Assignment

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Subject : Linear Algebra
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Chapter # 5:

Eigenvalues and Eigenvectors

Exercise # 5.4:

Question # 1:

Let $B = \{b_1, b_2, b_3\}$ and $D = \{d_1, d_2\}$, be bases for vector spaces ....

Solution:

$T(b_1) = 3d_1 - 5d_2$, $T(b_2) = -d_1 + 6d_2$, $T(b_3) = 4d_2$

$[T(b_1)]_D = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, $[T(b_2)]_D = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$, $[T(b_3)]_D = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

Thus, the matrix for relative to B and D is

$$[T(b_1)_D][T(b_2)_D][T(b_3)_D] = \begin{bmatrix} 3 & -1 & 0 \\ -5 & 6 & 4 \end{bmatrix}$$

Question # 2:

Let $D = \{d_1, d_2\}$ and $B = \{b_1, b_2\}$ be bases for vector spaces for V and W respectively............

Solution:

$T(d_1) = 2b_1 - 3b_2$, $T(d_2) = -4b_1 + 5b_2$

$[T(d_1)]_D = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, $[T(d_2)]_D = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$

Thus, The matrix for T relative to D and B is

$[T(d_1)]_D [T(d_2)_D] = \begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix}$ Ans

Question #3:

Let $\mathcal{E} = \{e_1, e_2, e_3\}$ be The standard bases for R^3, --------.

Solution:

$T(x_1, x_2, x_3) = (x_3 - x_2)b_1 - (x_1 + x_3)b_2 + (x_1 - x_2)b_3.$

(a) $T(e_1) = 0b_1 - 1b_2 + b_3$

$T(e_2) = -1b_1 - 0b_2 - 1b_3$

$T(e_3) = 1b_1 - 1b_2 + 0b_3$

(b) $[T(e_1)]_B = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, $[T(e_2)]_B = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$, $[T(e_3)_B] = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

(c) Matrix For T relative To E and B is

$T(e_1)_B + T(e_2)_B + T(e_3)_B = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ Ans

Question #4

Let $B = \{b_1, b_2, b_3\}$ be the basis for vector space V and T ---------

$$T(x_1b_1 + x_2b_2 + x_3b_3) = \begin{bmatrix} 2x_1 - 4x_2 + 5x_3 \\ -x_2 + 3x_3 \end{bmatrix}$$

Solution

Let $E = \{e_1, e_2\}$ be the standard bases for R^2.

$$[T(b_1)]_E = T(b_1) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$[T(b_2)]_E = T(b_2) = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$$

$$[T(b_3)]_E = T(b_3) = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Matrix for T relative to B and E is

$$[T(b_1)]_E\,[T(b_2)]_E\,[T(b_3)]_E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} \text{ Ans}$$

Question #5:

Let $T: P_2 \to P_3$ be the transformation that maps a polynomial ---------

Solution:

(a) $T(p) = (t+5)\,p(t)$

$p(t) = 2 - t + t^2$

$= (t+5)(2 - t + t^2)$

$= t^3 + 4t^2 - 3t + 10$

(b) Let, p and q be the polynomial in P_2,

$$T(p(t)+q(t)) = (t+5)[p(t)+q(t)]$$
$$= (t+5)p(t) + (t+5)q(t)$$
$$= T(p(t)) + T(q(t)).$$

$$T(c\cdot p(t)) = (t+5)[c\cdot p(t)]$$
$$= c\cdot(t+5)\,p(t)$$
$$= c\cdot T[p(t)].$$

So, T is a linear transformation.

(c) Let,

$$\beta = \{1, t, t^2\} \quad \text{and} \quad C = \{1, t, t^2, t^3\}$$

$$T(b_1) = T(1) = (t+5)(1) = t+5$$

$$[T(b_1)]_c = \begin{bmatrix} 5 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$T(b_2) = T(t) = (t+5)(t) = t^2+5t$$

$$[T(b_2)]_c = \begin{bmatrix} 0 \\ 5 \\ 1 \\ 0 \end{bmatrix}$$

$$T(b_3) = T(t^2) = (t+5)(t^2) = t^3+5t^2$$

$$[T(b_3)]_c = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

$$[T(b_1)]_c\,[T(b_2)]_c\,[T(b_3)]_c = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Question # 6:

Let $T: P_2 \to P_4$ be the Transformation that maps a polynomial.......

$p(t) \quad p(t) + t^2 p(t)$.

Solution:

(a). $p(t) = 2-t+t^2$

$p(t) = 2-t+t^2+t^2(2-t+t^2)$

$= 2-t+t^2+2t^2-t^3+t^4$

$= t^4-t^3+3t^2-t+2$

(b) $T(p(t)+q(t)) = [p(t)+q(t)]+t^2[p(t)+q(t)]$

$= [p(t)+t^2p(t)]+[q(t)+t^2q(t)]$

$= T(p(t))+T(q(t))$

$T(c\cdot p(t)) = c\cdot p(t)+t^2[c\cdot p(t)]$

$= c[p(t)+t^2(p(t))]$

$= cT[p(t)]$.

Hence, it is a Linear Transformation.

