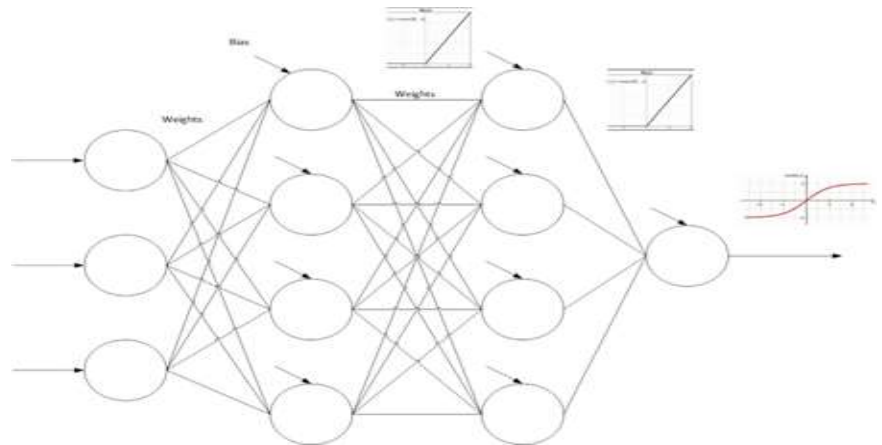


## SOLUCION PREGUNTA 1

$$Xs = \begin{bmatrix} 2.5 & 3.5 & -0.5 \\ 4 & -1 & 0.5 \\ 0.5 & 1.5 & 1 \\ 3 & 2 & -1.5 \end{bmatrix}$$

$$Ys = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$



### Estructura

#### Entrada:

3 Neuronas que reciben los valores de entrada (en este caso, 3 características o valores).

#### Capa Oculta 1:

4 Neuronas.

Función de activación ReLU: Aplica la activación ReLU

#### Capa Oculta 2:

4 Neuronas.

Función de activación ReLU.

#### Capa de Salida:

1 Neurona

Función de activación Tangente Hiperbólica (Tanh): (-1, 1)

#### Paso 1

Inicialización de Pesos y Sesos

$$W \sim \mathcal{U}\left(-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right) \quad \text{Se toma como referencia la inicialización de Xavier}$$

### Capa Oculta 1 (4 neuronas, 3 entradas)

Los pesos de la primera capa W1 deben moverse en el rango (−0.577,0.577)

$$\text{Pesos} \quad W1 = \begin{bmatrix} 0.1 & 0.5 & -0.2 \\ -0.3 & 0.4 & 0.1 \\ 0.2 & -0.1 & -0.5 \\ 0.05 & 0.3 & 0.2 \end{bmatrix}$$

$$\text{Sesgo} \quad b1 = \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix}$$

### Capa Oculta 2 (4 neuronas, 3 entradas)

Los pesos de la primera capa W2 deben moverse en el rango (−0.5, 0.5)

$$\text{Pesos} \quad W2 = \begin{bmatrix} 0.1 & 0.4 & -0.1 & 0.2 \\ -0.2 & 0.3 & 0.2 & 0.4 \\ 0.3 & -0.1 & -0.4 & -0.1 \\ 0.45 & 0.2 & 0.35 & 0.3 \end{bmatrix}$$

$$\text{Sesgo} \quad b2 = \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix}$$

