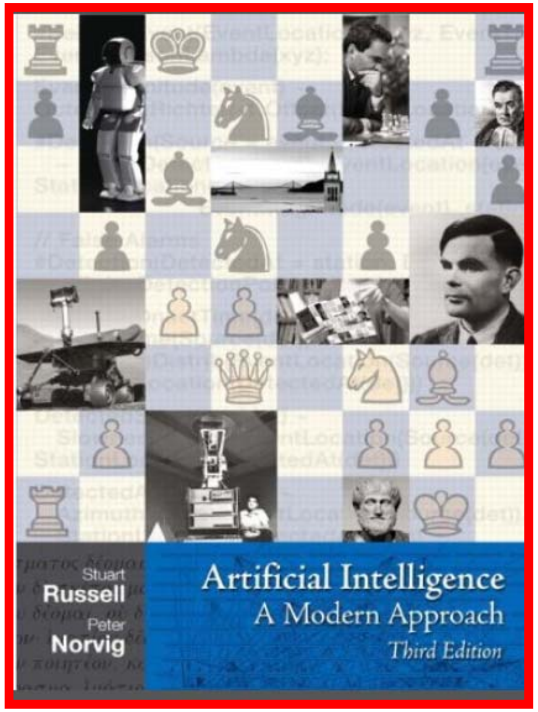


Missionary - savage problem

Yanmei Zheng

- There are M missionaries and C wildlings to cross the river, there is now a boat capable of carrying K men (including wildlings) K<M, K<C. If savages and missionaries are with each other at any time, the number of missionaries must be greater than or equal to the number of savages.
- Let M be the number of missionaries and C the number of wildlings. The process of solving this problem by using state space generation is as follows:

1) Set state variables and determine the range

- To establish the state space for this problem,
- Let the number of missionaries on the left bank be m. There are m={0,1,2,3}. The number of missionaries on the right bank is 3-m.
- The number of cannibals on the left bank c. There are c={0,1,2,3}. The number of cannibals on the right bank is 3-c.
- The number of ships on the left bank is b, so b={0,1}. The number of ships on the right bank is 1-b.
- M<=N, C< = N， boat = K， so that M>=C and M+C <= K
- The initial state

	L	R
M	3	0
C	3	0
B	1	0

The target state

	L	R
M	0	3
C	0	3
B	0	1

2)Determine the state group and list the initial state set and the target state set respectively.

The state of the problem can be described by a triplet, marked by the state of the left bank, that is, the state of the right bank can not be marked .

Sk=（m, c, b）

- There's only one initial state: S0=（3, 3, 1), the initial state indicates that all members are on the left bank of the river.
- There is only one target state: Sg=(0, 0, 0), it means that all members have crossed the river from the left bank.
- (3 , 3 , 1) $\longrightarrow$ (0 , 0 , 0)

（1） + （2）

- M<=N,C< = N， boat = K， ask M>=C and M+C <= K
- The initial state

	L	R
M	3	0
C	3	0
B	1	0

The target state

	L	R
M	0	3
C	0	3
B	0	1

- (1) It is represented as the triples （ML,CL,BL）
- The 0<=ML , CL <= 3 , BL ∈{ 0,1}
- (3,3,1) $\longrightarrow$ (0,0,0)

(3) Define and identify a set of operations

Still consider the left bank of the river as the base

Pij: Row the boat from the left bank to the right bank, where the first subscript i represents the number of missionaries on board, and the second subscript j represents the number of cannibals on board;

Qij: Row the boat back to the left bank from the right bank, subscript defined as before.

There are 10 operations, and the operation set is

- F={P01,P10,P11,P02,P20,Q01,Q10,Q11,Q02,Q20}

(3) The rules set

- **P10 if (ML ,CL , BL=1) then (ML–1 , CL , BL –1)**
- **P01 if (ML ,CL , BL=1) then (ML , CL–1 , BL –1)**
- **P11 if (ML ,CL , BL=1) then (ML–1 , CL–1 , BL –1)**
- **P20 if (ML ,CL , BL=1) then (ML–2 , CL , BL –1)**
- **P02 if (ML ,CL , BL=1) then (ML , CL–2 , BL –1)**
- **Q10 if (ML ,CL , BL=0) then (ML+1 , CL , BL+1)**
- **Q01 if (ML ,CL , BL=0) then (ML , CL+1 , BL +1)**
- **Q11 if (ML ,CL , BL=0) then (ML+1 , CL +1, BL +1)**
- **Q20 if (ML ,CL , BL=0) then (ML+2 , CL +2, BL +1)**
- **Q02 if (ML ,CL , BL=0) then (ML , CL +2, BL +1)**

(4) Estimate the total number of state Spaces

The total number of possible states for the missionary and cannibal problem is 32, as shown in the table .