(c) Let,

$\beta = \{1, t, t^2\}$, $C = \{1, t, t^2, t^3, t^4\}$;

$T(b_1) = T(1) = 1+t^2(1) = t^2+1$

$$[T(b_1)]_c = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$T(b_2) = T(t) = t+t^3 \Rightarrow [T(b_2)]_c = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$[T(b_3)]_c = T(t^2) = t^4 + t^2$

$$[T(b_3)]_c = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Hence, The matrix for T relative To B & C is

$$[T(b_1)_c][T(b_2)]_c [T(b_3)]_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question#7:

Assume The mapping $T: P_2 \to P_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$.....

Solution:

Since; $T(b_1) = T(1) = 3 + 5t$; $T(b_2) = T(t) = 4t^2 - 2t$

$T(b_3) = T(t^2) = t^2$

$$[T(b_1)]_B = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}, [T(b_2)]_B = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}, [T(b_3)]_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[T(b_1)_B][T(b_2)]_B [T(b_3)]_B = \begin{bmatrix} 3 & 0 & 0 \\ 5 & -2 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Question#8.

Let $B = \{b_1, b_2, b_3\}$ be The bases for vector space V.

Find $T(3b_1 - 4b_2)$ ....

$$[T]_B = \begin{bmatrix} 0 & -6 & 1 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$$

Solution:

$$[3b_1 - 4b_2]_B = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix},$$

$$[T(3b_1 - 4b_2)]_B = [T_B][(3b_1 - 4b_2)_B]$$

$$= \begin{bmatrix} 0 & -6 & 1 \\ 0 & 5 & -1 \\ +1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 24 \\ -20 \\ 11 \end{bmatrix}$$

$$T(3b_1 - 4b_2) = 24b_1 - 20b_2 + 11b_3$$

Question #9:

Define $T: P_2 \to R^3$ by $T(p) = \begin{bmatrix} P(-1) \\ P(0) \\ p(1) \end{bmatrix}$ .....

Solution:

(a) $T(p) = \begin{bmatrix} 5+3(-1) \\ 5+3(0) \\ 5+3(1) \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$

(b) Let, p and q be the polynomial in P_2, & c be the scalar.

$$T(p+q) = \begin{bmatrix} (p+q)(-1) \\ (p+q)(0) \\ (p+q)(1) \end{bmatrix} = \begin{bmatrix} p(-1)+q(-1) \\ p(0)+q(0) \\ p(1)+q(1) \end{bmatrix}$$

$$= \begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix} + \begin{bmatrix} q(-1) \\ q(0) \\ q(1) \end{bmatrix}$$

$$= T(p) + T(q).$$

$$T(c.p) = \begin{bmatrix} (c.p)(1) \\ (c.p)(0) \\ (c.p)(1) \end{bmatrix} = \begin{bmatrix} c.(p(-1)) \\ c.(p(0)) \\ c.(p(1)) \end{bmatrix} = c \begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix}$$

$$= c.T(p).$$

Hence,

T is a linear Transformation.

(c) Let,

$\beta = \{1, t, t^2\}$, $\mathcal{E} = \{e_1, e_2, e_3\}$ be the standard bases for $\mathbb{R}^3$.

$$[T(b_1)]_{\mathcal{E}} = T(b_1) = T(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$[T(b_2)]_{\mathcal{E}} = T(b_2) = T(t) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$[T(b_3)]_{\mathcal{E}} = T(b_3) = T(t^2) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Hence,

$$[T(b_1)]_E \; [T(b_2)]_E \; [T(b_3)]_E = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Question # 10:

Define $T: P_3 \to R^4$ by $T(p) = \begin{bmatrix} p(-3) \\ p(-1) \\ p(1) \\ p(3) \end{bmatrix}$

Solution-

(a) Let p and q be the polynomial in P_3 and let c be any scalar. Then.

$$T(p+q) = \begin{bmatrix} (p+q)(-3) \\ (p+q)(-1) \\ (p+q)(1) \\ (p+q)(3) \end{bmatrix} = \begin{bmatrix} p(-3) \\ p(-1) \\ p(1) \\ p(3) \end{bmatrix} + \begin{bmatrix} q(-3) \\ q(-1) \\ q(1) \\ q(3) \end{bmatrix}$$

$$= T(p) + T(q)$$

$$T(c \cdot p) = \begin{bmatrix} (c \cdot p)(-3) \\ (c \cdot p)(-1) \\ (c \cdot p)(1) \\ (c \cdot p)(3) \end{bmatrix} = c \begin{bmatrix} p(-3) \\ p(-1) \\ p(1) \\ p(3) \end{bmatrix}$$

$$= cT(p).$$

(b) Let,

$B = \{1, t, t^2, t^3\}$ & $\mathcal{E} = \{e_1, e_2, e_3, e_4\}$ be the standard basis for R^4. Since

$$[T(b_1)]_E = T(b_1) = T(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$[T(b_2)]_E = T(b_2) = T(t) = \begin{bmatrix} -3 \\ -1 \\ 1 \\ 3 \end{bmatrix}$$

$$[T(b_3)]_E = T(b_3) = T(t^2) = \begin{bmatrix} 9 \\ 1 \\ 1 \\ 9 \end{bmatrix}$$

$$[T(b_4)]_E = T(b_4) = T(t^3) = \begin{bmatrix} -27 \\ -1 \\ 1 \\ 27 \end{bmatrix}$$

The matrix T relative to B & E is

$$[T(b_1)]_E \, [T(b_2)]_E \, [T(b_3)]_E \, [T(b_4)]_E = \begin{bmatrix} 1 & -3 & 9 & -27 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 27 \end{bmatrix}$$

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