

1 Description Lab 1

1.1 Task 1

1.1.1 Subtask 1a

Insert $x_w = v_w = \frac{dv_w}{dt} = i_A = 0$ into equation 3 of the mathematical model.

$$\frac{dv_w}{dt} = 0 = \frac{1}{m_w + m_s(1 - \cos(\phi)^2)}(m_s l \omega^2 \sin(\phi) - m_s g \sin(\phi) \cos(\phi)) \quad (1)$$

$$m_s l \omega^2 \sin(\phi) = m_s g \sin(\phi) \cos(\phi) \quad (2)$$

$$l \omega^2 = g \cos(\phi) \quad (3)$$

Then change the fourth equation of the mathematical model to get the form $\dot{\omega}(t) = p_1 f_1(\phi(t), \omega(t), \dot{\omega}(t))$.

$$\frac{d\omega}{dt} = \frac{1}{l \left(1 - \frac{m_s}{m_w + m_s} \cos(\phi)^2\right)} \left(g \sin(\phi) - \frac{m_s}{m_w + m_s} \underbrace{l \omega^2}_{g \cos(\phi)} \sin(\phi) \cos(\phi) \right) \quad (4)$$

$$\frac{d\omega}{dt} = \frac{1}{l \left(1 - \frac{m_s}{m_w + m_s} \cos(\phi)^2\right)} \left(g \sin(\phi) \left(1 - \frac{m_s}{m_w + m_s} \cos(\phi)^2\right) \right) \quad (5)$$

$$\frac{d\omega}{dt} = \frac{g}{l} \sin(\phi) \quad (6)$$

1.1.2 Subtask 1b

1.1.3 Subtask 1c

1.1.4 Subtask 1d

1.2 Task 2

1.2.1 Subtask 2a

This is the 4th equation of the mathematical model:

$$\dot{\omega} = \frac{1}{l \left(1 - \frac{m_s}{m_w + m_s} \cos(\phi)^2\right)} \left[g \sin(\phi) - \frac{\cos(\phi) F}{m_w + m_s} - \frac{m_s}{m_w + m_s} l \omega^2 \sin(\phi) \cos(\phi) \right] \quad (7)$$

By multiplying with the denominator we get:

$$\begin{aligned}
 \dot{\omega}l \left(1 - \frac{m_s}{m_w + m_s} \cos(\phi)^2\right) &= g \sin(\phi) - \frac{\cos(\phi)F}{m_w + m_s} - \frac{m_s}{m_w + m_s} l \omega^2 \sin(\phi) \cos(\phi) \\
 \dot{\omega}l - g \sin(\phi) &= \frac{lm_s}{m_w + m_s} \dot{\omega} \cos(\phi)^2 - \frac{V}{m_w + m_s} \cos(\phi) i_A + \\
 &\quad \frac{k_1}{m_w + m_s} \cos(\phi) v_w - \frac{lm_s}{m_w + m_s} \omega^2 \sin(\phi) \cos(\phi) \quad (8) \\
 \dot{\omega}l - g \sin(\phi) &= \frac{lm_s}{m_w + m_s} (\dot{\omega} \cos(\phi)^2 - \omega^2 \sin(\phi) \cos(\phi)) \\
 &\quad - \frac{V}{m_w + m_s} \cos(\phi) i_A + \frac{k_1}{m_w + m_s} \cos(\phi) v_w
 \end{aligned}$$

The 3 additive functions with its constant prefactors are:

$$p_1 = \frac{lm_s}{m_w + m_s} ; f_1(\dot{\omega}, \phi, \omega) = \dot{\omega} \cos(\phi)^2 - \omega^2 \sin(\phi) \cos(\phi) \quad (9)$$

$$p_2 = \frac{V}{m_w + m_s} ; f_2(\phi, i_A) = -\cos(\phi) i_A \quad (10)$$

$$p_3 = \frac{k_1}{m_w + m_s} ; f_3(\phi, v_w) = \cos(\phi) v_w \quad (11)$$

1.2.2 Subtask 2b

1.2.3 Subtask 2c

$$\begin{aligned}
 p_1 &= \frac{lm_s}{m_w + m_s} \\
 p_2 &= \frac{V}{m_w + m_s} \\
 p_3 &= \frac{k_1}{m_w + m_s}
 \end{aligned} \quad (12)$$

→ three equations but 4 unknowns, thus not possible. Therefore, we use $m_s = 0.5kg$.

$$\begin{aligned}
 m_w &= \left(\frac{l}{p_1} - 1\right) m_s \\
 V &= (m_w + m_s) p_2 \\
 k_1 &= (m_w + m_s) p_3
 \end{aligned} \quad (13)$$

1.2.4 Subtask 2d

When using the small pendulum instead of the large one little to nothing changes for the parameter identification.

1 Description Lab 1

The equation for F in the system is simply replaced

$$F = Vi_A - k_1v_w \quad (14)$$

by the definition for the other force

$$F = \frac{k_M k_G}{r R_A} u - \frac{k_M^2 k_G^2}{r^2 R_A} v_w \quad (15)$$

Both forces are calculated by multiplying the input (u or i_A) and the velocity of the cart v_W with a constant factor. In the second case, the factor consists of multiple physical constants that can then be abbreviated again by a constant.

1.2.5 Subtask 2e

1.2.6 Subtask 2f

1.3 Task 3