State	m, c, b	State	m, c, b	State	m, c, b	State	m, c, b
S0	3,3,1	S8	1,3,1	S16	3,3,0	S24	1,3,0
S1	3,2,1	S9	1,2,1	S17	3,2,0	S25	1,2,0
S2	3,1,1	S10	1,1,1	S18	3,1,0	S26	1,1,0
S3	3,0,1	S11	1,0,1	S19	3,0,0	S27	1,0,0
S4	2,3,1	S12	0,3,1	S20	2,3,0	S28	0,3,0
S5	2,2,1	S13	0,2,1	S21	2,2,0	S29	0,2,0
S6	2,1,1	S14	0,1,1	S22	2,1,0	S30	0,1,0
S7	2,0,1	S15	0,0,1	S23	2,0,0	S31	0,0,0

（4） Handle an illegal state

First row to the shore where the cannibals outnumber the missionaries,that is S4、S8、S9、S20、S24、S25 and so on 6 states are illegal.

State	m, c, b	State	m, c, b	State	m, c, b	State	m, c, b
S0	3,3,1	S8	~~1,3,1~~	S16	3,3,0	S24	~~1,3,0~~
S1	3,2,1	S9	~~1,2,1~~	S17	3,2,0	S25	~~1,2,0~~
S2	3,1,1	S10	1,1,1	S18	3,1,0	S26	1,1,0
S3	3,0,1	S11	1,0,1	S19	3,0,0	S27	1,0,0
S4	~~2,3,1~~	S12	0,3,1	S20	~~2,3,0~~	S28	0,3,0
S5	2,2,1	S13	0,2,1	S21	2,2,0	S29	0,2,0
S6	2,1,1	S14	0,1,1	S22	2,1,0	S30	0,1,0
S7	2,0,1	S15	0,0,1	S23	2,0,0	S31	0,0,0

（4） Handle the illegal states

Row off the right bank where cannibals outnumber monks , that is S6,S7,S11,S22,S23,S27 and so on.（S6+S25, S7+S8, S11+S20, S22+S9, S23+S24, S27+S4）

State	m, c, b	State	m, c, b	State	m, c, b	State	m, c, b
S0	3,3,1	S8	~~1,3,1~~	S16	3,3,0	S24	~~1,3,0~~
S1	3,2,1	S9	~~1,2,1~~	S17	3,2,0	S25	~~1,2,0~~
S2	3,1,1	S10	1,1,1	S18	3,1,0	S26	1,1,0
S3	3,0,1	S11	~~1,0,1~~	S19	3,0,0	S27	~~1,0,0~~
S4	~~2,3,1~~	S12	0,3,1	S20	~~2,3,0~~	S28	0,3,0
S5	2,2,1	S13	0,2,1	S21	2,2,0	S29	0,2,0
S6	~~2,1,1~~	S14	0,1,1	S22	~~2,1,0~~	S30	0,1,0
S7	~~2,0,1~~	S15	0,0,1	S23	~~2,0,0~~	S31	0,0,0

