

• Entrada 1 [2.5 , 3.5 , -0.5]

Neurona 1 capa 1: 3

$$z_1 = \sum_{i=1}^3 w_{1i} x_i + b_1$$

$$z_1 = (2.5)(0.00418) + (3.5)(0.43075) + (-0.5)(-0.42825) + 0.32206$$

$$z_1 = 0.01045 + 1.50762 + 0.21412 + 0.32206$$

$$z_1 = 2.05425$$

Aplicamos la función de activación:

$$\tanh(z_1) = \tanh(2.05425) = 0.96766$$

Neurona 2 capa 1:

$$z_2 = \sum_{i=1}^3 w_{2i} x_i + b_2$$

$$z_2 = (2.5)(0.51585) + (3.5)(0.58195) + (-0.5)(-0.80580) + (-0.86865)$$

$$z_2 = 1.28962 + 2.03682 + 0.4044 - 0.86865$$

$$z_2 = 2.86219$$

$$\tanh(z_2) = \tanh(2.86219) = 0.99349$$

Neurona 3 capa 1 :

$$Z_3 = \sum_{i=1}^3 w_{3i} x_i + b_3$$

$$Z_3 = (2.5)(-0.44835) + (3.5)(0.38031) + (-0.5)(-0.56652) + 0.01807$$

$$Z_3 = -1.12087 + 1.33108 + 0.28326 + 0.01807$$

$$Z_3 = 0.51154$$

$$\tanh(Z_3) = \tanh(0.51154) = 0.47115$$

Neurona 4 capa 1 :

$$Z_4 = \sum_{i=1}^3 w_{4i} x_i + b_4$$

$$Z_4 = (2.5)(0.74294) + (3.5)(-0.47462) + (-0.5)(0.89499) + 0.17709$$

$$Z_4 = 1.85735 + (-1.66117) + (-0.44749) + 0.17709$$

$$Z_4 = -0.07422$$

$$\tanh(Z_4) = -0.07409$$

Capa 2 :

Neurona 1 capa 2 :

$$\text{entrada} = [0.96766, 0.99349, 0.47115, -0.07409]$$

$$Z_1 = \sum_{i=1}^4 w_{1i} x_i + b_1$$

$$Z_1 = (0.96766)(-0.80493) + (0.99349)(-0.47989) + (0.47115)(-0.1104) + (-0.07409)(0.64886) + (-0.11802)$$

$$z_1 = -0.77889 + (-0.49663) + (-0.05201) + (-0.04807) + (-0.11802)$$

$$z_1 = -1.49362$$

$$\tanh(z_1) = -0.90399$$

Neurona 2 capa 2

$$z_2 = \sum_{i=1}^3 w_{2i} x_i + b_2$$

$$z_2 = (0.96766)(0.91062) + (0.99349)(-0.01929) + (0.47115)(-0.36983) + (-0.07409)(-0.48958) + 0.22209$$

$$z_2 = 0.88117 + (-0.01916) + (-0.17424) + 0.03627 + 0.22209$$

$$z_2 = 0.94613$$

$$\tanh(z_2) = 0.73802$$

Neurona 3 capa 2:

$$z_3 = (0.96766)(-0.66095) + (0.99349)(-0.95604) + (0.47115)(0.10824) + (-0.07409)(0.61127) + 0.71648$$

$$z_3 = -0.63957 + (-0.94981) + 0.05099 - 0.04528 + 0.71648$$

$$z_3 = -0.86719$$

$$\tanh(z_3) = -0.69995$$

Neurona 4 capa 2:

$$z_4 = (0.96766)(0.84115) + (0.99349)(-0.01321) + (0.47115)(0.93422) + (-0.07409)(0.05702) - 0.99075$$

$$z_4 = 0.81394 - 0.01312 + 0.44015 - 0.00422 - 0.99075$$

$$z_4 = 0.246$$

$$\tanh(z_4) = 0.24117$$

Capa 3:

