1 Description Lab 1

1.1 Task 1

1.1.1 Subtask 1a

Insert $x_w = v_w = \frac{dv_w}{dt} = i_A = 0$ into equation 3 of the mathematical model.

$$\frac{dv_w}{dt} = 0 = \frac{1}{m_w + m_s(1 - \cos(\phi)^2)}(m_s l\omega^2 \sin(\phi) - m_s g \sin(\phi)\cos(\phi)) \tag{1}$$

$$m_s l\omega^2 \sin(\phi) = m_s g \sin(\phi)\cos(\phi) \tag{2}$$

$$l\omega^2 = g\cos(\phi) \tag{3}$$

Than change the fourth equation of the mathematical model to get the form $\dot{\omega}(t) = p_1 f1(\phi(t), \omega(t), \dot{\omega}(t))$.

$$\frac{d\omega}{dt} = \frac{1}{l\left(1 - \frac{m_s}{m_w+m_s}\cos(\phi)^2\right)}\left(g\sin(\phi) - \frac{m_s}{m_w + m_s}\underbrace{l\omega^2}_{g\cos(\phi)}\sin(\phi)\cos(\phi)\right) \tag{4}$$

$$\frac{d\omega}{dt} = \frac{1}{l\left(1 - \frac{m_s}{m_w+m_s}\cos(\phi)^2\right)}\left(g\sin(\phi)\left(1 - \frac{m_s}{m_w + m_s}\cos(\phi)^2\right)\right) \tag{5}$$

$$\frac{d\omega}{dt} = \frac{g}{l}\sin(\phi) \tag{6}$$

1.1.2 Subtask 1b

1.1.3 Subtask 1c

1.1.4 Subtask 1d

1.2 Task 2

1.2.1 Subtask 2a

This is the 4th equation of the mathematical model:

$$\dot{\omega} = \frac{1}{l\left(1 - \frac{m_s}{m_w+m_s}\cos(\phi)^2\right)}\left[g\sin(\phi) - \frac{\cos(\phi)F}{m_w + m_s} - \frac{m_s}{m_w + m_s}l\omega^2\sin(\phi)\cos(\phi)\right] \tag{7}$$

By multiplying with the denominator we get:

$$
\begin{aligned}
\dot{\omega} l \left(1 - \frac{m_s}{m_w + m_s}\cos(\phi)^2\right) =& g\sin(\phi) - \frac{\cos(\phi)F}{m_w + m_s} - \frac{m_s}{m_w + m_s} l\omega^2 \sin(\phi)\cos(\phi) \\
\dot{\omega} l - g\sin(\phi) =& \frac{l m_s}{m_w + m_s}\dot{\omega}\cos(\phi)^2 - \frac{V}{m_w + m_s}\cos(\phi) i_A + \\
& \frac{k_1}{m_w + m_s}\cos(\phi) v_w - \frac{l m_s}{m_w + m_s}\omega^2 \sin(\phi)\cos(\phi) \\
\dot{\omega} l - g\sin(\phi) =& \frac{l m_s}{m_w + m_s}\left(\dot{\omega}\cos(\phi)^2 - \omega^2\sin(\phi)\cos(\phi)\right) \\
& - \frac{V}{m_w + m_s}\cos(\phi) i_A + \frac{k_1}{m_w + m_s}\cos(\phi) v_w
\end{aligned} \tag{8}
$$

The 3 additive functions with its constant prefactors are:

$$p_1 = \frac{l m_s}{m_w + m_s} \; ; \; f_1(\dot{\omega}, \phi, \omega) = \dot{\omega}\cos(\phi)^2 - \omega^2 \sin(\phi)\cos(\phi) \tag{9}$$

$$p_2 = \frac{V}{m_w + m_s} \; ; \; f_2(\phi, i_A) = -\cos(\phi) i_A \tag{10}$$

$$p_3 = \frac{k_1}{m_w + m_s} \; ; \; f_3(\phi, v_w) = \cos(\phi) v_w \tag{11}$$

1.2.2 Subtask 2b

1.2.3 Subtask 2c

$$
\begin{aligned}
p_1 =& \frac{l m_s}{m_w + m_s} \\
p_2 =& \frac{V}{m_w + m_s} \\
p_3 =& \frac{k_1}{m_w + m_s}
\end{aligned} \tag{12}
$$

$\rightarrow$ three equations but 4 unknowns, thus not possible. Therefore, we use $m_s = 0.5kg$.

$$
\begin{aligned}
m_w =& \left(\frac{l}{p_1} - 1\right) m_s \\
V =& (m_w + m_s) p_2 \\
k_1 =& (m_w + m_s) p_3
\end{aligned} \tag{13}
$$

1.2.4 Subtask 2d

When using the small pendulum instead of the large one little to nothing changes for the parameter identification.

The equation for F in the system is simply replaced

$$F = V i_A - k_1 v_w \tag{14}$$

by the definition for the other force

$$F = \frac{k_M k_G}{r R_A} u - \frac{k_M^2 k_G^2}{r^2 R_A} v_w \tag{15}$$

Both forces are calculated by multiplying the input (u or i_A) and the velocity of the cart v_W with a constant factor. In the second case, the factor consists of multiple physical constants that can than be abbreviated again by a constant.

1.2.5 Subtask 2e

1.2.6 Subtask 2f

1.3 Task 3