### Capa Salida (1 neuronas, 4 entradas)

Los pesos de la primera capa W1 deben moverse en el rango (−0.577,0.577)

$$\text{Pesos} \quad Wout = \begin{bmatrix} 0.5 & -0.4 & 0.4 & -0.2 \end{bmatrix}$$

$$\text{Sesgo} \quad bout = 0.1$$

## Forward Propagation

**Primera entrada** Tomemos la primera fila de Xs (entrada [2.5,3.5,−0.5]):

**Capa Oculta 1:**

La salida de la primera capa oculta, Z1, se calcula como:

$$Z1 = W1 \cdot Xs + b1$$

Tomemos la primera fila de Xs (entrada [2.5, 3.5, -0.5]):

$$Z1 = \begin{bmatrix} 0.1 & 0.5 & -0.2 \\ -0.3 & 0.4 & 0.1 \\ 0.2 & -0.1 & -0.5 \\ 0.05 & 0.3 & 0.2 \end{bmatrix} * \begin{bmatrix} 2.5 \\ 3.5 \\ -0.5 \end{bmatrix} + \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix}$$

$$\begin{bmatrix} 2.1 \\ 0.6 \\ 0.4 \\ 1.075 \end{bmatrix} + \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 2.15 \\ 0.53 \\ 0.42 \\ 0.975 \end{bmatrix}$$

$$Z1 = [2.15, 0.53, 0.42, 0.975]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A1 = \text{ReLU}(Z1) = [2.15, 0.53, 0.42, 0.975]$$

## Capa Oculta 2:

La salida de la segunda capa oculta, Z2, se calcula como:

$$Z2 = W2 \cdot A1 + b2$$

$$Z2 = \begin{bmatrix} 0.1 & 0.4 & -0.1 & 0.2 \\ -0.2 & 0.3 & 0.2 & 0.4 \\ 0.3 & -0.1 & -0.4 & -0.1 \\ 0.45 & 0.2 & 0.35 & 0.3 \end{bmatrix} * \begin{bmatrix} 2.15 \\ 0.53 \\ 0.42 \\ 0.975 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix}$$

$$\begin{bmatrix} 0.58 \\ 0.20 \\ 0.33 \\ 1.51 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 0.63 \\ 0.30 \\ 0.63 \\ 1.41 \end{bmatrix}$$

$$Z2 = [0.63, 0.303, 0.6265, 1.413]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A2 = \text{ReLU}(Z2) = [0.63, 0.303, 0.6265, 1.413]$$

## Capa Salida:

$$Z_{out} = W_{out} * A2 + b_{out}$$

$$Z_{out} = \begin{bmatrix} 0.5 & -0.4 & 0.4 & -0.2 \end{bmatrix} * \begin{bmatrix} 0.63 \\ 0.303 \\ 0.6265 \\ 1.413 \end{bmatrix} + 0.1$$

$$Z_{out} = \begin{matrix} \text{Tanh} \\ 0.2618 & 0.2559 \end{matrix}$$

La activación final es:

$$y1 = 0.2559$$

**Segunda entrada** Tomemos la segunda fila de Xs (entrada [4, -1, 0.5]):

### Capa Oculta 1:

La salida de la primera capa oculta, Z1, se calcula como:

$$Z1 = W1 \cdot Xs + b1$$

Tomemos la segunda fila de Xs (entrada [4, -1, 0.5]):

$$Z1 \quad \begin{bmatrix} 0.1 & 0.5 & -0.2 \\ -0.3 & 0.4 & 0.1 \\ 0.2 & -0.1 & -0.5 \\ 0.05 & 0.3 & 0.2 \end{bmatrix} * \begin{bmatrix} 4 \\ -1 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix}$$

$$\begin{bmatrix} -0.2 \\ -1.55 \\ 0.65 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix} = \begin{bmatrix} -0.15 \\ -1.62 \\ 0.67 \\ -0.1 \end{bmatrix}$$

$$Z1 = [-0.15, -1.62, 0.67, -0.1]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A1 = \text{ReLU}(Z1) = [0, 0, 0.67, 0]$$