（4） Four impossible states

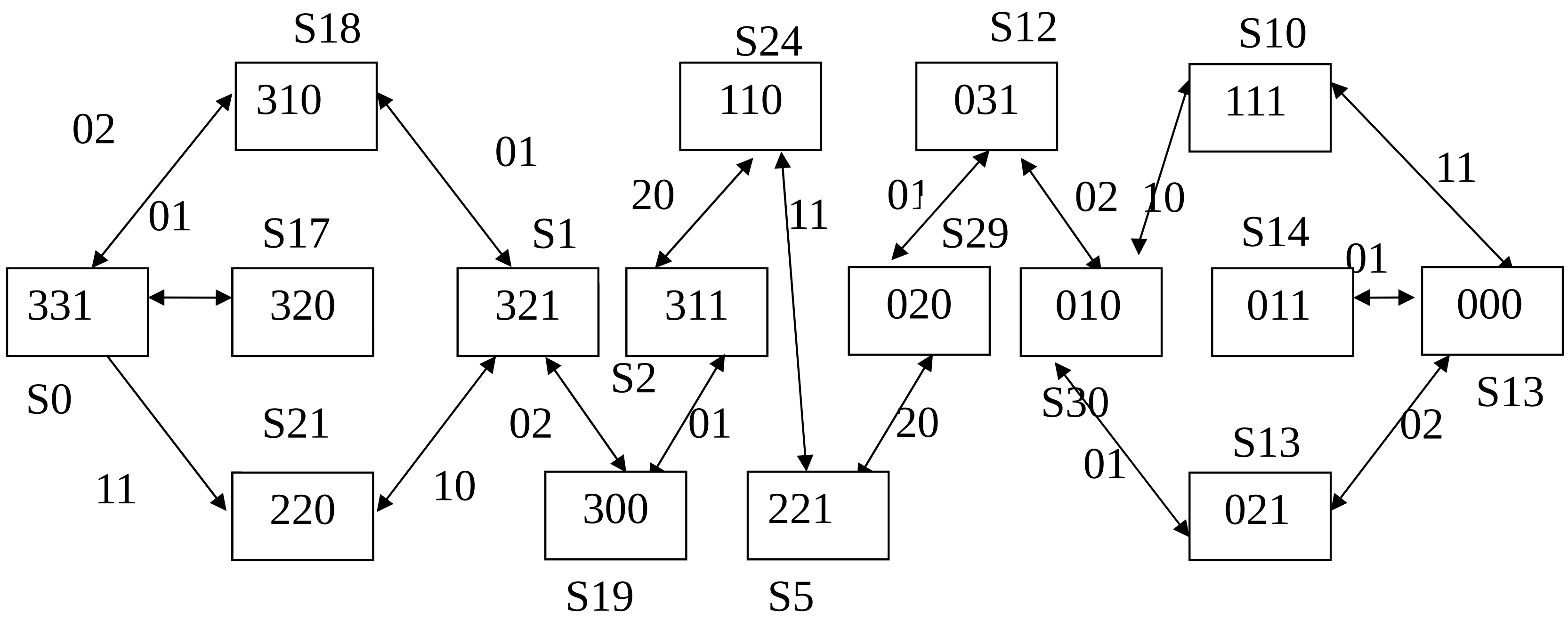
S15 and S16 could not have appeared, as ships cannot dock on uninhabited shores.

S3 could not have appeared because the missionaries could not safely row the boat back under the eyes of the dominant cannibals.

S28 could not have appeared, as the missionaries could not have rowed safely to the other side under the eyes of the superior cannibals.

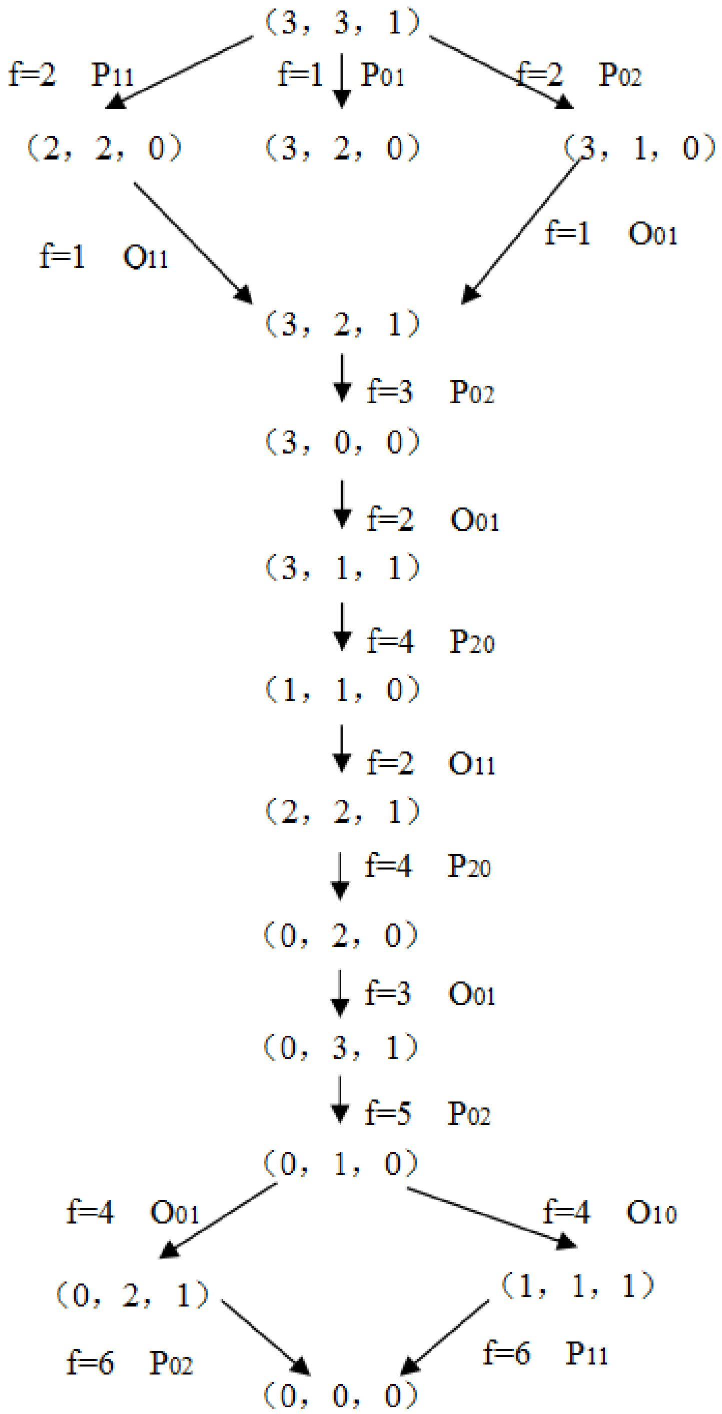
State	m, c, b	State	m, c, b	State	m, c, b	State	m, c, b
S0	3,3,1	S8	~~1,3,1~~	S16	~~3,3,0~~	S24	~~1,3,0~~
S1	3,2,1	S9	~~1,2,1~~	S17	3,2,0	S25	~~1,2,0~~
S2	3,1,1	S10	1,1,1	S18	3,1,0	S26	1,1,0
S3	~~3,0,1~~	S11	~~1,0,1~~	S19	3,0,0	S27	~~1,0,0~~
S4	~~2,3,1~~	S12	0,3,1	S20	~~2,3,0~~	S28	~~0,3,0~~
S5	2,2,1	S13	0,2,1	S21	2,2,0	S29	0,2,0
S6	~~2,1,1~~	S14	0,1,1	S22	~~2,1,0~~	S30	0,1,0
S7	~~2,0,1~~	S15	~~0,0,1~~	S23	~~2,0,0~~	S31	0,0,0

