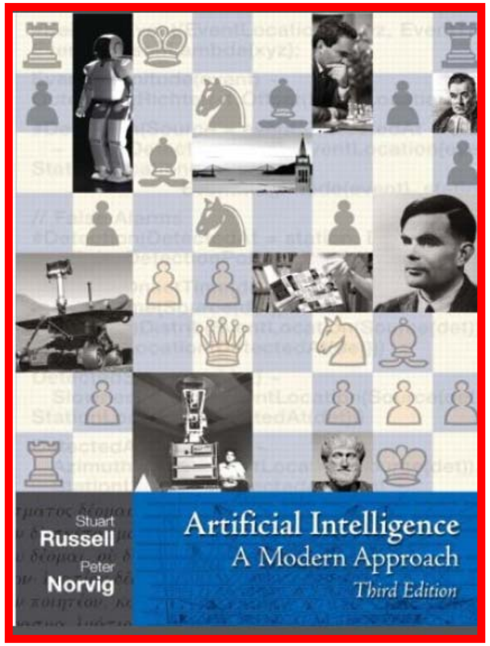



# Ant Colony Optimization

**Yanmei Zheng**

# Outline

- Introduction to Ant Colony Optimization (ACO)
- Ant Behavior
- Stigmergy
- Pheromones
- Basic Algorithm
- Example
- Advantages and Disadvantages
- References

# What is Ant Colony Optimization?

***Ant Colony Optimization (ACO) is a population-based, general search technique for the solution of difficult combinatorial problems which is inspired by the pheromone trail laying behavior of real ant colonies.***

# Ant Colony Optimization

Ant System was developed by Marco Dorigo (Italy) in his PhD thesis in 1992.

Max-Min Ant System developed by Hoos and Stützle in 1996

Ant Colony was developed by Gambardella Dorigo in 1997

# Ant Colony Optimization

- Proposed by **Marco Dorigo** in 1991
- Inspired in the behavior of real ants
- **Multi-agent approach** for solving complex combinatorial optimization problems
- Applications:
    - Traveling Salesman Problem  Scheduling
    - Network Model Problem
    - Vehicle routing
    -

# Biological Inspiration

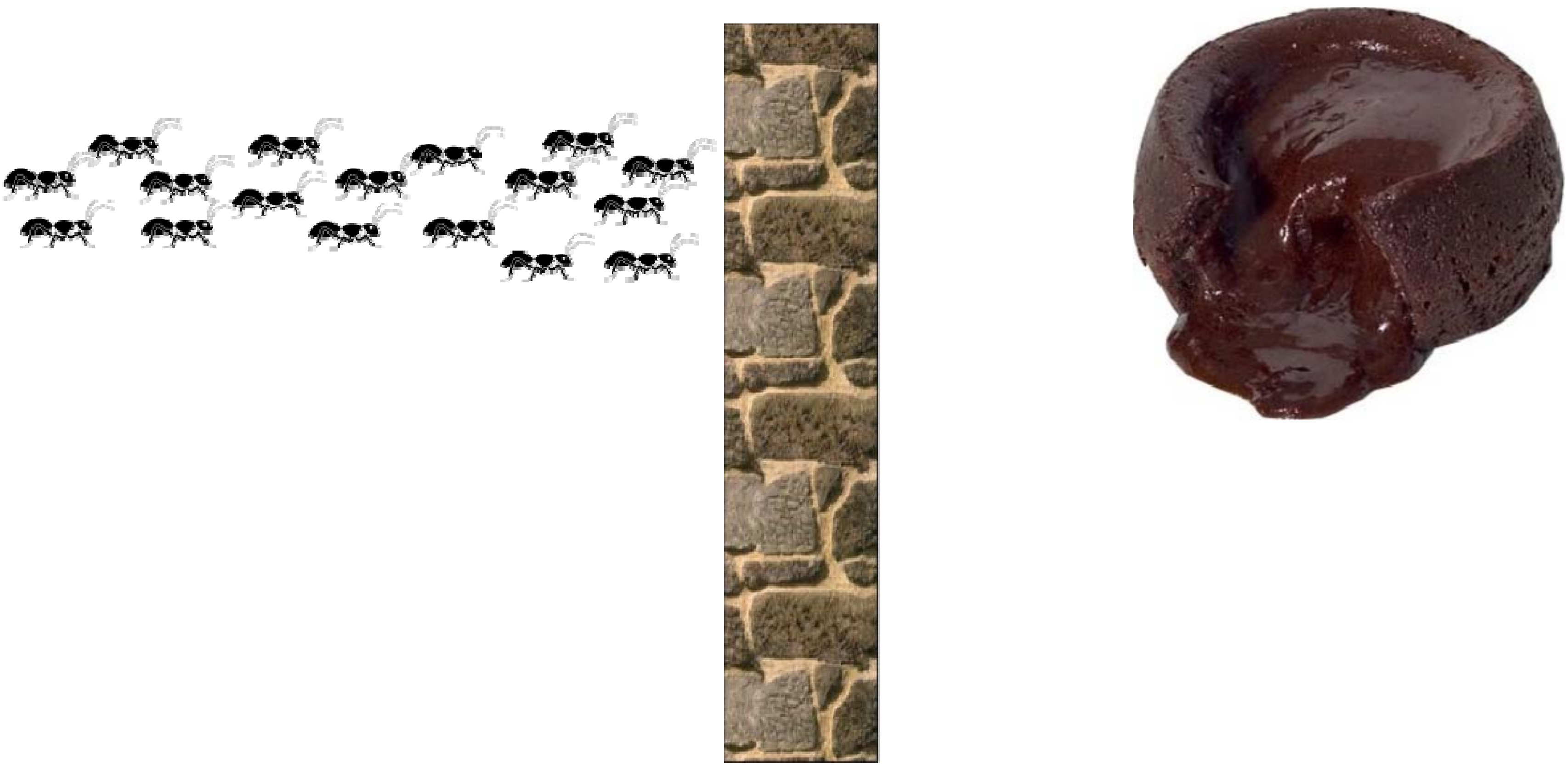
**Pheromone trail following behavior**

- In many ant species the visual perceptive faculty is only very rudimentarily developed
- Most communication among individuals is based on chemicals called pheromone
- A particular type in some ant species is *trail pheromone*, used for marking and following paths