### Capa Oculta 2:

La salida de la segunda capa oculta, Z2, se calcula como:

$$Z2 = W2 \cdot A1 + b2$$

$$Z2 = \begin{bmatrix} 0.1 & 0.4 & -0.1 & 0.2 \\ -0.2 & 0.3 & 0.2 & 0.4 \\ 0.3 & -0.1 & -0.4 & -0.1 \\ 0.45 & 0.2 & 0.35 & 0.3 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 0.67 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix}$$

$$\begin{bmatrix} -0.067 \\ 0.134 \\ -0.268 \\ 0.2345 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix} = \begin{bmatrix} -0.017 \\ 0.234 \\ 0.032 \\ 0.1345 \end{bmatrix}$$

$$Z1 = [-0.017, 0.234, 0.032, 0.1345]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A2 = \text{ReLU}(Z2) = [0, 0.234, 0.032, 0.1345]$$

### Capa Salida:

$$Z_{out} = W_{out} * A2 + b_{out}$$

$$Z_{out} = \begin{bmatrix} 0.5 & -0.4 & 0.4 & -0.2 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.234 \\ 0.032 \\ 0.1345 \end{bmatrix} + 0.1$$

$$Z_{out} = \begin{matrix} \text{Tanh} \\ -0.0077 & 0.0077 \end{matrix}$$

La activación final es:

$$y2 = 0.0077$$

**Tercera entrada** Tomemos la tercera fila de  $X_s$  (entrada  $[0.5, 1.5, 1.0]$ ):

### Capa Oculta 1:

La salida de la primera capa oculta,  $Z1$ , se calcula como:

$$Z1 = W1 \cdot Xs + b1$$

Tomemos la tercera fila de  $X_s$  (entrada  $[0.5, 1.5, 1.0]$ ):

$$Z1 = \begin{bmatrix} 0.1 & 0.5 & -0.2 \\ -0.3 & 0.4 & 0.1 \\ 0.2 & -0.1 & -0.5 \\ 0.05 & 0.3 & 0.2 \end{bmatrix} * \begin{bmatrix} 0.5 \\ 1.5 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix}$$

$$\begin{bmatrix} 0.6 \\ 0.55 \\ -0.55 \\ 0.675 \end{bmatrix} + \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 0.65 \\ 0.48 \\ -0.53 \\ 0.575 \end{bmatrix}$$

$$Z1 = [0.65, 0.48, -0.53, 0.575]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A1 = \text{ReLU}(Z1) = [0.65, 0.48, 0.0, 0.575]$$

## Capa Oculta 2:

La salida de la segunda capa oculta, Z2, se calcula como:

$$Z2 = W2 \cdot A1 + b2$$

$$Z2 = \begin{bmatrix} 0.1 & 0.4 & -0.1 & 0.2 \\ -0.2 & 0.3 & 0.2 & 0.4 \\ 0.3 & -0.1 & -0.4 & -0.1 \\ 0.45 & 0.2 & 0.35 & 0.3 \end{bmatrix} * \begin{bmatrix} 0.65 \\ 0.48 \\ 0 \\ 0.575 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix}$$

$$\begin{bmatrix} 0.372 \\ 0.244 \\ 0.0895 \\ 0.561 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 0.422 \\ 0.344 \\ 0.3895 \\ 0.461 \end{bmatrix}$$

$$Z2 = [0.422, 0.344, 0.3895, 0.461]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A2 = \text{ReLU}(Z2) = [0.422, 0.344, 0.3895, 0.461]$$

## Capa Salida:

$$Z_{out} = W_{out} * A2 + b_{out}$$

$$Z_{out} = \begin{bmatrix} 0.5 & -0.4 & 0.4 & -0.2 \end{bmatrix} * \begin{bmatrix} 0.422 \\ 0.344 \\ 0.3895 \\ 0.461 \end{bmatrix} + 0.1$$

$$Z_{out} = \begin{matrix} \text{Tanh} \\ 0.237 \end{matrix} \quad 0.2327$$

La activación final es:

$$y_3 = 0.2327$$

#### Cuarta entrada

Tomemos la cuarta fila de  $X_s$  (entrada [3.0, 2.0, -1.5]):