Neurona 1 capa 3

Entrada 5: $[-0.90399, 0.73802, -0.69995, 0.24117]$

$$z_1 = (-0.90399)(0.00543) + (0.73802)(-0.94891) + (-0.69995)(0.07198) + (0.24117)(-0.87837) - 0.96955$$

$$z_1 = -0.0049 - 0.70031 - 0.05038 - 0.21183 - 0.96955$$

$$z_1 = -1.93697$$

$$\tanh(z_1) = -0.95929 \leftarrow \text{predicción}$$

Entrada 2 $[4.0, -1.0, 0.5]$

Capa 1 neurona 1:

$$z_1 = (4.0)(0.00418) + (-1.0)(0.43075) + (0.5)(-0.42825) + 0.32206$$

$$z_1 = 0.01672 + (-0.43075) - 0.21412 + 0.32206$$

$$z_1 = -0.30609$$

$$\tanh(z_1) = -0.29686$$

Capa 1 neurona 2:

$$z_2 = (4.0)(0.51585) + (-1.0)(0.58195) + (0.5)(-0.8038) - 0.86865$$

$$z_2 = 2.0634 - 0.58195 - 0.4019 - 0.86865$$

$$z_2 = 0.2084$$

$$\tanh(z_2) = 0.20545$$

Capa 1 neurona 3:

$$z_3 = (4.0)(-0.44835) + (-1.0)(0.38031) + (0.5)(-0.56652) + 0.01807$$

$$= -1.7934 - 0.38031 - 0.28326 + 0.01807$$

$$z_3 = -2.4389$$

$$\tanh(z_3) = -0.98488$$

Capa 1 neurona 4

$$z_4 = (4.0)(0.74294) + (-10)(-0.47462) + (0.5)(0.89499) + 0.17709$$

$$z_4 = 2.97176 + 0.47462 + 0.44749 + 0.17709$$

$$z_4 = 4.07096$$

$$\tanh(z_4) = 0.99941$$

Capa 2 neurona 1

$$\text{Entradas } [-0.29686, 0.20545, -0.98488, 0.99941]$$

$$z_1 = (-0.29686)(-0.80493) + (0.20545)(-0.49989) + (-0.98488)(-0.1104) + (0.99941)(0.64886) - 0.11802$$

$$= 0.23895 - 0.1027 + 0.10873 + 0.64847 - 0.11802$$

$$z_1 = 0.77543$$

$$\tanh(z_1) = 0.65008$$

Capa 2 neurona 2

$$z_2 = (-0.29686)(0.91062) + (0.20545)(-0.07929) + (-0.98488)(-0.36983) + (0.99941)(-0.78958) + 0.22209$$

$$= -0.27032 - 0.00396 + 0.36424 - 0.7893 + 0.22209$$

$$z_2 = -0.17725$$

$$\tanh(z_2) = -0.17592$$

Capa 2 neurona 3:

$$z_3 = (-0.29686)(-0.66095) + (0.20545)(-0.95604) + (-0.98488)(0.10824) + (0.99941)(0.61127) + 0.71648$$

$$z_3 = 0.196213 - 0.19641 - 0.1066 + 0.61091 + 0.71648$$

$$z_3 = 1.22059$$

$$\tanh(z_3) = 0.83982$$

Capa 2 neurona 4:

$$z_4 = (0.29686)(0.84115) + (0.20545)(-0.01321) + (-0.98488)(0.93422) + (0.99941)(0.05702) - 0.99075$$

$$z_4 = -0.2497 - 0.00271 - 0.92010 + 0.05699 - 0.99075$$

$$z_4 = -2.10629$$

$$\tanh(z_4) = -0.97081$$

Capa 3:

Capa 3 neurona 1:

$$z_1 = (0.65008)(0.00543) + (-0.17542)(-0.94891) + (0.83982)(0.07198) + (-0.97081)(-0.87837) - 0.96955$$

$$z_1 = 0.00353 + 0.16646 + 0.06045 + 0.85273 - 0.96955$$

$$z_1 = 0.11363$$

$$\tanh(z_1) = 0.11314 \leftarrow \text{predicción}$$

Entrada 3 [0.5, 1.5, 1.0]