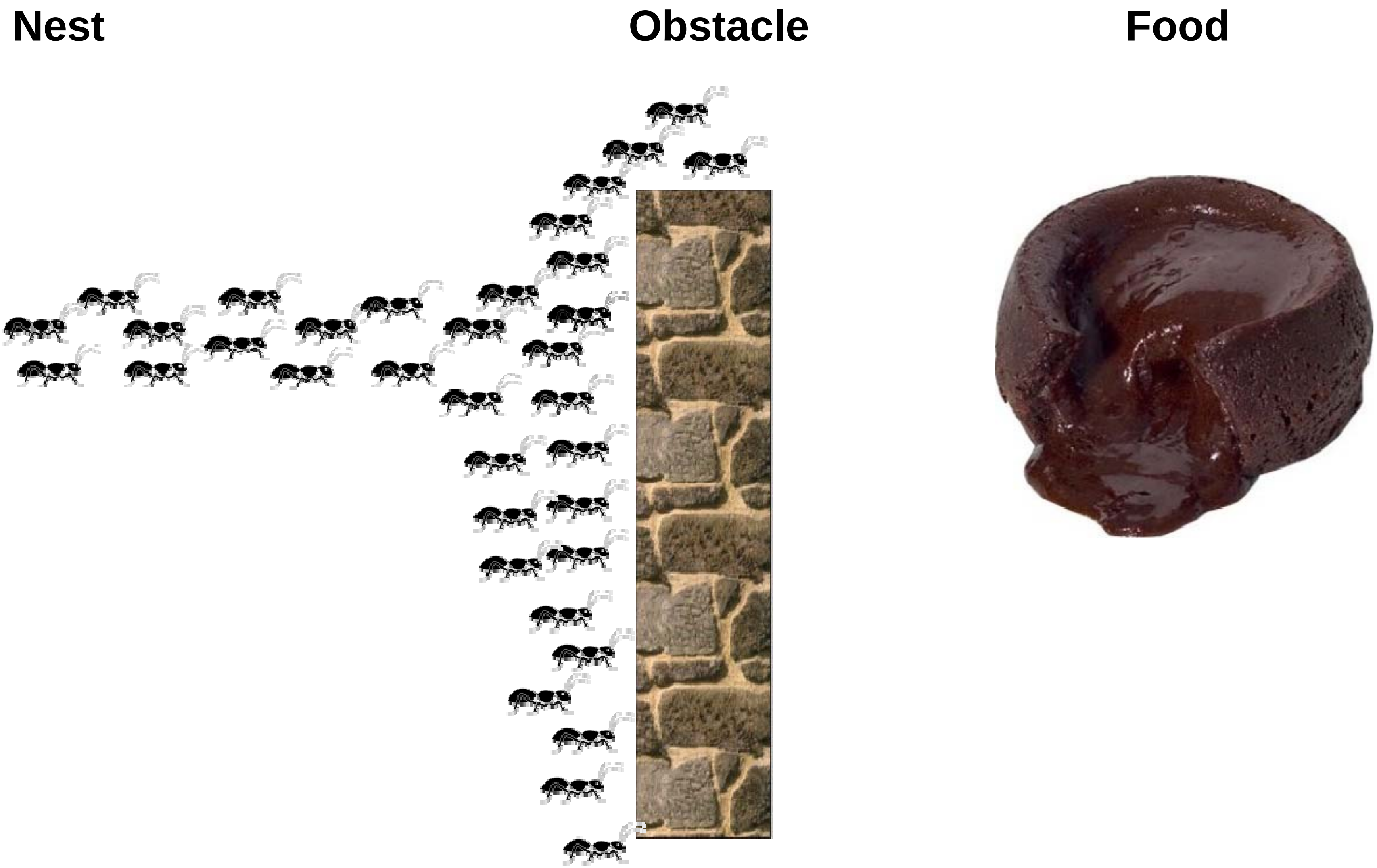
*collective trail laying / trail following behavior is the inspiring source of Ant Colony Optimization*