#### Capa Oculta 1:

La salida de la primera capa oculta,  $Z_1$ , se calcula como:

$$Z_1 = W_1 \cdot X_s + b_1$$

Tomemos la cuarta fila de  $X_s$  (entrada [3, 2, -1.5]):

$$Z_1 \begin{bmatrix} 0.1 & 0.5 & -0.2 \\ -0.3 & 0.4 & 0.1 \\ 0.2 & -0.1 & -0.5 \\ 0.05 & 0.3 & 0.2 \end{bmatrix} * \begin{bmatrix} 3 \\ 2 \\ -1.5 \end{bmatrix} + \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix}$$
$$\begin{bmatrix} 1.6 \\ -0.25 \\ 1.15 \\ 0.45 \end{bmatrix} + \begin{bmatrix} 0.05 \\ -0.07 \\ 0.02 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 1.65 \\ -0.32 \\ 1.17 \\ 0.35 \end{bmatrix}$$

$$Z_1 = [1.65, -0.32, 1.17, 0.35]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A_1 = \text{ReLU}(Z_1) = [1.65, 0, 1.17, 0.35]$$

#### Capa Oculta 2:

La salida de la segunda capa oculta,  $Z_2$ , se calcula como:

$$Z_2 = W_2 \cdot A_1 + b_2$$

$$Z_2 = \begin{bmatrix} 0.1 & 0.4 & -0.1 & 0.2 \\ -0.2 & 0.3 & 0.2 & 0.4 \\ 0.3 & -0.1 & -0.4 & -0.1 \\ 0.45 & 0.2 & 0.35 & 0.3 \end{bmatrix} * \begin{bmatrix} 1.65 \\ 0 \\ 1.17 \\ 0.35 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix}$$
$$\begin{bmatrix} 0.118 \\ 0.044 \\ -0.008 \\ 1.257 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.1 \\ 0.3 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 0.168 \\ 0.144 \\ 0.292 \\ 1.157 \end{bmatrix}$$

$Z_2 = [0.168, 0.144, 0.292, 1.157]$

ReLU:  $f(x) = \max(0,x)$ :

$A_2 = \text{ReLU} ( Z_2 ) = [ 0.168, 0.144, 0.292, 1.157 ]$

Capa Salida:

$Z_{out} = W_{out} * A_2 + b_{out}$

$Z_{out} = \begin{bmatrix} 0.5 & -0.4 & 0.4 & -0.2 \end{bmatrix} * \begin{bmatrix} 0.168 \\ 0.144 \\ 0.292 \\ 1.157 \end{bmatrix} + 0.1$

$Z_{out} = \begin{matrix} \text{Tanh} \\ 0.0118 & 0.0118 \end{matrix}$

La activación final es:

$y_4 = 0.0118$

Backward Propagation

Calculo del error de cada salida

	errori	=	yi'	-	yi	=	
Entrada 1	error 1	=	0.2559	-	1	=	-0.7441
Entrada 2	error 2	=	0.0077	-	-1	=	1.0077
Entrada 3	error 3	=	0.2327	-	-1	=	1.2327
Entrada 4	error 4	=	0.0118	-	1	=	-0.9882

Paso 1: Calculo del Error (Pérdida)

Función de pérdida

$L = (1/4) * ( (0.2559 -1)^2 + (0.0077-(-1))^2+(0.2327-(-1))^2+(0.0118-1)^2 )$

$L = 1.016308158$

Primera Entrada

Paso 2: Backward Propagation desde la Capa de Salida hacia la Segunda Capa Oculta

