$$

i_{in}: pump

$i_{out} = \mu A_V \cdot v_{out}$ "effective cross section"



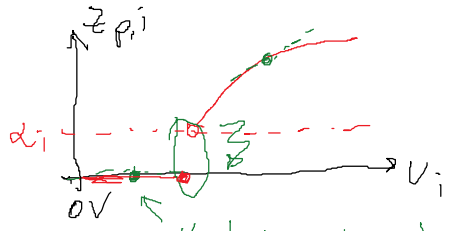
- "Top: Water is not moving"
  - ↳ only potential energy: mgx
  - ↳ Bottom: only kinetic energy: $\frac{m v_{out}^2}{2}$

$$mgx = \frac{m v_{out}^2}{2} \Rightarrow v_{out} = \sqrt{2g}\sqrt{x}$$

- $$\frac{dx}{dt} = \underbrace{\frac{1}{A_T} i_{in}}_{z_p} - \underbrace{\frac{1}{A_T} \mu A_V \sqrt{2g}}_{k} \sqrt{x}$$

z_p ... relative inflow

$$\frac{dx}{dt} = z_p - k\sqrt{x}$$



... Pump

v_i has no impact $\rightarrow$ $B = [b_1 \quad b_2]$, $b_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Task 1) • Equilibrium x_E , u_E such that $y_E = 15\,cm$

• $\dot{x} = f(x,u)$, $y = g(x)$ $\rightarrow$ $\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}$, $\tilde{y} = \tilde{C}\tilde{x}$ | $\tilde{x} = x - x_E$, $\tilde{u} = u - u_E$, $\tilde{y} = y - y_E$

Task 3) c2d (...) $\rightarrow$ $\tilde{x}_{n+1} = A\tilde{x}_n + B\tilde{u}_n$, $\tilde{y}_n = C\tilde{x}_n$

Task 4) MPC (unconstrained):

- Implementation $\Rightarrow$ see Quadcopter Example (lecture)
- Use a solver with variable step size and zero-order holds
- Include input constraints in the nonlin. model
- MPC: $\tilde{x}$, $\tilde{y}$, $\tilde{u}$ | Nonlin. Model: x, y, u
- Try f.e $x_0 = \begin{bmatrix} h_{0,1} \\ h_{0,2} \end{bmatrix}$, $r = y_E$ ... const.

Task 5) include constraints in plots.