# Ant Behavior

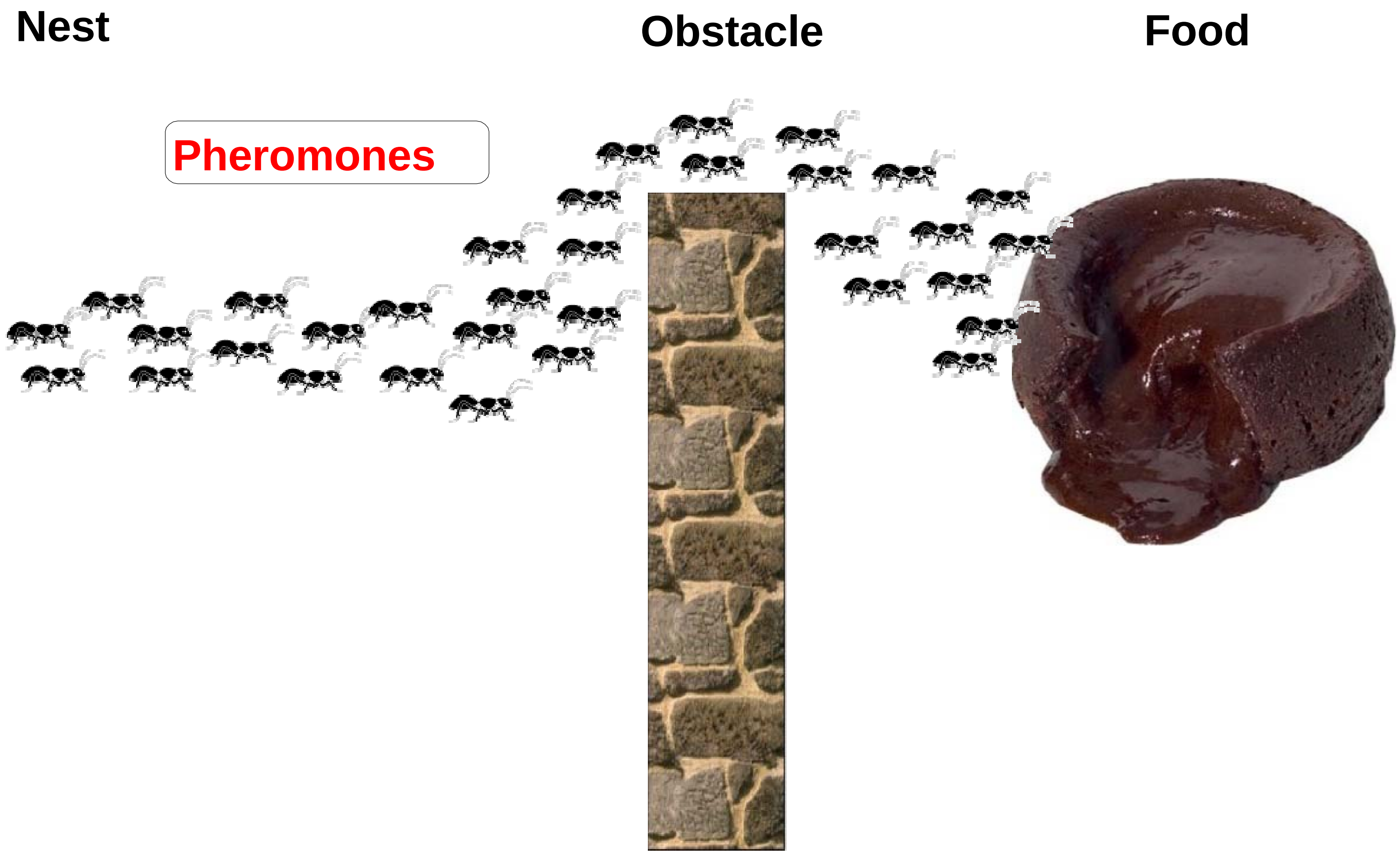
Nest Obstacle Food



# Ant Behavior

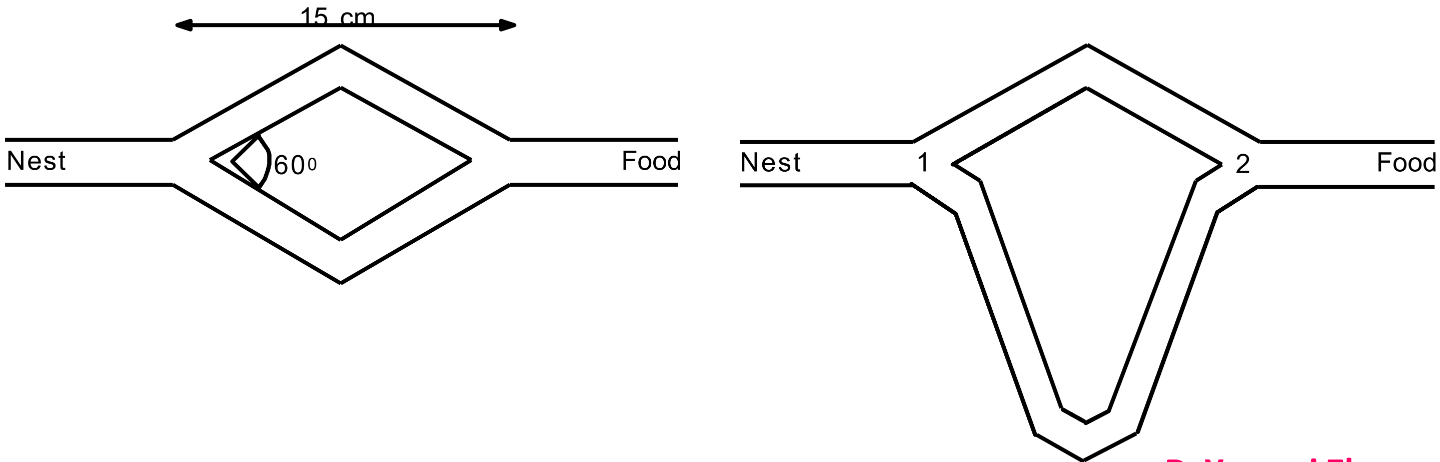


# Ant Behavior



# Double bridge experiments

- laboratory colonies of *Iridomyrmex humilis*
- ants deposit pheromone while walking from food sources to nest and vice versa
- ants tend to choose, in probability, paths marked by strong pheromone concentrations