1 Gradiente en la salida



$$(\partial L / \partial y_{out}) = (y_{out} - y) = (0.2559 - 1) = -0.7441$$

2 Derivada de la función tanh aplicada en la salida

$$(\partial L / \partial Z_{out}) = (\partial L / \partial y_{out}) * \tanh'(Z_{out})$$

$$\tanh'(Z_{out}) = 1 - \tanh^2(Z_{out}) = 1 - (0.2559)^2 = 0.93452$$

$$(\partial L / \partial Z_{out}) = -0.7441 * 0.9345 = -0.6954$$

3 Gradiente de los pesos W3 y las bias b3 en la capa de salida

Gradiente de los pesos Wout

$$(\partial L / \partial W_{out}) = (\partial L / \partial Z_{out}) * A2$$

$$(\partial L / \partial W_{out}) = -0.69536145 * \begin{bmatrix} 0.63 \\ 0.303 \\ 0.6265 \\ 1.413 \end{bmatrix} = \begin{bmatrix} -0.4381 \\ -0.2107 \\ -0.4356 \\ -0.9825 \end{bmatrix}$$

Gradiente de bias b out

$$(\partial L / \partial b_{out3}) = (\partial L / \partial Z_{out}) = -0.695$$

### Paso 3: Backward Propagation hacia la Segunda Capa Oculta

1 Cálculo del gradiente de la salida A2 de la segunda capa

$$(\partial L / \partial A2) = (\partial L / \partial Z_{out}) * W_{out}$$

$$(\partial L / \partial Z_{out}) = -0.69536145$$

$$W_{out} = [0.5, -0.4, 0.4, -0.2]$$

$$(\partial L / \partial A2) = -0.69536145 * \begin{bmatrix} 0.5 \\ -0.4 \\ 0.4 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -0.3477 \\ 0.27814 \\ -0.2781 \\ 0.13907 \end{bmatrix}$$

2 Derivada de la función tanh aplicada en Z2

$$(\partial L / \partial Z2) = (\partial L / \partial A2) * \tanh'(Z2)$$

$$\tanh'(x) = 1 - \tanh^2(x)$$

$$\tanh'(Z_2) = \begin{bmatrix} (1-(0.63)^2) \\ (1-(0.303)^2) \\ (1-(0.6265)^2) \\ (1-(1.413)^2) \end{bmatrix} = \begin{bmatrix} 0.603 \\ 0.908 \\ 0.607 \\ -0.997 \end{bmatrix}$$

$$(\partial L / \partial Z_2) = \begin{bmatrix} -0.348 \\ 0.278 \\ -0.278 \\ 0.139 \end{bmatrix} * \begin{bmatrix} 0.603 \\ 0.908 \\ 0.607 \\ -0.997 \end{bmatrix} = \begin{bmatrix} -0.210 \\ 0.253 \\ -0.169 \\ -0.139 \end{bmatrix}$$

3 Gradiente respecto a los pesos W2 y bias 2

Gradiente de los pesos W2

$$(\partial L / \partial W_2) = (\partial L / \partial Z_2) * A_1^T$$

$$A_1 = [2.15, 0.53, 0.42, 0.975]$$

$$(\partial L / \partial W_2) = \begin{bmatrix} -0.210 \\ 0.253 \\ -0.169 \\ -0.139 \end{bmatrix} * \begin{bmatrix} 2.15, 0.53, 0.42, 0.975 \end{bmatrix} =$$

$$= \begin{bmatrix} -0.452 & -0.1113 & -0.0882 & -0.20475 \\ 0.54395 & 0.13409 & 0.10626 & 0.246675 \\ -0.36335 & -0.08957 & -0.07098 & -0.164775 \\ -0.29885 & -0.07367 & -0.05838 & -0.135525 \end{bmatrix}$$

Gradiente bias b2

$$(\partial L / \partial b_2) = \begin{bmatrix} -0.210 \\ 0.253 \\ -0.169 \\ -0.139 \end{bmatrix}$$

#### Paso 4: Backward Propagation hacia la Primera Capa Oculta

1 Gradiente de A1

$$(\partial L / \partial A_1) = (\partial L / \partial Z_2) * W_{2T}$$

2 Propagado gradiente hacia atrás

$$(\partial L / \partial Z_2) = \begin{bmatrix} -0.210 \\ 0.253 \\ -0.169 \\ -0.139 \end{bmatrix}$$