The state space of the missionary and cannibal problem



(3) Define a heuristic function to guide the selection of rules

$$f = \begin{cases} \text{The number of the right bank 6} - ML - CL & \textbf{Number of missionaries>= Savage number} \\ -\infty & \text{others} \end{cases}$$


(3, 3, 1)
f=2 P11
f=1 P01
f=2 P02
(2, 2, 0)
(3, 2, 0)
(3, 1, 0)
f=1 O11
f=1 O01
(3, 2, 1)
f=3 P02
(3, 0, 0)
f=2 O01
(3, 1, 1)
f=4 P20
(1, 1, 0)
f=2 O11
(2, 2, 1)
f=4 P20
(0, 2, 0)
f=3 O01
(0, 3, 1)
f=5 P02
(0, 1, 0)
f=4 O01
f=4 O10
(0, 2, 1)
(1, 1, 1)
f=6 P02
f=6 P11
(0, 0, 0)

The Results

```
There is a solution, and the solution is :
[3, 3, 1]
[3, 1, 0]
[3, 2, 1]
[3, 0, 0]
[3, 1, 1]
[1, 1, 0]
[2, 2, 1]
[0, 2, 0]
[0, 3, 1]
[0, 1, 0]
[0, 2, 1]
[0, 0, 0]
```

```
There is a solution, and the solution is :
[3, 3, 1]
[3, 1, 0]
[3, 2, 1]
[3, 0, 0]
[3, 1, 1]
[1, 1, 0]
[2, 2, 1]
[0, 2, 0]
[0, 3, 1]
[0, 1, 0]
[1, 1, 1]
[0, 0, 0]
```

```
There is a solution, and the solution is :
[3, 3, 1]
[2, 2, 0]
[3, 2, 1]
[3, 0, 0]
[3, 1, 1]
[1, 1, 0]
[2, 2, 1]
[0, 2, 0]
[0, 3, 1]
[0, 1, 0]
[0, 2, 1]
[0, 0, 0]
```

```
There is a solution, and the solution is :
[3, 3, 1]
[2, 2, 0]
[3, 2, 1]
[3, 0, 0]
[3, 1, 1]
[1, 1, 0]
[2, 2, 1]
[0, 2, 0]
[0, 3, 1]
[0, 1, 0]
[1, 1, 1]
[0, 0, 0]
```

Thank you

End of
Chapter 1