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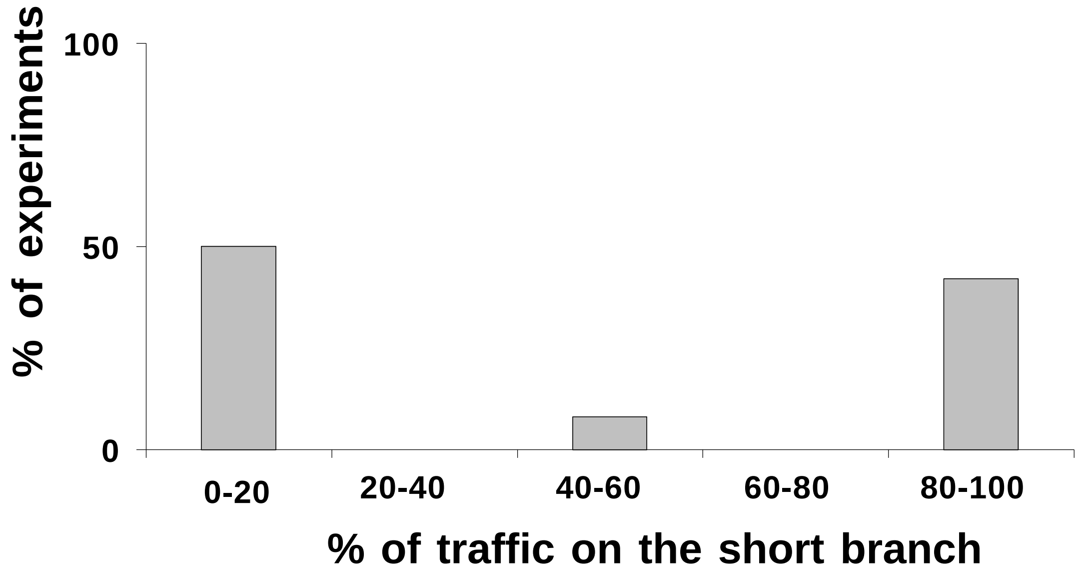
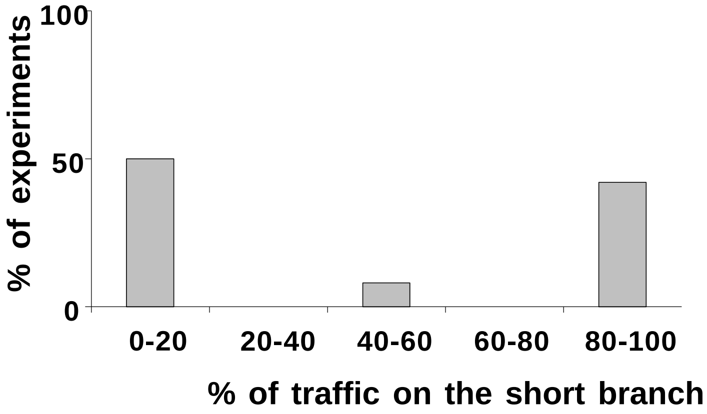
# Experimental results

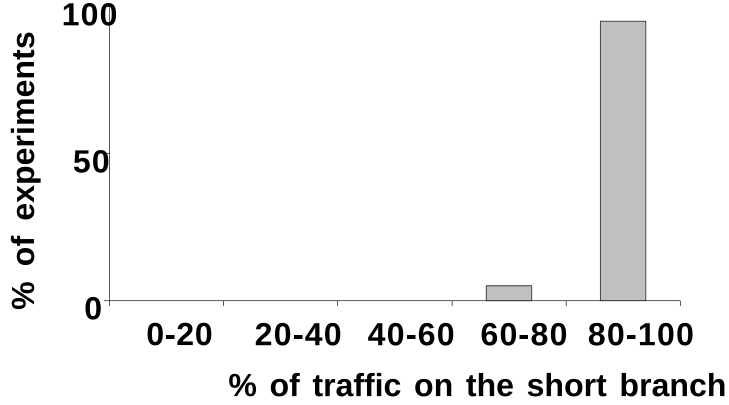
- equal length bridges: convergence to a single path