La Matriz de pesos W2

$$W_2 = \begin{bmatrix} 0.1 & 0.4 & -0.1 & 0.2 \\ -0.2 & 0.3 & 0.2 & 0.4 \\ 0.3 & -0.1 & -0.4 & -0.1 \\ 0.45 & 0.2 & 0.35 & 0.3 \end{bmatrix}$$

$$(\partial L / \partial A_1) = W_2^T * (\partial L / \partial Z_2)$$

$$(\partial L / \partial A_1) = \begin{bmatrix} 0.1 & -0.2 & 0.3 & 0.45 \\ 0.4 & 0.3 & -0.1 & 0.2 \\ -0.1 & 0.2 & -0.4 & 0.35 \\ 0.2 & 0.4 & -0.1 & 0.3 \end{bmatrix} * \begin{bmatrix} -0.210 \\ 0.253 \\ -0.169 \\ -0.139 \end{bmatrix}$$

$$(\partial L / \partial A_1) = \begin{bmatrix} -0.185 \\ -0.019 \\ 0.091 \\ 0.034 \end{bmatrix}$$

3 Gradiente de Z1 en la primera capa oculta

$$(\partial L / \partial Z_1) = (\partial L / \partial A_1) * \tanh'(Z_1)$$

$$\tanh'(Z_1) = 1 - \tanh^2(Z_1)$$

$$\tanh'(Z_1) = \begin{bmatrix} (1-(2.15)^2) \\ (1-(0.53)^2) \\ (1-(0.42)^2) \\ (1-(1.975)^2) \end{bmatrix} = \begin{bmatrix} -3.623 \\ 0.719 \\ 0.824 \\ -2.901 \end{bmatrix}$$

$$(\partial L / \partial Z_1) = \begin{bmatrix} -0.185 \\ -0.019 \\ 0.091 \\ 0.034 \end{bmatrix} * \begin{bmatrix} -3.623 \\ 0.719 \\ 0.824 \\ -2.901 \end{bmatrix} = \begin{bmatrix} 0.670 \\ -0.014 \\ 0.075 \\ -0.100 \end{bmatrix}$$

#### 4 Gradiente de W1

$$(\partial L / \partial W1) = (\partial L / \partial Z1) * X^T$$

Primera entrada de X1 = [2.5, 3.5, -0.5]

$$(\partial L / \partial W1) = \begin{bmatrix} 0.670 \\ -0.014 \\ 0.075 \\ -0.100 \end{bmatrix} * \begin{bmatrix} 2.5, 3.5, -0.5 \end{bmatrix}$$

$$(\partial L / \partial W1) = \begin{bmatrix} 1.675 & 2.345 & -0.335 \\ -0.035 & -0.049 & 0.007 \\ 0.1875 & 0.2625 & -0.0375 \\ -0.25 & -0.35 & 0.05 \end{bmatrix}$$

#### Paso 5: Gradiente de las bias b1 - Primera Capa Oculta

$$(\partial L / \partial b1) = (\partial L / \partial Z1) = \begin{bmatrix} 0.670 \\ -0.014 \\ 0.075 \\ -0.100 \end{bmatrix}$$

## Resumen de los Gradientes Finales: Primera Entrada

Gradiente W3

$$(\partial L / \partial Wout) = \begin{bmatrix} -0.438 \\ -0.211 \\ -0.436 \\ -0.983 \end{bmatrix}$$

Gradiente b3

$$(\partial L / \partial b \text{ out}) = -0.695$$

Gradiente W2

$$(\partial L / \partial W2) = \begin{bmatrix} -0.452 & -0.111 & -0.088 & -0.205 \\ 0.544 & 0.134 & 0.106 & 0.247 \\ -0.363 & -0.090 & -0.071 & -0.165 \\ -0.299 & -0.074 & -0.058 & -0.136 \end{bmatrix}$$