Capa 1 neurona 1:

$$z_3 = (0.5)(0.404184) + (1.5)(0.43075) + (1.0)(-0.42825) + 0.32206$$

$$z_3 = 0.00209 + 0.64612 - 0.42825 + 0.32206$$

$$Z_3 = 0.54202$$

$$\tanh(Z_3) = 0.49452$$

Capa 1 neurona 2:

$$Z_2 = (0.5)(0.51585) + (1.5)(0.58195) + (1.0)(-0.8082) - 0.96865$$

$$Z_2 = 0.25792 + 0.87293 - 0.8082 - 0.96865$$

$$Z_2 = -0.54659$$

$$\tanh(Z_2) = -0.49796$$

Capa 1 neurona 3:

$$Z_3 = (+0.5)(-0.44835) + (1.5)(0.38031) + (1.0)(-0.56652) + 0.01807$$

$$Z_3 = -0.22417 + 0.570478 - 0.56652 + 0.01807$$

$$Z_3 = -0.20214$$

$$\tanh(Z_3) = -0.19943$$

Capa 1 neurona 4:

$$Z_4 = (0.5)(0.74244) + (1.5)(-0.47462) + (1.0)(0.89494) + 0.17709$$

$$Z_4 = 0.37142 + (-0.71193) + 0.89499 + 0.17709$$

$$Z_4 = 0.33167$$

$$\tanh(Z_4) = 0.62405$$

Capa 2 neurona 1: $Z_1 = 0.15978$ $\tanh(Z_1) = 0.15844$

Capa 2 neurona 2:

$$Z_2 = (0.49452)(0.91062) + (-0.49796)(-0.01929) + (-0.19943)(-0.36983) + (0.62405)(-0.48958) + 0.22209$$

$$z_2 = 0.45632 + 0.0046 + 0.07375 - 0.30552 + 0.22209$$

$$z_2 = 0.45025$$

$$\tanh(z_2) = 0.42211$$

Capa 2 neurona 3

$$z_3 = (0.47452)(-0.66095) + (-0.49796)(-0.95604) + (-0.19943)(0.10824) + (0.62405)(0.61127) + 0.71648$$

$$z_3 = -0.32686 + 0.47607 - 0.02158 + 0.38146 + 0.71648$$

$$z_3 = 1.22558$$

$$\tanh(z_3) = 0.84129$$

Capa 2 neurona 4:

$$z_4 = (0.49452)(0.84115) + (-0.49796)(-0.01321) + (-0.19943)(0.93422) + 0.62405(0.05702) - 0.99075$$

$$z_4 = 0.41597 + 0.00658 - 0.18631 + 0.03558 - 0.99075$$

$$z_4 = -0.78929$$

$$\tanh(z_4) = -0.61624$$

Capa 3 neurona 1:

$$z_1 = (0.15844)(0.00543) + (0.42211)(-0.94891) + 0.84129(0.07198) + (-0.61624)(-0.87837) - 0.96955$$

$$z_1 = -0.76738$$

$$\tanh(z_1) = -0.6454 \leftarrow \text{predicción}$$

Entrada y $[3.0, 2.0, -1.5]$

Capa 1 neurona 1

$$z_1 = (3.0)(0.00418) + (2.0)(0.43075) + (-1.5)(-0.42825) + 0.32206$$

$$z_1 = 1.83851$$

$$\tanh(z_1) = 0.95065$$

Capa 1 neurona 2

$$z_2 = (3.0)(0.515585) + (2.0)(0.58195) + (-1.5)(-0.8088) - 0.86865$$

$$z_2 = 3.05603$$

$$\tanh(z_2) = 0.99557$$

Capa 1 neurona 3

$$z_3 = (3.0)(-0.44835) + (2.0)(0.38031) + (-1.5)(-0.56652) + 0.01807$$

$$z_3 = 0.28341$$

$$\tanh(z_3) = 0.27066$$

Capa 1 neurona 4:

$$z_4 = (2.0)(0.74294) + (2.0)(-0.47462) + (-1.5)(0.89449) + 0.17109$$

$$z_4 = 0.11418$$

$$\tanh(z_4) = 0.11369$$

Capa 2 neurona 1

$$z_1 = (0.95065)(-0.90493) + (0.99557)(-0.49989) + (0.27066)(-0.1104) + (0.11369)(+0.6886) - 0.118022$$

$$z_1 = -1.33763$$

$$\tanh(z_1) = -0.8711$$

$$z_2 = (0.95065)(0.91062) + (0.99557)(-0.01924) + (0.72606)(-0.36953) + (0.11369)(-0.4895) + 0.22209$$

$$z_2 = 0.91080$$

$$\tanh(z_2) = 0.21518$$

$$z_3 = (0.95065)(-0.66095) + (0.99557)(-0.95604) + (0.72606)(0.10824) + (0.11369)(0.61127) + 0.21648$$

$$z_3 = -0.76428$$

$$\tanh(z_3) = -0.64359$$

$$z_4 = (0.95065)(0.84115) + (0.99557)(-0.01321) + (0.72606)(0.93422) + (0.11369)(0.05202) - 0.99075$$

$$z_4 = 0.060126$$

$$\tanh(z_4) = 0.06005$$

Capa 3 neurona:

$$z_1 = (-0.8711)(0.005213) + (0.72151)(-0.9489) + (-0.643695)(0.0719) + (0.06005)(-0.9283) - 0.96955$$

$$z_1 = -1.75802$$

$$\tanh(z_1) = -0.99228 \leftarrow \text{predicción} = \hat{y}$$

Calculo de la perdida

$$L = \frac{1}{2} [(-0.95929 - 1)^2 + (0.11314 - (-1))^2 + (-0.6454 + 1)^2 + (-0.94228 - 1)^2]$$

$$L = 8.97613$$

Backpropagation

Calculo de gradientes

$$\frac{\partial L}{\partial w_{11}''} = \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial a_1''} \times \frac{\partial a_1''}{\partial z_1''} \times \frac{\partial z_1''}{\partial w_{11}''}$$

$$\frac{\partial L}{\partial w_{11}''} = 2(\hat{y} - y) \times (1 - \hat{y}^2) \times a_1''$$

$$\begin{aligned} \frac{\partial L}{\partial w_{11}''} &= 2(-0.94228 - 1)(1 - (-0.94228)^2)(-0.8711) \\ &= 0.379362 \end{aligned}$$

$$\frac{\partial L}{\partial w_{12}''} = \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial a_2''} \times \frac{\partial a_2''}{\partial z_2''} \times \frac{\partial z_2''}{\partial w_{12}''}$$

$$\begin{aligned} &= 2(-1.94228)(1 - (-0.94228)^2)(0.721518) \\ &= -0.31421 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial w_{13}''} &= (-0.43549)(0.64354) \\ &= 0.29027 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial w_{14}''} &= (-0.43549)(0.06003) \\ &= -0.02615 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial w_{11}'} &= \frac{\partial L}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial a_1''} \times \frac{\partial a_1''}{\partial z_1''} \times \frac{\partial z_1''}{\partial w_{11}''} \times \frac{\partial w_{11}''}{\partial z_1'} \times \frac{\partial z_1'}{\partial w_{11}'} \\ &= (0.379362)(1 - a_1''^2) \cdot a_1 \\ &= (0.379362)(1 - 0.8711^2)(0.45065) \end{aligned}$$

$$\frac{\partial L}{\partial w_{11}'} = 0.08698$$

$$\frac{\partial L}{\partial w_{12}} = (0.091496)(0.99552) = 0.09109$$

$$\frac{\partial L}{\partial w_{13}} = (0.091496)(0.22066) = 0.0202$$

$$\frac{\partial L}{\partial w_{14}} = (0.091496)(0.11369) = 0.010402$$