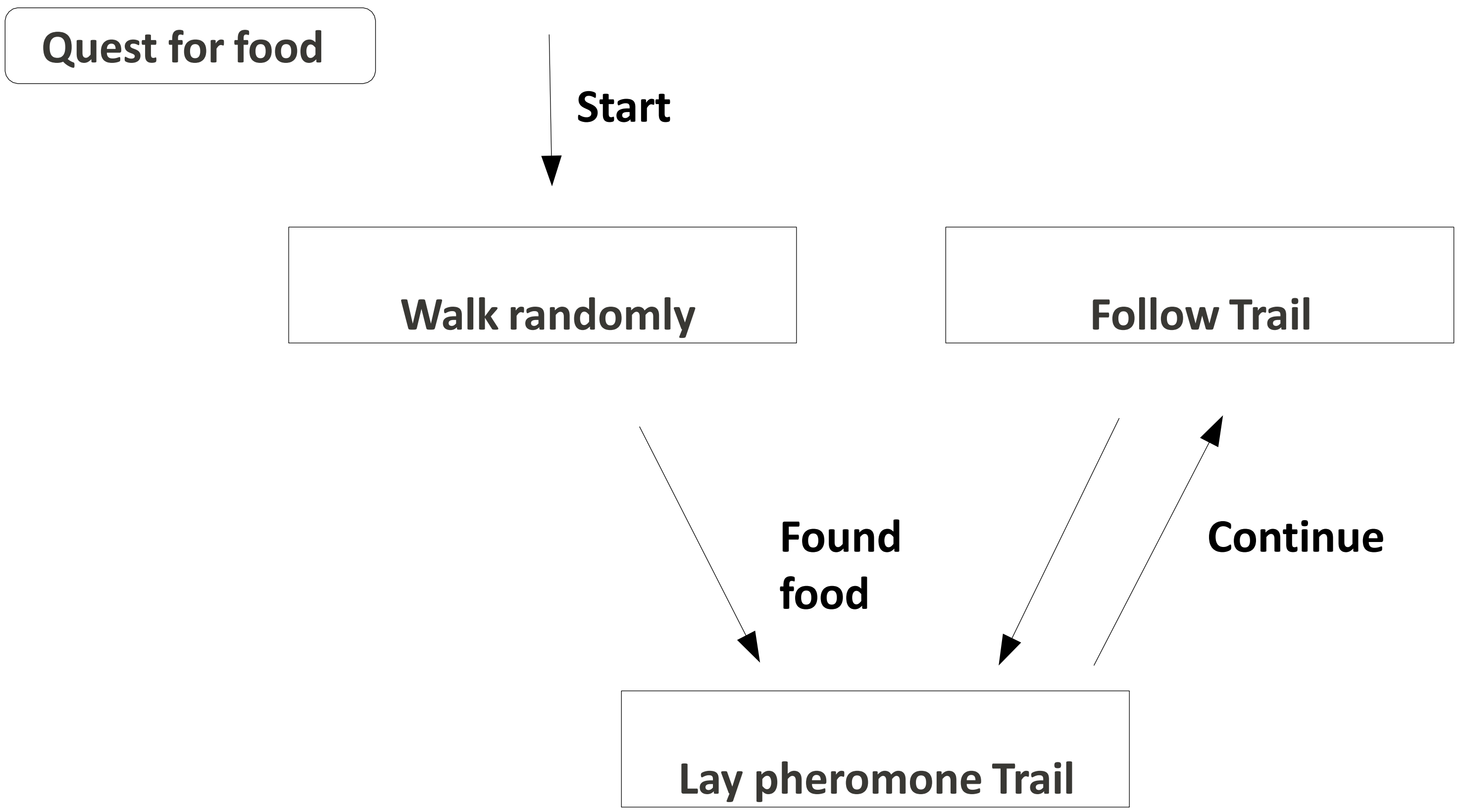
# Experimental results

- equal length bridges: convergence to a single path
- different length paths: convergence to short path



# Ant Behavior



# Ant Behavior

- Ant behavior is **stochastic**
- The behavior is induced by **indirect communication** (pheromone paths) - **Stigmergy**
- Ants explore the search space
- Limited ability to sense local environment  Act **concurrently** and
- **independently**
- High quality solutions emerge via **global cooperation**

# Stigmergy

- Term coined by French biologist Pierre-Paul Grasse, means interaction through the environment
- Indirect communication via **interaction with environment**
- Agents **respond to changes** in the environment
- Allows simpler agents
- Decreases direct communication

# Pheromones

- Ants lay pheromone trails while traveling
- Pheromones **accumulate** with multiple ants using a path This
- behavior leads to the appearance of **shortest paths**

Pheromones = **long-term memory** of an ant colony

- Pheromones **evaporate**
  - **Avoids being trapped** in local optima

$\rho$ small $\Rightarrow$ low evaporation $\Rightarrow$ **slow adaptation**

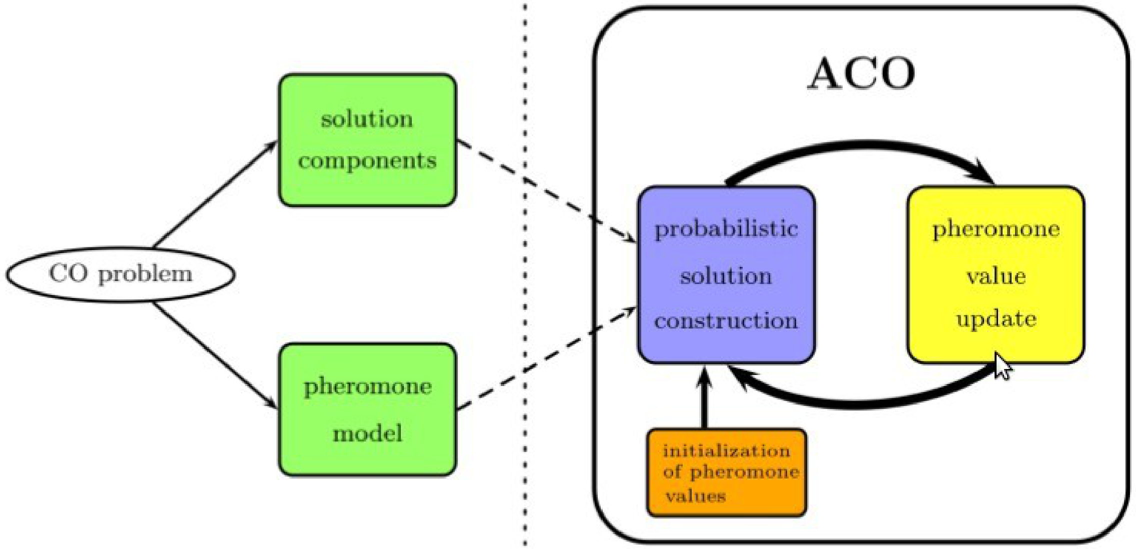
$\rho$ large $\Rightarrow$ high evaporation $\Rightarrow$ **fast adaptation**