Gradiente b2

$$(\partial L / \partial b2) = \begin{bmatrix} -0.210 \\ 0.253 \\ -0.169 \\ -0.139 \end{bmatrix}$$

Gradiente W1

$$(\partial L / \partial W1) = \begin{bmatrix} 1.675 & 2.345 & -0.335 \\ -0.035 & -0.049 & 0.007 \\ 0.188 & 0.263 & -0.038 \\ -0.250 & -0.350 & 0.050 \end{bmatrix}$$

Gradiente b1

$$(\partial L / \partial b1) = \begin{bmatrix} 0.670 \\ -0.014 \\ 0.075 \\ -0.100 \end{bmatrix}$$

## Actualización de pesos y bias con Descenso de Gradiente

$$W_{\text{nuevo}} = W_{\text{viejo}} - n * ((\partial L / \partial W))$$

$$b_{\text{nuevo}} = b_{\text{viejo}} - n * ((\partial L / \partial b))$$

### Primera Entrada

Revisar los Gradientes

**Gradiente W out**

$$(\partial L / \partial W_{\text{out}}) = \begin{bmatrix} -0.438 \\ -0.211 \\ -0.436 \\ -0.983 \end{bmatrix}$$

Gradiente b out

$$(\partial L / \partial b_{\text{out}}) = -0.695$$

Tasa de Aprendizaje (n)

$$n = 0.01$$

**Actualizar los pesos de W out**

$$W_{\text{out}} = \begin{bmatrix} 0.5 \\ -0.4 \\ 0.4 \\ -0.2 \end{bmatrix} - 0.01 * \begin{bmatrix} -0.438 \\ -0.211 \\ -0.436 \\ -0.983 \end{bmatrix}$$
$$W_{\text{out}} = \begin{bmatrix} 0.504 \\ -0.398 \\ 0.404 \end{bmatrix}$$

$$\begin{bmatrix} -0.190 \end{bmatrix}$$

**Actualizar los pesos de b out**

$$b_{out} = 0.107$$

**Gradiente W 2**

$$(\partial L / \partial W_2) = \begin{bmatrix} -0.452 & -0.111 & -0.088 & -0.205 \\ 0.544 & 0.134 & 0.106 & 0.247 \\ -0.363 & -0.090 & -0.071 & -0.165 \\ -0.299 & -0.074 & -0.058 & -0.136 \end{bmatrix}$$

**Gradiente b 2**

$$(\partial L / \partial b_2) = \begin{bmatrix} -0.210 \\ 0.253 \\ -0.169 \\ -0.139 \end{bmatrix}$$

Tasa de Aprendizaje (n)

$$n = 0.01$$

**Actualizar los pesos de W 2**

$$W_2 = \begin{bmatrix} 0.105 & 0.401 & -0.099 & 0.202 \\ -0.205 & 0.299 & 0.199 & 0.398 \\ 0.304 & -0.099 & -0.399 & -0.098 \\ 0.453 & 0.201 & 0.351 & 0.301 \end{bmatrix}$$

**Actualizar los pesos de b 2**

$$b_2 = \begin{bmatrix} 0.052 \\ 0.100 \\ 0.300 \\ -0.100 \end{bmatrix}$$

**Gradiente W 1**

$$(\partial L / \partial W_1) = \begin{bmatrix} 1.675 & 2.345 & -0.335 \\ -0.035 & -0.049 & 0.007 \\ 0.188 & 0.263 & -0.038 \\ -0.250 & -0.350 & 0.050 \end{bmatrix}$$

### Gradiente b 1

$$(\partial L / \partial b_1) = \begin{bmatrix} 0.670 \\ -0.014 \\ 0.075 \\ -0.100 \end{bmatrix}$$