# Towards artificial ants

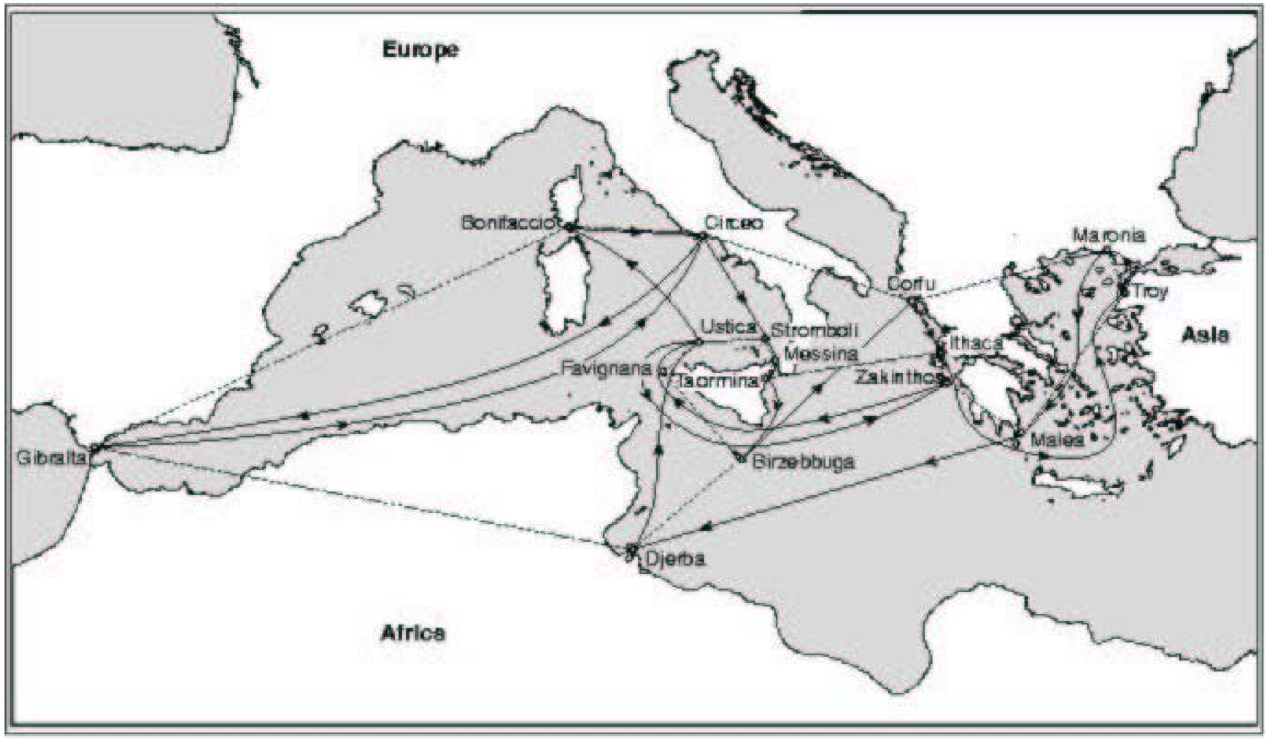
- Real ant colonies are solving *shortest path problems*
- Ant colony optimization takes elements from real ant behavior to solve more complex problems than real ants
- In ACO, artificial ants are *stochastic solution construction procedures* that probabilistically build solutions exploiting
  - (Artificial) *pheromone trails* which change dynamically at run time to reflect the agents' acquired search experience
  - *Heuristic information* on the problem instance being solved

# ACO Algorithm

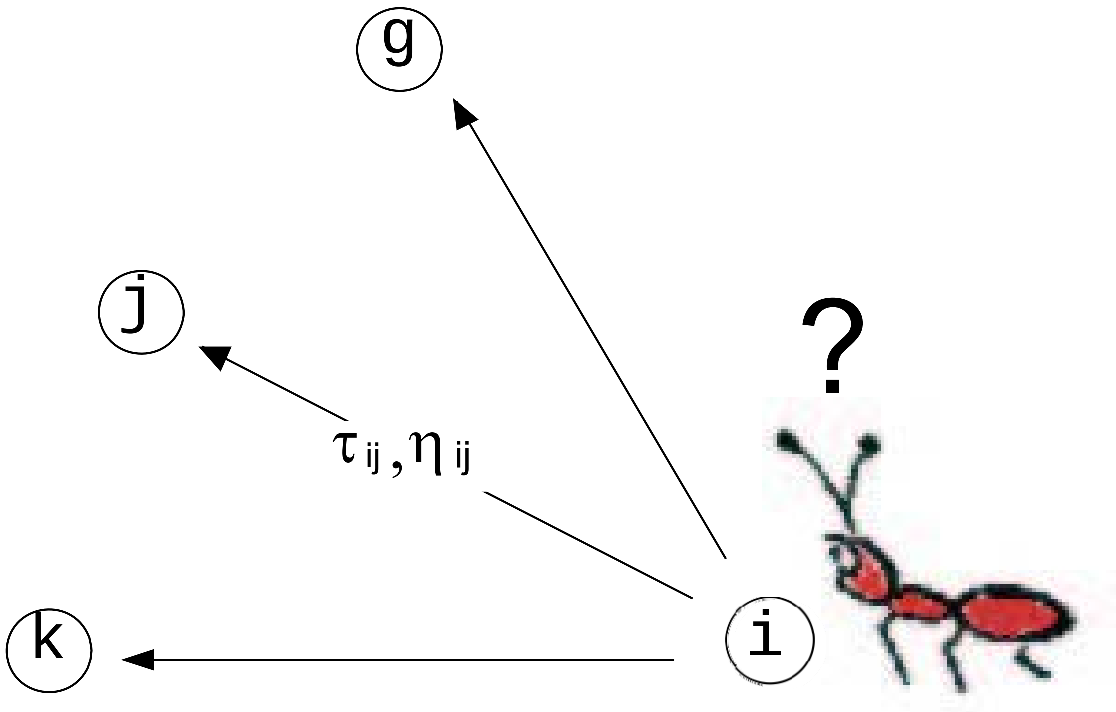
- **Construct solutions**
  - Explore the search space
  - Choose next step **probabilistically** according to the **pheromone model**
- Apply local search to constructed solutions (Optional)
- **Update pheromones** (add new + evaporate)



# Travelling Salesman Problem



# Stochastic solution construction



# Stochastic solution construction

- Use memory to remember partial tours
- Being at a city *i* choose next city *j* probabilistically among feasible neighbors
- Probabilistic choice depends on pheromone trails and heuristic information $\eta_{ij} = 1/d_{ij}$
- "Usual" action choice rule at node

$$p_{ij}^{k}(t) = \frac{[\tau_{ij}(t)]^{\alpha} \cdot [\eta_{ij}]^{\beta}}{\sum_{l \in \mathcal{N}_i^k} [\tau_{il}(t)]^{\alpha} \cdot [\eta_{il}]^{\beta}} \qquad \text{if } j \in \mathcal{N}_i^k$$

# Pheromone update

- Use positive feedback to reinforce components of good solutions
  - Better solutions give more feedback
- Use pheromone evaporation to avoid unlimited increase of pheromone trails and allow forgetting of bad choices
  - Pheromone evaporation $0 < \rho \leq 1$

# Pheromone update (2)

- Example of pheromone update

$$\tau_{xy}(t+1) = \rho\tau_{xy}(t) + \Delta\tau_{xy}(t)$$

$$\Delta\tau_{xy}(t) = \sum_{k=1}^{m} \Delta\tau_{xy}^{k}(t)$$

- $\Delta\tau_{xy}^{k}(t) = Q / L_k$ if arc *(i, j)* is used by ant *k* on its tour

  $L_k$: Tour length of ant *k*

  *m*: Number of ants

The resulting algorithm is called Ant System

# Ant System

- Ant System is the first ACO algorithm proposed in 1991 by Dorigo et al.
- encouraging initial results on TSP but inferior to state-of-the-art

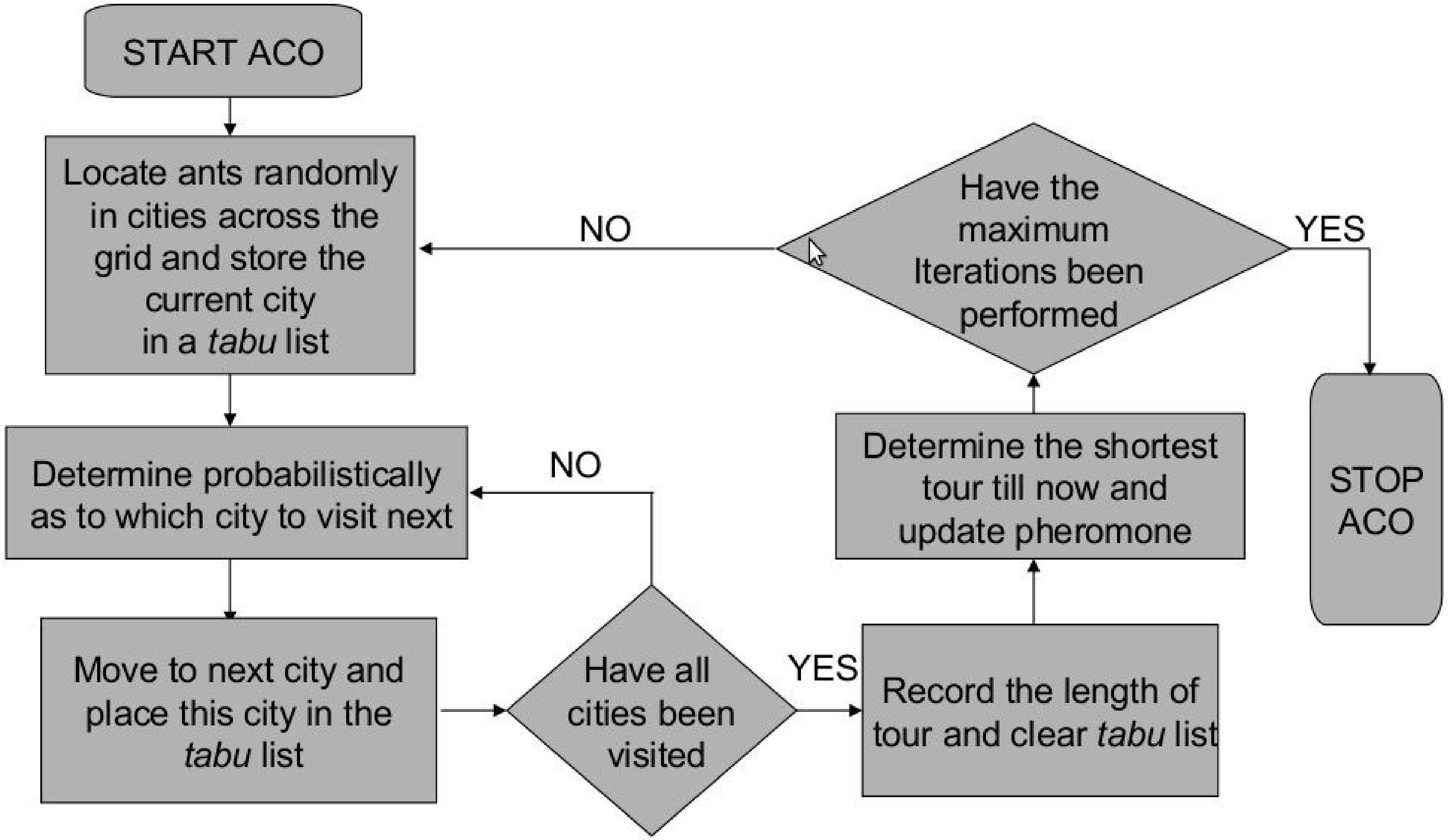
**Role of Ant System**

- “proof of concept”
- stimulation of further research on algorithmic variants and new applications