Tasa de Aprendizaje (n)

$$n = 0.01$$

### Actualizar los pesos de W 1

$$W_1 = \begin{bmatrix} 0.083 & 0.477 & -0.197 \\ -0.300 & 0.400 & 0.100 \\ 0.198 & -0.103 & -0.500 \\ 0.053 & 0.304 & 0.200 \end{bmatrix}$$

### Actualizar los pesos de b 1

$$b_1 = \begin{bmatrix} 0.043 \\ -0.070 \\ 0.019 \\ -0.099 \end{bmatrix}$$

## Forward Propagation Nuevos Pesos y Bias

### Primera entrada

Tomemos la primera fila de Xs (entrada [2.5, 3.5, -0.5]):

### Capa Oculta 1:

La salida de la primera capa oculta, Z1, se calcula como:

$$Z1 = W1 \cdot Xs + b1$$

Tomemos la primera fila de Xs (entrada [2.5, 3.5, -0.5]):

$$Z1 = \begin{bmatrix} 0.083 & 0.477 & -0.197 \\ -0.300 & 0.400 & 0.100 \\ 0.198 & -0.103 & -0.500 \\ 0.053 & 0.304 & 0.200 \end{bmatrix} * \begin{bmatrix} 2.5 \\ 3.5 \\ -0.5 \end{bmatrix} + \begin{bmatrix} 0.043 \\ -0.070 \\ 0.019 \\ -0.099 \end{bmatrix}$$
$$\begin{bmatrix} 1.9755 \\ 0.6 \\ 0.3845 \end{bmatrix} + \begin{bmatrix} 0.043 \\ -0.070 \\ 0.019 \end{bmatrix} = \begin{bmatrix} 2.019 \\ 0.530 \\ 0.404 \end{bmatrix}$$

$$\begin{bmatrix} 1.0965 \\ -0.099 \\ 0.998 \end{bmatrix}$$

$$Z1 = [2.019, 0.53, 0.404, 0.998]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A1 = \text{ReLU}(Z1) = [2.019, 0.53, 0.404, 0.998]$$

## Capa Oculta 2:

La salida de la segunda capa oculta, Z2, se calcula como:

$$Z2 = W2 \cdot A1 + b2$$

$$Z2 = \begin{bmatrix} 0.105 & 0.401 & -0.099 & 0.202 \\ -0.205 & 0.299 & 0.199 & 0.398 \\ 0.304 & -0.099 & -0.399 & -0.098 \\ 0.453 & 0.201 & 0.351 & 0.301 \end{bmatrix} * \begin{bmatrix} 2.019 \\ 0.53 \\ 0.404 \\ 0.998 \end{bmatrix} + \begin{bmatrix} 0.052 \\ 0.100 \\ 0.300 \\ -0.100 \end{bmatrix}$$

$$\begin{bmatrix} 0.59 \\ 0.22 \\ 0.30 \\ 1.46 \end{bmatrix} + \begin{bmatrix} 0.052 \\ 0.100 \\ 0.300 \\ -0.100 \end{bmatrix} = \begin{bmatrix} 0.64 \\ 0.32 \\ 0.60 \\ 1.36 \end{bmatrix}$$

$$Z2 = [0.64, 0.32, 0.60, 1.36]$$

ReLU:  $f(x) = \max(0, x)$ :

$$A2 = \text{ReLU}(Z2) = [0.64, 0.32, 0.60, 1.36]$$

## Capa Salida:

$$Z_{out} = W_{out} * A2 + b_{out}$$

$$Z_{out} = \begin{bmatrix} 0.5 & -0.4 & 0.4 & -0.2 \end{bmatrix} * \begin{bmatrix} 0.64 \\ 0.32 \\ 0.6 \\ 1.36 \end{bmatrix} + 0.1$$

$$Z_{out} = \begin{matrix} \text{Tanh} \\ 0.26 \end{matrix} \quad \begin{matrix} 0.254 \end{matrix}$$

La activación final es:

$$y1 = 0.254$$