# Ant System: The algorithm

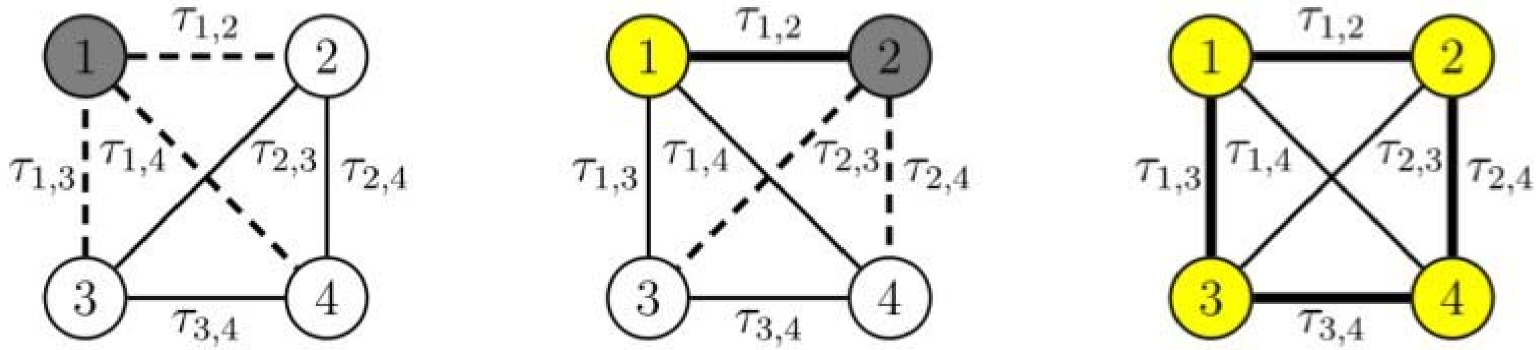
**procedure** *Ant System for TSP*
    Initialize pheromones
    **while** (termination condition not met) **do**
        **for i=1 to n-1 do**
            **for k=1 to m do**
                ApplyProbabilisticActionChoiceRule $(M^k, \tau, \eta)$
            **end end**
        GlobalPheromoneTrailUpdate
    **end**
**end** *Ant System for TSP*

# Example: TSP



START ACO
Locate ants randomly in cities across the grid and store the current city in a *tabu* list
Determine probabilistically as to which city to visit next
Move to next city and place this city in the *tabu* list
Have all cities been visited
NO
YES
Record the length of tour and clear *tabu* list
Determine the shortest tour till now and update pheromone
Have the maximum Iterations been performed
NO
YES
STOP ACO

# Example: TSP




$$\mathbf{p}(e_{1,j}) = \frac{\tau_{1,j}}{\tau_{1,2} + \tau_{1,3} + \tau_{1,4}}$$

$$\mathbf{p}(e_{2,j}) = \frac{\tau_{2,j}}{\tau_{2,3} + \tau_{2,4}}$$

(a) First step of the solution construction.

(b) Second step of the solution construction.

(c) The complete solution after the final construction step.

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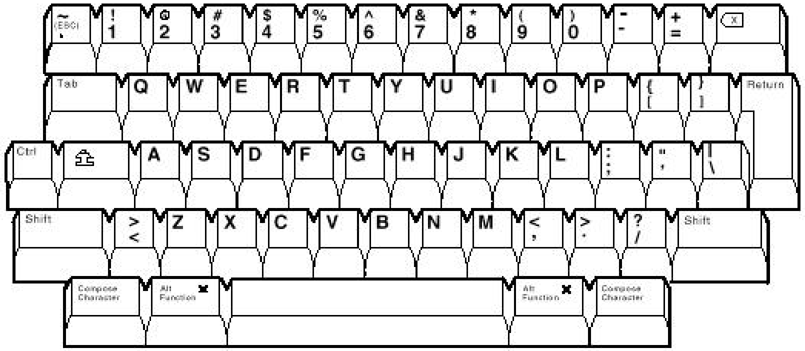
# ACO Algorithms: Overview

| *ACO algorithm* | *Authors* | *Year* | *TSP* |
|---|---|---|---|
| Ant System | Dorigo, Maniezzo & Colorni | 1991 | yes |
| Elitist AS | Dorigo | 1992 | yes |
| Ant-Q | Gambardella & Dorigo | 1995 | yes |
| Ant Colony System | Dorigo & Gambardella | 1996 | yes |
| AS | Stützle & Hoos | 1996 | yes |
| Rank-based AS | Bullnheimer, Hartl & Strauss | 1997 | yes |
| ANTS | Maniezzo | 1998 | no |
| Best-Worst AS | Cordón, et al. | 2000 | yes |
| Hyper-cube ACO | Blum, Roli, Dorigo | 2001 | no |

# ACO applications: QAP

Design a keyboard layout!



- Distance: time for pressing two keys consecutively
- Flow: frequency of a letter given another letter
- Objective function: average writing time

**Goal:**

find a keyboard layout that minimizes the average writing time!

$$\sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} f_{\phi_i \phi_j}$$

$\phi_k$ is the letter assigned to the key

# Advantages and Disadvantages

- Advantages
  - Can be used in **dynamic** applications
  - **Positive Feedback** leads to rapid discovery of good solutions
  - **Distributed computation** avoids premature convergence

- Disadvantages
  - Convergence is **guaranteed**, but **time** to convergence **uncertain**
  - Coding is not straightforward

# Recent Trends in ACO

- Continued interest in applications
- New applications
  - Multi-objective optimization
  - Dynamic optimization problems
  - Stochastic optimization problems
- Increasing interest in theoretical issues
- Methodology for applying ACO

# Conclusions

- **Ant Colony Optimization is becoming a mature field**
  - ✓ variety of algorithms available
  - ✓ many successful applications
  - ✓ theoretical results
  - ✓ dedicated workshops (ANTS'1998 — 2006) and journal special issues

**ACO is the most successful technique of the wider field of Swarm Intelligence**

# References


- Dorigo M., Blum C., *Ant colony optimization theory: A survey,* Theoretical Computer Science, Volume 344, Issues 2-3, November 2005
- Blum C., *Ant colony optimization: Introduction and recent trends*, Physics of Life Reviews, Volume 2, Issue 4, December 2005
- Dorigo M., Stutzle T., *Ant Colony Optimization*, Ant Colony Optimization, MIT Press 2004


# Thank you

## End of

## Ant Colony Optimization